# Leveraging Uncertainties to Infer Preferences: Robust Analysis of School Choice*

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**Abstract**

Strategic mistakes by applicants make it challenging to infer their preferences from school choice data. We propose a novel approach to address this issue in a deferred-acceptance matching environment. The key idea is to exploit the uncertainties applicants face, such as those arising from tie-breaking lotteries, which can incentivize the revelation of their true preferences. The proposed approach infers all preferences that can be robustly inferred in the presence of payoff-insignificant mistakes. We apply the approach to the school-choice data from Staten Island, NYC. Counterfactual analysis suggests that the effects of proposed desegregation reforms would be underestimated if applicants' mistakes were not accounted for. Still, the effects remain modest even under our proposed method, raising doubt about the reforms' effectiveness.

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## 1 Introduction

The design of school choice as a policy tool, particularly for promoting diversity and desegregation, remains a subject of ongoing debate.[1] The effectiveness of such policies hinges critically on understanding

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[1]For instance, nearly 80% of NYC's Black or Hispanic public high school students are concentrated in just half of the high schools, and in response, various desegregation policies have been proposed, including the elimination of admissions based on academic qualifications or residence locations (Shapiro, 2021).

student preferences. The ability to accurately infer and estimate these preferences from data is essential for evaluating the potential impact of policy changes.

In principle, preference inference should be straightforward in strategy-proof mechanisms like the widely used deferred acceptance (DA) algorithm, where applicants have a dominant strategy to report their preferences truthfully. However, recent studies have found that applicants often deviate from truthful reporting even in these settings. These "strategic mistakes" have been observed in lab settings (Chen and Sönmez, 2006; Li, 2017)[2] and high-stake real-world contexts such as school applications (Larroucau and Rios, 2020; Artemov, Che, and He, 2021; Hassidim, Romm, and Shorrer, 2021; Arteaga, Kapor, Neilson, and Zimmerman, 2022; Shorrer and Sóvágó, 2023), and medical resident matching (Rees-Jones, 2017; Rees-Jones and Skowronek, 2018).[3] Most of these mistakes are payoff-irrelevant, typically involving applicants omitting or mis-ranking highly sought-after schools that are beyond their reach. However, even these seemingly minor mistakes can lead to biased preference estimates and inaccurate policy predictions if applicant rankings are interpreted naively. For instance, the Weak Truth-Telling (WTT) hypothesis, which assumes students truthfully report their most preferred choices, might incorrectly infer that disadvantaged students have low preference for elite schools simply because they perceive these schools as unattainable and don't include them in their rankings. This could lead to underestimating the potential impact of policy reforms aimed at increasing access to such schools.

A common approach to address such mistakes in preference inference is to invoke a weaker assumption called Stability. The stability hypothesis posits that each student is assigned her most preferred school among those feasible for her, given her priority standings. Fack, Grenet, and He (2019) and Artemov, Che, and He (2023) justify this hypothesis by arguing that in a large economy, the uncertainty about schools' cutoffs vanishes, allowing students—even those prone to minor mistakes—to "recognize" feasible schools and secure their stable assignments (i.e., their favorite feasible schools). The stability hypothesis is less restrictive than WTT, as the latter implies the former under DA but not vice versa. It is also more robust to mistakes since it doesn't make assumptions about student preferences for unattainable schools. The stability hypothesis has been widely adopted in various studies.[4]

[2]We follow the existing literature and refer to such non-truthful reporting of preferences in strategy-proof environments as mistakes.

[3]For instance, Hassidim, Romm, and Shorrer (2021) find that 19% of Israeli postgraduate psychology program applicants either did not list a scholarship position for a program or ranked a non-scholarship position higher than the corresponding scholarship position against their interests. Artemov, Che, and He (2021) and Shorrer and Sóvágó (2023) find similar findings in college admissions in Australia and Hungary, respectively. Rees-Jones (2017) reports that 17% of the 579 surveyed US medical seniors indicate misrepresenting their preferences in the National Resident Matching Program.

[4]They include Aue, Klein, and Ortega (2020); Bertoni, Gibbons, and Silva (2020); Ngo and Dustan (2021); Otero, Barahona, and Dobbin (2021); Combe, Tercieux, and Terrier (2022); Combe, Dur, Tercieux, Terrier, and Ünver (2022); Hahm and Park

However, the preceding justification for stability crucially rests on the assumption that students face no uncertainties about their priorities. In reality, students often face priority uncertainties stemming from various sources. The widespread use of lotteries for tie-breaking in US public school districts introduces randomness in students' priorities. Even when lotteries are not used, priorities may not be known at the time of application, as seen in the NYC Specialized High School admissions.[5] Finally, uncertainties may also arise from students not knowing others' priorities or preferences. Table 1 lists a few examples of DA-based school assignments and college admission systems where students face priority uncertainties.[6]

The presence of uncertainties raises questions about the generalizability of the stability hypothesis. Moreover, it is unclear whether the previously discussed payoff-irrelevant mistakes would even occur when uncertainties are present. The uncertainty surrounding feasible school options generally makes mistakes more costly, prompting applicants to be more cautious. However, not *all* mistakes become costly in the presence of uncertainties. Some schools might remain out of reach for students due to factors like geographic zoning, regardless of lottery outcomes. Additionally, certain lottery mechanisms, such as Single-Tie-Breaking (STB), can still allow for payoff-irrelevant mistakes even when all schools are potentially within reach, as we illustrate below in Example 1. Therefore, even with uncertainties, payoff-irrelevant mistakes can persist and need to be accounted for.

Uncertainties also create opportunities for researchers to learn about student preferences. By making certain mistakes costly, uncertainties can incentivize applicants to reveal some preferences more accurately.[11] For example, a lottery system might bring previously unattainable elite schools within reach for some students. In such cases, neglecting to rank these schools becomes a costly mistake, as it could mean missing out on admission. Such a lottery would likely lead students to reveal their preferences for these schools more truthfully. The key question then becomes: *how can we systematically identify and leverage the information revealed by the specific uncertainties present in a given school choice environment?*

To systematically address this question, we employ the concept of *robust equilibrium* developed by Artemov, Che, and He (2023). This concept allows for the possibility that applicants might make mistakes

(2022); Chrisander and Bjerre-Nielsen (2023); Andersson, Kessel, Lager, Olme, and Reese (2024); Yang (2024).

[5]These schools use test scores for assignment, but students submit ROLs before knowing their scores. Similar uncertainty is present some Chinese provinces, where college applications precede entrance exams or occur before score release (Chen and Kesten, 2017).

[6]Online Appendix A provides many more examples of DA with and without priority uncertainties.

[7]https://www.bostonpublicschools.org

[8]https://www.cps.edu/gocps/high-school/hs-selection/

[9]https://www.schools.nyc.gov/enrollment

[10]https://enrolldcps.dc.gov/

[11]Example 1 also illustrates this point.

Table 1: Selected Examples of Deferred-Acceptance Mechanisms with Priority Uncertainties

| Education System | Priority Details | Sources |
|---|---|---|
| *Panel A: Primary and Secondary Education* | | |
| Boston (Open Enrollment High Schools) | Distance, other factors, random tie-breaking | Boston Public Schools[7] |
| Chicago (Non-selective Choice High Schools) | Distance, other factors, random tie-breaking | Chicago Public Schools[8] |
| NYC (Specialized High Schools) | Test score (unknown) | Abdulkadiroglu, Angrist, and Pathak (2014), NYC Public Schools portal[9] |
| NYC (Open High Schools) | Coarse priorities and random tie-breaking | Abdulkadiroğlu, Pathak, Schellenberg, and Walters (2020), NYC Public Schools[8] |
| Washington DC | Distance, siblings, other factors, random tie-breaking | Abdulkadiroğlu and Andersson (2023), DC Public Schools[10] |
| Chile (Schools with high-achieving student quota) | Coarse priorities (test result + siblings + working parents + former student) and random tie-breaking | Arteaga, Kapor, Neilson, and Zimmerman (2022) |
| Chile (Other schools) | Siblings, working parents, former student and random tie-breaking | Arteaga, Kapor, Neilson, and Zimmerman (2022) |
| Finland | Composite score (unknown): academic records (known) + exam score + other criteria | Salonen (2014) |
| France | Composite score (unknown): GPA + other factors | Hiller and Tercieux (2014); Grenet (2022) |
| Ghana | Nationwide test score (unknown) | Ajayi (2022) |
| *Panel B: Higher Education* | | |
| Chile | Composite score (unknown): GPA (known) + standardized test (known) | Hastings, Neilson, and Zimmerman (2013) |
| France | Composite score (unknown): GPA (known) + other criteria | Hakimov, Schmacker, and Terrier (2023) |
| Norway | Composite score (unknown): GPA + other factors | Kirkeboen, Leuven, and Mogstad (2016) |
| Tunisia | Nationwide test score (known) | Luflade (2019) |

but requires that the expected losses from mistakes become negligible as the market size increases.[12] We formally define a robust equilibrium and introduce an additional refinement requiring a small probability of truthful reporting. The main implication of this solution concept for DA is *asymptotic ex-post stability*: as the market size grows, with high probability, almost all students receive their most preferred feasible schools in *every possible realization of uncertainty* (see Theorem 1).[13]

[12]The robustness concept can be interpreted as applicants' inattention to low-probability options, although we remain agnostic about the specific reasons for such mistakes.

[13]This implicitly assumes that the underlying mechanism is strategy-proof and stable (under truthful reporting), which holds under a DA mechanism with unconstrained choice or with constraints that are unbinding for the vast majority of students.

The asymptotic ex-post stability result forms the basis for a new preference inference method called the *Transitive Extension of Preferences from Stability* (TEPS). TEPS applies to a wide range of DA-based assignment systems, both with and without priority uncertainties (including those listed in Table 1 and Table A.1). The method involves simulating the underlying priority uncertainty structure; for instance, in the DA-STB system, one can conduct the STB lottery and run the DA algorithm as in the actual assignment process. By invoking asymptotic ex-post stability, we infer that the assigned school is preferred to all other feasible schools under a given lottery realization. This process is repeated for multiple lottery draws to obtain preference relations for each draw. The final step connects these preference relations using the transitivity axiom, creating a transitive closure of all preference relations derived from the pairs of assigned and feasible schools.

The prediction of stable assignments for all uncertainty realizations holds primarily in large economies. In real-world scenarios, participants might make payoff-relevant mistakes, leading to deviations from stability.[14] To account for this, we introduce a generalized version of TEPS, denoted as TEPS$^\tau$, which incorporates an attention parameter $\tau \in [0, 100]$. This parameter restricts preference inference to only the most likely uncertainty realizations (or feasible sets), up to a cumulative probability of $\tau\%$. The remaining less likely uncertainties are disregarded due to their potential unreliability.

The choice of the attention parameter $\tau$ is inherently an empirical question that depends on the specific context and characteristics of the participants. To address this, we develop a data-driven testing procedure to select the most appropriate attention level, ranging from the WTT assumption to various levels of tolerance for mistakes within the TEPS$^\tau$ framework. Monte Carlo simulations confirm the potential for biased estimation when participant mistakes are ignored and demonstrate the effectiveness of our selection procedure in identifying the optimal level of tolerance.

To demonstrate the practical implications of our approach, we apply our methods to public high school choice data from Staten Island, NYC, where lotteries are used as tie-breakers, creating priority uncertainties. We evaluate the performance of various hypotheses in explaining the observed data and conduct counterfactual analyses to compare the predicted effects of several desegregation policies under different preference estimation assumptions. The accuracy of these predictions is crucial for assessing the effectiveness of policy reforms aimed at addressing school segregation, a pressing concern in large urban districts like NYC. The counterfactual analysis of such reforms necessitates estimating student preferences

Our empirical setting corresponds to the latter. A choice setting with binding constraints introduces strategic behavior, which is beyond the scope of the current paper.

[14]Payoff relevant mistakes are observed for a significant minority of applicants (e.g., Artemov, Che, and He, 2021).

for schools currently inaccessible due to priority restrictions. If student inattention towards these schools is not accounted for, the potential impact of these reforms could be significantly underestimated.

Our analysis reveals that the effects of desegregation policies are underestimated when the WTT hypothesis is used for preference inference. The WTT-based estimates predict a smaller decrease in the racial gap in the quality of assigned schools compared to the estimates derived from the TEPS method (selected from our procedure). Intuitively, WTT disregards student mistakes and assumes low preferences for unranked out-of-reach schools, leading to an underestimation of the potential impact of policies that make such schools more accessible. The underestimation is substantial; for example, WTT predicts only half the reduction in the racial gap in the proportion of Black or Hispanic students in assigned programs compared to the TEPS-based prediction.

The predicted effects of removing academic and geographic priorities on desegregation are relatively small, even under the TEPS estimates. This suggests that factors beyond school priorities, such as residential segregation, might play a more significant role in school segregation. The study provides descriptive evidence supporting this explanation, showing high residential segregation in Staten Island, with Black and Hispanic students concentrated in areas with schools that have higher proportions of Black/Hispanic, low-income, and lower-performing students. The preference estimates also indicate that minority students have a stronger aversion to commuting compared to non-minority students. The combination of residential segregation and varying commuting preferences could explain why school choice reforms focused solely on equalizing school access might not lead to substantial desegregation unless residential segregation is also addressed.

**Related Literature.** The current paper contributes to the growing body of research on robust preference inference from school choice data generated by the DA algorithm. The proposed TEPS procedure, while building upon the stability hypothesis of Fack, Grenet, and He (2019) and Artemov, Che, and He (2023), generalizes it to accommodate uncertainties arising from tie-breaking lotteries and other factors. This generalization, supported by Theorem 1, is crucial given the prevalence of lotteries and the inherent uncertainties in real-world school choice settings. The TEPS procedure, with its novel justification and tractable algorithm, offers a significant advancement in preference inference methodology.[15]

Kapor, Karnani, and Neilson (2024) propose an approach that assumes truthful reporting for a *subset* of programs on the ROL: those feasible under some lottery realizations and those ranked above the best *clearly-feasible* program. This approach, while motivated by robustness, differs from TEPS, which takes

[15]For example, one may be tempted to "implement" the stability hypothesis using the actual (if observed) or a single simulated lottery. However, such an approach lacks theoretical justification in an environment with uncertain priorities.

no stance on relative rankings within ROLs. The theoretical basis for their approach remains unclear; for example, the robust equilibrium concept cannot rationalize it. Further, their assumption does not accommodate certain types of mistakes observed in practice, such as "flips" among feasible schools (Hassidim, Romm, and Shorrer, 2021) or the mistakes arising from STB lotteries (e.g., Example 1), although they are are consistent with robust equilibria and thus accommodated by TEPS.

The current paper also connects with the literature that interprets an applicant's rank-order list (ROL) as an optimal portfolio choice under non-strategyproof mechanisms (see, as early examples, He, 2017; Agarwal and Somaini, 2018; Calsamiglia, Fu, and Güell, 2020). Notably, the approach by Agarwal and Somaini (2018), when applied to DA, exhibits robustness to payoff-irrelevant mistakes. However, it doesn't account for mistakes with minor but positive payoff consequences. Moreover, the practical implementation of this approach faces challenges due to the curse of dimensionality as the number of schools increases. In contrast, our proposed empirical method is computationally efficient and robust to the strategic mistakes commonly observed in real-world settings.

Several papers have sought to address the computational challenges associated with the portfolio choice approach. Larroucau and Rios (2020) demonstrate that considering a subset of ROLs ("one-shot swaps") can mitigate the curse of dimensionality, but their result is limited to settings where cutoffs (and thus admission probabilities) are independent across schools (e.g., Multiple Tie-Breaking, or MTB). Idoux (2022) modifies Agarwal and Somaini (2018) approach by incorporating application costs, limited rationality,[16] and behavioral heuristics into applicant decision-making. This modification circumvents the curse of dimensionality and allows for some payoff-insignificant mistakes. However, it still doesn't account for other common mistakes, such as mis-ranking schools with low admission probabilities (Hassidim, Romm, and Shorrer (2021)) or ranking schools that are irrelevant under STB (as illustrated in Example 1), even those with high admission probabilities. In contrast, TEPS is robust to a wider range of mistakes, as long as their impact on payoffs remains limited.

The concept of stability has played a crucial role in identifying preferences in the two-sided matching literature (see Chiappori and Salanié, 2016, for a survey). In this context, the preferences of both sides of the market (e.g., students and schools) are unknown to the researcher. This scenario encompasses decentralized school choice systems, like the one in Chile (He, Sinha, and Sun, 2021), where private schools' preferences are unobserved. In contrast, our framework focuses on centralized school choice systems where school priorities are observed.

[16] Relatedly, Lee and Son (2023) consider a model where applicants may hold inaccurate beliefs about admission probabilities and consider only a limited subset of schools during the application process.

## 2 Theoretical Analysis

We consider a large matching market under the Deferred Acceptance (DA) algorithm. We first illustrate how payoff-irrelevant mistakes can persist even in lottery-based assignments and how uncertainty can help uncover student preferences.

Example 1. Consider three schools, $a$, $b$, and $c$, using a Single Tie-Breaking (STB) lottery system that uses a single lottery number for each student across all schools.[17] The schools' cutoffs consistently follow the order $p_a > p_b > p_c$. Now consider a student with the preference order $b > a > c$. For such a student, school $a$ is never pivotal. Whenever $a$ is feasible, $b$ is also feasible, and the student prefers $b$. Thus, omitting $a$ or misplacing it in the ROL (e.g., $b$-$c$ or $b$-$c$-$a$) does not impact the student's assignment. This illustrates that even under STB, payoff-irrelevant mistakes can occur.

However, uncertainty aids in preference revelation and, thus, inference. In Example 1, the student must rank $b$ above $a$ and $c$ in their ROL, or else risk payoff loss. This contrasts with a scenario without uncertainty, where if the student "knows" only $c$ is feasible, the stability hypothesis reveals no further preferences as *any* ROL listing $c$ results in the same assignment.

### 2.1 Model Primitives

We begin with a continuum economy, which will serve as a benchmark for finite economies—the object of our central interest.

**Continuum Economy.** The *continuum economy*, denoted by $E$, comprises a finite set of schools $C$ and a unit mass of students with types $\widetilde{\theta} \in \widetilde{\Theta}$. We use $C$ to represent both the set of schools and its cardinality. Let $\widetilde{C} := C \cup \{\emptyset\}$, where $\emptyset$ denotes the outside option. The schools have capacities represented by $S = (S_1, ..., S_C)$, where $S_j \in (0, 1]$ and $S_\emptyset = \infty$.

Each student has an **ex-ante type** $\widetilde{\theta} = (u, t)$, where $u^{\widetilde{\theta}} = (u_1^{\widetilde{\theta}}, ..., u_C^{\widetilde{\theta}}) \in [\underline{u}, \overline{u}]^C$ represents their von Neumann-Morgenstern utilities for attending schools in $C$, with $\underline{u} < 0 < \overline{u}$. We normalize $u_\emptyset^{\widetilde{\theta}} = 0$. The ex-ante priority type, $t$, reflects the student's "intrinsic" priority or perceived merit. The set of all ex-ante priority types is denoted by $T$. Students know their ex-ante type $(u, t)$ before applying.

Given $t \in T$, a student's **scores** $s \in [0, 1]^C$ are drawn from a distribution function $\Phi_t \in \Delta([0, 1]^C)$, which may depend on $t$. These scores determine the student's ex-post priorities used for school assignments. Students do not observe these scores when submitting applications.[18] The randomness in scores stems

[17] Hence, all schools are 'within reach', and the usual reason for mistakes—clearly out of reach schools—does not apply.

[18] In NYC school district, lottery draws have been recently disclosed to the families upon request before their application (see here). However, this practice remains an exception rather than the norm. Moreover, such score revelation does not invalidate our assumption, as these revealed scores can be incorporated into the ex-ante priority type $t$.

from tie-breaking lotteries or test score uncertainty. Let $S^t := \prod_c [\underline{s}_c^t, \overline{s}_c^t]$ be the support of scores for type $t$, where $\underline{s}_c^t$ and $\overline{s}_c^t$ are the infimum and supremum scores, respectively. The pair $\theta = (u, s)$ constitutes a student's **ex-post type**. The measure $\widetilde{\eta}$ of ex-ante types, along with $(\Phi_t)_t$, induces a probability measure $\eta$ on the ex-post type $(u, s) \in \Theta := [u, u]^C \times [0, 1]^C$. We assume $\eta$ is atomless.[19] (By contrast, $\tilde{\eta}$ can, and typically will, have atoms.) The continuum economy is summarized by $E = [\widetilde{\eta}, S, (\Phi^t)_t]$.

While our main result is established under a general priority structure in Appendix A, we focus on a specific structure for clarity. The schools in $C$ are partitioned into three types: $C_1$, $C_2$, and $C_3$, each with distinct priority rules. We assume $T = T_1 \times T_2 \times T_3$, where schools in $C_i$ apply priorities in $T_i$, for $i = 1, 2, 3$.

(a) *Priority structure $T_1$–Non-lottery-based assignment with known scores*: Here, $T_1 = [0, 1]^{|C_1|}$, and for each $t \in T_1$, $\phi_t$ is degenerate with $\underline{s}_c^t = \overline{s}_c^t = t_c$ for all $c$. This represents scenarios where priorities are determined by merit scores known to students at the time of application, such as in Australian college admissions and Paris high school assignments. This case was studied by Artemov, Che, and He (2023).

(b) *Priority structure $T_2$–Coarse priorities*: The set $T_2$ is finite, and ties are broken by lotteries. Each school $c \in C_2$ has a finite number $n_c$ of intrinsic priorities $\mathcal{T}_c = \{0, \frac{1}{n_c}, ..., \frac{n_c - 1}{n_c}\}$. A student's ex-ante type is a profile $t = (t_c)_{c \in C_2} \in T_2 = \prod_{c \in C_2} \mathcal{T}_c$. The student's score for school $c$ is $s_c = t_c + \lambda_c \frac{1}{n_c}$, where $\lambda_c$ is the lottery score for tie-breaking. The lottery scores $(\lambda_c)_{c \in C_2}$ are either uniform on $[0, 1]^{|C_2|}$ (MTB) or $\lambda_c = \lambda$ for all $c \in C_2$ with $\lambda$ uniform on $[0, 1]$ (STB). This structure is commonly used in US public schools.

(c) *Priority structure $T_3$–Non-lottery-based assignment with unknown scores*: Here, $T_3 = [0, 1]$, and for each $t \in T_3$, $\Phi^t$ is absolutely continuous with a strictly positive density over scores $s \in [0, 1]$. This corresponds to scenarios like NYC's Specialized High School assignment, which uses a serial dictatorship based on applicants' SHSAT scores (unknown at application time). The ex-ante type $t \in [0, 1]$ represents the student's belief about their score.

If $C = C_1$, the model reduces to that of Artemov, Che, and He (2023), with no student uncertainty. The other extreme is **full-support uncertainty**, where $\Phi^t$ has full support $[0, 1]^C$ for all $t$, which occurs when there are no priorities and MTB lotteries are used.

**Finite Economies.** We consider a sequence of finite random economies, $F^k$, converging to the continuum economy $E$. Each *$k$-economy*, $F^k$, consists of $k$ students with types drawn independently from

[19] This assumption ensures that indifferences either in student preferences or in schools' scores arise only for a measure zero set of students.

$\widetilde{\eta}^k$, and normalized capacities $S^k = [k \cdot S]/k$, where $[x]$ rounds $x$ to the nearest integer vector (rounding down in case of ties). The distributions $\widetilde{\eta}^k$ and $\eta^k$ represent the empirical measures of ex-ante and ex-post types, respectively.

**Matching and Mechanism.** A *matching* is a mapping $\mu : C \cup \Theta \to 2^\Theta \cup (C \cup \Theta)$, describing student assignments to schools based on their ex-post types. It satisfies standard two-sidedness, consistency, and "open on the right" as defined in Azevedo and Leshno (2016) (henceforth AL, see p. 1241). Stability is a central concept defined as follows. Consider an economy, either the continuum economy $E$ or a *realization* of a finite $k$-economy. Fix any profile $p \in [0, 1]^C$, interpreted as **cutoffs** of schools. We say a school $c$ is **ex-post feasible** for type $\theta = (u, s)$ if $s_c > p_c$. Demand for $c$, $D_c(p)$, is then the measure of students for whom $c$ is the most preferred feasible school. An **ex-post stable matching** assigns all students their best feasible school at market-clearing cutoffs $p$, namely the $p$ such that $D_c(p) \leq (=) S_c$ for all $c$ (if $p_c > 0$).[20] In finite economies, cutoffs and assignments are random, so stability requires this condition to hold with probability one.

The student-proposing deferred acceptance (DA) algorithm takes students' reported ROLs and scores as input and produces a student-optimal stable matching. The resulting cutoffs are referred to as **DA cutoffs**.[21] While DA incentivizes truthful ROLs, we allow for dominated strategies in line with our focus on student mistakes.

## 2.2 Robust Equilibria

For each $k$-economy, $F^k$, we let $i = 1, ..., k$ index a student present in that economy. Each student $i$ observes her own ex-ante type $\widetilde{\theta}$ but not others'. A student $i$'s (mixed) strategy maps her ex-ante type to a probability distribution over all possible ROLs $\mathcal{R}$ of $\widetilde{C}$. The **truthful reporting strategy (TRS)**, $\rho$, ranks schools strictly according to the student's preferences.[22]

We focus on **robust equilibria** developed by Artemov, Che, and He (2023), which permits dominated strategies (mistakes) but requires that their payoff losses vanish in large economies. This concept encompasses common mistakes like omitting or mis-ranking out-of-reach schools.

The concept of robust equilibria captures applicant inattention, akin to rational inattention (Matějka

[20]Equivalently, a stable matching is defined by two properties: *individual rationality* and *no blocking*. In priority structure $T_2$, the scores may result from tie-breaking, so no blocking, and therefore stability, should ideally be defined based on ex-ante type $\widetilde{\theta}$. Nevertheless, we focus on ex-post stability since it, and not the stability, directly underpins our empirical preference inference method. Moreover, (asymptotic) ex-post stability implies (asymptotic) ex-ante stability, so establishing the former will deliver the latter.

[21]More precisely, they are the lowest market-clearing cutoffs, which are well-defined by the well-known lattice property.

[22]How $\rho$ breaks a tie is immaterial since $\eta$ is assumed to be atomless, so a tie will occur with zero probability. Hence, we leave this unspecified.

and McKay, 2015) and quantile response equilibria (Goeree, Holt, and Palfrey, 2002). Individuals may rationally allocate less attention to choices with lower payoff consequences, leading to more mistakes. However, unlike those theories, such inattention diminishes as the economy grows.

To operationalize this, we consider an infinite strategy profile $\sigma = (\sigma_i)_{i \in \mathbb{N}}$, with the interpretation that the participatants of each $k$-economy $F^k$ use its $k$-truncation $\sigma^k = (\sigma_1, ..., \sigma_k)$ as their strategies. This framework allows for asymmetric strategies; i.e., students with the same ex-ante type may submit different ROLs. The solution concept is defined as follows:

Definition 1. *An infinite profile $\sigma$ is a **robust equilibrium** if, for any $\epsilon > 0$, there exists $K \in \mathbb{N}$ such that for all $k > K$, the $k$-truncation $\sigma^k$ is an interim $\varepsilon$-Bayes Nash equilibrium. That is, for each student $i$, $\sigma_i$ yields a payoff within $\epsilon$ of her highest possible payoff, given that all other students employ $\sigma^k_{-i}$.*

The concept of robust equilibrium does not require exact optimality for the strategies but their near optimality in a sufficiently large economy. For our main result, we introduce a slight refinement:

Definition 2. *For any $\gamma \in (0, 1)$, a strategy is $\gamma$-**regular** if it coincides with TRS with probability at least $\gamma$. A profile $\sigma$ is a **regular robust equilibrium** if it is robust and there exists a $\gamma \in (0, 1)$ such that $\sigma_i$ is $\gamma$-regular for all $i \in \mathbb{N}$.*[23]

### 2.3 Analysis of Robust Equilibria

As illustrated in Example 1, applicants may not use TRS in a regular robust equilibrium.[24] Hence, one cannot use truth-telling as a behavioral restriction. Instead, we establish that the outcome of a regular robust equilibrium is asymptotically ex-post stable, which will give rise to a method for inferring preferences under the robust equilibrium.

We introduce a few concepts. For any *deterministic* cutoffs $p \in [0, 1]^C$, a strategy $\sigma_i$ is a **stable-response strategy (SRS) against** $p$ if student $i$ receives her best feasible school for all ex-ante types, with probability one. The probabilistic qualification accounts for potential randomization in $\sigma_i$ and the randomness of the ex-post type (e.g., due to lotteries). A strategy is **non-SRS against** $p$ if it is not an SRS against $p$, meaning the student might not receive her most-preferred feasible school with positive probability given $p$.

The SRS requirement, ensuring stable assignments for *almost all* uncertainty realizations, is stringent. However, TRS is an SRS against all $p$. The existence of non-TRS strategies as SRS hinges on the degree of

[23]This does not mean that all students randomize on TRS. The play of TRS could equivalently be encoded as a student's ex-ante type: namely, a measure $\gamma$ of students in the continuum economy is honest and plays TRS.

[24]While the example in Example 1 suggests only one preference type who may deviate from TRS, it is not difficult to imagine that there is a significant fraction of applicants with the same preference type.

uncertainty students face. In the extreme case of full-support uncertainty, TRS is the only SRS against any interior cutoffs.

Lemma 1 (Full-support uncertainty). *Under full-support uncertainty ($\Phi^t$ has support $[0,1]^C$ for all $t$), any SRS against interior cutoffs $p \in (0,1)^C$ must be the TRS.*

However, full-support uncertainty is rare in school choice. Schools often have priorities and/or use STB lotteries, leading to uncertainties without full support. In such cases, multiple non-TRS can be SRS against deterministic cutoffs. Recall Example 1, where for any $p_a > p_b > p_c$, reporting $b$-$c$ or $b$-$c$-$a$ (when true preferences are $b$-$a$-$c$) always yields a stable assignment, despite the random assignment due to the lottery.[25]

In finite economies, DA cutoffs are random. Thus, even an SRS against the deterministic limit cutoffs $\bar{p}$ (shown to be well-defined later) may not be an SRS against the random DA cutoffs, unless it is the TRS. The fraction of applicants playing SRS against DA cutoffs is then random. However, if this fraction converges to 1 in probability, then in large economies, almost all students receive their most preferred feasible school under DA cutoffs with high probability. The formal definition follows:

Definition 3. *A profile $\sigma$ is* ***asymptotically ex-post stable*** *if the fraction of students who employ SRS against DA cutoffs in each $\sigma^k$ converges to 1 in probability as $k \to \infty$.*

Asymptotic ex-post stability enables preference inference from DA choice data. Theorem 1 below justifies this, stating that under certain conditions, any regular robust equilibrium is asymptotically ex-post stable.

Artemov, Che, and He (2023) established this result when students face no score uncertainty ($C = C_1$), and the measure $\eta$ has full support. The full support assumption, crucial for guaranteeing a unique stable matching in their proof, must be relaxed to accommodate scenarios like STB without priorities and coarse priorities with STB. We introduce a weaker condition:

Assumption 1 (Marginal Full Support). *For each ordinal preference $r \in \mathcal{R}$ and for each school $c \in C$, the marginal density for score $s_c$*

$$\bar{\eta}(s_c, r) := \int_{\rho(u)=r, s_{-c}} \eta(u, s_c, s_{-c}) du ds_{-c}$$

*is strictly positive for all $s_c \in [0,1]$.*

This condition is weaker than the full support assumption and allows for low-dimensional score support. For example, Marginal Full Support holds in the case of STB without priorities; the support of the

[25] It is important that an SRS is defined only in conjunction with some deterministic cutoffs $p$, interpreted either as the cutoffs of the limit continuum economy (which will be shown to be deterministic) or as realizations of random cutoffs.

uncertainty/lottery is then diagonal and one-dimensional, yet the marginal of its lottery has full support. With nontrivial priorities, the condition states that each ordinal preference type has fully supported priorities. We can generalize the uniqueness result by Azevedo and Leshno (2016) and Azevedo (2014) under this weaker condition:[26]

Lemma 2. *If $\eta$ satisfies Marginal Full Support, then $E = [\eta, S]$ admits a unique stable matching.*

We are now ready to state the main result.

Theorem 1. *Suppose $\eta$ satisfies Marginal Full Support. Then, any regular robust equilibrium is asymptotically ex-post stable.*

This theorem implies that we can invoke stability for almost all uncertainty realizations in a large enough market; this is precisely what we will do in our TEPS procedure.

The theorem's key insight is that in sufficiently large economies, almost all students play SRS against the unique stable cutoffs of the limit continuum economy (well-defined by Lemma 2). We prove the theorem by contradiction. Assuming a regular robust equilibrium that is not asymptotically ex-post stable, we identify a subsequence of economies where DA cutoffs converge, and a non-vanishing fraction of students play non-SRS against these cutoffs. We then show that some of these students would gain significantly by deviating to TRS, even in large economies. This contradicts the robustness of the equilibrium.

While this argument builds on the proof of Artemov, Che, and He (2023), the ex-ante uncertainty presents a novel challenge of ensuring that the deviation from non-SRS yields discrete changes in assignment probabilities even in large economies. The proof is highly nontrivial,[27] and also requires assumptions on priority structure (those embodied by $T_1, T_2, T_3$ in Section 2.1). In Appendix A, we prove the theorem under weaker conditions.

Recall that a robust equilibrium, despite Theorem 1, is consistent with multiple behaviors regarding students' ROLs. This multiplicity raises the question of how to conduct counterfactual analysis. The next corollary of Theorem 1 addresses this, justifying the assumption of truth-telling in counterfactual scenarios for predicting assignment outcomes.

Corollary 1. *Suppose $\eta$ satisfies Marginal Full Support. Then, in any regular robust equilibrium, the fraction of students whose assigned schools coincide with the ones that would arise if everyone employs TRS converges to one in probability as $k \to \infty$.*

[26] Relatedly, Agarwal and Somaini (2018) and Grigoryan (2022) provide other weak conditions for guaranteeing the uniqueness of stable matching in the continuum economy.

[27] See Lemma 4 and its subsequent use in the proof of Theorem 1′.

## 3 Inferring Student Preferences from ROLs: Transitive Extension of Preferences from Stability

Building upon Theorem 1, we introduce the Transitive Extension of Preferences from Stability (TEPS) procedure to infer students' ordinal preferences from school choice data. We will show that TEPS extracts the maximal information about preferences that is consistent with our theoretical framework.

Consider a standard dataset from a centralized DA assignment system, in which the analyst has access to applicants' submitted ROLs, their intrinsic priorities, and school capacities.

### 3.1 Preference Relations Inferred by Stability and Transitivity

As preference inference is conducted individually for each applicant, we can focus on a single applicant without loss of generality. Assume the applicant submits a ROL $R$ and has priorities $t$. We will simplify the notation by omitting $R$ and $t$ when the context is clear. For now, we will assume no outside option exists, though incorporating outside options is straightforward.[28] Let $\Omega$ represent the set of all assignment-relevant states, essentially, all possible profiles of priorities for all applicants at all schools. Practically, each state $\omega \in \Omega$ encompasses lottery draws, entrance exam scores, and other uncertainties affecting school cutoffs.

Our objective is to utilize the data and Theorem 1 to make an inference about the set of preference relations $\mathcal{P} := \{(x, y) \in C^2 : x \in C$ is inferred more preferred to $y \in C\}$ for the applicant. The interpretation of Theorem 1 depends on market size and the payoff relevance of mistakes. To simplify the initial explanation, we initially assume stable matching outcomes for all uncertainty realizations $\omega$, implying no payoff-relevant mistakes. While this strictly holds only in the limit continuum economy, it is a useful starting point to explain our method. We will relax this assumption later in Section 3.2.

The TEPS procedure comprises three steps to generate preference relations $\mathcal{P}$. We will use the following example to illustrate the procedure.

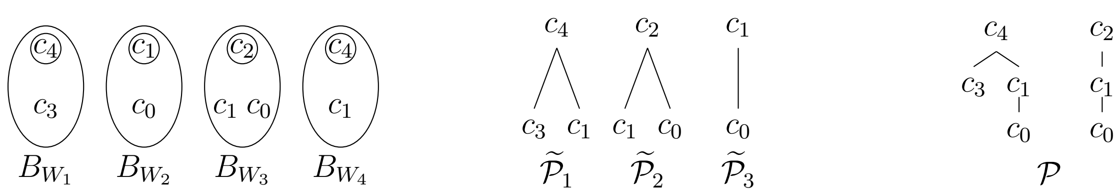
Example 2. *There are six schools,* $\{c_0, c_1, c_2, c_3, c_4, c_5\}$. *A student has submitted ROL,* $R = c_4$-$c_3$-$c_2$-$c_1$. *Random priorities facing her then entail four possible feasible sets: (a)* $\{c_3, c_4\}$ *with probability* $0.4$*; (b)* $\{c_0, c_1\}$ *with probability* $0.3$*; (c)* $\{c_0, c_1, c_2\}$ *with probability* $0.25$*; (d)* $\{c_1, c_4\}$ *with probability* $0.05$.

**Step 1. Simulating uncertainties and compiling choice data:**

We first simulate all possible priority profiles by simulating uncertainties. For instance, in lottery-based systems, uncertainties can be simulated by conducting lotteries and executing DA, mirroring the

[28] The procedure can be easily extended to include an outside option (ø), representing non-assignment or other always-feasible alternatives, by redefining the set of schools as $\widetilde{C} = C \cup \{ø\}$.

actual mechanism.[29] Each simulated state $\omega \in \Omega$ captures the students' realized priorities, allowing us to determine school cutoffs based on their submitted ROLs. From this, we can identify the *feasible schools*, $B(\omega)$, and the *assigned schools*, $\alpha(\omega)$,[30] for each student in each state. Since multiple $\omega$'s can lead to the same feasible set, we next partition $\Omega$ so that any two $\omega, \omega'$ giving rise to the same feasible set belongs to the same partition cell. Let $P_\Omega$ denote the resulting partition and $W \in P_\Omega$ a typical partition cell. In the example, we have four partition cells, $W_1, W_2, W_3, W_4$ with distinct feasible sets, $B_{W_1} = \{c_4, c_3\}$, $B_{W_2} = \{c_1, c_0\}$, $B_{W_3} = \{c_2, c_1, c_0\}$, and $B_{W_4} = \{c_4, c_1\}$, and the assigned schools $\alpha_{W_1} = c_4$, $\alpha_{W_2} = c_1$, $\alpha_{W_3} = c_2$, and $\alpha_{W_4} = c_4$. The resulting pairs $(B_W, \alpha_W)_{W \in P_\Omega}$ form the *choice data* for the student, visually represented in Figure 1(a).[31]




(a) Step 1 (b) Step 2 (c) Step 3

Figure 1: An Example of TEPS

**Step 2. Inferring preference relations in each realized uncertainty:**

Theorem 1 states that in the limit economy, almost all students are stably assigned for every state $\omega \in \Omega$. This implies that, for each partition cell $W$, the assigned school $\alpha_W$ is the student's most preferred choice among the feasible schools $B_W$. This yields preference relations $\widetilde{\mathcal{P}}_W \subset C^2$ for each $W \in P_\Omega$.[32] The collection of all these inferred preference relations is denoted as $\widetilde{\mathcal{P}} := \cup_{W \in P_\Omega} \widetilde{\mathcal{P}}_W$. We then partition $\widetilde{\mathcal{P}}$ into ordered sets $\{\widetilde{\mathcal{P}}_1, \widetilde{\mathcal{P}}_2, ...., \widetilde{\mathcal{P}}_m\}$ , ensuring that within each set, the same school is assigned, and across sets, the assigned schools follow the order they appear in the student's ROL.[33] In our example, this step produces $\widetilde{\mathcal{P}}_1 = \{(c_4, c_3), (c_4, c_1)\}$, $\widetilde{\mathcal{P}}_2 = \{(c_2, c_1), (c_2, c_0)\}$, and $\widetilde{\mathcal{P}}_3 = \{(c_1, c_0)\}$. In Figure 1(b), those are represented by three ordered trees with one length, where the roots correspond to the schools the student may be assigned in some realizations of the uncertainties.

[29]To simulate the student composition uncertainty, we may bootstrap from the observed ROL to create multiple economies/applicant compositions. For uncertainties arising from other sources, like unknown test scores, an empirical model can be built to simulate the score distribution based on observable factors (e.g., academic performance measures).

[30]The highest-ranked choice within $B(\omega)$ according to the student's ROL.

[31]Since we do not consider outside options, we omit $(\alpha_W, B_W)$ if $\alpha_W = \emptyset$, a non-assignment. If we include an outside option, we should include $(\alpha_W, B_W)$, where $\alpha_W = \emptyset$ and $B_W$ is the set of (unranked) schools that were feasible when the student was not assigned.

[32]Recall that each element in $\widetilde{\mathcal{P}}$ is an ordered pair of schools, $(c, c')$, where $c$ is inferred preferred to $c'$.

[33]More precisely, for any $(c, d) \in \widetilde{\mathcal{P}}_i$ and $(c', d') \in \widetilde{\mathcal{P}}_j$, (i) $c = c'$ if $i = j$, and (ii) $c$ is ranked ahead of $c'$ on the ROL if $i < j$.

**Step 3. Extending preference relations by transitivity axiom:**

The last step employs the transitivity axiom to connect the preference relations $\widetilde{\mathcal{P}}$ derived in Step 2. The algorithm begins with the preference sets associated with the worst-ranked assigned schools and progressively incorporates any preferences implied by transitivity.[34] Starting with $\widetilde{\mathcal{P}}_m$ and $\widetilde{\mathcal{P}}_{m-1}$ (where $m$ is the highest index), a new set $\mathcal{P}_{m-1}$ is constructed by including $\widetilde{\mathcal{P}}_m \cup \widetilde{\mathcal{P}}_{m-1}$ and adding preferences relations implied by transitivity: If $(y, z) \in \widetilde{\mathcal{P}}_m$ and $(x, y) \in \widetilde{\mathcal{P}}_{m-1}$ then, we infer that $x$ is revealed preferred to $z$ and add $(x, z)$ to $\mathcal{P}_{m-1}$. This process is repeated with $\mathcal{P}_{m-1}$ and $\widetilde{\mathcal{P}}_{m-2}$ to obtain $\mathcal{P}_{m-2}$, and so forth, until the final output $\mathcal{P} := \mathcal{P}_1$ is obtained. In our example, $\mathcal{P} = \{(c_1, c_0), (c_2, c_1), (c_2, c_0), (c_4, c_1), (c_4, c_3), (c_4, c_0)\}$, depicted in Figure 1(c) as a collection of trees.

Importantly, TEPS procedure does not infer all preference rankings stated in a student's ROL. In our example, TEPS procedure does not establish a relationship between $c_4$ and $c_2$ despite the ROL listing $c_4$-$c_3$-$c_2$-$c_1$. This is because these two schools are never feasible simultaneously,[35] so their relative ranking is neither directly revealed, nor are they indirectly revealed by transitivity. Thus, TEPS deems this part of the ROL unreliable for inferring true preferences. Additionally, the unranked schools $c_5$ is never feasible (i.e., out of reach) under any uncertainty realizations, and thus, arbitrarily ranking or omitting it has no impact on the student's assignment. Consequently, TEPS does not infer any preference relation involving $c_5$. In essence, TEPS extracts a subset of preference relations of what the submitted ROL (or WTT, as will be seen later) states, focusing on those supported by stability and transitivity.

Nevertheless, TEPS infers the maximum preference relations that are consistent with Theorem 1. More generally, consider any $\Omega' \subset \Omega$, and let $\mathcal{P}^{\text{ST}}(\Omega')$ denote the preference relations inferred by applying stability to all uncertainties in $\Omega'$ and the transitivity axiom. In practice, $\Omega'$ is the set of uncertainties the analyst considers "relevant." We will elaborate on how $\Omega'$ is selected from simulated uncertainties later. The following result holds:

Proposition 1. *Fix a student's ROL $R$ and her observable priorities $t$. Suppose $\Omega' \subset \Omega$ is used in TEPS, then*

[34]Alternative fast algorithms for computing transitive closures exist, often based on shortest path or breadth-/depth-first search, with polynomial time complexity in the number of choices (see https://en.wikipedia.org/wiki/Transitive_closure.) The computational burden is further reduced in practice as the number of schools on a student's ROL is typically much smaller than the total number of available schools. This is because only schools ranked on the ROL and revealed to be preferred to other ranked schools are involved in the transitive closure procedure.

[35]Such cases occur under MTB, which uses distinct lotteries to break ties at different schools. Even with STB, such cases may occur if the cutoffs are not degenerate (e.g., in a finite economy) as the relative ranking of school cutoffs may vary across STB lottery realizations. Practically, applicants may struggle to identify simultaneously feasible schools. We adopt a conservative approach, avoiding inferences about non-simultaneously feasible schools to prevent potential reliance on unreliable preference information.

$\mathcal{P} = \mathcal{P}^{ST}(\Omega')$. *That is, the TEPS procedure infers the preference relations if and only if they can be inferred by the stability conditions on possible uncertainties in* $\Omega' \subset \Omega$ *and transitivity.*

### 3.2 Allowing Violations of Stability in TEPS

In principle, Step 1 of TEPS could simulate all uncertainties by setting $\Omega' = \Omega$. This amounts to requiring ex-post stability for all uncertainties, which is justified when the applicant makes no payoff-relevant mistakes. Theorem 1 guarantees this in a robust equilibrium only for the infinite economy. In any finite economy, a robust equilibrium allows for some payoff-relevant mistakes that lead to deviations from ex-post stability, as long as the payoff loss is minor. To allow for such mistakes, we define "sufficiently likely" states $\Omega' \subset \Omega$ such that TEPS is applied only to $\Omega'$. The modified procedure is as follows.

Recall in Step 1, we partitioned $\Omega$ into cells based on the applicant's feasible sets $B_W$. Now, we order these feasible sets in descending order of likelihood. In our example, the four feasible sets are already indexed this way, with $B_{W_1}$ being the most likely and $B_{W_4}$ being the least likely. We consider the cumulative likelihood of these sets, starting from the most likely. We then introduce an "attention parameter" $\tau \in [0, 100]$, which acts as a cumulative likelihood threshold (in percentage) for including feasible sets in our TEPS procedure. In our example, setting $\tau = 95$ includes only $B_{W_1}$, $B_{W_2}$, and $B_{W_3}$, as their combined likelihood reaches 95%. TEPS inference is then focused solely on $\Omega' = \{B_{W_1}, B_{W_2}, B_{W_3}\}$.[36]

We call the adjusted procedure TEPS$^\tau$, and let $\mathcal{P}^\tau$ denote the resulting preference relations. We label TEPS$^{100}$ and $\mathcal{P}^{100}$ as TEPS$^{all}$ and $\mathcal{P}^{all}$, respectively, as they utilize *all* feasible set realizations.[37] Decreasing the attention parameter $\tau$ makes TEPS$^\tau$ more tolerant of stability violations. At $\tau = 0$, TEPS$^0$ infers student preferences solely based on the most likely feasible set, amounting to her considering only the most likely feasible set. Thus, TEPS$^0$ is also referred to as TEPS$^{top}$.

The rationale for the attention parameter is straightforward. Students may not fully consider low-probability feasible sets such as $B_{W_4}$ when forming their ROLs, potentially leading to unreliable inferences from these sets. TEPS$^\tau$ mitigates this by focusing only on the most likely feasible sets. In our example, TEPS$^{95}$ produces preference relations $\mathcal{P}^{95}$ (Figure 2) which is a subset of $\mathcal{P}^{100}$ (Figure 1(c)).

### 3.3 Comparison of TEPS with WTT and Stability

The Weak Truth-Telling (WTT) assumption has traditionally been used to infer student preferences (e.g., Abdulkadiroglu, Agarwal, and Pathak, 2017; Laverde, 2022). WTT involves two assumptions: (a) the

[36]Formally, Step 1 of TEPS with threshold $\tau$ always includes the most likely feasible set $B_{W_1}$, and additionally includes the feasible set up to $B_{W_{\bar{T}}}$ such that $\sum_{t=1}^{\bar{T}} Prob(W_t) \leq \tau\%$ and $\sum_{t=1}^{\bar{T}+1} Prob(W_t) > \tau\%$.

[37]Using this notation, the TEPS procedure and the resulting preference relations in Section 3.1 correspond to TEPS$^{all}$ and $\mathcal{P}^{all}$, respectively.

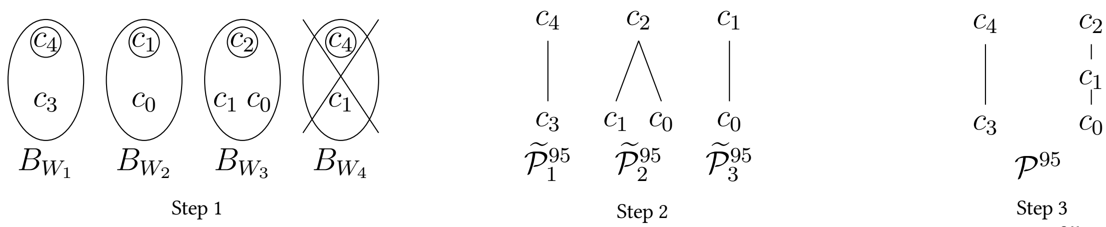


Figure 2: An Example of Allowing Violations of Stability in TEPS: TEPS$^{95}$

number of choices ranked in any ROL is exogenous to student preferences, and (b) students rank their top choices truthfully but may omit some least preferred schools. Let $\mathcal{P}^{\text{WTT}}$ represent the preferences of a student inferred by WTT. In Example 2, for the student with $R$=$c_4$-$c_3$-$c_2$-$c_1$, WTT infers $u_{c_4} > u_{c_3} > u_{c_2} > u_{c_1} > u_{c_0}, u_{c_5}$, or $\mathcal{P}^{\text{WTT}} = \{(c_4, c_3), (c_4, c_2), (c_4, c_1), (c_4, c_0), (c_4, c_5), (c_3, c_2), (c_3, c_1), (c_3, c_0), (c_3, c_5), (c_2, c_1), (c_2, c_0), (c_2, c_5), (c_1, c_0), (c_1, c_5)\}$. Notably, unranked schools, $c_0$ and $c_5$ are inferred as less preferred to all ranked schools in $R$.

WTT remains justifiable by asymptotic ex-post stability of robust equilibria if applicants face full-support uncertainty (Lemma 1). In this case, uncertainty makes mistakes costly, discouraging them in any robust equilibrium (in a large market). However, mistakes can persist in a robust equilibrium without full-support uncertainty, potentially leading to biased preference estimates under WTT, as have been argued in Fack, Grenet, and He (2019) and Artemov, Che, and He (2023). TEPS$^\tau$, for $\tau \in [0, 100]$, addresses this issue by offering a flexible range of methods for identifying preferences from ROL data, even with uncertainties and potential mistakes. Importantly, TEPS does not inherently contradict WTT; rather, it focuses on extracting more reliable information from ROLs. WTT and TEPS$^\tau$ with varying $\tau$ form a nesting structure:

Proposition 2. *Fix any student with a ROL $R$ and intrinsic priority $t$. We have $\mathcal{P}^{top} \subseteq \mathcal{P}^{\tau} \subseteq \mathcal{P}^{\tau'} \subseteq \mathcal{P}^{all} = \mathcal{P}^{ST}(\Omega) \subseteq \mathcal{P}^{WTT}$ for any $0 < \tau < \tau' < 100$.*

To account for strategic mistakes, Fack, Grenet, and He (2019) and Artemov, Che, and He (2023) suggested stability as an alternative to WTT. However, this approach assumes no uncertainties in ex-post cutoffs or applicants' priorities/scores, which is often unrealistic. Moreover, their method is not even defined for the uncertain priority structure.[38] Our method generalizes the stability-based approach to

[38]A naive approach to extend the method could be to impose stability using ex-post cutoff obtained from a single, arbitrary lottery draw. While seemingly similar to TEPS$^{top}$, this approach has two key problems. First, this ad-hoc adaptation of stability lacks theoretical grounding from Theorem 1. The ex-post cutoffs from a single lottery draw could very well be an outlier, a very low-probability event neglected by the applicant, making the "inferred" preference potentially unreliable about the applicant's true preference. Second, if the actual lottery results are not observed, researchers must simulate them, introducing the possibility of researchers selectively choosing lottery realizations that support desired conclusions. In contrast, TEPS$^{top}$ is

accommodate uncertainties.

### 3.4 Identification and Estimation

The assumptions of stability and transitivity underlying TEPS do not uniquely pin down a student's best response or the ROL she submits. This may raise concerns about the completeness of our model as defined by Tamer (2003), where the mapping from a student's type, $(u, t)$, to TEPS inferred preferences could be a correspondence, potentially hindering the point identification of the distribution of $u$. These concerns, however, do not apply to our method. As formally proven in Online Appendix C, TEPS ensures a unique mapping from each student type to a preference—i.e., a set of inferred preference relations. The intuition is that TEPS, by considering all uncertainties, derives a unique distribution of assignment outcomes using stability and transitivity. TEPS is complete since it relies solely on this unique distribution, even though multiple best-response ROLs may be consistent with it.

Once preferences are inferred, a parametric utility function can be fitted for parameter estimation. A common approach involves using random utility models (e.g., with EVT1 or normal errors) and estimating parameters via maximum likelihood estimation, Markov Chain Monte Carlo methods (e.g., Gibbs sampling), or Expectation-Maximization algorithms. We recommend a multinomial probit model (with or without random coefficients) estimated via Gibbs sampling for its flexibility in drawing cardinal utilities consistent with inferred preferences. For example, with the inferred preferences $c_1 \succ c_2$ and $c_1 \succ c_3 \succ c_4$, writing the exact likelihood function is challenging even with EVT1 errors. However, the inferred preferences provide utility bounds for each school, which can be easily incorporated into the Gibbs sampler.[39]

### 3.5 Selecting from among Alternative Preference Hypotheses

Proposition 2 presents a nested family of preference inference hypotheses, each offering a tradeoff between utilizing more data and being robust to potential mistakes. The optimal method depends on the extent and the payoff-significance of these mistakes, which is an empirical question. Thus, we devise a data-driven testing procedure using Wald tests to select the most suitable method, ranging from WTT to various tolerance levels for mistakes within the TEPS$^\tau$ framework. We assume that TEPS$^{top}$, the most robust inference, yields consistent parameter estimates, serving as the alternative hypothesis to sequentially test the consistency of more informationally demanding models, starting with WTT and then TEPS$^\tau$ in the descending order of $\tau$'s.

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both well-founded from theoretical justification (Theorem 1) and is ex-post verifiable.

[39]In the example, assuming no outside options, the bounds are given by $u_1 \in (\max\{c_2, c_3\}, \infty)$, $u_2 \in (-\infty, u_1)$, $u_3 \in (u_4, u_1)$, $u_4 \in (-\infty, u_3)$. Each Gibbs sampler iteration sequentially draws school utilities for each student, subject to these bounds. The detailed procedure is outlined in Online Appendix E.

Concretely, suppose we wish to estimate a parameter vector $\beta$ in a parametric model. Each method produces an estimator for $\beta$: the WTT-based estimator ($\widehat{\beta}^{\text{WTT}}$) and the TEPS$^{\tau}$-based estimator ($\widehat{\beta}^{\tau}$). As an example, consider TEPS$^{top}$ and 10 other $\tau$ values, $\tau_1 = 10, \tau_2 = 20, \ldots, \tau_9 = 90, \tau_{10} = 100$ (note that TEPS$^{\tau_{10}}$=TEPS$^{all}$). The following procedure selects the consistent and efficient estimator among those considered.[40]

**Step 1: Test TEPS$^{top}$ vs. WTT.** We formulate two hypotheses:

- $H_0^{\text{WTT}}$: $\widehat{\beta}^{\text{WTT}}$ and $\widehat{\beta}^{top}$ are both consistent, while $\widehat{\beta}^{\text{WTT}}$ is efficient.
- $H_1^{\text{WTT}}$: only $\widehat{\beta}^{top}$ is consistent.

Since WTT leads to an efficient estimator under the null, we conduct a Wald test based on the statistic,

$$\left(\widehat{\beta}^{top} - \widehat{\beta}^{\text{WTT}}\right)' \left(V(\widehat{\beta}^{top}) - V(\widehat{\beta}^{\text{WTT}})\right)^{-1} \left(\widehat{\beta}^{top} - \widehat{\beta}^{\text{WTT}}\right)$$

where $V(\cdot)$ denotes the covariance matrix of its argument. Under the null, the test statistic follows a $\chi^2_{|\beta|}$ distribution. Justified by the nesting structure (Proposition 2) and the maintained assumption on TEPS$^{top}$, if $H_0^{\text{WTT}}$ is not rejected, we select $\widehat{\beta}^{\text{WTT}}$ and exit the procedure; otherwise, we perform the next test.

**Step 2: Test TEPS$^{top}$ vs. TEPS$^{\tau}$.** We start with the largest $\tau$, $\tau_{10} = 100$.

- $H_0^{\tau}$: $\widehat{\beta}^{\tau}$ and $\widehat{\beta}^{\text{top}}$ are both consistent, while $\widehat{\beta}^{\tau}$ is efficient;
- $H_1^{\tau}$: only $\widehat{\beta}^{\text{top}}$ is consistent.

We conduct a Wald test based on the statistic,

$$\left(\widehat{\beta}^{\text{top}} - \widehat{\beta}^{\tau}\right)' \left(V(\widehat{\beta}^{top}) - V(\widehat{\beta}^{\tau})\right)^{-1} \left(\widehat{\beta}^{\text{top}} - \widehat{\beta}^{\tau}\right).$$

Under the null, the test statistic follows a $\chi^2_{|\beta|}$ distribution. If $H_0^{\tau}$ is not rejected, we select $\widehat{\beta}^{\tau}$; otherwise, we proceed to test the next largest $\tau$, $\tau_9$. We continue until we reach a value of $\tau$ such that $H_0^{\tau}$ is not rejected, in which case we select $\widehat{\beta}^{\tau}$, or we reject $H_0^{\tau_1}$, in which case we select $\widehat{\beta}^{top}$.

The order of these steps guarantees the desired size of the tests and makes it appropriate to use the Hausman-type test statistics.[41]

[40] One drawback of the testing procedure is its reliance on correct parametric model specification, potentially susceptible to misspecification. In such cases, nonparametric tests for nested models may offer a more general approach, if one is willing to consider nonparametric models.

[41] The power of the testing procedure, however, is affected by the number of $\tau$ values considered. The sequential nature of the tests precludes the use of multiple hypotheses testing corrections like Bonferroni or Benjamini and Hochberg (1995).

### 3.6 Performance of TEPS: Monte Carlo Simulations

We use Monte Carlo simulations to evaluate the performance of TEPS (see Online Appendix D for details.)[42] We simulate 100 finite economies with 1,000 students applying to 12 schools under a student-proposing DA algorithm with coarse priorities and single tie-breaking, similar to the NYC public school choice ($T_2$ in Section 2.1). Each economy contains independently randomly generated priorities and preferences following a parametric random utility model. We consider three data generating processes (DGP); (i) Truth-telling (TT) where students submit truthful ROLs; (ii) Payoff Irrelevant Mistakes (MIS-IRR) where students may skip or flip (i.e., arbitrarily rank) out-of-reach schools, and (iii) Payoff Relevant Mistakes (MIS-REL) where students may additionally skip schools with small but positive admission chances.[43] Table 2 summarizes these scenarios. The fraction of students making mistakes rises from 0% in TT to 74.4% in MIS-IRR and MIS-REL, resulting in only 27.0% and 28.8% WTT-consistent preferences in MIS-IRR and MIS-REL, respectively. Stability holds for all students in TT and MIS-IRR, but not in MIS-REL, as some students skip schools within potential reach.

Table 2: Mistakes in Monte Carlo Simulations (%)

| | Scenarios: DGP w/ Different Student Strategies | | |
|---|---|---|---|
| | Truth-Telling | Payoff Irrelevant Mistakes | Payoff Relevant Mistakes |
| | TT | MIS-IRR | MIS-REL |
| Average length of submitted ROLs | 12 | 6.1 | 5.0 |
| WTT: *Weak-Truth-Telling* | 100 | 27.0 | 28.8 |
| Matched w/ favorite feasible school | 100 | 100 | 96.2 |
| Make Mistakes | 0 | 74.4 | 74.4 |

*Note:* Each entry reported is a percentage that is averaged over the 100 estimation samples. A student is WTT if 1) ROL is in the true preference order, and 2) all ranked schools are more preferred to all unranked schools.

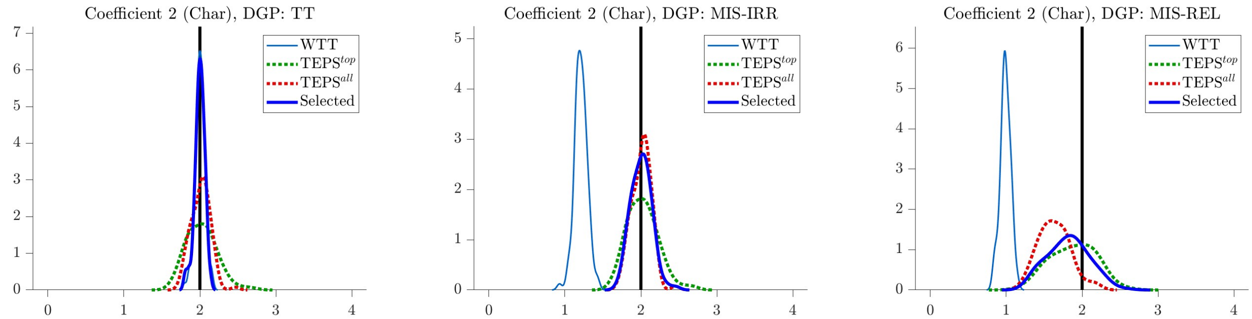
For each of 300 scenarios (3 DGPs $\times$ 100 copies), we estimate preference parameters using a Gibbs sampler, based on WTT and TEPS with attention parameters $\tau = 0, 20, 40, 60, 80, 100$. After estimation, we apply the testing procedure from Section 3.5 to choose the optimal estimate, referred to as the "selected" estimate.

Figure 3 summarizes the results. First, under the TT DGP, all estimates are consistent, while the WTT-based estimator has the smallest variance since it utilizes the maximum possible information from the observed ROLs, even though some of this information might be potentially unreliable due to mistakes. Second, with payoff-irrelevant or -relevant mistakes (MIS-IRR, MIS-REL), WTT shows significant bias due

[42] Online Appendix D additionally presents results from TEPS incorporating finite market uncertainty. These results closely resemble those reported in this section.

[43] Specifically, students may omit schools with admission probabilities below 10%.

(a) Distribution of Estimates




(b) Test Results: Fraction of Each Method Being Selected

| Data Generating Process: | | TT | MIS-IRR | MIS-REL |
|---|---|---|---|---|
| Estimation method | WTT | **0.94** | 0 | 0 |
| | TEPS$^{top}$ | 0.01 | 0 | 0 |
| | TEPS$^{20}$ | 0 | 0 | 0 |
| | TEPS$^{40}$ | 0 | 0 | 0.01 |
| | TEPS$^{60}$ | 0 | 0.01 | 0.14 |
| | TEPS$^{80}$ | 0.03 | 0.12 | **0.68** |
| | TEPS$^{all}$ | 0.01 | **0.87** | 0.17 |

Figure 3: Monte Carlo Simulations: Performance of TEPS and WTT

*Note:* See Online Appendix D for the exact description of our Monte Carlo simulation. In panel (a), we plot the kernel density plots of one parameter estimate ($\beta_2$ in Equation (D.4)) from 100 Monte Carlo samples. The left subfigure corresponds to TT DGP, the center subfigure corresponds to MIS-IRR DGP, and the right subfigure corresponds to MIS-REL DGP. The black vertical line at 2 denotes the true value of the parameter. We depict only WTT, TEPS$^{top}$, TEPS$^{all}$, and the Selected estimates for conciseness. In panel (b), we report the fraction of each estimate being chosen as the 'Selected' estimate among 100 Monte Carlo samples, according to the testing procedure in Section 3.5.

to its susceptibility to mistakes. In contrast, TEPS-based estimators are robust to payoff-irrelevant mistakes. A two-sided two-sample Kolmogorov-Smirnov (KS) test confirms this, rejecting the null hypotheses of identical distributions for WTT and TEPS$^{top}$/TEPS$^{all}$ estimates ($p$ <0.001), but not rejecting it for TEPS$^{top}$ and TEPS$^{all}$ ($p$ =0.193). Third, with payoff-relevant mistakes (MIS-REL), TEPS$^{all}$ becomes inconsistent as it does not account for such mistakes. However, TEPS$^{top}$ and other TEPS$^{\tau}$ with $\tau < 100$ remain consistent (see Online Appendix D.) In this case, the KS test rejects the null hypothesis of identical distributions for TEPS$^{top}$ and TEPS$^{all}$ ($p$ <0.001).

Aligned with these findings, our testing procedure selects WTT-based estimates 94% of the time under the TT DGP but never under MIS-IRR or MIS-REL. With no payoff-relevant mistakes (MIS-IRR), the procedure selects TEPS$^{all}$ 87% of the time, as it utilizes the maximal information and hence has the smallest variance among TEPS-based methods robust to payoff-irrelevant mistakes (Proposition 2). Finally, with payoff-relevant mistakes (MIS-REL), TEPS$^{80}$ is selected 68% of the time, as TEPS$^{all}$ may no longer be robust to such mistakes.

# 4 High School Choice in Staten Island, New York City

We now apply our framework to high school choice in NYC with two purposes. First, we showcase the practical implementation of our methodology on real-world data. We will estimate preferences under various assumptions about student behavior and let our testing procedure select the most appropriate one. Second, we highlight the importance of choosing the right assumption for preference inference by comparing predicted effects of desegregation policies under different assumptions on mistakes, demonstrating how these assumptions can lead to different policy predictions.[44]

NYC public high school admissions use the student-proposing DA (Abdulkadiroglu, Pathak, and Roth, 2005). Students apply to up to 12 programs, each with its own capacity and independent admissions process. Eligible students are grouped into priority groups, with ties broken by a single lottery draw across all programs (STB). This random tie-breaking generates priority uncertainties (of type $T_2$ in Section 2.1), making NYC public high school admissions a suitable candidate for our methodology.[45]

We focus on Staten Island (SI) students and programs participating in Round 1 (the main round) of NYC public high school choice in the 2016–17 academic year. SI is one of the five boroughs of NYC, and its geographic isolation arguably makes it an independent matching market of a typical medium-sized US city.[46] Further details on the institutional background and data are in the Online Appendix F.[47]

Column (1) of Table 3 provides summary statistics of the students, and Table 4 describes the programs and schools. Most students participate in Round 1 and enroll in their assigned school. Our data covers 3,731 students and 50 programs at nine schools. The ROL length limitation is not binding in SI, with an average length of 4.05 and only about 3% of the students exhausting the list. Nearly all middle school students residing in SI attend SI middle schools (about 95%) and apply only to SI high schools (see panel D of Table 3). Compared to NYC overall, SI is wealthier and has a higher proportion of white students, with median household income of $74,580 vs. $55,191 and 53% vs. 15% white students, respectively.

[44]There is a substantive interest among the public and policymakers in diversifying and desegregating student bodies of NYC schools. The NYC DOE recently proposed eliminating test-based screening and geographic preferences in admissions (Shapiro, 2021). Some versions of these proposals were implemented but later reversed (Closson, 2022).

[45]In the 2016-17 academic year, approximately 63% of the 769 NYC programs used lotteries for tie-breaking, while others employed non-random tie-breakers (e.g., test or audition scores). In our main sample from SI, about 68% of programs utilized lotteries.

[46]SI is connected with the rest of the city only by the Staten Island Ferry or the Verrazzano-Narrow Bridge. While direct policy implications for NYC may be limited, SI's demographics and size better reflect those of typical medium-sized US cities, enhancing the relevance of our findings for other urban areas.

[47]We do not model students' preferences for 9 specialized high schools (e.g., Stuyvesant, Bronx Science, etc.) in NYC, as they use a separate admission procedure using SHSAT.

Table 3: Summary Statistics: Sample Means across Students

| | (1) Total | (2) Cell 1 | (3) Cell 2 | (4) Cell 3 | (5) Cell 4 | (6) Cell 5 | (7) Cell 6 | (8) Cell 7 | (9) Cell 8 |
|---|---|---|---|---|---|---|---|---|---|
| *Panel A. Cell characteristics* | | | | | | | | | |
| Female | 0.49 | no | yes | no | yes | no | yes | no | yes |
| Free or reduced-price lunch (FRPL) | 0.53 | no | no | yes | yes | no | no | yes | yes |
| Black/Hispanic | 0.36 | no | no | no | no | yes | yes | yes | yes |
| *Panel B. Student characteristics* | | | | | | | | | |
| Asian | 0.10 | 0.10 | 0.09 | 0.25 | 0.22 | 0.00 | 0.00 | 0.00 | 0.00 |
| White | 0.53 | 0.90 | 0.90 | 0.73 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 |
| Black | 0.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.27 | 0.33 | 0.34 |
| Hispanic | 0.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.70 | 0.74 | 0.67 | 0.66 |
| Median Income ($1,000) | 74.58 | 85.21 | 86.09 | 77.72 | 78.42 | 65.97 | 66.56 | 59.55 | 57.32 |
| Grade 7 ELA | 310.87 | 317.38 | 326.83 | 306.23 | 316.91 | 299.04 | 319.93 | 289.67 | 301.48 |
| Grade 7 Math | 310.21 | 326.72 | 326.11 | 313.36 | 315.34 | 296.88 | 308.43 | 285.54 | 287.27 |
| *Panel C. Admission outcomes* | | | | | | | | | |
| Matched with top choice | 0.57 | 0.61 | 0.60 | 0.56 | 0.55 | 0.55 | 0.54 | 0.54 | 0.56 |
| Unassigned | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 |
| *Panel D. Submitted ROLs* | | | | | | | | | |
| # of SI Choices | 4.05 | 4.03 | 3.97 | 4.01 | 4.06 | 4.14 | 3.69 | 4.11 | 4.20 |
| # of Choices (all) | 4.06 | 4.05 | 3.98 | 4.02 | 4.07 | 4.16 | 3.70 | 4.12 | 4.21 |
| Observations | 3731 | 710 | 743 | 471 | 455 | 178 | 136 | 545 | 493 |

Table 4: Summary Statistics: Sample Means across Programs/Schools

| | Program | | School | |
|---|---|---|---|---|
| | Mean | Std. Dev. | Mean | Std. Dev. |
| Capacity | 134.06 | (215.47) | 744.78 | (551.49) |
| 9th Grade Size | 71.30 | (108.79) | 396.78 | (293.48) |
| % High Performer (ELA) | 31.08 | (26.82) | 34.12 | (15.01) |
| % High Performer (Math) | 26.21 | (25.46) | 25.83 | (12.19) |
| % White | 36.32 | (22.14) | 43.93 | (21.80) |
| % Asian | 7.94 | (7.39) | 7.61 | (3.40) |
| % Black | 19.93 | (13.82) | 17.51 | (12.79) |
| % Hispanic | 34.45 | (16.76) | 29.57 | (12.72) |
| % Free or Reduced-Price Lunch (FRPL) | 63.40 | (18.08) | 60.50 | (13.65) |
| $\mathbb{1}$(STEM) | 0.20 | (0.40) | | |
| Observations | 50 | | 9 | |

*Note:* Programs that are 'For Continuing 8th Graders' and zoned do not have capacity restrictions. We count the total number of students who are eligible for those programs and report that as their capacity.

## 4.1 Preference Estimation

We use a random utility model to represent student preferences over programs.[48] The sample is divided into eight mutually exclusive cells based on gender, free or reduced-price lunch (FRPL) status, and

[48]We do not model outside options. Our data shows nearly all students (3,724 out of 3,731) received assignments (Table 3), with less than 2% declining enrollment.

Black/Hispanic ethnicity, allowing model parameters to vary freely across these cells à la Abdulkadiroğlu, Pathak, Schellenberg, and Walters (2020). Columns (2)–(9) of Table 3 present the summary statistics for each cell.

Let $u_{icps}$ denote the utility of student $i$ in cell $c$ when matched with program $p$ at school $s$. The utility is parameterized as follows:

$$u_{icps} = \alpha_{cs} + \beta_{1,c} D_{is} + \beta_{2,c} \textit{Nearest}_{is} + \beta_{3,c} z_p + \sum_{l} \gamma_c^l x_i^l Z_s^l + \sum_{k} \delta_c^k x_i^k z_p^k + \sigma_{\tau(p),c} \epsilon_{icps}, \quad (1)$$

where $\alpha_{cs}$'s are cell-specific school fixed effects; $D_{is}$ is the distance from student $i$'s residence to school $s$;[49] $\textit{Nearest}_{is}$ is a binary variable which equals 1 if $D_{is}$ is the smallest for $i$ among all schools and 0 otherwise; $x_i$ is a vector of student characteristics to be interacted with program characteristics $z_p$ and school characteristics $Z_s$. Specifically, $x_i$ includes 7th grade standardized ELA and Math scores, their average, and median neighborhood income. $z_p$ and $Z_s$ include the proportions of high performers in 7th grade standardized ELA and Math (a high performer is defined as being above the 75th percentile), the size of the 9th grade, the proportion of each race/ethnicity group, the proportion of students on FRPL, and a STEM program indicator. $\epsilon_{icps}$ is i.i.d. standard normal conditional on the above observables. $\sigma_{\tau(p),c}$ allows for heterogeneous variances of the unobserved idiosyncratic preference shocks based on program type $\tau(p)$ (STEM, or others), with $\sigma^2_{others,c}$ normalized to 1 for all $c$.

We estimate equation (1) separately for the eight cells, which exhibit substantial heterogeneity in observables (Table 3). For example, median neighborhood income is $85,207 for Cell 1 (male, non-FRPL, non-Black/Hispanic) versus $57,321 for Cell 8 (female, FRPL, Black/Hispanic). For each cell, we employ a Gibbs sampling procedure for Hierarchical Bayesian estimation, detailed in Online Appendix E. Full preference estimates are reported in Online Appendix H.1. For TEPS, we consider TEPS$^{top}$, TEPS$^{10}$, TEPS$^{20}$, $\cdots$, TEPS$^{90}$, and TEPS$^{all}$.[50]

**Test Results: Selected Estimates.** Table 5 presents the student behavior assumptions chosen by our testing procedure. WTT is consistently rejected, implying that students in our data across all the cells tend to make some, possibly payoff-insignificant, mistakes. The selected version ranges from TEPS$^{20}$ to TEPS$^{all}$ across the cells, indicating heterogeneity in the degree to which such mistakes are made across students' observable characteristics. Notably, mistakes seem more pronounced for low SES and minority students.[51]

[49]We use the exact address for schools. For students, the centroid of their residential census tract serves as their residence. The Haversine formula is employed to calculate the straight-line distance between these points.

[50]As stated in Section 3.3, we do not consider the existing stability method (Fack, Grenet, and He, 2019), due to its inapplicability under priority uncertainties.

[51]Using the Least Absolute Shrinkage and Selection Operator (LASSO), we find that among the cell characteristics that are used to define the cells, FRPL status has the most predictive of the selected $\tau$.

Table 5: Test Results: Selected Estimates

| Cells | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Selected Estimates | $TEPS^{70}$ | $TEPS^{all}$ | $TEPS^{40}$ | $TEPS^{20}$ | $TEPS^{70}$ | $TEPS^{80}$ | $TEPS^{50}$ | $TEPS^{20}$ |

**Descriptive Analysis.** TEPS differentiates unranked programs based on feasibility, whereas WTT treats all unranked schools as less preferred than all ranked schools regardless of their feasibility. To see which approach fits the data better, we examine if program characteristics systematically vary with feasibility status. Figure 4 presents the percentage of high-performing students (by average score) for each program type, categorized by feasibility.[52]

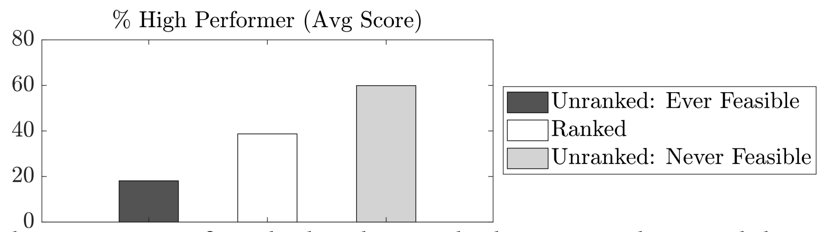



Figure 4: Characteristics of Ranked and Unranked Programs by Feasibility Status

*Note:* For each student, we classify the programs into three types—ever-feasible-unranked, never-feasible-unranked, and ranked. Ranked programs are those included in the student's ROL, and Step 1 of $TEPS^{all}$ procedure determines the feasibility of each unranked program. We focus on the fraction of high-performing students (measured by the average of ELA and math scores) in a program. In Online Appendix H.3, we report results on additional program characteristics. The figure reports the average across all students for each type of program.

There is a clear order among the three types of programs: ever-feasible unranked programs have the lowest percentage of high-performing students, never-feasible unranked programs have the highest, and ranked programs fall in between. The lower percentage in ever-feasible programs likely stems from feasibility itself—highly competitive programs with higher cutoffs are less likely to be feasible. Nevertheless, this observation aligns with TEPS being selected over WTT in all cells.[53]

**Model Fit.** To assess the model fit of different estimation methods, we predict matching outcomes and calculate average assigned program characteristics. Using each estimate set, we simulate student ordinal preferences (assuming truthful reporting, supported by Corollary 1 in analyzing matching *outcomes*), and obtain a matching for each lottery draw. We then calculate the average of each program characteristic over 40,000 simulations for each set of estimates and compare it to the actual data. Table 6 reports how close the average is to the data, using estimates from all eight cells combined.

[52]A 'high performer' is defined as a student whose average 8th-grade statewide standardized ELA and math scores exceeds the citywide 75th percentile.

[53]We obtain the same conclusion when we look at other program characteristics, such as the percentages of high performers in ELA and math separately and the proportion of non-FRPL students (see Online Appendix H.3.)

Table 6: Model Fit: Average Characteristics of Assigned Programs

| | Black and Hispanic Students | | | | | White, Asian, and Other Students | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Data | Difference b/t model prediction & data | | | | Data | Difference b/t model prediction & data | | | |
| | | WTT | TEPS$^{all}$ | TEPS$^{top}$ | Selected | | WTT | TEPS$^{all}$ | TEPS$^{top}$ | Selected |
| % High Performer (ELA) | 27.66 | -1.12 | 0.24 | 0.01 | 0.11 | 46.00 | -0.36 | -0.25 | -0.39 | -0.33 |
| | | (0.46) | (0.49) | (0.51) | (0.52) | | (0.26) | (0.24) | (0.27) | (0.28) |
| % High Performer (Math) | 20.66 | -0.76 | 0.30 | 0.09 | 0.21 | 39.68 | -0.56 | -0.49 | -0.44 | -0.51 |
| | | (0.43) | (0.43) | (0.47) | (0.49) | | (0.27) | (0.23) | (0.25) | (0.27) |
| % FRPL (Program Level) | 67.06 | 1.33 | 0.55 | 0.75 | 0.86 | 49.86 | 1.02 | 0.63 | 0.68 | 0.62 |
| | | (0.31) | (0.36) | (0.43) | (0.40) | | (0.23) | (0.17) | (0.21) | (0.19) |
| % White | 34.19 | -2.58 | -0.88 | -1.27 | -1.30 | 58.41 | -1.06 | -0.58 | -0.89 | -0.65 |
| | | (0.45) | (0.48) | (0.57) | (0.55) | | (0.33) | (0.26) | (0.29) | (0.26) |
| % Asian | 7.29 | 0.16 | 0.47 | 0.48 | 0.52 | 9.27 | -0.09 | -0.05 | -0.13 | -0.12 |
| | | (0.15) | (0.17) | (0.17) | (0.16) | | (0.08) | (0.09) | (0.09) | (0.09) |
| % Black | 21.40 | 0.82 | -0.04 | 0.13 | 0.23 | 9.75 | 0.23 | 0.20 | 0.46 | 0.30 |
| | | (0.29) | (0.32) | (0.36) | (0.34) | | (0.16) | (0.17) | (0.18) | (0.16) |
| % Hispanic | 35.91 | 1.59 | 0.42 | 0.63 | 0.53 | 21.61 | 0.83 | 0.36 | 0.49 | 0.41 |
| | | (0.33) | (0.34) | (0.39) | (0.38) | | (0.22) | (0.17) | (0.19) | (0.18) |
| Size of 9th Grade | 135.54 | -15.89 | -3.08 | 2.69 | 1.38 | 287.77 | -13.32 | -3.18 | -7.57 | -5.98 |
| | | (4.50) | (4.75) | (5.16) | (5.02) | | (3.96) | (2.70) | (2.93) | (2.79) |
| $\mathbb{1}$(STEM) | 15.67 | -0.89 | -1.89 | -2.81 | -1.73 | 14.56 | 0.77 | 0.84 | 1.13 | 0.84 |
| | | (0.98) | (1.76) | (3.28) | (1.98) | | (0.56) | (0.79) | (1.39) | (0.96) |
| Average Standardized Difference | | 3.03 | 1.16 | 1.16 | 1.28 | | 2.47 | 2.18 | 1.67 | 1.93 |

Notes: We sample 200 draws from the posterior distribution of each parameter and, for each draw, draw 200 sets of lotteries and run DA $200 \times 200 = 40,000$ times. The mean and standard deviations across the preference estimates draws are reported. The last row calculates the average standardized differences, i.e., the absolute value of the mean difference divided by the standard deviation, across all rows.

We report the model fit based on WTT, TEPS$^{all}$, TEPS$^{top}$, and the selected estimates. We observe two key patterns. First, WTT-based estimates fit the data poorly, while the TEPS-based and the selected estimates closely match the actual data. This is reflected in the average standardized differences reported in the last row of the table, suggesting WTT's vulnerability to payoff-insignificant mistakes and its potential unreliability in preference inference. Second, TEPS$^{top}$ predictions tend to be less precise (i.e., have higher standard deviations) than TEPS$^{all}$ or the selected estimates (recall that TEPS$^{\tau}$ with large enough attention parameter $\tau$ were selected in Table 5). This is because TEPS$^{top}$ uses less information about student preferences than other TEPS-based estimates, leading to less precise preference estimates (Tables H.6-H.9).

## 4.2 Counterfactual Analysis

We proceed to evaluate the desegregation effects of three counterfactual policies based on alternative preference estimates, motivated by recent debates and policy implementations by the NYC DOE:[54]

(i) No screening: tie-breaking is solely lottery-based, eliminating academic screening.

[54] Idoux (2022); Hahm and Park (2022) study similar policies in the context of middle schools and a dynamic setting, respectively.

(ii) No zoning: no programs prioritize students based on their residence. This amounts to removing zone priorities at zoned programs in SI.

(iii) No priorities: all priority rules are eliminated, leaving only the tie-breaking lottery

We assess each scenario's desegregation effects compared to the status quo.[55] For each set of estimates (WTT; TEPS$^{\tau}$ with $\tau = top, 10, \cdots, 90, all$; or selected), we randomly sample 200 draws of $u_{icps}$ for each $(i, c, p, s)$ from its respective posterior distribution and simulate student ROLs based on their true ordinal preferences per Corollary 1. We then draw 200 sets of random lotteries, construct students' priority scores under each scenario, and run DA to obtain counterfactual outcomes. We report the average of each measure we consider across 40,000 simulation results for each scenario and each set of estimates, focusing on WTT, TEPS$^{top}$, and the selected estimates for conciseness.

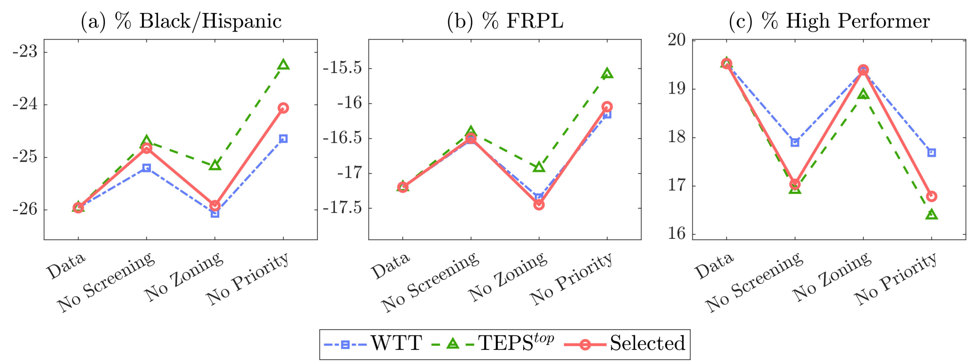



Figure 5: Racial Gap in Characteristics of Assigned Programs

We assess the effects of the three policies on the racial gap between Black or Hispanic students and others, focusing on average assigned program characteristics (Figure 5).[56] The current system exhibits a significant gap. Black or Hispanic students are assigned to programs with 57% Black or Hispanic students on average, compared to 31% for others, resulting in a gap of $-26$ percentage points (panel a). Similar gaps exist in the proportion of FRPL students (panel b) and high performers (panel c).

We find two prominent patterns in the predicted policy effects. Firstly, WTT-based estimates consistently underestimate the desegregation impact of the policies. For example, WTT predicts only a 1 percentage point reduction in the racial gap for Black or Hispanic student proportions, compared to a 2 percentage point reduction predicted by the selected estimates. These differences, though seemingly small,

[55]Throughout the simulations, we assume that the characteristics of schools and programs remain unchanged at their baseline values (as measured in 2015-16). Thus, reported results represent the policies' short-term impacts.

[56]For each racial group, we calculate the mean of three assigned program characteristics—proportion of Black and Hispanic students, proportion of FRPL students, and average 7th-grade standardized test score—, and report them in Figure 5. The full table, along with the standard deviations of the predictions, is available in Table H.5.

are statistically significant due to the high precision of our predictions (see Table H.5). This underestimation is expected as WTT disregards students' mistakes/inattention, assuming that students do not prefer unranked, out-of-reach schools, even when they become within reach.

Second, even under our selected estimates, the predicted effects appear relatively small given the substantial nature of the policies. While removing all priorities reduces racial gaps, the magnitude is modest, suggesting factors beyond school priorities (e.g., residential segregation) may significantly contribute to segregation. This raises doubts about the effectiveness of solely removing priorities for desegregation.

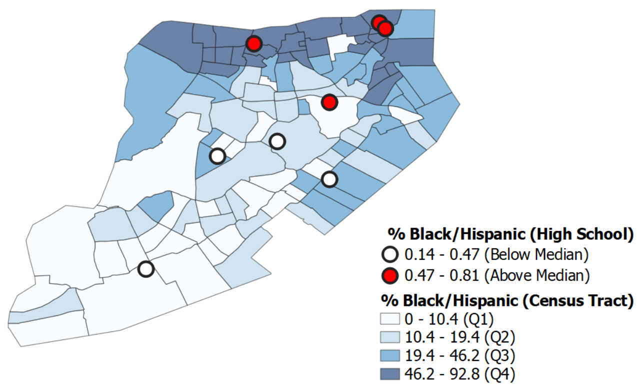



Figure 6: Percentage of Black or Hispanic: Census Tracts and High Schools

Source: 2017 American Community Survey: 5-Year Data, US Census TIGER/Line Shapefiles.

Descriptive evidence supports the role of residential segregation in explaining the limited impact of policy changes (Monarrez, 2020; Laverde, 2022; Park and Hahm, 2022). Figure 6 illustrates high residential segregation in Staten Island, with Black and Hispanic students concentrated in areas with schools serving higher proportions of Black/Hispanic, low-income, and lower-performing students. Furthermore, our estimates indicate a stronger aversion to commuting among minority students. For instance, our selected estimates indicate that the willingness to travel for a 10 pp increase in the proportion of high performers in math is 0.45 miles for Cell 1 (male/non-FRPL/White or Asian) students but only 0.10 miles for Cell 7 (male/FRPL/Black or Hispanic) students (see Appendix Tables H.6-H.9).[57] This interplay of residential segregation and varying commuting preferences underscores the limitations of school choice reforms in achieving desegregation without addressing residential patterns. A more holistic approach encompassing both school and residential choices is necessary, as explored in Park and Hahm (2022); Agostinelli, Luflade,

[57]The average commuting distance of SI students is 2.3 miles. The willingness to travel for school/program characteristics $X$ is calculated by dividing the coefficient on $X$ (the marginal utility of $X$) by the negative of the coefficient on commuting distance (the marginal utility of traveling) in the estimated utility function.

and Martellini (2024).

## 5 Conclusion

This paper explores how to leverage uncertainties faced by applicants to infer their preferences in the presence of payoff-insignificant mistakes. We study a general DA matching market in which students may face uncertainties about their priorities, and consider a robust equilibrium in which no applicant makes payoff-significant mistakes. We show that a robust equilibrium is asymptotically ex-post stable—the proportion of students assigned their favorite feasible schools converges to one in probability. Based on this theoretical finding, we develop the *Transitive Extension of Preferences from Stability*, a novel, theoretically grounded, and computationally efficient method that extracts robust preference information from observed ROLs.

Our empirical application to NYC high school choice data confirms the importance of accounting for student application mistakes in preference estimation and policy counterfactuals, and demonstrates the effectiveness of TEPS as a remedy. Substantively, our counterfactual analysis calls into question the effectiveness of removing geographic and academic priorities in achieving the City's desegregation goals.

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# A Proofs from Sections 2 and 3

We first introduce a general priority structure that nests the priority structure assumed in the main text as a special case. We then establish asymptotic ex-post stability using that general priority structure in Theorem 1′ below, which will in turn imply Theorem 1.

## A.1 A General Priority Structure

Here, we consider a more general priority structure than those consisting of $T_1, T_2$, and $T_3$ assumed in the text. Recall the ex-ante priority types $T$. Let $T_d := \{t \in T : \underline{s}_c^t = \overline{s}_c^t, \forall c\}$ be the types whose scores are all degenerate, and let $T_n := T \setminus T_d$ denote types whose scores for some schools are non-degenerate. We assume that $T$ is compact, that $T_n$ is closed, and that there exists some $\kappa > 0$ such that $\overline{s}_c^t - \underline{s}_c^t > \kappa$ if $\overline{s}_c^t - \underline{s}_c^t > 0$. Further, for each $t \in T_n$, $\Phi^t$ is absolutely continuous and admits density $\phi^t$ on $S^t$. As in the text, let $\widetilde{\eta}$ denote the probability measure of $\widetilde{\theta} \in \widetilde{\Theta} := [\underline{u}, \overline{u}]^C \times T$. In summary, the continuum economy is summarized by $E = [\widetilde{\eta}, S, (\Phi^t)_t]$.

In addition to Marginal Full Support, we introduce two additional conditions, which will be later shown to hold under the priority structure satisfying $T_2$ and $T_3$.

Assumption 2 (Finite Atoms [at Extremal Ex-post Scores]). *The distribution of $(\underline{s}_c^t, \overline{s}_c^t)_t$, viewed as a random function of $t$, has at most a finite number of atoms.*

This assumption can be seen as weakening the assumption of AL model. First, for priority structure $T_1$, with no uncertainty on ex-post scores, we have $T = T_d$, with $\underline{s}_c^t = \overline{s}_c^t$ for all $t \in T$. In this case, the atomlessness of ex-post measure $\eta$ (which is also assumed in AL) implies that the infimum and supremum scores have no point mass, so Finite Atoms will be trivially satisfied. However, this condition does allow point mass for infimum and supremum scores but only finitely many of them; this is the case of coarse priorities in priority structure $T_2$. A mass of students belong to each of finitely many priorities, so there will be a finite number of atoms in the infima and suprema of their ex-post scores. Last, in the priority structure priority structure $T_3$, the infimum and supremum of $s_c^t$ are 0 and 1, respectively, so the finite atoms condition is clearly satisfied.

For the last condition, for any $\delta \in (0, 1)$, we say a school $c$ is $\delta$-**feasible** for type $t$ given $p$ if $\overline{s}_c^t - \delta > p_c$ and $\delta$-**infeasible** for type $t$ given $p$ if $\underline{s}_c^t + \delta < p_c$. Plainly, $\delta$-feasibility and $\delta$-infeasibility mean feasibility and infeasibility, respectively, with probabilities that are bounded away from zero.

Assumption 3 (Rich Uncertainty). *Fix any $p \in [0, 1]^C$. Then, for any $\delta < \bar{\delta}$, for some $\bar{\delta} > 0$, there exists $\beta(\delta) > 0$ such that, for any $t \in T$, for any $\delta$-feasible schools $a, b \in C$, and for any set $C^t \subset C \setminus \{a, b\}$ of $\delta$-infeasible schools,*

$$\Pr\{s_c^t < p_c, \forall c \in C^t, \text{ and } s_a^t > p_a, s_b^t > p_b\} > \beta(\delta), \tag{A.1}$$

*whenever* $\Pr\{s_c^t < p_c, \forall c \in C^t, s_a^t > p_a\} > 0$ *and* $\Pr\{s_c^t < p_c, \forall c \in C^t, s_b^t > p_b\} > 0$.[A-1]

[A-1] Rich Uncertainty is vacuously satisfied if schools with the specified restrictions do not exist. Also, if $C^t = \emptyset$, the required condition reduces to $\Pr\{s_a^t > p_a, s_b^t > p_b\} > \beta(\delta)$ whenever $\Pr\{s_a^t > p_a\} > 0$ and $\Pr\{s_b^t > p_b\} > 0$.

The last condition, which we argue is quite weak, is most technical and thus requires unpacking. In words, the condition states that whenever school $a$ is feasible or $b$ is feasible, while all $\delta$-infeasible schools in $C^t$ are infeasible, each with positive probability, then $a$ and $b$ are *both simultaneously* feasible and $C^t$ are infeasible with probability at least of $\beta(\delta)$, for some $\beta(\delta) > 0$.[A-2] Essentially, this rules out the case in which a student's scores for $a$ and $b$ are extremely negatively correlated. Hence, the condition is quite weak and is satisfied in all realistic environments in the main text. [A-3]

### A.2 Proof of Lemma 1

*Proof.* Suppose a student $i$ adopts a strategy $\sigma_i \neq \rho$. Then, there exists an ex ante type $\tilde{\theta} = (u, t)$ such that $\sigma_i(\tilde{\theta}) \neq \rho(\tilde{\theta})$ with positive probability. Let $k \geq 1$ be the first rank at which $\sigma_i(\tilde{\theta})$ differs from $\rho(\tilde{\theta})$. More precisely, the student ranks school $b$ for her $k$-th rank, even though $a \neq b$ is her $k$-th best school. Full support uncertainty means that with positive probability, her scores have $s_a > p_a$, $s_b < p_b$, and $s_c < p_c$, for all $c \neq a, b$. In that case, $\sigma_i(\tilde{\theta})$ yields an assignment at $b$, differing from her stable assignment $a$. Therefore, $\sigma_i$ fails to be an SRS against $p$. □

### A.3 Proof of Lemma 2

*Proof.* We first show that the following condition, called *strict gross substitutes*, guarantees the uniqueness: $\eta$ is such that for any $p, p' \in [0, 1]^C$ with $p < p'$,

$$(SGS) \qquad \sum_{c \in J} D_c(p') < \sum_{c \in J} D_c(p),$$

where $J := \{c \in C : p_c < p'_c\}$. Suppose to the contrary there are two stable matchings characterized by two cutoffs $p$ and $p'$. By the lattice property, we can assume without loss that $p' > p$. Since both $p$ and $p'$ clear the markets and since $p'_c > p_c \geq 0$ for each $c \in J$, for each $c \in J$,

$$D_c(p) \leq S_c \text{ and } D_c(p') = S_c.$$

Summing across $c \in J$, we get

$$\sum_{c \in J} D_c(p') \geq \sum_{c \in J} D_c(p),$$

which contradicts $(SGS)$.

To prove the statement, it now suffices to show that $\eta$ satisfies $(SGS)$. To this end, fix any $p, p' \in [0, 1]^C$ with $p < p'$ and the corresponding set $J = \{c \in C : p_c < p'_c\}$. Observe that any ex-post type $\theta$ who demands a school in $J$ at cutoffs $p'$ never switches its demand to a school in $C \setminus J$ or to the outside option

[A-2]One non-obvious condition is the existence of a lower bound probability $\beta(\delta)$ that is independent of $t$. The condition is still reasonable given the compactness of $T$ and $T_n$.

[A-3]See Online Appendix B.1 for the formal proof.

$\emptyset$, when the cutoffs shift to $p$. Hence, any such type continues to demand a (possibly different) school in $J$ at cutoffs $p$. Next, consider the type $(u, s)$ with $u_c > 0 > u_{c'}$, $c \in J, \forall c' \neq c$, and $s_c \in (p_c, p'_c)$. By Marginal Full Support, there is a positive mass of such types. These types could not demand any school at $p'$ but now demand $c$ at $p$. Collecting our observations, we conclude that $D_c(p') < D_c(p)$. Since the same result holds for each $c \in J$, $(SGS)$ holds. $\square$

## A.4 Proof of Asymptotic Ex-post Stability of Robust Equilibria

We now prove our main result under the general priority structure satisfying Marginal Full Support, Finite Atoms, and Rich Uncertainty:

THEOREM $1'$. *Suppose $\eta$ satisfies Marginal Full Support, Finite Atoms, and Rich Uncertainty. Then, any regular robust equilibrium is asymptotically ex-post stable.*

Note that THEOREM 1 follows from THEOREM $1'$ in light of the fact that the priority structures in the former satisfy Finite Atoms and Rich Uncertainty.

Before proceeding with the proof, we need to perform a few preliminary analyses. Specifically, for each $k$-economy $F^k$, we study the strategy profile for that economy, or the $k$-truncation of $\sigma$, denoted by $\sigma^k := (\sigma_1, ..., \sigma_k)$. A later analysis requires us to consider the consequence of an arbitrary student $i$ deviating to truthful reporting $\rho$. We denote the resulting profile by $\sigma_{(i)} := (\rho, \sigma_{-i})$ which is obtained by replacing the $i$-th component of $\sigma$ with $\rho$. Likewise, $\sigma^k_{(i)}$ denotes the $k$-truncation of $\sigma_{(i)}$. Note that if $i > k$, then $\sigma^k_{(i)} = \sigma^k$. Let $P^k_{(i)}$ denote the cutoffs that would prevail if the students employ $\sigma^k_{(i)}$. Finally, let $\overline{p}$ be the unique, deterministic cutoff under the unique stable matching in the continuum economy $E$ (LEMMA 2). We first establish a desirable limit behavior of $(P^k_{(i)})_{i \in \mathbb{N}_0}$ as $k \to \infty$, where $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$.

LEMMA 3. *Let $\sigma$ be any $\gamma$-regular strategy profile. There exists a subsequence $\{F^{k_\ell}\}_\ell$ such that*

$$\sup_{0 \leq i \leq k_\ell} \|P^{k_\ell}_{(i)} - \overline{p}\| \xrightarrow{p} 0 \text{ as } \ell \to \infty.$$

*Proof.* The proof follows exactly the same argument as ACH, upon noting that the uniqueness of stable matching follows from (SGS) (instead of ACH's full support assumption). $\square$

Next, define a set

$$\tilde{\Theta}^\delta := \left\{ (u, t) \in \tilde{\Theta} : \inf_{c, c' \in \tilde{C}, c \neq c'} |u_c - u_{c'}| > \delta;\ \forall c,\ \overline{s}^t_c = \underline{s}^t_c \text{ or } \overline{s}^t_c \neq \bar{p}_c \Rightarrow |\overline{s}^t_c - \bar{p}_c| > \delta;\ \underline{s}^t_c \neq \bar{p}_c \Rightarrow |\underline{s}^t_c - \bar{p}_c| > \delta \right\}.$$

The Finite Atoms condition, together with atomlessness of $\eta$, implies that $\tilde{\eta}(\tilde{\Theta}^\delta) \to 1$ as $\delta \to 0$.[A-4]

[A-4]We note that the limit set $\tilde{\Theta}^- := \tilde{\Theta} - \cup_{\delta > 0} \tilde{\Theta}^\delta$ contains no mass point. This is seen by the fact that the only possibility of a point mass in $\tilde{\Theta}^-$ may arise from a point mass occurring at some $\overline{s}^t_c = \underline{s}^t_c = p^t_c$ for some $c$. (Note that any type $t$ with $\overline{s}^t_c > \underline{s}^t_c$ belongs to $\cup_{\delta > 0} \tilde{\Theta}^\delta$, so it does not belong to $\tilde{\Theta}^-$.) But if there were such a point mass, then there must be a positive

Further, we introduce a few notations. Define $B_\nu(p) := \{p' \in [0,1]^C : ||p' - p|| < \nu\}$. Consider a student $\tilde{\theta}$ and fix any arbitrary economy with other students playing some arbitrary reporting strategies. If she adopts any arbitrary reporting strategy $\sigma_i'(\tilde{\theta})$, this induces set of assignment probability $(X_c')_{c\in\tilde{C}}$, where $X_c'$ is the probability of the student being assigned to a school that she weakly prefers to $c$. Suppose she switches to TRS which induces assignment probability $(X_c^*)_{c\in\tilde{C}}$, where $X_c^*$ represents the probability of the student being assigned a school that she weakly prefers to $c$. The strategyproofness of DA implies that TRS yields better assignment than any other strategy in the sense of first-order stochastic dominance:

Fact 1. *For each $c \in \tilde{C}$, $X_c^* - X_c' \geq 0$.*

Naturally, we define the **probability gain** from the switch to be $\max_{c\in\tilde{C}}(X_c^* - X_c')$. The following preliminary result is useful.

Lemma 4. *Fix any $\bar{p} \in [0,1]^C$ and any $\delta \in (0, \frac{1}{2}\min\{\kappa, \zeta\})$, where $\zeta := \inf_{c,c'\in C\cup\{x,y\},\bar{p}_c\neq\bar{p}_{c'}} |\bar{p}_c - \bar{p}_{c'}|$. Then, there exist $\nu(\delta) > 0$ and $\alpha(\delta) > 0$ such that deviating from any non-SRS against $\bar{p}$, say $\sigma'$, to TRS yields a probability gain of at least $\alpha(\delta)$, and thus a payoff gain of at least $\delta\alpha(\delta)$, for any student with type $\tilde{\theta} \in \Theta^\delta$, provided that $\sigma'$ and TRS induce cutoffs $p'$ and $p''$ both in $B_{\nu(\delta)}(p)$.*

*Proof.* Let $\beta(\delta)$ be the probability lower bound defined in Rich Uncertainty. Set $\nu(\delta) = \min\{\frac{\delta}{4}, \frac{\beta(\delta)}{4\upsilon}\}$, where $\upsilon := \max_{t\in T_n, s\in S^t} \phi^t(s)$. Fix a student with type $\tilde{\theta} = (u,t) \in \tilde{\Theta}^\delta$. Without loss, we index schools $\tilde{C} = \{1, ..., C+1\}$ (including the outside option $\emptyset$) so that $u_j > u_{j'}$ if and only if $j < j'$. Let $C_j$ denote the set of schools that student $\tilde{\theta}$ prefers to $j$. Let $C^F \subset \tilde{C}$ be a set of *feasible* schools that the student would get with positive probabilities when she plays SRS against $\bar{p}$. It means that for each $j \in C^F$, we have

$$\Pr\{s_c < \bar{p}_c, \forall c \in C_j, s_j > \bar{p}_j\} > 0. \tag{A.2}$$

There could be a feasible, and less preferred, school outside $C^F$ that she is not assigned but would have been with positive probability if she had ranked it sufficiently favorably. Since $\tilde{\theta} \in \tilde{\Theta}^\delta$, all these feasible schools are in fact $\delta$-feasible.

Now take any non-SRS against $\bar{p}$ for that student, say $\sigma'$. $\sigma'$ must rank some less-preferred, feasible—and thus $\delta$-feasible—school $b$ ahead of some school $a \in C^F$, where she prefers $a$ to $b$, and let $a$ be the most preferred school in $C^F$ that suffers from such a ranking-reversal by $\sigma'$. Let $C_a$ be the set of schools that the student prefers to $a$ among $\tilde{C}$. For the reversal to make a difference (which follows from $\sigma'$ being non-SRS against $\bar{p}$), we must have

$$\Pr\{s_c < \bar{p}_c, \forall c \in C_a, s_b > \bar{p}_b\} > 0. \tag{A.3}$$

mass at score $s_c = p_c^t$, which contradicts the atomlessness of $\eta$.

Suppose as hypothesized that $\sigma'$ in the hypothesized economy induces a cutoff $p' \in B_{\nu(\delta)}$. We wish to compute the probability $X'_a$ of the student receiving $a$ or better according to the true preferences. Let $C' \subset C_a$ be the set of schools among $C_a$ that $\sigma'$ ranks ahead of $a$. Then,

$$\begin{aligned}
X'_a \leq & \Pr\{\exists c \in C' \text{ s.t. } s_c > p'_c\} + \Pr\{s_c < p'_c, \forall c \in C', s_b < p'_b, s_a > p'_a\} \\
\leq & \Pr\{\exists c \in C_a \text{ s.t. } s_c > p'_c\} + \Pr\{s_c < p'_c, \forall c \in C_a, s_b < p_b, s_a > p'_a\} \\
\leq & \Pr\{\exists c \in C_a \text{ s.t. } s_c > p_c\} + \Pr\{s_c < \bar{p}_c, \forall c \in C_a, s_b < \bar{p}_b, s_a > \bar{p}_a\} + \upsilon\|\bar{p}' - \bar{p}\| \\
\leq & \Pr\{\exists c \in C_a \text{ s.t. } s_c > \bar{p}_c\} + \Pr\{s_c < \bar{p}_c, \forall c \in C_a, s_b < \bar{p}_b, s_a > \bar{p}_a\} + \frac{\beta(\delta)}{4},
\end{aligned}$$

where the first inequality holds since it is possible that $\sigma'$ ranks some other less preferred school other than $b$ ahead of $a$, the second follows from the fact that $C' \subset C_a$, the third follows from the fact that for type $\tilde{\theta} \in \tilde{\Theta}^\delta$, $B_{\delta/4}(\bar{p})$ contains no point mass of scores and that $p', p \in B_{\nu(\delta)}(\bar{p}) \subset B_{\delta/4}(\bar{p})$, and the last inequality follows from the fact that $\nu(\delta) \leq \frac{\beta(\delta)}{4\upsilon}$ and $p' \in B_{\nu(\delta)}(\bar{p})$.

Suppose next the student switches to TRS and as a result faces $p'' \in B_{\nu(\delta)}(p)$ as cutoffs. The probability of getting $a$ or better schools from the switch is given by:

$$\begin{aligned}
X^*_a = & \Pr\{\exists c \in C_a \text{ s.t. } s_c > p''_c\} + \Pr\{s_c < p''_c, \forall c \in C_a, s_a > p''_a\} \\
\geq & \Pr\{\exists c \in C_a \text{ s.t. } s_c > \bar{p}_c\} + \Pr\{s_c < \bar{p}_c, \forall c \in C_a, s_a > \bar{p}_a\} - \upsilon\|p'' - \bar{p}\| \\
\geq & \Pr\{\exists c \in C_a \text{ s.t. } s_c > \bar{p}_c\} + \Pr\{s_c < \bar{p}_c, \forall c \in C_a, s_a > \bar{p}_a\} - \frac{\beta(\delta)}{4}.
\end{aligned}$$

Again, the second inequality follows from the fact that for type $\tilde{\theta} \in \tilde{\Theta}^\delta$, $B_{\delta/4}(\bar{p})$ contains no point mass of scores and that $p'', p \in B_{\nu(\delta)}(\bar{p}) \subset B_{\delta/4}(\bar{p})$, and the last inequality follows from the fact that $\nu(\delta) \leq \frac{\beta(\delta)}{4\upsilon}$ and $p'' \in B_{\nu(\delta)}(\bar{p})$.

Consequently, the probability gain from switching from $\sigma'$ to TRS is at least:

$$\begin{aligned}
\max_{c \in \tilde{C}} (X^*_c - X'_c) \geq & X^*_a - X'_a \\
\geq & \Pr\{s_c < \bar{p}_c, \forall c \in C_a, s_a > \bar{p}_a\} - \Pr\{s_c < p_c, \forall c \in C_a, s_b < \bar{p}_b, s_a > \bar{p}_a\} - \frac{\beta(\delta)}{2} \\
= & \Pr\{s_c < \bar{p}_c, \forall c \in C_a, s_a > \bar{p}_a, s_b > \bar{p}_b\} - \frac{\beta(\delta)}{2} \\
\geq & \beta(\delta) - \frac{\beta(\delta)}{2} = \frac{\beta(\delta)}{2}.
\end{aligned}$$

The last inequality follows from Rich Uncertainty. To see this, note first that each school in $C_a \setminus C^F$ is infeasible at $p$ to $\tilde{\theta} = (u, t)$; then, since $\tilde{\theta} \in \tilde{\Theta}^\delta$, it is $\delta$-infeasible at $p$ for $\tilde{\theta}$. Consider next any $c \in C^F \cap C_a$. The school is feasible given $p$ to $\tilde{\theta} = (u, t)$, but we must have $\underline{s}^t_c < \bar{p}$; otherwise, the student would not have been assigned $a$ with positive probability under SRS given $p$. Then, since $\tilde{\theta} \in \tilde{\Theta}^{\delta}$, $\underline{s}_c^{\delta} < \bar{p} - \delta$, so it is $\delta$-infeasible. Finally, recall that both $a$ and $b$ are feasible for $\tilde{\theta}$; again since $\tilde{\theta} \in \tilde{\Theta}^{\delta}$, both $a$ and $b$ are $\delta$-feasible for $\tilde{\theta}$. Therefore, (A.2) and (A.3) imply the inequality (A.1), leading to that last inequality.

Setting $\alpha(\delta) = \frac{\beta(\delta)}{2}$, we have established that the switch results in the probability gain of at least $\alpha$. Since $\tilde{\theta} \in \tilde{\Theta}_p^{\delta}$, the associated payoff gain is

$$\sum_i (X_i^* - X_{i-1}^*) u_i^{\tilde{\theta}} - \left[\sum_i (X_i' - X_{i-1}') u_i^{\tilde{\theta}}\right] \geq \sum_i (X_i^* - X_i')(u_i^{\tilde{\theta}} - u_{i+1}^{\tilde{\theta}}) \geq (X_a^* - X_a')(u_a^{\tilde{\theta}} - u_{a+1}^{\tilde{\theta}}) > \alpha\delta,$$

where $X_0^* = X_0' \equiv 0$. We note that this bound does not depend on $\tilde{\theta} \in \tilde{\Theta}^{\delta}$. □

*Proof of* Theorem 1. For any sequence $\{F^k\}$ induced by $E$, fix any arbitrary regular robust equilibrium $\{(\sigma^k_{1 \leq i \leq k})\}_k$. The strategies induce a random ROL, $R_i$, for each player $i$. We prove that the fraction of students who play non-SRS against (random) DA cutoffs in $\left\{ (\sigma_i^k)_{1 \leq i \leq k} \right\}_k$ converges in probability to zero as $k \to \infty$. Suppose not. Then, there exists $\varepsilon > 0$ and a subsequence of finite economies $\left\{F^{k_j}\right\}_j$ such that

$$\Pr\left(\text{The fraction of students playing SRS against } p^{k_j} \text{ is no less than } 1 - \varepsilon\right) < 1 - \varepsilon. \qquad (*)$$

By Lemma 3, there exists a sub-subsequence of economies $\left\{F^{k_{j_\ell}}\right\}_\ell$, such that the associated cutoffs $p^{k_{j_\ell}}$ converge to $\bar{p}$ in probability, where $\bar{p}$ are the deterministic cutoffs stated in Lemma 3.

We choose $\delta > 0$ small enough so that $\eta(\tilde{\Theta}^{\delta}) > (1-\varepsilon)^{1/3}$ (this can be done since $\eta$ is absolutely continuous). We then choose $\nu(\delta)$ and $\alpha(\delta)$ according to Lemma 4.

By WLLN, we know that $\eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta})$ converges to $\eta(\tilde{\Theta}^{\delta})$ in probability, and therefore there exists $L_1$ such that for all $\ell > L_1$ we have

$$\Pr\left(\eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}\right) \geq (1-\varepsilon)^{1/2}.$$

For each economy $F^{k_{j_\ell}}$, consider the event

$$A^{k_{j_\ell}} := \left\{ \sup_{0 \leq i \leq k_{j_\ell}} ||P_{(i)}^{k_{j_\ell}} - \bar{p}|| < \nu(\delta) \right\}.$$

Since $P_{(i)}^{k_{j_\ell}} \xrightarrow{p} \bar{p}$ uniformly over $i \in \mathbb{N}_0$, there exists $L_2$ such that, for all $\ell > L_2$ s.t. for all $\ell > L_2$ we have

$$\Pr\left(A^{k_{j_\ell}}\right) \geq \max\left\{(1-\varepsilon)^{1/6}, 1 - (1-\varepsilon)^{1/2}\left[(1-\varepsilon)^{1/3} - (1-\varepsilon)^{1/2}\right]\right\}. \qquad (**)$$

Because $\left\{ (\sigma_i^k)_{1 \leq i \leq k} \right\}_k$ is a robust equilibrium, there exists $L_3$ s.t. for all $\ell > L_3$ the strategy profile $(\sigma_i^{k_{j_\ell}})_{i=1}^{k_{j_\ell}}$ is a $\delta \cdot \alpha(\delta)\left[(1-\varepsilon)^{1/6} - (1-\varepsilon)^{1/3}\right]$-BNE for economy $F^{k_{j_\ell}}$.

By WLLN, there exists a sufficiently large $\hat{L}$ such that $\hat{L}$ i.i.d. Bernoulli random variables with probability $(1-\varepsilon)^{1/3}$ have a sample mean greater than $(1-\varepsilon)^{1/2}$ with probability no less than $(1-\varepsilon)^{1/3}$. Define next $L_4$ such that $\ell > L_4$ implies $(1-\varepsilon)^{1/2} k_{j_\ell} > \hat{L}$.

Now we fix an arbitrary $\ell > \max\{L_1, L_2, L_3, L_4\}$ and wish to show that in economy $F^{k_{j_\ell}}$

$$\Pr\left(\text{The fraction of students playing SRS against } p^{k_{j_\ell}} \text{ is no less than } 1-\varepsilon\right) \geq 1-\varepsilon,$$

which would contradict $(*)$ and complete the proof.

We first prove that in economy $F^{k_{j_\ell}}$, a student with $\tilde{\theta} \in \tilde{\Theta}^\delta$ plays SRS against $\bar{p}$ with probability no less than $(1-\varepsilon)^{1/3}$. To see this, suppose to the contrary that there exists some student $i$ and some type $\tilde{\theta} \in \tilde{\Theta}^\delta$ such that

$$\Pr\left(\sigma_i^{k_{j_\ell}}(\tilde{\theta}) \text{ plays SRS against } \bar{p}\right) < (1-\varepsilon)^{1/3}.$$

Then deviating to truthful reporting will give student $i$ with type $\tilde{\theta} \in \tilde{\Theta}^\delta$ at least a gain of

$$\begin{aligned}
&\mathbb{E}\left[i\text{'s gain from deviation } \middle| \sigma_i^{k_{j_l}} \text{ plays non-SRS against } p^{k_{j_\ell}}\right] \Pr\{\sigma_i^{k_{j_\ell}} \text{ plays non-SRS against } p^{k_{j_\ell}}\} \\
\geq &\mathbb{E}\left[i\text{'s gain from deviation } \middle| \{\sigma_i^{k_{j_l}} \text{ plays non-SRS against } p^{k_{j_\ell}}\} \wedge A^{k_{j_l}}\right] \\
&\cdot \Pr\left(\sigma_i^{k_{j_\ell}}(\tilde{\theta}) \text{ plays non-SRS against } p^{k_{j_\ell}} \textit{ and } \text{event } A^{k_{j_\ell}}\right) \\
\geq &\delta\alpha(\delta) \cdot \Pr\left(\sigma_i^{k_{j_\ell}}(\tilde{\theta}) \text{ plays non-SRS against } p^{k_{j_\ell}} \textit{ and } \text{event } A^{k_{j_\ell}}\right) \\
\geq &\delta\alpha(\delta) \cdot \Pr\left(\sigma_i^{k_{j_\ell}}(\tilde{\theta}) \text{ plays non-SRS against } \bar{p} \textit{ and } \text{event } A^{k_{j_\ell}}\right) \\
\geq &\delta\alpha(\delta)\left[\Pr\left(A^{k_{j_\ell}}\right) - \Pr\left(\sigma_i^{k_{j_\ell}}(\tilde{\theta}) \text{ plays SRS against } \bar{p}\right)\right] \\
\geq &\delta\alpha(\delta)\left[(1-\varepsilon)^{1/6} - (1-\varepsilon)^{1/3}\right],
\end{aligned}$$

where the inequalities are explained as follows. The first inequality holds since the gains from the deviation is nonnegative whether the event $A^{k_{j_\ell}}$ occurs or not. The second inequality results from Lemma 4, upon noting that by Lemma 3 the cutoffs under the non-SRS against $\bar{p}$ and those that would prevail if $i$ deviates to TRS are within $\nu(\delta)$-distance from $\bar{p}$. The third inequality holds since, given our choice $\delta$, conditional on event $A^{k_{j_\ell}}$, a non-SRS against $\bar{p}$ is a non-SRS against $p^{k_{j_\ell}}$. The above inequalities contradict $\ell > L_3$, which implies that the strategy profile $\left(\sigma_i^{k_{j_\ell}}\right)_{i=1}^{k_{j_\ell}}$ is a $\delta\alpha(\delta)\left[(1-\varepsilon)^{1/6} - (1-\varepsilon)^{1/3}\right]$-BNE for the economy $F^{k_{j_\ell}}$.

Therefore, in economy $F^{k_{j_\ell}}$, for each student $i = 1, \ldots, k_{j_\ell}$ and each $\tilde{\theta} \in \tilde{\Theta}^\delta$, we have

$$\begin{aligned}
&\Pr\left(\sigma_i^{k_{j_\ell}}(\tilde{\theta}) \text{ plays SRS against } \bar{p} \middle| \eta^{k_{j_\ell}}(\tilde{\Theta}^\delta) \geq (1-\varepsilon)^{1/2}\right) \\
= &\Pr\left(\sigma_i^{k_{j_\ell}}(\tilde{\theta}) \text{ plays SRS against } \bar{p}\right) \geq (1-\varepsilon)^{1/3}, \qquad (***)
\end{aligned}$$

where the first equality holds because student $i$'s random report according to her mixed strategy is independent of random draws of the students' type.

Then we have

$$\begin{aligned}
&\Pr\left(\begin{array}{c}\text{The fraction of students with } \tilde{\theta} \in \tilde{\Theta}^{\delta} \\ \text{playing SRS against } \bar{p} \text{ is no less than } (1-\varepsilon)^{1/2}\end{array}\middle|\, \eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}\right) \\
&\geq \Pr\left(\begin{array}{c}\eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \cdot k_{j_\ell} \text{ i.i.d. Bernoulli random variables with} \\ \text{probability } (1-\varepsilon)^{1/3} \text{ have a sample mean no less than } (1-\varepsilon)^{1/2}\end{array}\middle|\, \eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}\right) \\
&\geq \Pr\left(\begin{array}{c}\hat{L} \text{ i.i.d. Bernoulli random variables with probability } (1-\varepsilon)^{1/3} \\ \text{have a sample mean no less than } (1-\varepsilon)^{1/2}\end{array}\right) \\
&\geq (1-\varepsilon)^{1/3},
\end{aligned} \tag{A.4}$$

where the first inequality follows from (***) and that $\sigma_i$'s are independent across students conditioning on the event $\eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}$, and the second inequality holds since, for $\ell > L_4$, $\eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}$ implies that $\eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \cdot k_{j_l} > \hat{L}$.

Comparing the finite economy random cutoff $p^{k_{j_\ell}}$ with the deterministic cutoff $\bar{p}$, we have

$$\begin{aligned}
&\Pr\left(\begin{array}{c}\text{The fraction of students with } \tilde{\theta} \in \tilde{\Theta}^{\delta} \\ \text{playing SRS against } p^{k_{j_\ell}} \text{ is no less than } (1-\varepsilon)^{1/2}\end{array}\middle|\, \eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}\right) \\
&\geq \Pr\left(\begin{array}{c}\text{The fraction of students with } \tilde{\theta} \in \tilde{\Theta}^{\delta} \\ \text{playing SRS against } \bar{p} \text{ is no less than } (1-\varepsilon)^{1/2}, \\ \text{and event } A^{k_{j_\ell}}\end{array}\middle|\, \eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}\right) \\
&\geq \Pr\left(\begin{array}{c}\text{The fraction of students with } \tilde{\theta} \in \tilde{\Theta}^{\delta} \\ \text{playing SRS against } \bar{p} \text{ is no less than } (1-\varepsilon)^{1/2}\end{array}\middle|\, \eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}\right) \\
&- \Pr\left(A^{k_{j_\ell}} \text{ does not occur} \,\middle|\, \eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}\right) \\
&\geq (1-\varepsilon)^{1/3} - \frac{1-\Pr\left(A^{k_{j_\ell}}\right)}{\Pr\left(\eta^{k_{j_\ell}}(\tilde{\Theta}^{\delta}) \geq (1-\varepsilon)^{1/2}\right)} \\
&\geq (1-\varepsilon)^{1/3} - \frac{(1-\varepsilon)^{1/2}\left[(1-\varepsilon)^{1/3} - (1-\varepsilon)^{1/2}\right]}{(1-\varepsilon)^{1/2}} \\
&= (1-\varepsilon)^{1/2}.
\end{aligned}$$

The first inequality follows since in event $A^{k_{j_\ell}}$, the strategy $\sigma_i(\theta)$ is SRS against $P^{k_{j_\ell}}$ if and only if $\sigma_i(\theta)$ is SRS against $\bar{p}$ for type $\theta \in \tilde{\Theta}^{\delta}$. The third inequality follows from (A.4). The fourth inequality follows from (**).

The construction of $L_1$ implies $\Pr\left(\eta^{k_{j_\ell}}(\tilde{\Theta}^\delta) \geq (1-\varepsilon)^{1/2}\right) \geq (1-\varepsilon)^{1/2}$, so finally we have in economy $F^{k_{j_\ell}}$,

$$\begin{aligned}
&\Pr\left(\text{The fraction of students playing SRS against } p^{k_{j_\ell}} \text{ is no less than } 1-\varepsilon\right)\\
&\geq \Pr\left(\begin{array}{c}\text{At least } (1-\varepsilon)^{1/2} \text{ of students with } \tilde{\theta} \in \tilde{\Theta}^\delta \text{ play SRS against } p^{k_{j_\ell}} \\ \textit{and } \eta^{k_{j_\ell}}(\tilde{\Theta}^\delta) \geq (1-\varepsilon)^{1/2}\end{array}\right)\\
&= \Pr\left(\eta^{k_{j_\ell}}(\tilde{\Theta}^\delta) \geq (1-\varepsilon)^{1/2}\right) \cdot \Pr\left(\begin{array}{c}\text{At least } (1-\varepsilon)^{1/2} \text{ of students} \\ \text{with } \tilde{\theta} \in \tilde{\Theta}^\delta \text{ play SRS against } p^{k_{j_\ell}}\end{array} \middle| \eta^{k_{j_\ell}}(\tilde{\Theta}^\delta) \geq (1-\varepsilon)^{1/2}\right)\\
&\geq (1-\varepsilon)^{1/2} \cdot (1-\varepsilon)^{1/2}\\
&= 1-\varepsilon,
\end{aligned}$$

where the last inequality follows from the above string of inequalities. Therefore, we have obtained a contradiction to $(*)$, and the statement of Theorem 1 follows. □

### A.5 Proof of Proposition 1

*Proof.* First, as TEPS only uses stability and transitivity to infer preferences, it must be that $\mathcal{P} \subseteq \mathcal{P}^{\text{ST}}(\Omega')$. Second, we show that for any $(c, c') \in \mathcal{P}^{\text{ST}}(\Omega')$, $(c, c') \in \mathcal{P}$. As $c'$ is inferred worse than $c$ by stability and transitivity, there must be a sequence of schools, $c^1, \ldots, c^J$ for $1 < J \leq C$ with $c^1 = c$ and $c^J = c'$, such that $c^{j-1}$ and $c^j$, for $1 < j \leq J$, are both feasible in some realized uncertainty in $\Omega'$ when $c^{j-1}$ is the assigned school. Since TEPS uses all uncertainties in $\Omega'$ in Stage 1, it must be that $(c_{j-1}, c_j) \in \widetilde{\mathcal{P}}$ for $1 < j \leq J$ in Stage 2. By transitivity at Stage 3 of TEPS, $(c, c') = (c_1, c_J) \in \mathcal{P}$. We thus have $\mathcal{P} = \mathcal{P}^{\text{ST}}(\Omega')$. □

### A.6 Proof of Proposition 2

*Proof.* First, for any $0 < \tau < \tau' < 100$, $\mathcal{P}^{top} \subseteq \mathcal{P}^{\tau} \subseteq \mathcal{P}^{\tau'} \subseteq \mathcal{P}^{all}$ directly follows from Stage 1 of each TEPS$^\tau$, and $\mathcal{P}^{all} = \mathcal{P}^{\text{ST}}(\Omega)$ directly follows from Proposition 1 . We only need to prove that $\mathcal{P}^{all} \subseteq \mathcal{P}^{\text{WTT}}$. Consider some $(c, c') \in \mathcal{P}^{all}$. We show that $(c, c') \in \mathcal{P}^{\text{WTT}}$.

First, since TEPS$^{all}$ infers only an assigned school in some realized uncertainty is preferred to other schools, and an unranked school can never be a student's assigned school, $c$ should be a ranked school in $R$. Next, notice that $\mathcal{P}^{all}$ can never include any $(c, c')$ such that $c'$ is ranked above $c$ in $R$. To see this, assume to the contrary that $(c, c') \in \mathcal{P}^{all}$ and that $c'$ is ranked above $c$ in $R$. Then at Stage 1 of TEPS$^{all}$, there must be a sequence of schools, $c^1, \ldots, c^J$ for $1 < J \leq C$ with $c^1 = c$ and $c^J = c'$, such that $c^{j-1}$

and $c^j$, for $1 < j \leq J$, are both feasible in some realized uncertainty when $c^{j-1}$ is the assigned school. Then, for each $1 < j \leq J$, it must be that $c^{j-1}$ is ranked above $c^j$, which implies that $c^1 = c$ is ranked above $c^J = c'$ in $R$, a contradiction.

Hence, $c'$ is either (i) a school ranked lower than $c$ on $R$, or (ii) an unranked school. Since WTT infers that $c$ is more preferable than *every* school ranked below $c$ and *every* unranked school, it should be that $(c, c') \in \mathcal{P}^{\text{WTT}}$. Therefore, $\mathcal{P}^{all} \subseteq \mathcal{P}^{\text{WTT}}$. $\square$

(For Online Publication)

Supplemental Appendix to

# Leveraging Uncertainties to Infer Preferences: Robust Analysis of School Choice

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## A Priority Structures of Deferred-Acceptance Mechanism: Examples

Table A.1: Priority Structures of Deferred-Acceptance Mechanism: Examples

| Education System | Mechanism | Priority Structure | Priority Details | Sources |
|---|---|---|---|---|
| *Panel A: Primary and Secondary Education* | | | | |
| Boston (Open Enrollment High Schools) | Student-proposing DA | $T_2$ | Distance, other factors, random tie-breaking | Boston Public Schools[A-1] |
| Boston (Exam Schools) | Student-proposing DA | $T_3$ | Composite score (unknown): GPA (unknown when submitting ROL) + test score (known) | Abdulkadiroglu, Angrist, and Pathak (2014), Boston Public Schools[A-1] |
| Chicago (Non-selective Choice High Schools) | Student-proposing DA | $T_2$ | Distance, other factors, random tie-breaking | Chicago Public Schools[A-2] |
| Chicago (Selective Enrollment Programs) | DA (serial dictatorship) | $T_3$ | Composite score (unknown): GPA (known) + test score (known) | Pathak and Sönmez (2013), Chicago Public Schools[A-2] |
| Denver | Student-proposing DA | $T_2$ | Distance, siblings, other factors, random tie-breaking | Abdulkadiroğlu and Andersson (2023), Denver Public Schools[A-3] |
| NYC (Specialized High Schools) | DA (serial dictatorship) | $T_3$ | Test score (unknown) | Abdulkadiroglu, Angrist, and Pathak (2014), NYC Public Schools portal[A-4] |
| NYC (Screened High Schools) | Student-proposing DA | $T_3$ / $T_2$[A-5] | Coarse priorities (test results + distance) and random tie-breaking | Abdulkadiroğlu, Pathak, Schellenberg, and Walters (2020), NYC Public Schools[A-4] |
| NYC (Open High Schools) | Student-proposing DA | $T_2$ | Coarse priorities and random tie-breaking | Abdulkadiroğlu, Pathak, Schellenberg, and Walters (2020), NYC Public Schools[A-4] |
| Washington DC | School-proposing DA | $T_2$ | Distance, siblings, other factors, random tie-breaking | Abdulkadiroğlu and Andersson (2023), DC Public Schools[A-6] |

[A-1] https://www.bostonpublicschools.org

[A-2] https://www.cps.edu/gocps/high-school/hs-selection/

[A-3] https://schoolchoice.dpsk12.org/o/schoolchoice

[A-4] https://www.schools.nyc.gov/enrollment

[A-5] $T_3/T_2$ denotes the case in which an unknown merit-based measure is used by schools but does not lead to a strict ranking. Random tie-breaking is thus used when several students have the same merit-based ranking.

[A-6] https://enrolldcps.dc.gov/

| | | | | |
|---|---|---|---|---|
| Chile (Schools with high-achieving student quota) | Student-proposing DA | $T_3$ / $T_2$[A-5] | Coarse priorities (test result + siblings + working parents + former student) and random tie-breaking | Arteaga, Kapor, Neilson, and Zimmerman (2022) |
| Chile (Other schools) | Student-proposing DA | $T_2$ | Siblings, working parents, former student and random tie-breaking | Arteaga, Kapor, Neilson, and Zimmerman (2022) |
| England: Non-selective state schools | Student-proposing DA | $T_2$ | Distance, siblings, other factors, random tie-breaking | Carter, Pathak, and Terrier (2020), UK School Choice Portal[A-7] |
| England: Grammar schools | Student-proposing DA | $T_1$ or $T_3$ | Test score only (known); or composite score (unknown): test score (known) + other factors | UK Department for Education (2021) |
| Estonia (Inter-district applications) | Decentralized school-proposing DA | $T_3$ | School-specific test score (unknown) | Lauri, Põder, and Veski (2014) |
| Finland | School-proposing DA | $T_3$ | Composite score (unknown): academic records (known) + exam score + other criteria | Salonen (2014) |
| France | School-proposing DA | $T_3$ | Composite score (unknown): GPA + other factors | Hiller and Tercieux (2014); Grenet (2022) |
| Ghana | DA (serial dictatorship) | $T_3$ | Nationwide test score (unknown) | Ajayi (2022) |
| Hungary | Student-proposing DA | $T_1$ or $T_3$ | Previous grades (known); or composite score (unknown): grades + test score + interview | Biró (2012) |
| Mexico City | DA (serial dictatorship) | $T_3$ | Nationwide test score (unknown) | Chen and Sebastián Pereyra (2019) |
| Romania | DA (serial dictatorship) | $T_1$ | Composite score (known): GPA + nationwide test result | Pop-Eleches and Urquiola (2013) |
| Singapore | DA (serial dictatorship) | $T_1$ | Nationwide test score (known) | Teo, Sethuraman, and Tan (2001), Singapore Ministry of Education[A-8] |
| *Panel B: Higher Education* | | | | |
| Australia (Victoria) | College-proposing DA | $T_1$ | Nationwide test score (known) | Artemov, Che, and He (2021) |
| Brazil (Last phase) | DA (serial dictatorship) | $T_1$ | Nationwide test score (known) | Otero, Barahona, and Dobbin (2021) |
| Chile | Student-proposing DA | $T_3$ | Composite score (unknown): GPA (known) + standardized test (known) | Hastings, Neilson, and Zimmerman (2013) |
| France | Decentralized college-proposing DA | $T_3$ | Composite score (unknown): GPA (known) + other criteria | Hakimov, Schmacker, and Terrier (2023) |
| Germany (DoSV) | College-proposing DA [A-9] | $T_3(\sim T_1)$[A-10] | Composite score: GPA (known, main criterion) + other factors | Kübler (2019) |
| Hungary | Student-proposing DA | $T_3$ | Composite score (unknown):GPA + nationwide test score + other factors | Biró (2011) |
| Ireland | College-proposing DA | $T_3$ / $T_2$[A-5] | nationwide test score (unknown) and random tie-breaking | Chen (2012) |
| Israel (Psychology Master's Match) | Student-proposing DA | $T_3$ | Composite score (unknown): past transcripts + test results + interview results + recommendation letters | Hassidim, Romm, and Shorrer (2021) |

[A-7] https://www.gov.uk/schools-admissions/admissions-criteria

[A-8] https://www.moe.gov.sg/secondary/s1-posting

[A-9] Implemented in 2 phases: A first decentralized phase, and a second centralized one.

[A-10] Formally, DoSV is $T_3$ because schools use a composite score that is unknown to the students. However, the grade to the national end-of-high school test is the primary determinant of the final score and other factors only have a marginal effect on the final score. Hence, DoSV is very close to $T_1$.

| | | | | |
|---|---|---|---|---|
| Norway | College-proposing DA | $T_3$ | Composite score (unknown): GPA + other factors | Kirkeboen, Leuven, and Mogstad (2016) |
| Spain | Student-proposing DA | $T_3$ | Composite score (unknown): previous grades + test score (unknown) | Mora and Romero-Medina (2001) |
| Taiwan (Individual Application Process) | College-proposing DA | $T_1$ | Composite score (known): nationwide test score + school-specific evaluation | Hu and Wang (2024) |
| Tunisia | College-proposing DA | $T_3$ | Nationwide test score (known) | Luflade (2019) |
| Turkey | College-proposing DA | $T_1$ | Nationwide test score (known) | Saygin (2016) |
| Ukraine | College-proposing DA [A-11] | $T_1$ | Composite score (known): nationwide test score + previous grades | Kiselgof (2011) |

# B Supplementary Materials for Section 2

## B.1 Rich Uncertainty Assumption

We show that Assumption 3, Rich Uncertainty, is satisfied in all realistic environments (a), (b), and (c) in the main text.

Lemma B.1. *Rich Uncertainty holds for priority structures, (a), (b), and (c).*

*Proof.* For case (a), $\underline{s}_c^t = \overline{s}_c^t$ for all $t \in T$, so the probability in the LHS of Equation (A.1) is equal to one. Hence, Rich Uncertainty holds trivially. We thus focus on (b) and (c).

**Structure (b): STB version.** The cutoff $p_c$ for each school $c$ corresponds to a lottery threshold $\ell_c^t \in [0, 1]$, possibly dependent on the type $t$ of the student. To see this, the cutoff can be expressed as: $p_c = \hat{t}_c + \frac{\lambda_c}{n_c}$, where the $\hat{t}_c$ is the intrinsic priority level that one needs for admission at $c$ and $\lambda_c$ is the lottery cutoff score for $c$. If $\overline{s}_c^t - \delta > p_c$ for some $\delta > 0$, either the student has an intrinsic priority level of $\hat{t}_c$ or a higher priority level. In the former, $s_c > p_c$ amounts to having a lottery draw above $\lambda_c$, so $\ell_c^t = \lambda_c$. In the latter, $s_c > p_c$ regardless of the lottery draw, so $\ell_c^t = 0$. Similarly, if $\underline{s}_c^t + \delta < p_c$ for some $\delta > 0$, either the student has an intrinsic priority level of $\hat{t}_{c,i}$ or a lower priority level. In the former, $s_c < p_c$ amounts to having a lottery draw below $\lambda_c$, in which case $\ell_c^t = \lambda_c$. In the latter, $s_c < p_c$ regardless of the lottery draw, so $\ell_c^t = 1$.

Given the STB structure, the LHS of (A.1) then reduces to

$$\Pr\left\{\max\{\ell_a^t, \ell_b^t\} < \lambda^t < \min_{c \in C^t} \ell_c^t\right\}, \tag{B.1}$$

namely, the probability that the student draws an STB lottery number $\lambda^t \in (\max\{\ell_a^t, \ell_b^t\}, \min_{c \in C^t} \ell_c^t)$. Further, if $\Pr\{s_c^t < p_c, \forall c \in C^t, s_a^t > p_a\} > 0$ and $\Pr\{s_c^t < p_c, \forall c \in C^t, s_b^t > p_b\} > 0$, it must be that

[A-11] Only the first three phases of the DA mechanism are implemented.

$\max\{\ell_a^t, \ell_b^t\} < \min_{c\in C^t} \ell_c^t$. Note that $\max\{\ell_a^t, \ell_b^t\} = \lambda_i$ for some $i \in \{a, b, x\}$ and $\min_{c\in C^t} \ell_c^t = \lambda_j$ for some $j \in C^t \cup \{y\}$ such that $\lambda_i < \lambda_j$, where $\lambda_x := 0$ and $\lambda_y := 1$. Hence, (B.1) equals

$$\Pr\left\{\max\{\ell_a^t, \ell_b^t\} < \lambda^t < \min_{c\in C^t} \ell_c^t\right\} = \Pr\{\lambda_i < U < \lambda_j\}$$
$$\geq \min\left\{|\lambda_c - \lambda_{c'}| : c, c' \in \hat{C}, \lambda_c \neq \lambda_{c'}\right\} =: \beta(\delta),$$

where $U$ is uniform random variable on $[0, 1]$ and $\hat{C} := C \cup \{x, y\}$. Importantly, this lower bound $\beta(\delta)$ does not depend on $t$ or the particular pair $(a, b)$ or other $c \in C^t$. It only depends on $(\lambda_c)_c$, which is determined uniquely by $p$.

**Structure (b): MTB version.** The approach is similar to that of STB. In particular, the first part (mapping cutoffs $p$ to lottery cutoffs $\lambda = (\lambda_c)_c$) is exactly the same. The difference is that the LHS of (A.1) is now equal to

$$\Pr\left\{\lambda_c^t < \ell_c^t, \forall c \in C^t, \ell_i^t < \lambda_i^t, \forall i = a, b\right\}, \tag{B.2}$$

where $\lambda_i^t$ is $t$'s MTB draw for school $i$. Clearly, given the hypothesis, this probability must be positive, which implies that $\ell_i^t < 1$ for $i = a, b$ and $\ell_c^t > 0$ for $c \in C^t$. Since each $\ell_j^t = \lambda_j$ for some $j \in C \cup \{x, y\}$, we have

$$\Pr\left\{\lambda_c^t < \ell_c^t, \forall c \in C^t, \ell_i^t < \lambda_i^t, \forall i = a, b\right\} \geq \Pr\left\{U^{(C-2:1)} < \lambda_* \text{ and } U^{(2:2)} > \lambda^*\right\} =: \beta(\delta),$$

where $\lambda_* := \min\{\lambda_i : i \in C \cup \{x, y\}, \lambda_i > 0\}$, $\lambda^* := \max\{\lambda_i : i \in C \cup \{x, y\}, \lambda_i < 1\}$, and $U^{(n:m)}$ is the $m$-th highest value of $n$ independent draws of $U[0, 1]$. The proof is complete upon noting that the lower bound $\beta(\delta)$ is independent of $t$ and of particular pair $(a, b)$ or other $c \in C^t$.

**Structure (c):** The argument is similar to that of Structure (b).

We first consider a model similar to the NYC Specialized High Schools, where the ex-post score is one-dimensional. The argument is similar to that of Structure (b): STB. In particular, one can show that the LHS of (A.1) is lower bounded by the probability that one's score lies within a certain interval:

$$\Pr\left\{\max\{p_a^t, p_b^t\} < s^t < \min_{c\in C^t} p_c^t\right\}. \tag{B.3}$$

Similarly to (b), if $\Pr\{s_c^t < p_c, \forall c \in C^t, s_a^t > p_a\} > 0$ and $\Pr\{s_c^t < p_c, \forall c \in C^t, s_b^t > p_b\} > 0$, then we must have $\max\{p_a^t, p_b^t\} < \min_{c\in C^t} p_c^t$. Hence, (B.3) is lower bounded by

$$\beta(\delta) := \varrho \cdot \min\{\delta \wedge |p_c - p_{c'}| : c, c' \in \hat{C}, p_c \neq p_{c'}\} > 0,$$

where $\hat{C} := C \cup \{x, y\}$ with $p_x := 0, p_y := 1$ and $\varrho := \min_{t \in T} \min_{s \in [\underline{s}_c^t, \overline{s}^t]} \phi^t(s) > 0$. The positivity of $\varrho$ follows from the full and compact support assumption for each $t$ and the compactness of $T$. In words, the probability lower bound is given by the shortest length of distinct score cutoffs or by $\delta$, whichever is smaller. The relevance of $\delta$ here comes from the fact that, due to the $\delta$-rejectability or $\delta$-acceptability of the schools, the support of $t$'s score in the interval $(\max\{p_a^t, p_b^t\}, \min_{c \in C^t} p_c^t)$ spans at least the length of $\delta$, whenever that interval exceeds $\delta$ in length. The proof is complete upon noting that the lower bound $\beta(\delta)$ is independent of $t$ and of particular pair $(a, b)$ or other $c \in C^t$.

We next consider a model where ex-post scores are not perfectly correlated. The argument is similar to that of Structure (b): MTB. Again, one can show that

$$\begin{aligned} &\Pr\{s_c^t < p_c, \forall c \in C^t, \text{ and } s_a^t > p_a, s_b^t > p_b\} \\ \geq &\Pr\{s_c^t < p_*, \forall c \in C^t, \text{ and } s_a^t > p^*, s_b^t > p^*\} \\ \geq &\hat{\varrho}^C \cdot (\delta \wedge p_*)^{C-2} (\delta \wedge (1 - p^*))^2 =: \beta(\delta), \end{aligned}$$

where $p_* := \min\{p_i : i \in C \cup \{x, y\}, p_i > 0\}$, $p^* := \max\{p_i : i \in C \cup \{x, y\}, p_i < 1\}$, and $\hat{\varrho} := \min_{t \in T} \min_{s \in S^t} \phi^t(s) > 0$. Again, the appearance of $\delta$ in the lower bound follows from the fact that the support of $t$'s score for $i = a, b$ in the interval $[0, p_i]$ and its scores for $c \in C^t$ in the interval $[p_c, 1]$ each spans $\delta$ in length, whenever each interval exceeds $\delta$ in length, again due to the $\delta$-acceptability and $\delta$-rejectability of these schools. The proof is complete upon noting that the lower bound $\beta(\delta)$ is independent of $t$ and of particular pair $(a, b)$ or other $c \in C^t$. □

# C Multiple Equilibria, Completeness, and Coherency

This appendix shows that our TEPS procedure does not suffer from incompleteness or incoherence in the sense of Tamer (2003). Incompleteness in our context would mean that the mapping from a student's type, $(u, t)$, to TEPS inferred preferences is a correspondence, which may cause the point identification of the distribution of $u$ to fail. Meanwhile, incoherency would imply that the model does not have a well-defined likelihood for TEPS inferred preferences given exogenous variables, implying certain logical inconsistency.

We start with some definitions. Recall that we consider a student with submitted ROL $R$ and intrinsic priorities $t$. We also make it explicit that the preferences inferred by TEPS depend on $R$ and $t$, i.e., $\mathcal{P}(R, t)$.

We say ROL $R'$ is *consistent* with $\mathcal{P}(R, t)$ if $R'$ satisfies two conditions: (i) every ever-assigned school $c = \alpha_W$ for some $W \subseteq \Omega$ is included in $R'$, and (ii) for any $c'$ ranked above $c$ in $R'$, $c'$ is *not* inferred worse than than $c$, i.e., $(c, c') \notin \mathcal{P}(R, t)$. This implies that we may exclude a never-feasible school from $R'$ or insert it in $R'$ at any position; however, for an ever-feasible school that is included in $R'$, its position in $R'$ must respect the inferred preferences. Let $\mathcal{R}^*(R, t)$ be the set of all ROLs that are consistent with $\mathcal{P}(R, t)$. It can be verified that $\mathcal{R}^*(R, t)$ includes the student's true preference order given the assumption of stability and transitivity.

Further, let $\mathcal{U}^*(R,t) \subseteq [\underline{u}, \overline{u}]^C$ be all the utility types that are consistent with $\mathcal{P}(R,t)$. That is, if $u \in \mathcal{U}^*(R,t)$, the associated true preference order $\rho(u)$ is in $\mathcal{R}^*(R,t)$.

Proposition C.1. *Assume that in every realized uncertainty, each student cannot change her own set of feasible schools and that the stable matching is unique and achieved. Suppose that a student of type $(u,t)$ submits a ROL $R$. We have the following results:*
*(i)* ***Equivalent class of ROLs.*** *When her ROL $R$ is replaced by $R'$, the student receives her stable assignment in every realized uncertainty if and only if $R' \in \mathcal{R}^*(R,t)$.*
*(ii)* ***Completeness.*** *$\mathcal{P}(R,t) = \mathcal{P}(R',t), \forall R' \in \mathcal{R}^*(R,t)$. That is, given stability, TEPS infers a unique set of preference relations for the student even if she submitted any ROL in $\mathcal{R}^*(R,t)$.*
*(iii)* ***Coherency.*** *TEPS infers $\mathcal{P}(R,t)$ if and only if $u \in \mathcal{U}^*(R,t)$.*

*Proof.* We prove the three statements one by one.
(i) We first prove sufficiency and then necessity.

**Sufficiency.** Suppose $R' \in \mathcal{R}^*(R,t)$. In any given realized uncertainty, by the definition of $\mathcal{R}^*(R,t)$, $R'$ must rank the assigned school above any other simultaneously feasible schools, while the assigned school is included in $R'$. The student must be assigned the same school regardless of submitting $R$ or $R'$, because she cannot change her set of feasible schools in this realized uncertainty. Hence, she always obtains her stable assignment.

**Necessity.** Suppose that $R$ and $R'$ give the student the same stable assignment in every realized uncertainty. We argue that $R'$ is in $\mathcal{R}^*(R,t)$. For any $c$ that is the assigned school in a realized uncertainty, $c$ must be ranked in $R'$. Therefore, to show $R' \in \mathcal{R}^*(R,t)$, we only need to prove that for any $c'$ ranked above $c$ in $R'$, $c'$ is *not* inferred worse than $c$, or $(c,c') \notin \mathcal{P}(R,t)$. Suppose on the contrary that $c'$ ranked above $c$ in $R'$ and $(c,c') \in \mathcal{P}(R,t)$. For TEPS to infer that $c'$ is worse than $c$ given that $R$ is submitted, there must exist $c^1, \ldots, c^J$ for $1 < J \leq C$ with $c^1 = c$ and $c^J = c'$ such that, for each $1 < j \leq J$, there exists a realized uncertainty in which $c^{j-1}$ is the assigned school while $c^j$ is feasible. This implies that $c^1, \ldots, c^{J-1}$ must be included in $R'$. Since $c^J = c'$ is ranked above $c^1 = c$ in $R'$, $R'$ must rank $c^{j^*}$ above $c^{j^*-1}$ for some $1 < j^* \leq J$. Because the student cannot influence her own feasible set, there must exist a realized uncertainty in which the student's assignment when submitting $R'$ is $c^{j^*}$, in contrast to $c^{j^*-1}$ which is the assignment when submitting $R$. This contradiction lets us conclude that $R'$ must be consistent with $\mathcal{P}(R,t)$, or equivalently, $R' \in \mathcal{R}^*(R,t)$.

(ii) By part (i), the set of ever-assigned schools is the same under $R$ or $R'$. Hence, for any $c$ which is never assigned to the student, there does not exist $(c,c'), \forall c' \in C$ such that $(c,c') \in \mathcal{P}(R,t)$ or $(c,c') \in \mathcal{P}(R',t)$.

Consider $c'$ such that $(c, c') \in \mathcal{P}(R, t)$ for some $c$ that is assigned to the student in some realization of uncertainty. When $R$ is submitted, there must exist $c^1, \ldots, c^J$ for $1 < J \leq C$ with $c^1 = c$ and $c^J = c'$ such that, for each $1 < j \leq J$, there exists a realized uncertainty in which $c^{j-1}$ is the assigned school while $c^j$ is feasible. Together with the assumption that $i$ cannot change her own feasible set, part (i) implies that in each of those same realized uncertainties associated with $R$, $c^{j-1}$ remains the assigned school while $c^j$ is still feasible for each $1 < j \leq J$ even when $R'$ is submitted. Hence, $(c, c') \in \mathcal{P}(R', t)$. Similarly, we can show that for any $c'$ such that $(c, c') \in \mathcal{P}(R', t)$ for some $c$ that is assigned to the student in some realization of uncertainty, $(c, c') \in \mathcal{P}(R, t)$.

Taken together, $\mathcal{P}(R, t) = \mathcal{P}(R', t)$.

(iii) We first prove sufficiency and then necessity.

**Sufficiency.** For $u \in \mathcal{U}^*(R, t)$, to guarantee the unique stable matching in each realized uncertainty, part (i) implies that the student must submit a ROL in $\mathcal{R}^*(R, t)$. By part (ii), TEPS infers $\mathcal{P}(R, t)$ for the student.

**Necessity.** Suppose that TEPS infers $\mathcal{P}(R, t)$ for the student while $u \notin \mathcal{U}^*(R, t)$. By the definition of $\mathcal{U}^*(R, t)$, the student's true preference order, $\rho(u)$, is not in $\mathcal{R}^*(R, t)$. Part (i) implies that the student's assignment from submitting $\rho(u)$ is not the same as her assignment from submitting $R$, while $R$ gives her the unique stable assignment. This implies that reporting truthfully leads to an unstable assignment for her, contradicting the property of the DA mechanism.

□

The assumptions in Proposition C.1 are justified by our Theorem 1 in large markets. When the market becomes large, a student's impact on cutoffs and thus her own feasible sets becomes negligible and the matching in any realized uncertainty is virtually stable and unique.

Part (i) implies that, although stability does not predict a unique ROL for every student, it predicts a unique class of outcome-equivalent ROLs, $\mathcal{R}^*(R, t)$. Further, part (ii) shows that TEPS$^{all}$ maps a student's true preferences into a unique set of inferred preferences, regardless of which ROL from $\mathcal{R}^*(R, t)$ the student submits. Finally, part (iii) indicates that our model is coherent and that the likelihood of a set of TEPS$^{all}$ inferred preferences can be written as the likelihood of the student's cardinal preferences satisfying a set of conditions.

Figure C.1 illustrates the logic behind the proposition. Following part (i) of Proposition C.1, we divide the action space into equivalent classes of ROLs that lead to the same distribution of stable assignments (middle panel). As part (ii) implies, each equivalence class of ROLs uniquely maps into a set of preference relations inferred by TEPS (right panel). Finally, although the mapping between the utility space and the action space is a correspondence, the mapping between the utility space and the space of possible sets

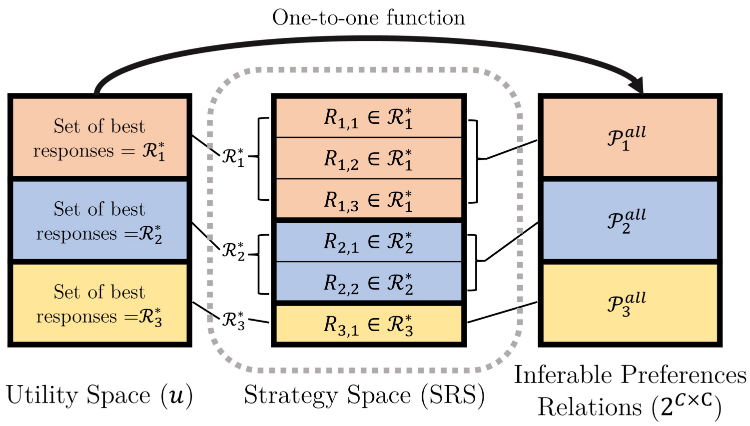


Figure C.1: Completeness and Coherency

TEPS inferred preferences is a function as implied by part (iii).[A-12]

# D Performance of Transitive Extension of Preferences from Stability: Monte Carlo Simulation

This section describes the Monte Carlo simulations that we perform to analyze the implications of our theoretical results.

## D.1 Model Specification

Consider a finite economy in which $k = 1000$ students apply to $C = 12$ schools for admission. The vector of school capacities is specified as follows:

$$\{S_c\}_{c=1}^{12} = \{110, 50, 100, 100, 50, 100, 100, 50, 100, 100, 50, 100\}$$

The total capacity is set to be larger than the total number of students by 10 in order to ensure all students are matched to some school in the simulations.

The economy is located in an area within a circle of radius 1. Students are uniformly distributed in the circle and schools are evenly located on another circle of radius 1/2 around the center. Denote the Euclidean distance between student $i$ and school $c$ as $d_{i,c}$.

Students are matched with schools through a student-proposing DA algorithm with single tie-breaking (DA-STB), similar to the NYC high school choice. Students submit a ROL of schools that can include any number of available schools. We assume all schools are acceptable to all students; hence, student $i$ submits a ROL including all 12 schools if she truthfully reports her preferences.

[A-12] In the version of TEPS that allows for ignoring small probability events, flips of near-indifferent preferences (i.e., flips that are payoff-relevant schools) are ruled out by assumption. However, incorrectly ruling out flips of near-indifferent preferences (e.g., $100 over $100.01) would not bias the estimation significantly.

School priorities over students are coarse. Each school has 4 categories of school-specific (intrinsic) priority groups 0, 1, 2, and 3, with a larger number indicating a higher priority. Denote the priority group that student $i$ belongs at school $c$ as $t_{i,c} \in \{0, 1, 2, 3\}$. Therefore, school $c$ prioritizes student $i$ over $i'$ if $t_{i,c} > t_{i',c}$. A student's priority at a school is drawn independently and uniformly from the four groups. All students are eligible/acceptable at each school. Each student knows her priority group at each school at the time of submitting ROL.

To break ties in priorities, every student is assigned a random lottery number drawn from *Uniform*$[0, 1]$, $l_i$ for all $i$. Lottery numbers are not known at the time of submitting ROL. The score of student $i$ at school $c$ is $s_{i,c} = \frac{t_{i,c}+l_i}{4} \in [0, 1]$. School $c$ prioritizes student $i$ over $i'$ if and only if $s_{i,c} > s_{i',c}$.

Student preferences over schools follow a random utility model without an outside option. Student $i$'s utility from being matched with school $c$ is specified as follows:

$$u_{i,c} = \beta_1 \times c + \beta_2(D_i \times A_c) + \beta_3 d_{i,c} + \beta_4 Small_c + \epsilon_{i,c}, \quad \forall i, c \tag{D.4}$$

where $\beta_1 \times c$ is school $c$'s baseline quality; $d_{i,c}$ is the distance between student $i$'s and school $c$; $D_i = 1$ or $0$ is student $i$'s type (e.g., disadvantaged or not); $A_c = 1$ or $0$ is school $c$'s type (e.g., known for resources for disadvantaged students); $Small_c = 1$ if $S_c = 50$, 0 otherwise; and $\epsilon_{i,c}$ is distributed as $N(0, \sigma_c^2)$ where $\sigma_c^2 = 1$ for $c = 1, \cdots, 6$ and $\sigma_c^2 = 2$ for $c = 7, \cdots, 12$. $\epsilon_{i,c}$ are independent across all $i$ and $c$.

The type of school $c$, $A_c$, equals 1 if $c$ is an odd number and otherwise 0. The type of student $i$, $D_i$, is 1 with probability 2/3 among the lowest priority group of school 1 ($t_{i,1} = 0$); and $D_i = 0$ for all students in highest three priority groups ($t_{i,1} \in \{1, 2, 3\}$).

The coefficients of interest are $(\beta_1, \beta_2, \beta_3, \beta_4)$ which are fixed at $(0.3, 2, -1, 0)$ in the simulations. By this specification, schools with larger indices are of higher quality, and $Small_c$ does not play a role in student preferences over schools. The purpose of estimation is to recover these coefficients and therefore the distribution of preferences.

## D.2 Data Generating Process

Each simulation sample contains an independent preference profile obtained by randomly drawing $D_i$ and $\{t_{i,c}, d_{i,c}, \epsilon_{i,c}\}_c$ for all $i$ from the distributions specified above. In all samples, school capacities and school types ($A_c$ and $Small_c$) are kept constant.

The first set of simulation samples, *cutoff* samples, are used to simulate the joint distribution of the 12 schools' cutoffs (in terms of score, $s_{i,c}$) by letting every student submit a ROL ranking all schools according to her true preferences. To do so, we simulate 100 samples each consisting of 12 schools and 1,000 students. Each sample contains 1,000 sets of independent draws of tie-breaking lotteries $l_i$. After running the DA algorithm, we calculate the cutoffs in each simulation sample with each draw of the lottery. Figure D.2 shows the marginal distribution of each school's cutoff from $100 \times 1000 = 100,000$ simulated realizations.

Note that schools with smaller capacities tend to have higher cutoffs. For example, school 11 with 50 seats often has the highest cutoff, although school 12 with 100 seats has the highest baseline quality. Since every student is guaranteed a seat at some school, school 1 which has the lowest baseline quality has a cutoff equal to 0 (not depicted in the graph).

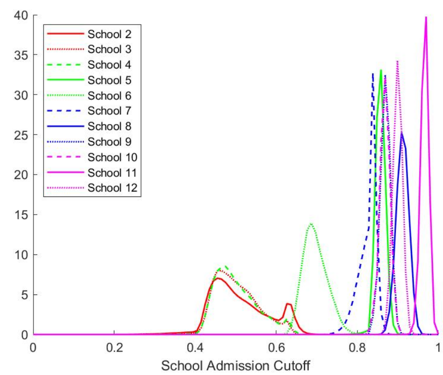


Figure D.2: Simulated Cutoff Distribution

To generate data on student behavior and admission outcomes for preference estimation, we simulate another $100$ samples, the *estimation* samples, with new independent draws of $D_i$ and $\{t_{i,c}, d_{i,c}, \epsilon_{i,c}\}_c$. For each of the 100 estimation samples, we calculate the distribution of cutoffs which represents the uncertainties in cutoffs that students face. By drawing a set of tie-breaking lotteries, we simulate the uncertainty due to random tie-breaking.[A-13] The resulting cutoff distributions are used in the Transitive Extension of Preferences from Stability procedure to determine the feasible sets for each realization of the uncertainty. The *estimation* samples are used for the estimation, and in each of them, we consider three types of data-generating processes (DGPs) with different student behaviors.

(i) **TT (Truth-Telling)** : Every student submits a ROL ranking all 12 schools according to her true preferences.

(ii) **MIS-IRR ((Almost) Payoff Irrelevant Mistakes)**: A fraction of students skip schools with which they are never matched according to the simulated distribution of cutoffs. For a given student, a skipped school can have a high (expected) cutoff and thus be "out of reach" (i.e., never feasible.) Alternatively, the school may also have a low cutoff, but the student is always accepted by one of her more preferred schools. To specify the fraction of skippers, we first randomly choose about 25.6% of the students to be never-skippers who always rank all schools truthfully. All other students are potential skippers, and we make all of them skip. Students with $T_i = 1$ are more likely to skip than those with $T_i = 0$, as

[A-13] One may additionally simulate uncertainties in cutoffs due to the finiteness of the economy. To do so, we can randomly generate economies in the spirit of the bootstrap by randomly resampling a set of students and then use a set of tie-breaking lotteries for each resample.

their scores tend to be lower: 95.6 percent of $T_i = 1$ are potential skippers, compared to 70.1 percent of $T_i = 0$. Finally, we introduce flips by adding back the most preferred school (according to each skipper's true preference) at the end of the ROL if it is never-feasible and thus skipped. It is important to note that some of the mistakes may turn out to be payoff-relevant, because a never-matched school is determined by the simulated cutoff distributions (which may not exhaust all uncertainties in cutoffs).

(iii) **MIS-REL (Payoff Relevant Mistakes)**: In addition to MIS-IRR, i.e., given all the potential skippers have skipped the never-matched schools, we now let them make payoff-relevant mistakes. That is, students skip some of the schools with which they have a small chance of being matched according to the simulated distribution of cutoffs. Recall that the joint distribution of cutoffs is only simulated once under the assumption that everyone is truth-telling. We specify a threshold and make the skippers omit the schools at which they have an admission probability lower than the threshold, where the threshold is equal to 10 percent. We allow for flips in the same fashion as in MIS-IRR.

In summary, for each of the 100 estimation samples, we simulate the matching games 3 times: TT, MIS-IRR, and MIS-REL. Table 2 summarizes the scenarios under each DGP. The fraction of students who make mistakes increases from 0% in TT to 74.4% in MIS-IRR and MIS-REL. As a result, the fraction of students reporting preferences consistent with WTT is only 27.0% in MIS-IRR and 28.8% in MIS-REL. Also, note that stability is satisfied for all students in TT and MIS-IRR, but not in MIS-REL since some students skip schools that are not completely out-of-reach for them.

### D.3 Estimation and Results

The random utility model described by Equation (D.4) is estimated under two methods, WTT and TEPS using Gibbs Sampler, where the procedure is described in the Appendix E. For TEPS estimators, we use TEPS$^{top}$, TEPS$^{20}$, TEPS$^{40}$, TEPS$^{60}$, TEPS$^{80}$, and TEPS$^{all}$. Table D.2 presents the mean and standard deviation of the posterior mean of each parameter across the 100 samples.[A-14]

We evaluate the performance of the two sets of estimators along two dimensions. The first is the bias-variance tradeoff, focusing on $\beta_2$ which measures how students of type $T_i = 1$ value schools of type $A_c = 1$. The second dimension is the performance of the TEPS estimators relative to the TEPS estimators that use smaller sets of inferred preferences, which are both robust to (some) strategic mistakes.

**Bias-Variance Tradeoff** Figure D.3 plots the distributions of the estimates of $\beta_2$ given each DGP; the true value of $\beta_2$ is 2. The figures plot WTT, TEPS$^{top}$, and TEPS$^{all}$.

There are a few notable patterns. First, when the DGP is TT, all estimates are consistent, while the WTT-based estimator has the smallest variance as shown in Panel (a). This is expected since no students make strategic mistakes violating the WTT or stability assumptions. Furthermore, the WTT-based estimator having the smallest variance is not only true under DGP TT but also under other DGPs. Intuitively, this is

[A-14] We iterate through the MCMC 100,000 times and discard the first 75,000 for mixing. We calculated the Potential Scale Reduction Factor (PSRF) (Gelman and Rubin (1992)) to ensure enough convergence of the posterior distributions.

Table D.2: Estimation with Different Identifying Assumptions: Monte Carlo Results

| DGPs | Identifying Condition | Quality ($\beta_1 = 0.3$) | | | Interaction ($\beta_2 = 2$) | | | Distance ($\beta_3 = -1$) | | | Small ($\beta_4 = 0$) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | mean | s.d. | $\sqrt{MSE}$ | mean | s.d. | $\sqrt{MSE}$ | mean | s.d. | $\sqrt{MSE}$ | mean | s.d. | $\sqrt{MSE}$ |
| TT | WTT | 0.30 | 0.00 | 0.00 | 2.00 | 0.06 | 0.06 | -1.00 | 0.03 | 0.03 | 0.00 | 0.02 | 0.02 |
| | $\text{TEPS}^{top}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.03 | 0.11 | 0.12 | 0.00 | 0.07 | 0.07 |
| | $\text{TEPS}^{20}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.02 | 0.11 | 0.12 | 0.00 | 0.07 | 0.07 |
| | $\text{TEPS}^{40}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.03 | 0.12 | 0.12 | 0.00 | 0.07 | 0.07 |
| | $\text{TEPS}^{60}$ | 0.30 | 0.02 | 0.02 | 2.03 | 0.20 | 0.20 | -1.02 | 0.11 | 0.11 | -0.01 | 0.07 | 0.07 |
| | $\text{TEPS}^{80}$ | 0.30 | 0.01 | 0.01 | 2.02 | 0.17 | 0.17 | -1.02 | 0.10 | 0.10 | 0.00 | 0.06 | 0.06 |
| | $\text{TEPS}^{all}$ | 0.30 | 0.01 | 0.01 | 2.01 | 0.12 | 0.12 | -1.01 | 0.07 | 0.07 | 0.00 | 0.04 | 0.04 |
| | Selected | 0.30 | 0.01 | 0.01 | 2.00 | 0.07 | 0.07 | -1.00 | 0.04 | 0.04 | 0.00 | 0.03 | 0.03 |
| MIS-IRR | WTT | 0.08 | 0.00 | 0.22 | 1.21 | 0.08 | 0.79 | -0.49 | 0.04 | 0.51 | -0.19 | 0.02 | 0.19 |
| | $\text{TEPS}^{top}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.03 | 0.11 | 0.12 | 0.00 | 0.07 | 0.07 |
| | $\text{TEPS}^{20}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.03 | 0.11 | 0.12 | 0.00 | 0.07 | 0.07 |
| | $\text{TEPS}^{40}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.03 | 0.11 | 0.12 | 0.00 | 0.07 | 0.07 |
| | $\text{TEPS}^{60}$ | 0.30 | 0.02 | 0.02 | 2.03 | 0.20 | 0.20 | -1.02 | 0.11 | 0.11 | -0.01 | 0.07 | 0.07 |
| | $\text{TEPS}^{80}$ | 0.30 | 0.01 | 0.01 | 2.02 | 0.17 | 0.17 | -1.02 | 0.09 | 0.10 | 0.00 | 0.06 | 0.06 |
| | $\text{TEPS}^{all}$ | 0.30 | 0.01 | 0.01 | 2.01 | 0.13 | 0.13 | -1.01 | 0.07 | 0.07 | 0.00 | 0.04 | 0.04 |
| | Selected | 0.30 | 0.01 | 0.01 | 2.01 | 0.13 | 0.13 | -1.01 | 0.08 | 0.08 | 0.00 | 0.04 | 0.04 |
| MIS-REL | WTT | 0.13 | 0.00 | 0.17 | 0.99 | 0.07 | 1.01 | -0.44 | 0.03 | 0.56 | -0.11 | 0.02 | 0.11 |
| | $\text{TEPS}^{top}$ | 0.29 | 0.02 | 0.02 | 1.88 | 0.29 | 0.31 | -0.98 | 0.14 | 0.14 | 0.01 | 0.09 | 0.09 |
| | $\text{TEPS}^{20}$ | 0.29 | 0.02 | 0.02 | 1.88 | 0.29 | 0.31 | -0.97 | 0.14 | 0.14 | 0.01 | 0.09 | 0.09 |
| | $\text{TEPS}^{40}$ | 0.29 | 0.02 | 0.02 | 1.88 | 0.29 | 0.31 | -0.97 | 0.14 | 0.14 | 0.01 | 0.09 | 0.09 |
| | $\text{TEPS}^{60}$ | 0.29 | 0.02 | 0.02 | 1.87 | 0.28 | 0.31 | -0.96 | 0.13 | 0.14 | 0.01 | 0.09 | 0.09 |
| | $\text{TEPS}^{80}$ | 0.28 | 0.02 | 0.03 | 1.82 | 0.25 | 0.30 | -0.92 | 0.12 | 0.14 | 0.00 | 0.10 | 0.10 |
| | $\text{TEPS}^{all}$ | 0.24 | 0.01 | 0.06 | 1.64 | 0.21 | 0.42 | -0.77 | 0.08 | 0.24 | -0.07 | 0.10 | 0.12 |
| | Selected | 0.28 | 0.03 | 0.04 | 1.82 | 0.27 | 0.32 | -0.91 | 0.13 | 0.16 | -0.01 | 0.09 | 0.09 |

*Note:* The results are from the 100 Monte Carlo samples.

due to the fact that WTT uses the maximal (but possibly unreliable) information that one can infer from observed ROLs.

Second, Panel (b) shows the results from MIS-IRR in which some students make (almost) payoff-irrelevant mistakes. The WTT-based estimator is susceptible to strategic mistakes. For example, the WTT-based estimates have a mean 1.21 (standard deviation 0.08). That is, the WTT-based estimator is no longer consistent and the bias is sizable. On the other hand, the estimators based on TEPS are robust to payoff-irrelevant mistakes.

Finally, Panel (c) show the results from MIS-REL in which some students make payoff-relevant mistakes. Recall that when students make payoff-relevant mistakes, first, WTT is not satisfied and thus WTT-based estimator is inconsistent, and second, stability is also not 100% satisfied, and thus $\text{TEPS}^{all}$ which does not take care of payoff relevant mistakes is also inconsistent. However, the bias of $\text{TEPS}^{all}$ remains a lot smaller than that of the WTT-based estimator since the violation of WTT (74.4%) is much more severe than the violation of stability (3.8%) (see Table 2).

The bias-variance tradeoffs are summarized by the square root of the mean squared errors in Table

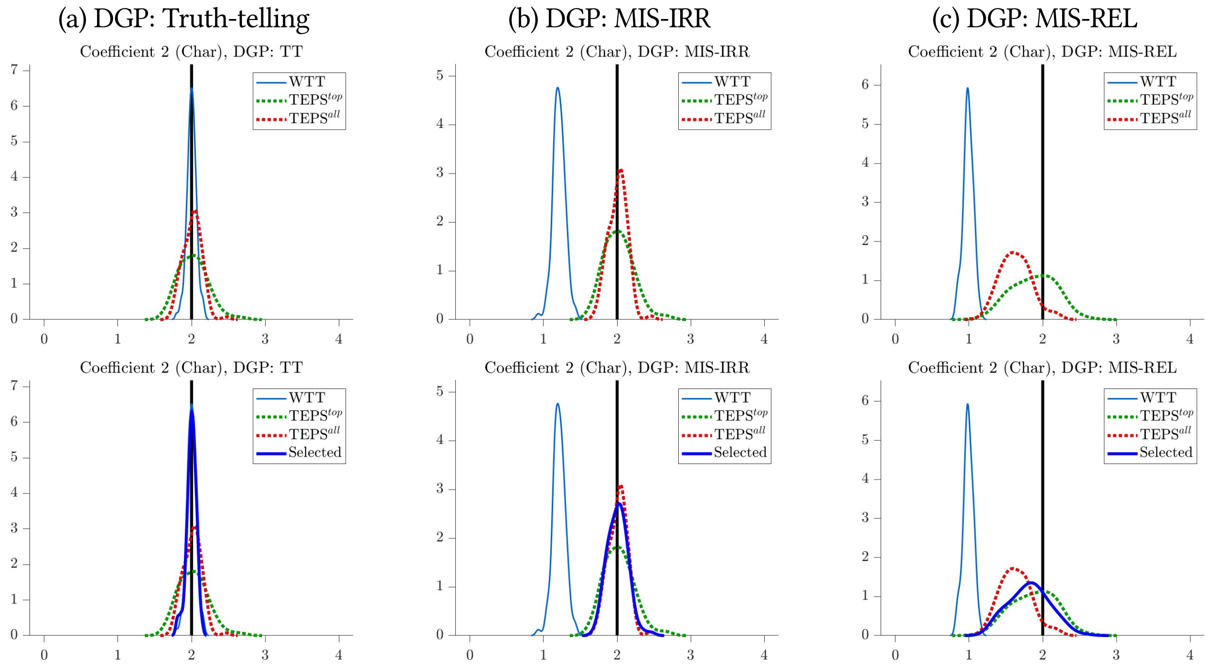


Figure D.3: Distribution of Estimates based on WTT and TEPS ($\beta_2 = 2$)

*Note:* We plot the kernel density plots of the estimates of $\beta_2$ from 100 Monte Carlo samples. The first column corresponds to when the data-generating process is TT, the second column corresponds to when the data-generating process is MIS-IRR, and the third column corresponds to when the data-generating process is MIS-REL. The black vertical line at 2 denotes the true value of the parameter.

D.2. In DGP TT, where all estimators are consistent, the WTT-based estimator attains the minimum mean squared error, as it has the minimum variance where the biases of all estimators are close to zero. However, in DGP MIS-IRR with payoff-irrelevant mistakes, the mean squared error of the WTT-based estimator is larger than that of any other estimator. This is due to the fact that even though the WTT-based estimator has the smallest variance, it is significantly inconsistent. The mean squared errors of the WTT-based estimator are even larger with DGP MIS-REL with payoff-relevant mistakes.

**Performance of the TEPS estimator.** We now compare the relative performance among TEPS-based estimators. As discussed earlier, to the extent that stability is satisfied, all TEPS-based estimators are consistent. However, the procedures by which the TEPS estimators are constructed imply that TEPS$^{top}$ uses less (but potentially more reliable) information contained in the observed ROLs compared to other TEPS-based estimators. Therefore, TEPS$^{\tau}$, $\tau = 20, 40, 60, 80, 100$, should have higher precision compared to TEPS$^{top}$. Similarly, TEPS$^{\tau}$ is expected to have higher precision compared to TEPS$^{\tau'}$ for all $\tau' < \tau$.

Figure D.3 shows the results, in which we only present TEPS$^{top}$ and TEPS$^{all}$ for clear comparison. TEPS$^{all}$ is more precise than TEPS$^{top}$, having a higher level of concentration around the true value $\beta_2 = 2$. As reported in Table D.2, TEPS$^{\tau}$, $\tau = 20, 40, 60, 80, 100$ always have (weakly) smaller standard deviations compared to TEPS$^{top}$ and TEPS$^{\tau'}$ for all $\tau' > \tau$. For example, in MIS-IRR in which approximately 74.4%

students make payoff-irrelevant mistakes, the standard deviation of TEPS$^{top}$ is 0.20, while that of TEPS$^{all}$ is 0.13.

Next, in DGP MIS-REL where students skip some schools with a positive admission probability and hence make payoff-relevant mistakes, the choice of TEPS threshold (i.e., how much payoff irrelevant mistakes are to be tolerated) becomes important. The bias of TEPS$^{all}$ is larger than TEPS$^{top}$, TEPS$^{20}$, TEPS$^{40}$, TEPS$^{60}$, and TEPS$^{80}$. For example, for $\beta_2 = 2$, the bias of TEPS$^{all}$ estimator is 0.36 where those of TEPS$^{top}$ is 0.12 on average (Table D.2). As expected, the bias decreases as we tolerate fewer payoff-relevant mistakes (i.e., as we decrease $\tau$ in TEPS$^{\tau}$).

Table D.3: Determining the Selected Estimator: Test Results (at the $5\%$ significance level)

| Data Generating Process: | | TT | MIS-IRR | MIS-REL |
|---|---|---|---|---|
| Estimation method | WTT | **0.94** | 0 | 0 |
| | TEPS$^{top}$ | 0.01 | 0 | 0 |
| | TEPS$^{20}$ | 0 | 0 | 0 |
| | TEPS$^{40}$ | 0 | 0 | 0.01 |
| | TEPS$^{60}$ | 0 | 0.01 | 0.14 |
| | TEPS$^{80}$ | 0.03 | 0.12 | **0.68** |
| | TEPS$^{all}$ | 0.01 | **0.87** | 0.17 |

*Note:* The results are from the 100 estimation samples.

**Choosing among the estimation methods.** Recall the main motivation that led us to introduce the TEPS-based estimator is to "correct" the strategic mistakes that students might make in a school choice environment in order to obtain robust estimates. Without the information on how students make mistakes, we follow the procedure described in Section 3.5 to determine the selected estimator that corrects (almost) all the mistakes (thus consistent) and uses the maximum information from the data (thus has the highest precision among all consistent estimators we consider.) Table D.3 reports the test results based on the testing procedure with a size equal to 0.05, and the resulting 'chosen' estimates are reported in the second row of Figure D.3 and 'Selected' rows of Table D.2. As expected, WTT is chosen 94% out of 100 estimation samples when the DGP is TT since all estimators are consistent while WTT uses the maximum information (Proposition 2). When the DGP is MIS-IRR, TEPS$^{all}$ is chosen 87%. Note that TEPS$^{top}$, TEPS$^{20}$, TEPS$^{40}$ are never chosen since all TEPS-based estimators are consistent, and TEPS$^{\tau}$'s with larger $\tau$ use more information relative to that with smaller $\tau$'s. Finally, when the DGP is MIS-REL, TEPS$^{80}$ is chosen 68%. Note that the testing procedure selects TEPS$^{\tau}$ with $\tau < 100$ most of the time since students make payoff-relevant mistakes in MIS-REL and TEPS$^{all}$ cannot handle payoff-relevant mistakes and thus is inconsistent.

**Accounting for Finite Market Uncertainty** The previously reported Monte Carlo simulation results do not consider finite market uncertainties. However, our TEPS procedure can naturally address this by adjusting its first step. We can calculate the cutoff distribution by bootstrapping from the observed ROL, creating various economy/applicant compositions. Then, using the original economy's applicant profile, we compute the distribution of feasible sets against the cutoff distribution. The subsequent steps (2 and 3) remain as described in the main text. Table D.4 presents results that are nearly identical to those shown in Table D.2.

Table D.4: Estimation with Different Identifying Assumptions: Monte Carlo Results with Bootstrapping

| DGPs | Identifying Condition | Quality ($\beta_1 = 0.3$) | | | Interaction ($\beta_2 = 2$) | | | Distance ($\beta_3 = -1$) | | | Small ($\beta_4 = 0$) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | mean | s.d. | $\sqrt{MSE}$ | mean | s.d. | $\sqrt{MSE}$ | mean | s.d. | $\sqrt{MSE}$ | mean | s.d. | $\sqrt{MSE}$ |
| TT | WTT | 0.30 | 0.00 | 0.00 | 2.00 | 0.06 | 0.06 | -1.00 | 0.03 | 0.03 | 0.00 | 0.02 | 0.02 |
| | TEPS$^{top}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.03 | 0.11 | 0.12 | 0.00 | 0.07 | 0.07 |
| | TEPS$^{20}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.03 | 0.11 | 0.12 | 0.00 | 0.07 | 0.07 |
| | TEPS$^{40}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.03 | 0.11 | 0.12 | 0.00 | 0.07 | 0.07 |
| | TEPS$^{60}$ | 0.30 | 0.02 | 0.02 | 2.03 | 0.20 | 0.20 | -1.02 | 0.11 | 0.11 | -0.01 | 0.07 | 0.07 |
| | TEPS$^{80}$ | 0.30 | 0.01 | 0.01 | 2.02 | 0.17 | 0.17 | -1.02 | 0.09 | 0.10 | 0.00 | 0.06 | 0.06 |
| | TEPS$^{all}$ | 0.30 | 0.01 | 0.01 | 2.01 | 0.12 | 0.12 | -1.01 | 0.07 | 0.07 | 0.00 | 0.04 | 0.04 |
| | Selected | 0.30 | 0.01 | 0.01 | 2.00 | 0.07 | 0.07 | -1.01 | 0.05 | 0.05 | 0.00 | 0.03 | 0.03 |
| MIS-IRR | WTT | 0.08 | 0.00 | 0.22 | 1.21 | 0.08 | 0.79 | -0.50 | 0.04 | 0.51 | -0.19 | 0.02 | 0.19 |
| | TEPS$^{top}$ | 0.30 | 0.02 | 0.02 | 2.03 | 0.20 | 0.20 | -1.03 | 0.11 | 0.11 | 0.00 | 0.07 | 0.07 |
| | TEPS$^{20}$ | 0.30 | 0.02 | 0.02 | 2.03 | 0.20 | 0.20 | -1.02 | 0.11 | 0.11 | 0.00 | 0.07 | 0.07 |
| | TEPS$^{40}$ | 0.30 | 0.02 | 0.02 | 2.02 | 0.20 | 0.20 | -1.02 | 0.11 | 0.11 | 0.00 | 0.07 | 0.07 |
| | TEPS$^{60}$ | 0.30 | 0.01 | 0.02 | 2.03 | 0.19 | 0.19 | -1.02 | 0.11 | 0.11 | 0.00 | 0.07 | 0.06 |
| | TEPS$^{80}$ | 0.30 | 0.01 | 0.01 | 2.02 | 0.17 | 0.17 | -1.02 | 0.09 | 0.09 | 0.00 | 0.06 | 0.06 |
| | TEPS$^{all}$ | 0.26 | 0.02 | 0.04 | 1.95 | 0.15 | 0.16 | -0.92 | 0.07 | 0.10 | 0.04 | 0.06 | 0.08 |
| | Selected | 0.30 | 0.01 | 0.01 | 2.00 | 0.16 | 0.16 | -1.00 | 0.09 | 0.09 | 0.00 | 0.06 | 0.06 |
| MIS-REL | WTT | 0.13 | 0.00 | 0.17 | 0.99 | 0.07 | 1.01 | -0.44 | 0.03 | 0.56 | -0.11 | 0.02 | 0.11 |
| | TEPS$^{top}$ | 0.29 | 0.02 | 0.02 | 1.88 | 0.29 | 0.31 | -0.98 | 0.14 | 0.14 | 0.01 | 0.09 | 0.09 |
| | TEPS$^{20}$ | 0.29 | 0.02 | 0.02 | 1.88 | 0.29 | 0.31 | -0.97 | 0.14 | 0.14 | 0.01 | 0.09 | 0.09 |
| | TEPS$^{40}$ | 0.29 | 0.02 | 0.02 | 1.88 | 0.28 | 0.31 | -0.97 | 0.14 | 0.14 | 0.01 | 0.09 | 0.09 |
| | TEPS$^{60}$ | 0.29 | 0.02 | 0.02 | 1.82 | 0.26 | 0.32 | -0.94 | 0.12 | 0.14 | 0.01 | 0.09 | 0.09 |
| | TEPS$^{80}$ | 0.27 | 0.02 | 0.03 | 1.74 | 0.23 | 0.35 | -0.87 | 0.11 | 0.17 | -0.01 | 0.10 | 0.10 |
| | TEPS$^{all}$ | 0.19 | 0.01 | 0.11 | 1.25 | 0.14 | 0.76 | -0.62 | 0.06 | 0.39 | -0.09 | 0.07 | 0.11 |
| | Selected | 0.28 | 0.02 | 0.03 | 1.79 | 0.25 | 0.32 | -0.92 | 0.12 | 0.15 | 0.00 | 0.10 | 0.10 |

*Note:* The results are from the 100 Monte Carlo samples.

# E Markov Chain Monte Carlo Procedure for Preference Estimation

## E.1 Setup

There are $k$ students competing for admissions to $C$ schools/programs. Each school $c$ has a type $\tau(c) \in \{1, \cdots, \bar{T}\}$ where $\bar{T} \leq C$. Denote the number of schools with type $\tau$ by $C_\tau$. WLOG, let schools be ordered in increasing order of type. Student $i$'s utility when being admitted to school $c$ is given by,

$$U_{i,c} = X_{i,c}\beta + \epsilon_{i,c}, \tag{E.5}$$

where $\epsilon_{i,c}$ is i.i.d. $N(0, \sigma^2_{\tau(c)})$.[A-15] Then $\Sigma \equiv Var(\epsilon_i)$ is a diagonal matrix with $(\sigma^2_1, \cdots, \sigma^2_1, \cdots, \sigma^2_{\bar{T}}, \cdots, \sigma^2_{\bar{T}})$ on the diagonal. We use $\Sigma$ wherever possible for notational simplicity.

Let $\mathcal{P}_i^{method}$ be the set of all preference relations inferred by some method (for example, WTT, TEPS$^{top}$, or TEPS$^{\tau}$ for some $\tau \in (0, 100]$) for student $i$. The procedure we describe below applies to any method for inferring preference relation from choice data. When there is no ambiguity, we simply use $\mathcal{P}_i$. Denote the schools that are inferred to be less preferred than $c$ as $\mathcal{L}_{i,c}$ and those are inferred to be preferred to $c$ as $\mathcal{M}_{i,c}$:

$$\mathcal{L}_{i,c} = \{c' : (c, c') \in \mathcal{P}_i \text{ i.e., } c' \text{ is revealed to be less preferred than } c\}$$
$$\mathcal{M}_{i,c} = \{c' : (c', c) \in \mathcal{P}_i \text{ i.e., } c' \text{ is revealed preferred to } c\}$$

## E.2 A Gibbs sampling procedure

We specify the following diffuse priors:

$$\beta \sim N(0, A^{-1})$$
$$\sigma^2_\tau \sim IW(\nu_\tau, V_{0\tau}), \ \ \tau = 1, 2, \cdots, \bar{T} - 1$$

where

$$A^{-1} = 100 \cdot I_{dim(\beta)},$$
$$\nu_\tau = 3 + C_\tau, \ \ V_{0\tau} = 3 + C_\tau, \ \ \forall \tau$$

We go through the following iterative process, a Gibbs sampler.

**Initialization.**

1. Draw $\Sigma^0$ from its prior distribution.
2. Draw $\beta^0$ from its prior distribution.
3. Draw $U^0$: Following the index of students, $i = 1, \ldots, k$, we draw $\{U^0_{i,c}\}_c$ sequentially as follows: for a given $i$,

   (a) start with $c = 1$. Draw $U^0_{i,1}$ from $N(X_{i,1}\beta^0, \Sigma^0_{1,1})$;
   (b) for each $c = 2, \ldots, C$, draw $U^0_{i,c}$ from $N(X_{i,c}\beta^0, \Sigma^0_{c,c})$ with truncations imposed by $\mathcal{P}_i$. To be specific, let

$$\widetilde{\mathcal{L}}_{i,c} = \{c' : c' < c \text{ and } (c, c') \in \mathcal{P}_i \text{ i.e., } c' \text{ is revealed to be less preferred than } c\} \subseteq \mathcal{L}_{i,c}$$

[A-15]Note that $\sigma^2_\tau$ for some $\tau$ has to be normalized to 1 for identification. WLOG, we set $\sigma^2_{\bar{T}} = 1$ for the following.

$$\widetilde{\mathcal{M}}_{i,c} = \{c' : c' < c \text{ and } (c', c) \in \mathcal{P}_i \text{ i.e., } c' \text{ is revealed preferred to } c\} \subseteq \mathcal{M}_{i,c}$$

That is, $\widetilde{\mathcal{L}}_{i,c}$ ($\widetilde{\mathcal{M}}_{i,c}$) is the set of schools whose utility is already drawn,[A-16] and at the same time less (more) preferred than school $c$ in $\mathcal{P}_i$ respectively. Then, we draw $U^0_{i,c}$ with truncations from below at $\underline{u}^0_c$, and from above at $\bar{u}^0_c$ where

$$\underline{u}^0_c = \begin{cases} \max\{U_{i,c'} : c' \in \widetilde{\mathcal{L}}_{i,c}\} & \text{if } \widetilde{\mathcal{L}}_{i,c} \neq \emptyset \\ -\infty & \text{if } \widetilde{\mathcal{L}}_{i,c} = \emptyset \end{cases}, \quad \bar{u}^0_c = \begin{cases} \min\{U_{i,c'} : c' \in \widetilde{\mathcal{M}}_{i,c}\} & \text{if } \widetilde{\mathcal{M}}_{i,c} \neq \emptyset \\ \infty & \text{if } \widetilde{\mathcal{M}}_{i,c} = \emptyset \end{cases}$$

Note that if both $\underline{u}^0_c$ and $\bar{u}^0_c$ are finite, we draw from a two-sided truncated distribution, and if only either one of them is finite, we draw from a one-sided truncated distribution, and if none of them is finite, we draw from the untruncated distribution.

**Iteration $r \geq 1$.**

1. Following the index of students, $i = 1, \ldots, k$, we draw $\{U^r_{i,c}\}_c$ sequentially as follows.
   For a given $i$, for $c = 1, \ldots, C$, draw $U^r_{i,c}$ from $N(X_{i,c}\beta^{r-1}, \Sigma^{r-1}_{c,c})$ with truncations imposed by $\mathcal{P}_i$. To be specific, we draw $U^r_{i,c}$ with truncations from below at $\underline{u}^r_c$, and from above at $\bar{u}^r_c$ where

$$\underline{u}^r_c = \begin{cases} \max\left\{\{U^r_{i,c'} : c' \in \widetilde{\mathcal{L}}_{i,c}\} \cup \{U^{r-1}_{i,c'} : c' \in \mathcal{L}_{i,c} \setminus \widetilde{\mathcal{L}}_{i,c}\}\right\} & \text{if } \mathcal{L}_{i,c} \neq \emptyset \\ -\infty & \text{if } \mathcal{L}_{i,c} = \emptyset \end{cases}$$

$$\bar{u}^r_c = \begin{cases} \min\left\{\{U^r_{i,c'} : c' \in \widetilde{\mathcal{M}}_{i,c}\} \cup \{U^{r-1}_{i,c'} : c' \in \mathcal{M}_{i,c} \setminus \widetilde{\mathcal{M}}_{i,c}\}\right\} & \text{if } \mathcal{M}_{i,c} \neq \emptyset \\ \infty & \text{if } \mathcal{M}_{i,c} = \emptyset \end{cases}$$

   where $\mathcal{L}_{i,c}, \widetilde{\mathcal{L}}_{i,c}, \mathcal{M}_{i,c}, \widetilde{\mathcal{M}}_{i,c}$ are defined above. In words, for example, if $c$ is revealed preferred to some school by $\mathcal{P}_i$ (i.e., $\mathcal{L}_{i,c} \neq \emptyset$), the lower bound $\underline{u}^r_c$ is given by the maximum among the $r$-th draws of utilities of schools that are less preferred than $c$ and are already drawn in the current step (i.e., $\{U^r_{i,c'} : c' \in \widetilde{\mathcal{L}}_{i,c}\}$) and the $(r-1)$-th draws of utilities of schools that are less preferred than $c$ but not drawn in the current step yet (i.e., $\{U^{r-1}_{i,c'} : c' \in \mathcal{L}_{i,c} \setminus \widetilde{\mathcal{L}}_{i,c}\}$.) $\bar{u}^r_c$ is defined analogously but with schools that are more preferred than $c$ by $\mathcal{P}_i$.
2. Draw $\beta^r$ from the posterior distribution $N(\tilde{\beta}, V)$[A-17] where

$$V = (X^{*\top} X^* + A)^{-1}$$
$$\tilde{\beta} = V X^{*\top} U^*$$

[A-16]Note that we draw $c = 1, \cdots, C$ sequentially so that $U^0_{i,c'}, c' < c$ is already drawn in this step when drawing $U^0_{i,c}$.
[A-17]Note that $\tilde{\beta}$ is specific to the $r$-th iteration where we omit the dependence on $r$ for notational simplicity.

$$
\begin{aligned}
(\Sigma^{r-1})^{-1} &= \Lambda^\top \Lambda \\
X_i^* &= \Lambda^\top X_i \\
U_i^* &= \Lambda^\top U_i^r \\
X &= \begin{bmatrix} X_1 \\ \vdots \\ X_k \end{bmatrix}, \quad U^r = \begin{bmatrix} U_1^r \\ \vdots \\ U_k^r \end{bmatrix}
\end{aligned}
$$

and $X_i$ is $C$ by $dim(\beta)$ matrix and $U_i \in \mathbb{R}^C$.

This is a standard result for Bayesian regression with normal errors.

3. For each $\tau = 1, \cdots, \bar{T} - 1$, draw $(\sigma_\tau^2)^r$ from the posterior distributions $IW(\nu_\tau + kC(\tau), V_{0\tau} + S_\tau)$ where

$$
\begin{aligned}
S_\tau &= \sum_{c:\tau(c)=\tau} \sum_{i=1}^{k} \varepsilon_{i,c}^r (\varepsilon_{i,c}^r)^\top \\
\varepsilon_{i,c}^r &= U_{i,c}^r - X_{i,c}\beta^r
\end{aligned}
$$

where $\varepsilon_{i,c}^r$, $U_{i,c}^r$ and $X_{i,c}$ denote the part of $\varepsilon_i^r$, $U_i^r$ and $X_i$ corresponding to school $c$.

4. Save and pass $(\beta^r, \Sigma^r, U^r)$ to the next iteration.

# F Data on NYC High School Choice

## F.1 Institutional Background

NYC public high school system consists of two sectors: specialized high schools and regular high schools. There are nine specialized high schools in NYC.[A-18] We do not consider nine specialized high schools in our analysis because they use different admission methods from the regular high schools, and students submit a separate ROL of specialized high schools.

Regular high schools are traditional public schools and have six types of programs differentiated in their admission method: *Unscreened*, *Limited unscreened*, *Screened*, *Audition*, *Educational option*, and *Zoned* programs. Multiple programs of different types may be offered by a single school. *Unscreened* programs admit students by a random lottery number attached to each student. *Limited unscreened* programs operate similarly as *Unscreened* programs but give higher priority to students who attended an information session

[A-18]They are Stuyvesant High School; Brooklyn Technical High School; Bronx High School of Science; High School of American Studies at Lehman College; The Brooklyn Latin School; High School for Mathematics, Science and Engineering at City College; Queens High School for the Sciences at York College; Staten Island Technical High School; and Fiorello H. LaGuardia High School of Music & Art and Performing Arts. These schools, except for Fiorello H. LaGuardia High School, use Specialized High School Admission Test (SHSAT) as the sole criterion of admission, which is a required exam for students wanting to attend any of the specialized high schools. Fiorello H. LaGuardia High School uses audition as its admission criterion.

or open houses. *Screened* programs as well as *Audition* programs rank students by individual assortment of criteria. For example, *Screened* programs use several criteria such as final report-card grade, statewide standardized test scores, and attendance and punctuality. *Audition* programs hold school/program-specific auditions to admit students. *Educational option* is a mixture of unscreened and screened programs. They have the purpose of serving students at diverse academic performance levels and divide students into high (the highest 16%), middle (the middle 68%), and low (the lowest 16%) levels in terms of English Language Arts (ELA) scores. 50% of the seats in each group are filled using school-specific criteria similarly as a *Screened* program, and the other 50% are filled randomly similarly as an *Unscreened* program. *Zoned* programs give priority or guarantee admission to students who apply and live in the zoned area of the school.

Each program has its own eligibility and priority group criteria. For example, in the academic year 2016–17, Young Women's Leadership School in Astoria opened its seats only to female students, i.e., being a female student was the eligibility criterion. Besides, they gave the highest admission priority to continuing eighth graders, then to students or residents in Queens, and then to other NYC residents.

The number of priority groups is a lot smaller than the number of applicants to each program. Hence students who apply to programs that do not actively rank students —*Unscreened*, *Limited Unscreened*, *Zoned* and the unscreened part of *Educational option*—are often in the same priority group. Hence the ties need to be broken for the SPDA algorithm to run. For this purpose, a random lottery number is drawn and attached to each student, which is used to break ties at all programs that require tie-breaking in the same fashion—single tie-breaking (STB) rule. The lottery number is unknown to the student at the moment of application.

The timeline of the admission process is as follows (Corcoran and Levin (2011)). In October and November, students may apply to specialized high schools for which they should take SHSAT or audition at LaGuardia High School. In December, they are required to submit up to 12 ranked non-specialized high school programs regardless of their application status to specialized high schools. In March, the SPDA algorithms for specialized high schools (SHSAT takers) and non-specialized high schools (all students) are separately run, which is called Round 1. The Department of Education sends each student a letter with an offer from regular schools and an offer from specialized high schools, if any. If a student receives offers from both regular and specialized high schools, he/she must choose one. All students who accept a Round 1 offer have a finalized admission decision. If a student did not submit an application in Round 1, did not receive an offer in Round 1, or wants to apply to a program with availability, he/she can participate in Round 2. Round 2 takes place in March and operates with students who submitted Round 2 applications and programs that still have seat availability. Round 2 offers automatically replace Round 1 offers, if any. Students who are unassigned in Rounds 1 and 2 or reject the assignment go to the administrative round in which students are administratively assigned a school on a case-by-case basis.

## F.2 Data Source

The main data that we use is the administrative data from the New York City Department of Education (NYC DOE) for the academic year 2016–17. There are four sets of data used to construct information on the applicants. First, high school application (HSAP) data contains the submitted ROLs and student information such as ELA and math standardized test scores, English-language Learner (ELL) status, and student priorities at programs (including priority rank, priority criteria, and eligibility). Second, June biographic data provides comprehensive student biographic information, including ethnicity, gender, disability status, as well as information on attendance and punctuality. Third, standardized test data contains more detailed information on statewide standardized exams. Fourth, zoned DBN data provides information on the zoned school of each student, the census tract, and the school district of each student's residence. Finally, Middle School Course and Grade data contains all of the courses and information on credits and grades for each student in a given year. There exists a unique scrambled student ID variable that enables merging all NYC DOE datasets while personally identifying a student is impossible. Lastly, we use the information on each census tract and zip code in NYC obtained from the 2016 American Community Survey 5-Year Estimates from the US Census database.

School information is constructed using the NYC High School Directory which is published every year before the application process starts. This includes each program's capacity in the previous year, the number of students who applied in the previous year, eligibility and priority criteria, accountability data such as progress reports, graduation rate and college enrollment rate, and types of language classes provided. Other variables about the current 9th graders, such as ethnicity composition and the fraction of high-performing students, are constructed using the high school application data from the previous year, the academic year 2015–2016.

## F.3 Sample Restrictions

We focus on students from Staten Island. There are two different samples—one for tracing out the uncertainties, the other for estimation of student preferences: *Priority Construction* sample and *Estimation* sample. They differ because of missing values in some variables. *Priority Construction* sample is used to reconstruct the priority scores of each student at each school/program. *Estimation* sample is a strict subset of *Priority Construction* sample and is used for preference estimation and counterfactual simulations.

First, *Priority Construction* sample consists of students who applied to at least one Staten Island school/program. Among such 4,824 students, 785 did not have information on variables needed to reconstruct priority and were dropped leaving us 4,039 students. Next, *Estimation* sample consists of students who applied to at least one Staten Island school/program, went to a middle school in Staten Island, and resided in Staten Island at the point of application. Among such 4,480 students, 741 did not have information on variables needed for priority reconstruction or estimation and were dropped. Finally, 8 students had invalid ROLs such as containing an invalid program code on their ROLs, and were dropped, leaving us 3,731 students.

Finally, we adjust the capacities of each school/program whenever we restrict our sample. Specifically, we treat the Round 1 assignment in the data (or the Round 2 assignment if applicable) of each dropped student as fixed whenever we resimulate DA using the restricted sample. This is in order to ensure that we do not overestimate the probability of school/programs being feasible for each student in our TEPS procedure.

# G TEPS for NYC High School Choice

## G.1 Constructing Priority Scores

Before describing how to simulate uncertainty, let us first specify the procedure we take to prepare ingredients for the procedure. The main inputs for the SPDA algorithm are the capacities of programs, students' preferences, and programs' preferences (priority scores). First, we use adjusted programs' capacities as described in Appendix F. Next, we use students' submitted ROL in the data as students' preferences.

School preferences are a bit more involved than the other two inputs. As described in Appendix F, NYC public high schools have coarse priority rules. First, for eligibility criteria and admission priority groups which are publicly available before the admission process starts, we use the information listed on the High School Directory.

Next, we estimate the priority ranks for *Screened*, *Audition*, and the screened part of *Educational option* programs that actively rank students. While there is information on the priority rank of students in the data set provided by NYC DOE, it is limited only to students who ranked that program. Furthermore, how each program ranks its applicants is not public information, and each program has its individual assortment of criteria. For our purposes to calculate the probability of each program being feasible to a student regardless of whether she ranked it or not, we need to construct a priority rank for all students at each program that actively ranks students. To do so, we assume that there exists a program-specific latent variable $v_{ij}$ for student $i$ at actively ranking program $j$ which determines the priority ranks:

$$v_{ij} = \beta_j X_i + \varepsilon_{ij} \quad \text{and} \quad i \succ_j i' \text{ if and only if } v_{ij} > v_{i'j},$$

where $X_i$ is a vector of student characteristics including 7th Standardized Math and ELA scores, middle school Math, Social Studies, English, Science GPA, days absent and days late, and $\varepsilon_{ij}$ is independent and identically distributed as extreme value type I conditional on $X_i$. We form a log-likelihood by considering all possible pairs of applicants to each program $j$ and estimate via MLE separately for each program. That is for $\mathcal{I}_j$, the set of applicants to program $j$,

$$\hat{\beta}_j = \arg\max_{\beta_j} l(\beta_j) \equiv \sum_{i > i' : i, i' \in \mathcal{I}_j} \log \left( \frac{\exp(v_{ij}) 1\{i \succ_j i'\} + \exp(v_{i'j}) 1\{i' \succ_j i\}}{\exp(v_{ij}) + \exp(v_{i'j})} \right)$$

With estimates $\hat{\beta}_j$, we predict $\hat{v}_{ij} = \hat{\beta}_j X_i$ for all students (not limited to applicants) and reconstruct priority ranks based on $\hat{v}_{ij}$. The average rank correlation between reconstructed and actual priority ranks at programs ranked by students is 0.72.

### G.2 Simulation of Uncertainties

We describe how we simulate uncertainties present in the matching environment, the first step of TEPS. After reconstructing priority scores for each student at each program, we simulate $B_L = 10,000$ lotteries from a uniform distribution to break ties at non-actively ranking programs and run the SPDA 10,000 times. This procedure would give us an empirical distribution of cutoffs of all programs $P = (P_1, \cdots, P_C)$ where the cutoff of a program is defined to be the lowest priority score of admitted students if the seats are filled, and zero if the seats are not filled.

Next, we draw $L$ number of lotteries ($L = 5,000$) from a uniform distribution in order to account for the fact that a student's own ex-post score is uncertain due to the random tie-breaking rules. For each lottery draw, we use the cutoff distribution simulated above to figure out the set of feasible schools and its probability for each student in each realization of uncertainties. Together with the submitted ROLs, we can also compute the assigned program in each realization of uncertainties.

## H Analysis of the NYC High School Choice Data

### H.1 Preference Estimates

Tables H.6–H.9 present preference estimates for each covariate cell. We report the mean and standard deviation of the posterior distribution as the point estimate and the standard error. We iterate through the MCMC 1 million times and discard the first 90% to ensure mixing. We calculate the Potential Scale Reduction Factor (PSRF) using the draws that we keep following Gelman and Rubin (1992). For those that did not converge, we additionally iterate 1 million times and keep the last 0.1 million. The resulting PSRFs for all parameters for all cells are below 1.1 which ensures convergence.

### H.2 Counterfactual Analysis

In Table H.5, we report the full table of the mean and standard deviations of the counterfactual policy predictions reported in Figure 5. In particular, for TEPS$^{top}$-based and the selected estimates, we calculate the $p$-value for a two-sided mean comparison t-test which tests the null hypothesis that the prediction based on each method is equal to that based on WTT. Due to the high precisions of the predictions (as demonstrated by the low standard deviations), we largely confirm that the WTT-based estimates result in a systematic underprediction of the effects of desegregation policies compared to our TEPS-based estimates.

Table H.5: Racial Gap in Characteristics of Assigned Programs

| | Data | No Screening | No Zoning | No Priority |
|---|---|---|---|---|
| *Panel A: % Black/Hispanic* | | | | |
| WTT | -25.96 | -25.20 | -26.07 | -24.64 |
| | | (0.09) | (0.15) | (0.10) |
| TEPS$^{top}$ | | -24.70 | -25.17 | -23.25 |
| | | (0.53) | (0.55) | (0.66) |
| $p$-value v.s. WTT | | $<$0.001 | $<$0.001 | $<$0.001 |
| Selected | | -24.82 | -25.92 | -24.06 |
| | | (0.39) | (0.30) | (0.43) |
| $p$-value v.s. WTT | | $<$0.001 | $<$0.001 | $<$0.001 |
| *Panel B: % FRPL* | | | | |
| WTT | -17.20 | -16.52 | -17.35 | -16.15 |
| | | (0.05) | (0.10) | (0.06) |
| TEPS$^{top}$ | | -16.41 | -16.92 | -15.58 |
| | | (0.37) | (0.35) | (0.45) |
| $p$-value v.s. WTT | | $<$0.001 | $<$0.001 | $<$0.001 |
| Selected | | -16.50 | -17.45 | -16.05 |
| | | (0.27) | (0.19) | (0.29) |
| $p$-value v.s. WTT | | 0.303 | $<$0.001 | $<$0.001 |
| *Panel C: % High Performer* | | | | |
| WTT | 19.53 | 17.90 | 19.37 | 17.69 |
| | | (0.07) | (0.12) | (0.07) |
| TEPS$^{top}$ | | 16.92 | 18.88 | 16.39 |
| | | (0.69) | (0.33) | (0.74) |
| $p$-value v.s. WTT | | $<$0.001 | $<$0.001 | $<$0.001 |
| Selected | | 17.04 | 19.40 | 16.79 |
| | | (0.60) | (0.19) | (0.60) |
| $p$-value v.s. WTT | | $<$0.001 | 0.059 | $<$0.001 |

*Note:* We sample 200 draws from the posterior distribution of each parameter and, for each draw, draw 200 sets of lotteries and run DA $200 \times 200 = 40,000$ times. For each simulation, we calculate three observable characteristics of the assigned program—proportion of Black and Hispanic students, proportion of FRPL students, and average 7th-grade standardized test score for each student in each racial group. We then calculate the mean across all students in each racial group for each preference estimate draw. The mean and standard deviations across the preference estimate draws are reported. For TEPS$^{top}$ and the selected estimates, we calculate the $p$-value for a two-sided mean comparison t-test testing the null hypothesis that the prediction based on each method is equal to that based on WTT.

## H.3 Additional Figures

### H.3.1 Program Characteristics by Feasibility Status

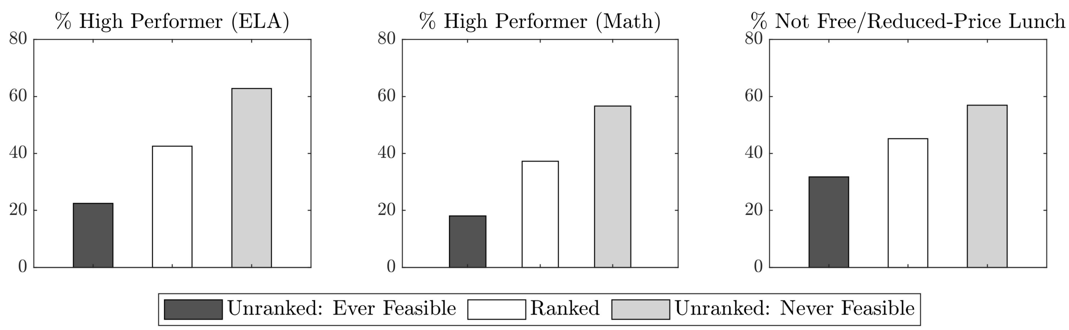


Figure H.4: Characteristics of Ranked and Unranked Programs by Feasibility Status

Notes: For each student, we classify the programs into three types—ever-feasible-unranked, never-feasible-unranked, and ranked. Ranked programs are those included in the student's ROL and Stages 1 and 2 of the TEPS procedure determine the feasibility of each unranked program. We use the fraction of high-performing students (measured by ELA and math scores) and the fraction of those who are not eligible for free/reduced-price lunch in each program. The figure reports the average across all students for each type of program.

## Table H.6: Preference Estimates: Cell 1–2

| | Cell 1 | | | | | | | | Cell 2 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | WTT | | $TEPS^{all}$ | | $TEPS^{top}$ | | Selected | | WTT | | $TEPS^{all}$ | | $TEPS^{top}$ | | Selected | |
| | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. |
| School FE 4 | -0.26 | 0.05 | -0.45 | 0.08 | -0.86 | 0.11 | -0.55 | 0.08 | 0.11 | 0.05 | -0.12 | 0.08 | -0.38 | 0.13 | -0.12 | 0.08 |
| School FE 5 | -0.65 | 0.08 | -1.05 | 0.12 | -1.45 | 0.17 | -1.18 | 0.13 | -0.45 | 0.09 | -1.3 | 0.15 | -1.55 | 0.21 | -1.3 | 0.15 |
| School FE 6 | -0.24 | 0.08 | -0.48 | 0.13 | -0.64 | 0.18 | -0.53 | 0.14 | 0.18 | 0.09 | -0.44 | 0.15 | -0.61 | 0.22 | -0.44 | 0.15 |
| School FE 7 | 0 | 0.09 | -0.7 | 0.15 | -0.87 | 0.21 | -0.73 | 0.16 | -0.4 | 0.1 | -1.43 | 0.18 | -1.58 | 0.25 | -1.43 | 0.18 |
| School FE 8 | -0.29 | 0.06 | -0.39 | 0.09 | -0.47 | 0.12 | -0.43 | 0.09 | -0.2 | 0.06 | -0.48 | 0.09 | -0.58 | 0.13 | -0.48 | 0.09 |
| School FE 9 | -0.03 | 0.08 | -0.27 | 0.13 | -0.59 | 0.18 | -0.39 | 0.14 | -0.69 | 0.11 | -1.41 | 0.21 | -1.51 | 0.3 | -1.41 | 0.21 |
| $\mathbb{1}$(STEM) | | | | | | | | | | | | | | | | |
| Main Effect | 0.22 | 0.06 | 0.33 | 0.11 | 0.44 | 0.16 | 0.3 | 0.13 | -0.09 | 0.07 | 0.07 | 0.14 | -0.26 | 0.25 | 0.07 | 0.14 |
| ×ELA Score | -0.22 | 0.05 | -0.19 | 0.08 | -0.14 | 0.1 | -0.14 | 0.08 | -0.06 | 0.06 | -0.13 | 0.1 | -0.2 | 0.15 | -0.13 | 0.1 |
| ×Math Score | 0.27 | 0.06 | 0.24 | 0.09 | 0.29 | 0.12 | 0.21 | 0.1 | 0.16 | 0.06 | 0.08 | 0.1 | 0.4 | 0.16 | 0.08 | 0.1 |
| % High Perf. (ELA) | | | | | | | | | | | | | | | | |
| Main Effect | -0.13 | 0.15 | -0.86 | 0.23 | -1.07 | 0.34 | -0.73 | 0.26 | 0.16 | 0.16 | -0.37 | 0.25 | -0.48 | 0.35 | -0.37 | 0.25 |
| ×ELA Score | 0.32 | 0.07 | 0.4 | 0.15 | 0.49 | 0.2 | 0.38 | 0.16 | 0.42 | 0.08 | 0.35 | 0.18 | 0.56 | 0.25 | 0.35 | 0.18 |
| % High Perf. (Math) | | | | | | | | | | | | | | | | |
| Main Effect | 0.46 | 0.2 | 1.49 | 0.33 | 1.58 | 0.51 | 1.36 | 0.36 | 0.7 | 0.22 | 2.31 | 0.4 | 2.96 | 0.6 | 2.31 | 0.4 |
| ×Math Score | 0.91 | 0.08 | 0.71 | 0.17 | 0.84 | 0.25 | 0.77 | 0.19 | 1 | 0.09 | 1.12 | 0.21 | 0.7 | 0.27 | 1.12 | 0.21 |
| % FRPL (program) | | | | | | | | | | | | | | | | |
| Main Effect | 1.07 | 0.18 | -0.59 | 0.35 | -0.23 | 0.52 | -0.64 | 0.39 | 0.08 | 0.2 | -2 | 0.37 | -1.89 | 0.52 | -2 | 0.37 |
| ×Med Income | -0.06 | 0.15 | 0.17 | 0.28 | 0.17 | 0.4 | 0.32 | 0.31 | 0.06 | 0.15 | -0.03 | 0.3 | -0.23 | 0.42 | -0.03 | 0.3 |
| ×Avg Score | -0.62 | 0.14 | -0.22 | 0.28 | -0.03 | 0.44 | -0.18 | 0.31 | 0.04 | 0.14 | 0.19 | 0.32 | 0.04 | 0.46 | 0.19 | 0.32 |
| % FRPL (school) | | | | | | | | | | | | | | | | |
| ×Med Income | 0.08 | 0.22 | 0.35 | 0.35 | 0.44 | 0.48 | 0.25 | 0.39 | 0.58 | 0.24 | 0.53 | 0.4 | 1.05 | 0.56 | 0.53 | 0.4 |
| ×Avg Score | 0.39 | 0.18 | -0.42 | 0.32 | -0.83 | 0.45 | -0.38 | 0.34 | -0.43 | 0.2 | -0.92 | 0.39 | -1.03 | 0.51 | -0.92 | 0.39 |
| 9th Grade Size (100s) | 0.16 | 0.01 | 0.19 | 0.02 | 0.23 | 0.02 | 0.19 | 0.02 | 0.17 | 0.01 | 0.25 | 0.02 | 0.29 | 0.03 | 0.25 | 0.02 |
| % Asian | 0.68 | 0.31 | 0.75 | 0.51 | 0.64 | 0.71 | 0.75 | 0.55 | -2.27 | 0.35 | -3.84 | 0.63 | -3.5 | 0.89 | -3.84 | 0.63 |
| % Black | -1.25 | 0.21 | -0.62 | 0.33 | -0.49 | 0.46 | -0.6 | 0.36 | -0.81 | 0.23 | 1.17 | 0.41 | 1.13 | 0.62 | 1.17 | 0.41 |
| % Hispanic | -0.75 | 0.21 | -0.66 | 0.33 | -0.71 | 0.45 | -0.51 | 0.36 | -1.05 | 0.22 | 0.25 | 0.38 | 0.68 | 0.53 | 0.25 | 0.38 |
| 1(Nearest School) | 0.28 | 0.04 | 0.26 | 0.07 | 0.26 | 0.09 | 0.27 | 0.07 | 0.2 | 0.05 | 0.26 | 0.07 | 0.24 | 0.11 | 0.26 | 0.07 |
| Distance | -0.26 | 0.01 | -0.3 | 0.02 | -0.3 | 0.02 | -0.3 | 0.02 | -0.34 | 0.01 | -0.35 | 0.02 | -0.42 | 0.04 | -0.35 | 0.02 |
| $\sigma^2_{STEM}$ | 0.86 | 0.07 | 0.75 | 0.11 | 0.73 | 0.15 | 0.85 | 0.15 | 0.99 | 0.08 | 1.06 | 0.17 | 1.53 | 0.34 | 1.06 | 0.17 |

Notes: We report the full estimates of the parameters in (1). All % variables are within $[0, 1]$. For conciseness, we report only the estimates based on WTT, $TEPS^{all}$, $TEPS^{top}$, and the selected estimates obtained by following the procedure described in Section 3.5. We report the mean and standard deviation of the posterior distribution as the point estimate and the standard error.

## Table H.7: Preference Estimates: Cell 3–4

| | Cell 3 | | | | | | | | Cell 4 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | WTT | | $\text{TEPS}^{all}$ | | $\text{TEPS}^{top}$ | | Selected | | WTT | | $\text{TEPS}^{all}$ | | $\text{TEPS}^{top}$ | | Selected | |
| | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. |
| School FE 4 | -0.35 | 0.06 | -0.44 | 0.08 | -0.55 | 0.13 | -0.49 | 0.1 | -0.01 | 0.06 | -0.12 | 0.08 | -0.42 | 0.14 | -0.39 | 0.13 |
| School FE 5 | -0.53 | 0.09 | -1.06 | 0.13 | -1.18 | 0.19 | -1.21 | 0.16 | -0.42 | 0.09 | -1.17 | 0.14 | -1.36 | 0.21 | -1.38 | 0.2 |
| School FE 6 | -0.5 | 0.09 | -0.61 | 0.14 | -0.72 | 0.2 | -0.8 | 0.17 | -0.08 | 0.09 | -0.72 | 0.15 | -0.86 | 0.22 | -0.95 | 0.22 |
| School FE 7 | 0.18 | 0.11 | -0.47 | 0.17 | -0.43 | 0.24 | -0.51 | 0.21 | -0.27 | 0.11 | -1.03 | 0.2 | -1.3 | 0.29 | -1.19 | 0.27 |
| School FE 8 | -0.34 | 0.07 | -0.42 | 0.1 | -0.38 | 0.14 | -0.44 | 0.12 | -0.15 | 0.06 | -0.54 | 0.1 | -0.58 | 0.15 | -0.62 | 0.15 |
| School FE 9 | -0.42 | 0.09 | -0.57 | 0.14 | -0.53 | 0.2 | -0.64 | 0.17 | -0.76 | 0.11 | -1.63 | 0.21 | -1.7 | 0.31 | -1.79 | 0.3 |
| $\mathbb{1}$(STEM) | | | | | | | | | | | | | | | | |
| Main Effect | 0.19 | 0.09 | 0.27 | 0.14 | -0.02 | 0.28 | -0.07 | 0.24 | -0.26 | 0.1 | -0.29 | 0.2 | -0.54 | 0.35 | -0.41 | 0.32 |
| ×ELA Score | -0.04 | 0.07 | 0.01 | 0.09 | 0.09 | 0.15 | 0.13 | 0.13 | -0.02 | 0.08 | -0.01 | 0.14 | -0.22 | 0.19 | -0.1 | 0.18 |
| ×Math Score | 0.13 | 0.07 | 0.01 | 0.1 | 0.07 | 0.16 | -0.05 | 0.13 | 0.1 | 0.08 | -0.04 | 0.13 | 0.38 | 0.2 | 0.12 | 0.18 |
| % High Perf. (ELA) | | | | | | | | | | | | | | | | |
| Main Effect | 0.21 | 0.17 | -0.26 | 0.26 | -0.2 | 0.36 | 0 | 0.3 | 0.13 | 0.17 | -0.2 | 0.26 | -0.47 | 0.38 | -0.26 | 0.35 |
| ×ELA Score | 0.43 | 0.09 | 0.69 | 0.17 | 0.76 | 0.24 | 0.72 | 0.21 | 0.31 | 0.09 | 0.14 | 0.19 | 0.41 | 0.27 | 0.3 | 0.25 |
| % High Perf. (Math) | | | | | | | | | | | | | | | | |
| Main Effect | -0.34 | 0.24 | 0.65 | 0.37 | 0.65 | 0.56 | 0.69 | 0.44 | 0.8 | 0.23 | 2.27 | 0.41 | 3.12 | 0.63 | 2.95 | 0.58 |
| ×Math Score | 0.79 | 0.09 | 0.73 | 0.18 | 0.97 | 0.27 | 0.92 | 0.23 | 0.59 | 0.09 | 0.9 | 0.21 | 0.23 | 0.29 | 0.43 | 0.28 |
| % FRPL (program) | | | | | | | | | | | | | | | | |
| Main Effect | 1.17 | 0.21 | -0.39 | 0.35 | -1.13 | 0.48 | -0.66 | 0.43 | 0.29 | 0.2 | -0.94 | 0.38 | -1.01 | 0.55 | -0.52 | 0.52 |
| ×Med Income | 0 | 0.16 | 0.14 | 0.27 | 0.17 | 0.39 | 0.1 | 0.32 | 0.03 | 0.14 | -0.04 | 0.29 | 0.04 | 0.4 | 0.03 | 0.4 |
| ×Avg Score | -0.25 | 0.15 | 0.08 | 0.27 | 0.08 | 0.37 | 0.19 | 0.31 | -0.15 | 0.15 | 0.05 | 0.33 | -0.23 | 0.49 | -0.5 | 0.48 |
| % FRPL (school) | | | | | | | | | | | | | | | | |
| ×Med Income | -0.4 | 0.27 | -0.38 | 0.37 | -0.32 | 0.52 | -0.29 | 0.43 | 0.06 | 0.24 | 0.17 | 0.36 | 0.27 | 0.53 | -0.03 | 0.57 |
| ×Avg Score | 0.29 | 0.21 | -0.3 | 0.31 | -0.06 | 0.43 | -0.37 | 0.38 | 0.12 | 0.21 | 0.01 | 0.4 | -0.27 | 0.59 | 0.24 | 0.56 |
| 9th Grade Size (100s) | 0.14 | 0.01 | 0.19 | 0.02 | 0.25 | 0.03 | 0.23 | 0.03 | 0.15 | 0.01 | 0.23 | 0.02 | 0.29 | 0.04 | 0.28 | 0.03 |
| % Asian | 2.3 | 0.38 | 1.7 | 0.6 | 2.41 | 0.87 | 1.88 | 0.72 | -1.31 | 0.38 | -3.27 | 0.67 | -2.99 | 0.95 | -3.36 | 0.9 |
| % Black | -1.29 | 0.26 | -0.59 | 0.38 | 0.08 | 0.59 | 0.03 | 0.47 | -0.48 | 0.26 | 1.18 | 0.43 | 1.85 | 0.64 | 1.66 | 0.62 |
| % Hispanic | -0.7 | 0.25 | -0.08 | 0.38 | 0.73 | 0.54 | 0.46 | 0.45 | -0.51 | 0.25 | 0.45 | 0.4 | 0.71 | 0.56 | 0.91 | 0.55 |
| 1(Nearest School) | -0.01 | 0.06 | 0.06 | 0.08 | 0.01 | 0.11 | 0.03 | 0.09 | 0.26 | 0.06 | 0.22 | 0.08 | 0.22 | 0.12 | 0.3 | 0.11 |
| Distance | -0.31 | 0.02 | -0.33 | 0.02 | -0.35 | 0.03 | -0.35 | 0.03 | -0.26 | 0.01 | -0.29 | 0.02 | -0.3 | 0.03 | -0.28 | 0.03 |
| $\sigma^2_{STEM}$ | 1.17 | 0.12 | 0.97 | 0.17 | 1.28 | 0.36 | 1.42 | 0.33 | 1.21 | 0.14 | 1.53 | 0.3 | 1.66 | 0.47 | 1.7 | 0.47 |

Notes: We report the full estimates of the parameters in (1). All % variables are within $[0, 1]$. For conciseness, we report only the estimates based on WTT, $\text{TEPS}^{all}$, $\text{TEPS}^{top}$, and the selected estimates obtained by following the procedure described in Section 3.5. We report the mean and standard deviation of the posterior distribution as the point estimate and the standard error.

Table H.8: Preference Estimates: Cell 5–6

| | Cell 5 | | | | | | | | Cell 6 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | WTT | | $TEPS^{all}$ | | $TEPS^{top}$ | | Selected | | WTT | | $TEPS^{all}$ | | $TEPS^{top}$ | | Selected | |
| | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. |
| School FE 4 | -0.29 | 0.1 | -0.5 | 0.12 | -0.5 | 0.24 | -0.64 | 0.13 | -0.13 | 0.11 | -0.23 | 0.15 | -0.41 | 0.25 | -0.32 | 0.16 |
| School FE 5 | -0.8 | 0.12 | -1.47 | 0.19 | -1.72 | 0.33 | -1.61 | 0.21 | -0.58 | 0.14 | -1.19 | 0.24 | -0.98 | 0.34 | -1.2 | 0.25 |
| School FE 6 | -0.45 | 0.12 | -0.83 | 0.19 | -0.54 | 0.33 | -0.85 | 0.2 | -0.07 | 0.14 | -0.56 | 0.23 | -0.22 | 0.37 | -0.54 | 0.24 |
| School FE 7 | 0.22 | 0.18 | -0.62 | 0.29 | -1.37 | 0.55 | -0.74 | 0.31 | 0.14 | 0.2 | -0.55 | 0.37 | -1.16 | 0.53 | -0.62 | 0.39 |
| School FE 8 | -0.3 | 0.1 | -0.44 | 0.15 | -0.33 | 0.26 | -0.48 | 0.16 | -0.07 | 0.11 | -0.48 | 0.19 | -0.7 | 0.29 | -0.56 | 0.2 |
| School FE 9 | -0.3 | 0.12 | -0.64 | 0.17 | -0.78 | 0.33 | -0.8 | 0.19 | -0.85 | 0.16 | -1.22 | 0.27 | -0.88 | 0.39 | -1.14 | 0.27 |
| $\mathbb{1}$(STEM) | | | | | | | | | | | | | | | | |
| Main Effect | 0.28 | 0.11 | 0.46 | 0.18 | 0.02 | 0.43 | 0.41 | 0.22 | -0.15 | 0.16 | -0.48 | 0.34 | -0.85 | 0.54 | -0.45 | 0.37 |
| ×ELA Score | -0.02 | 0.07 | 0.07 | 0.09 | 0.17 | 0.2 | 0.08 | 0.1 | -0.17 | 0.12 | -0.43 | 0.21 | -0.4 | 0.33 | -0.36 | 0.22 |
| ×Math Score | 0 | 0.07 | -0.08 | 0.1 | 0.11 | 0.23 | -0.09 | 0.11 | 0.14 | 0.11 | 0.38 | 0.23 | 0.64 | 0.43 | 0.3 | 0.24 |
| % High Perf. (ELA) | | | | | | | | | | | | | | | | |
| Main Effect | 0.4 | 0.25 | -0.12 | 0.37 | -0.64 | 0.6 | -0.14 | 0.4 | 0.74 | 0.3 | 0.24 | 0.48 | -0.09 | 0.66 | 0.38 | 0.51 |
| ×ELA Score | 0.52 | 0.12 | 0.61 | 0.23 | 1.19 | 0.42 | 0.7 | 0.25 | 0.37 | 0.15 | 0.33 | 0.31 | 0.38 | 0.41 | 0.29 | 0.31 |
| % High Perf. (Math) | | | | | | | | | | | | | | | | |
| Main Effect | -0.75 | 0.34 | 0.85 | 0.5 | 1.56 | 0.88 | 0.87 | 0.55 | -0.36 | 0.39 | 1.48 | 0.69 | 2.91 | 0.96 | 1.38 | 0.74 |
| ×Math Score | 0.51 | 0.13 | 0.61 | 0.25 | 0.44 | 0.47 | 0.57 | 0.27 | 0.35 | 0.14 | 0.69 | 0.37 | 0.45 | 0.51 | 0.81 | 0.39 |
| % FRPL (program) | | | | | | | | | | | | | | | | |
| Main Effect | 0.78 | 0.3 | -1.02 | 0.51 | -2.79 | 0.81 | -1.12 | 0.56 | 0.6 | 0.35 | 0.19 | 0.68 | -0.18 | 0.95 | 0.09 | 0.73 |
| ×Med Income | -0.14 | 0.18 | -0.18 | 0.33 | -0.82 | 0.56 | -0.16 | 0.36 | -0.09 | 0.24 | -0.36 | 0.48 | -0.69 | 0.69 | -0.56 | 0.5 |
| ×Avg Score | -0.05 | 0.21 | 0.46 | 0.36 | 1.02 | 0.54 | 0.42 | 0.39 | -0.51 | 0.26 | -0.86 | 0.56 | -1.46 | 0.88 | -0.59 | 0.6 |
| % FRPL (school) | | | | | | | | | | | | | | | | |
| ×Med Income | 0.44 | 0.26 | 0.12 | 0.44 | 1.23 | 0.75 | 0.18 | 0.46 | -0.54 | 0.37 | -0.21 | 0.59 | 1.24 | 0.93 | 0.09 | 0.64 |
| ×Avg Score | -0.07 | 0.31 | -0.66 | 0.46 | -1.14 | 0.75 | -0.44 | 0.48 | 0.84 | 0.36 | 1.42 | 0.65 | 1.98 | 0.96 | 1.16 | 0.66 |
| 9th Grade Size (100s) | 0.1 | 0.02 | 0.16 | 0.04 | 0.33 | 0.07 | 0.17 | 0.04 | 0.09 | 0.03 | 0.18 | 0.04 | 0.33 | 0.07 | 0.19 | 0.05 |
| % Asian | 2.34 | 0.51 | 1.28 | 0.75 | 0.45 | 1.34 | 1.1 | 0.82 | -0.41 | 0.63 | -2.23 | 1.08 | -3.27 | 1.6 | -1.81 | 1.16 |
| % Black | -0.96 | 0.34 | 0.11 | 0.47 | 0.67 | 0.84 | -0.06 | 0.52 | -0.46 | 0.41 | 0.12 | 0.64 | 0.35 | 0.96 | 0.16 | 0.67 |
| % Hispanic | -0.24 | 0.32 | 0.68 | 0.48 | 1.84 | 0.82 | 0.84 | 0.54 | -0.98 | 0.39 | -0.32 | 0.65 | -0.16 | 0.88 | -0.22 | 0.68 |
| 1(Nearest School) | -0.12 | 0.08 | -0.22 | 0.11 | -0.07 | 0.18 | -0.15 | 0.12 | 0.13 | 0.09 | -0.01 | 0.15 | -0.02 | 0.21 | -0.01 | 0.16 |
| Distance | -0.3 | 0.02 | -0.29 | 0.03 | -0.35 | 0.05 | -0.31 | 0.03 | -0.23 | 0.02 | -0.25 | 0.04 | -0.3 | 0.06 | -0.25 | 0.04 |
| $\sigma^2_{STEM}$ | 1.18 | 0.18 | 0.86 | 0.21 | 2.17 | 0.88 | 1.03 | 0.29 | 1.1 | 0.2 | 1.31 | 0.42 | 1.39 | 0.58 | 1.3 | 0.45 |

Notes: We report the full estimates of the parameters in (1). All % variables are within $[0, 1]$. For conciseness, we report only the estimates based on WTT, $TEPS^{all}$, $TEPS^{top}$, and the selected estimates obtained by following the procedure described in Section 3.5. We report the mean and standard deviation of the posterior distribution as the point estimate and the standard error.

## Table H.9: Preference Estimates: Cell 7–8

| | Cell 7 | | | | | | | | Cell 8 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | WTT | | $\text{TEPS}^{all}$ | | $\text{TEPS}^{top}$ | | Selected | | WTT | | $\text{TEPS}^{all}$ | | $\text{TEPS}^{top}$ | | Selected | |
| | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. | Coef. | S.E. |
| School FE 4 | -0.11 | 0.06 | -0.25 | 0.08 | -0.28 | 0.13 | -0.31 | 0.09 | 0.32 | 0.06 | 0.13 | 0.08 | 0.16 | 0.14 | 0.06 | 0.12 |
| School FE 5 | -0.5 | 0.07 | -0.98 | 0.1 | -1.03 | 0.17 | -1.03 | 0.12 | -0.22 | 0.07 | -0.85 | 0.11 | -0.74 | 0.17 | -0.79 | 0.16 |
| School FE 6 | -0.17 | 0.08 | -0.44 | 0.12 | -0.42 | 0.19 | -0.48 | 0.14 | 0.09 | 0.08 | -0.51 | 0.12 | -0.43 | 0.19 | -0.48 | 0.18 |
| School FE 7 | 0.66 | 0.1 | 0.24 | 0.16 | -0.33 | 0.28 | 0.08 | 0.18 | 0.06 | 0.11 | -0.6 | 0.19 | -1.1 | 0.37 | -1.04 | 0.35 |
| School FE 8 | 0.19 | 0.06 | 0.07 | 0.09 | 0.03 | 0.15 | 0.08 | 0.1 | 0.17 | 0.06 | -0.17 | 0.1 | 0.04 | 0.17 | -0.04 | 0.15 |
| School FE 9 | -0.04 | 0.07 | -0.24 | 0.1 | -0.25 | 0.17 | -0.33 | 0.12 | -0.48 | 0.08 | -0.89 | 0.12 | -0.73 | 0.2 | -0.79 | 0.18 |
| $\mathbb{1}$(STEM) | | | | | | | | | | | | | | | | |
| Main Effect | 0.18 | 0.07 | -0.19 | 0.18 | -0.99 | 0.55 | -0.15 | 0.2 | -0.39 | 0.1 | -0.92 | 0.29 | -0.93 | 0.55 | -0.7 | 0.41 |
| ×ELA Score | -0.04 | 0.04 | -0.08 | 0.07 | -0.09 | 0.12 | -0.03 | 0.07 | -0.03 | 0.06 | 0.02 | 0.1 | -0.1 | 0.16 | -0.09 | 0.14 |
| ×Math Score | -0.01 | 0.04 | -0.1 | 0.07 | -0.11 | 0.14 | -0.16 | 0.08 | -0.07 | 0.06 | -0.27 | 0.11 | -0.1 | 0.19 | -0.2 | 0.17 |
| % High Perf. (ELA) | | | | | | | | | | | | | | | | |
| Main Effect | 0.31 | 0.15 | 0.23 | 0.23 | 0.39 | 0.34 | 0.38 | 0.26 | 0.33 | 0.14 | 0.17 | 0.22 | -0.28 | 0.31 | -0.08 | 0.29 |
| ×ELA Score | 0.44 | 0.07 | 0.57 | 0.15 | 0.66 | 0.2 | 0.54 | 0.15 | 0.4 | 0.08 | 0.5 | 0.17 | 0.27 | 0.23 | 0.34 | 0.21 |
| % High Perf. (Math) | | | | | | | | | | | | | | | | |
| Main Effect | -0.57 | 0.19 | 0.18 | 0.3 | 0.61 | 0.47 | 0.24 | 0.33 | 0.62 | 0.19 | 1.34 | 0.34 | 1.68 | 0.48 | 1.43 | 0.45 |
| ×Math Score | 0.3 | 0.07 | 0.41 | 0.15 | 0.49 | 0.22 | 0.54 | 0.16 | 0.66 | 0.08 | 1.05 | 0.2 | 1.08 | 0.28 | 1.25 | 0.28 |
| % FRPL (program) | | | | | | | | | | | | | | | | |
| Main Effect | 1.29 | 0.18 | 0.5 | 0.34 | 0.18 | 0.47 | 0.53 | 0.36 | 0.76 | 0.18 | 0.07 | 0.34 | -0.48 | 0.49 | -0.3 | 0.49 |
| ×Med Income | -0.14 | 0.1 | -0.3 | 0.18 | -0.67 | 0.3 | -0.46 | 0.19 | 0.07 | 0.1 | 0.15 | 0.2 | 0.03 | 0.3 | -0.01 | 0.29 |
| ×Avg Score | 0.01 | 0.12 | 0.12 | 0.21 | 0.17 | 0.31 | 0.24 | 0.23 | -0.12 | 0.13 | -0.26 | 0.27 | -0.5 | 0.38 | -0.26 | 0.35 |
| % FRPL (school) | | | | | | | | | | | | | | | | |
| ×Med Income | 0.27 | 0.15 | 0.26 | 0.23 | 0.89 | 0.42 | 0.37 | 0.27 | -0.5 | 0.16 | -0.83 | 0.25 | -0.71 | 0.42 | -0.67 | 0.41 |
| ×Avg Score | 0.22 | 0.18 | 0.13 | 0.25 | 0.45 | 0.39 | 0.25 | 0.28 | 0.71 | 0.18 | 1.02 | 0.32 | 0.94 | 0.49 | 0.89 | 0.44 |
| 9th Grade Size (100s) | 0.01 | 0.01 | 0.05 | 0.02 | 0.16 | 0.03 | 0.08 | 0.02 | 0.04 | 0.02 | 0.11 | 0.03 | 0.24 | 0.04 | 0.22 | 0.04 |
| % Asian | 1.24 | 0.28 | 0.78 | 0.45 | 0.2 | 0.73 | 0.59 | 0.5 | -1.78 | 0.31 | -2.92 | 0.52 | -1.76 | 0.76 | -1.97 | 0.73 |
| % Black | -1.01 | 0.2 | -0.78 | 0.28 | -0.42 | 0.46 | -0.65 | 0.31 | 0.19 | 0.2 | 0.71 | 0.31 | 1.74 | 0.48 | 1.49 | 0.44 |
| % Hispanic | -0.58 | 0.17 | -0.17 | 0.26 | 0.27 | 0.41 | 0.14 | 0.3 | -0.28 | 0.18 | 0.23 | 0.28 | 0.5 | 0.39 | 0.37 | 0.37 |
| 1(Nearest School) | 0.13 | 0.05 | 0.12 | 0.06 | 0.15 | 0.1 | 0.12 | 0.07 | 0.14 | 0.05 | 0.15 | 0.07 | 0.29 | 0.1 | 0.28 | 0.1 |
| Distance | -0.19 | 0.01 | -0.22 | 0.02 | -0.25 | 0.03 | -0.23 | 0.02 | -0.14 | 0.01 | -0.15 | 0.02 | -0.14 | 0.03 | -0.12 | 0.02 |
| $\sigma^2_{STEM}$ | 1.14 | 0.11 | 1.43 | 0.25 | 2.75 | 1.03 | 1.43 | 0.29 | 1.13 | 0.13 | 1.6 | 0.36 | 1.68 | 0.69 | 1.42 | 0.49 |

Notes: We report the full estimates of the parameters in (1). All % variables are within $[0, 1]$. For conciseness, we report only the estimates based on WTT, $\text{TEPS}^{all}$, $\text{TEPS}^{top}$, and the selected estimates obtained by following the procedure described in Section 3.5. We report the mean and standard deviation of the posterior distribution as the point estimate and the standard error.