Foreign Direct Investment and Foreign Reserve Accumulation*

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Abstract

Why do emerging market economies hold high levels of foreign exchange reserves? What is the optimal level of reserves, and do economies accumulate reserves in excess of this optimal level? I argue that foreign exchange reserves help emerging markets attract foreign direct investment. This incentive can play an important role when analyzing central banks' reserve accumulation. I study the interaction between foreign exchange reserves and foreign direct investment to explain the level of reserves using a small open economy model. The model puts the domestic entities and international investors in the same picture. The optimal level of the reserve-to-GDP ratio generated by the model is close to the level observed in East Asian economies. Additionally, the model generates positive co-movement between technology growth and the current account. This feature suggests that high technology growth corresponds to net capital outflow, because of the outflow of foreign exchange reserves in attracting the inflow of foreign direct investment, thus providing a rationale to the 'allocation puzzle' in cross-economy comparisons. The model also generates positive co-movement between foreign exchange reserves and foreign debt, thus relating to the puzzle of why economies borrow and save simultaneously.

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1 Introduction

In recent decades, there has been rapid growth in foreign exchange reserves (RX)[1] held by emerging market (EM) economies. The average level of foreign exchange reserves for EM economies has increased from below 10% of GDP in the 1980s to more than 20% of GDP in recent years.

The widely accepted traditional motives for reserve accumulation are to cushion exchange rate volatility and prevent extreme events, for example, the 1994 Mexico peso crisis and the 1997 Asian Crisis. These motives have been explored in multiple strands of literature, for instance, Jeanne and Rancière (2008) and Obstfeld, Shambaugh and Taylor (2009). However, although the literature finds that reserves help stabilize the exchange rate and reduce the probability of a crisis, the general conclusion is that certain economies, such as the East Asian economies, hold excessive RX. In other words, based on the existing traditional motives, the probability of a crisis needs to be unrealistically high to justify the reserve level observed in some economies. Because reserve assets are liquid and pay low returns, holding reserves incurs a substantial opportunity cost. Whether we can explain the large accumulation of foreign reserves for some economies remains a puzzle.

In this paper, I study a novel channel to rationalize the accumulation of foreign exchange reserves and discuss the macroeconomic implications. My argument is based on the idea that reserves can reduce an economy's perceived risk among foreign investors and thus have first-order effects in attracting foreign investment, specifically in the form of foreign direct investment (FDI). In other words, both foreign investors and recipient economies know that reserves can serve as a buffer against a currency crisis or exchange rate movement. Provided that foreign investors have limited information on private firms in EM economies, which is common for EM economies with underdeveloped financial markets and less transparent regulations, reserves can be a tool that central banks can employ to send positive signals to foreign investors and thus impact the latter's investment decisions. Central banks choose to place domestic resources in low-yield assets and attract external investment because they value the productivity gain provided by foreign investors, which is a commonly discussed feature of FDI.

I first present a set of stylized facts regarding the relationships connecting reserves and FDI. I show that reserve assets and FDI liabilities have both increased tremendously for EM economies in recent decades. Reserve assets have grown to be the most significant component of the external asset position of EM economies, and FDI liabilities have risen to be the most significant component of their external liability position. There are robust positive correlations between reserves and FDI across different measures.

[1] Foreign exchange reserves refer to cash or cash equivalents, liquid assets held by economies' central banks, mostly denominated in USD or EURO.

I then build a small open economy model to specifically study the incentive of holding reserves to attract FDI inflow in explaining the level of foreign exchange reserves. The model features a two-way interaction between domestic recipient firms seeking foreign investment and foreign investors seeking yield. Since foreign investors do not have complete information on the recipient economies, they expect a high level of reserves to reduce the volatility of the risk of the economy, which I show is present in the data by using reserve levels and stock returns. The inflow of foreign investment contributes to a higher level of domestic productivity. The information conveyed by the reserve assets to foreign investors and the benefits of foreign investment provide central banks incentives to allocate a large amount of domestic resources to reserve assets. I calibrate the model to the average level of EM economies and show that the optimal reserves generated by this incentive can be more than 20% of GDP, suggesting that this incentive can make up for the gap between the reality and the optimal level of reserves reported in the previous literature.

Another key feature of this mechanism described in this paper is that it can speak to an international macroeconomics puzzle called the 'allocation puzzle' identified by Gourinchas and Jeanne (2013). The allocation puzzle refers the empirical observation of a negative correlation between the net flow of capital into countries and the technology growth of economies. Economies with higher average technology growth over the 1980-2000 period experienced lower average net capital inflows, which contradicts the standard neoclassic growth model. However, this phenomenon can be explained by the incentive for accumulating reserve assets and attracting foreign investment. Given a technology improvement, there is indeed larger capital inflows into an economy for the purpose of accumulating capital stock. However, there is simultaneously an outflow coming from the accumulation of reserve assets to encourage more private capital inflow. The public capital outflow and private capital inflow yield a net negative correlation between net capital inflow and technology growth. This is also observed in the data if I decompose the capital flows into FDI inflow and reserve outflow. The key mechanism in the model can generate the same pattern as cited in the allocation puzzle, both at the aggregate level and after decomposing the capital flows.

In the following sections of the paper, I first review the status quo of the research on foreign reserves and FDI in section 2. Then, I present empirical motivations for this work in section 3, where I examine the relationships between foreign reserves and FDI in the data. Next, I present the model in section 4 and provide evidence for the model's key mechanism. The calibration of the model is explained in section 5. The equilibrium level of reserves and its relationship with the key mechanism in the model are discussed in section 6. Section 7 provides the dynamics generated by the model following shocks. Section 8 discusses the allocation puzzle in detail by comparing the pattern generated in the model and the pattern in the data. Finally, section 9 discusses the general business cycle moments of the model.

2 Literature review

This paper mainly relates to the strands of literature on foreign exchange reserves, FDI, and the correlations between capital flows in open economies. I will briefly summarize each strand of research and discuss their connections with this paper.

The literature on foreign reserves has focused on the precautionary saving motive, reducing the probability of a currency crisis and a capital flow crisis and stabilizing the exchange rate. Reducing the probability of a crisis and precautionary saving are often intertwined and examined jointly in the literature. Frankel and Saravelos (2010) claim that reserve accumulation and past movements in the real exchange rate were the two leading indicators of the varying incidence of the Global Financial Crisis. Adopting a broader focus, Gourinchas and Obstfeld (2011) use panel analysis across countries and across time to conclude that higher foreign reserves are associated with a reduced probability of crisis in EMs. Obstfeld, Shambaugh and Taylor (2009) documents that having reserves in excess of predicted levels is associated with smaller subsequent nominal exchange rate depreciation after 2008. Jeanne and Rancière (2008) uses a model to discuss the optimal level of international reserves for a small open economy that is vulnerable to sudden stops in capital flows. Reserves allow a country to smooth domestic absorption in response to sudden stops. They conclude that existing theory can explain the level of reserves observed in many countries, but the trend of accumulating reserves observed in EM economies appears excessive. The purpose of my paper is to propose a new explanation of holding reserves for EM economies, in an effort to reconcile the optimal reserve level predicted by theory and that observed in the data, and simultaneously explain other patterns of capital flows in the data.

Another strand of literature focuses on the question of why countries save through reserves while simultaneously accumulating external debt. Although previous works argue that high levels of reserve reduce the risk of crisis, EM economies can naturally avoid the risk of potential sudden stops and insolvent situations by reducing the level of debt instead of holding reserves. Alfaro and Kanczuk (2009) constructs a model in which both debt and reserves are imperfect substitutes to shift resources from repayment states to default states. However, the optimal strategy in that model is not to hold reserves at all. Bianchi, Hatchondo and Martinez (2018) add borrowing through long-term debt to the former model. In this setup, the country has incentives to borrow when the borrowing cost is low and deleverage when the borrowing cost is high. In my model, I show that without resorting to long-maturity debt, there is co-movement between external debt and reserves. The central bank has the incentive to finance the accumulation of reserves through high-spread external debt because of the reserves' impact on private capital flows.

The paper further relates to the literature on reserves and exchange rate volatility. The signaling effect of reserves in reducing investment risk in EM economies can work through stabilizing the exchange rate, which leads to less volatile returns. There is vast literature suggesting that reserves stabilize the exchange rate. Nowak, Hviding and Ricci (2004) study the role of an increase in foreign exchange reserves in reducing currency volatility for EM countries. Their results provide support for the proposition that holding adequate reserves reduces exchange rate volatility. The effect is nonlinear and appears to operate through a signaling effect. Cady and Gonzalez-Garcia (2007) find that reserve adequacy is significant in determining exchange rate volatility for both EM and industrialized countries. Aizenman and Riera-Crichton (2008) observe that international reserves cushion the impact of terms-of-trade shocks on the real exchange rate and that this effect is essential for developing but not for industrialized countries. Eichler and Littke (2018) find that country pairs with higher levels of foreign exchange reserves exhibit less exchange rate volatility.

This paper relates to the FDI literature by addressing one of the main characteristics studied by that literature: FDI increases the productivity of recipient countries. Aitken and Harrison (1999) finds that foreign equity participation is positively correlated with plant productivity for small enterprises. There is a productivity gain from foreign ownership but the little spillover effect. Blonigen and Wang (2004) and Melitz, Helpman and Yeaple (2004) find that export platform FDI may have a potentially larger effect on growth. Bloom, Sadun and Van Reenen (2012) study US multinationals operating in the EU in sectors that intensively use information technology (IT). These sectors in the EU experienced the same level of productivity growth as the US. In general, the literature has observed an increase in productivity from FDI due to the transfer of the investors' technology to the firms of the recipient economies.

There is little literature on the interaction of FDI and RX. Matsumoto (2018) builds on Benigno and Fornaro (2012) in modeling that accumulating reserves depreciates the real exchange rate, which leads to a shift in domestic production to the tradable sector. This will attract FDI into the tradable sector, which leads to higher productivity gains. My paper does not rely on the distinction between the tradable and non-tradable sectors and focuses more on the signaling gain due to reserves without relating it to the first-order effect of devaluating the exchange rate. I isolate the channel of the potential second-order effect by stabilizing the return on investment for foreign investors through reserve accumulation. Huang (2018) discusses the joint impact of a high level of reserves and a high level of FDI stock in China from the perspective of the investment return of the country. He gathered data from top banks in China and stock market information to calculate the market value of reserve assets and FDI liability. He then showed that the return on equity in China is higher than the return from reserve asset investment. My paper argues that reserves do not serve as an investment device but rather as a signaling device.

This paper also speaks to the research on capital flows of EMs, mainly the 'allocation puzzle', and can provide an explanation for that puzzle based on the main mechanism of the model. Gourinchas and Jeanne (2013) discuss the phenomenon of a negative correlation between the net flow of capital into an economy and its technology growth, which they call the 'allocation puzzle'. Alfaro, Kalemli-Ozcan and Volosovych (2014) further documents the public and private flows and demonstrates that sovereign-to-sovereign transactions account for upstream capital flows and global imbalances. Dooley, Folkerts-Landau and Garber (2004) describes how international reserves can be interpreted as a form of collateral for FDI flows to less developed economies. This causes the outflow of capital for EMs in the form of reserve assets and the inflow of capital in the form of FDI liability, which is called the 'Bretton Woods 2' system. The Bretton Wood 2 system is not quantified and is based only on the rough description of potential channels. However, it can be falsified by decomposing the reserve assets flowing into a certain economy and the FDI liabilities coming from a certain economy to see if they match each other proportionally, that is, whether the economies that receive larger reserve inflows provide more FDI. My paper differs from the Bretton Woods 2 concept in that it does not impose the insurance role of reserve assets but models reserves as a coordinating device for both investors and recipient economies and provides evidence for this mechanism. The signals do not work by directly providing collateral to a certain economy but generally indicate the economic conditions of the recipient country. Aguiar and Amador (2011) adopts a political economy setup that can also speak to the allocation puzzle in which there is a limited commitment from the governments' debt. Foreign investors would only let capital flow to countries with lower debt levels. Since political parties prefer spending to occur when they are in office to reduce the chance of losing their positions, there is a trade-off between borrowing to consume today and paying off external debt to let capital flow into the country and allow higher consumption in the future. This gives rise to the phenomenon whereby economies that grow rapidly tend to increase their net foreign asset position to encourage other forms of capital inflow. Thus, there is a net reduction in public debt combined with an inflow of private capital in fast-growing economies and the reverse in shrinking economies. My paper differs in that an increased net asset position means higher reserve accumulation instead of a lower debt level. Borrowing is used to accumulate reserves to encourage more FDI instead of more consumption.

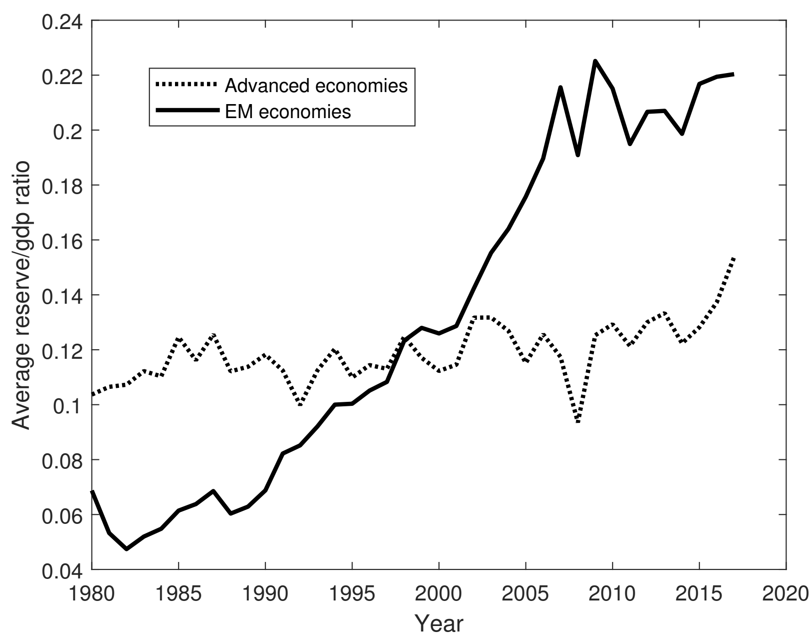
3 Empirical analysis

In this section, I will show some empirical evidence on the status quo of the levels of reserves and levels of FDI liabilities for emerging market economies. I will also put the two capital flows in the same pictures and show the connections between the two.

3.1 Aggregate trend of reserves and FDI

The amount of foreign reserves held by EM economies has grown the past 30 years. Figure 1 shows the average reserves-to-GDP ratio[2] for EM economies and advanced economies over the period 1980-2017. The average reserves-to-GDP ratio for EM economies tripled in recent decades, from below 0.1 in the early 1980s to more than 0.2 in recent years. The ratio has remained relatively stable for advanced economies, with an average of approximately 0.1-0.15 in recent years.

Figure 1: Reserves-to-GDP ratio, average for advanced economies and EM economies



Note: This figure plots the average reserves-to-GDP ratio for 33 EM economies and 47 advanced economies from 1980 to 2017
Source: World Bank WDI

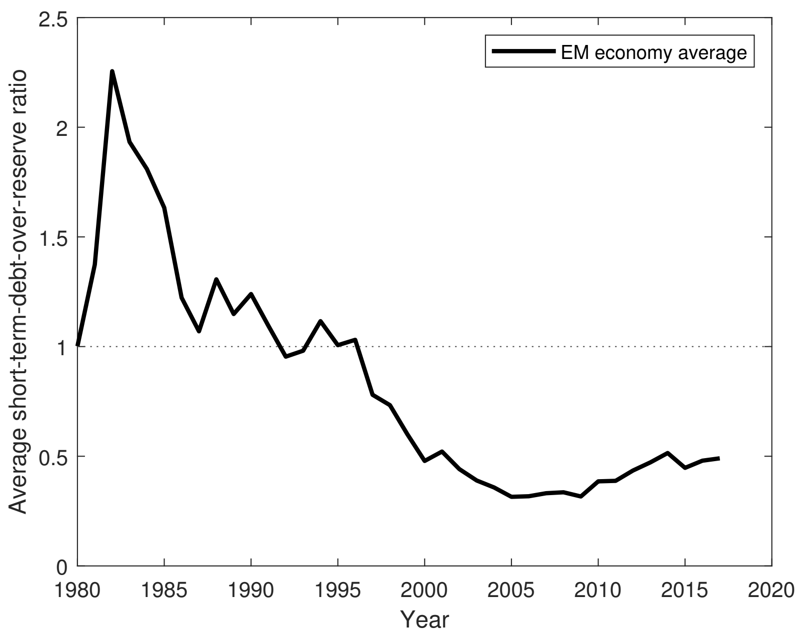
Another indicator that reveals the adequacy of EM economies' reserves is the short-term-debt-to-reserves ratio. Although policymakers have long discussed the optimal amount of reserves that EM economies should hold, there is no consensus on this question. The IMF has rule-of-thumb guidelines for reserve holdings according to which a country should hold foreign reserves in an amount equal to their short-term debt liability. This rule can ensure that the economies are able to repay their short-term debt and avoid insolvency when facing sudden

[2]The average for 33 EM economies and 47 advanced economies. The full set of countries is listed in the appendix

stops in financing. These sudden stops refer to periods when economies are not able to secure capital inflow or roll over debt. However, the amount of reserves held by EM economies has far exceeded their short-term debt. Their average level of reserves is nearly four times that of their total short-term external debt.

Figure 2 plots the average short-term debt-to-reserves ratio for EM economies. A value greater than one indicates that the short-term debt exceeds the amount of reserves, while a value smaller than one value indicates the opposite. Short-term debt exceeded reserves before 2000. By contrast, reserves have exceeded short-term debt for the past ten years, and the ratio suggests that the amount of reserves is three times that of short-term debt.

Figure 2: Short-term-debt-to-reserves ratio, average for all EM economies

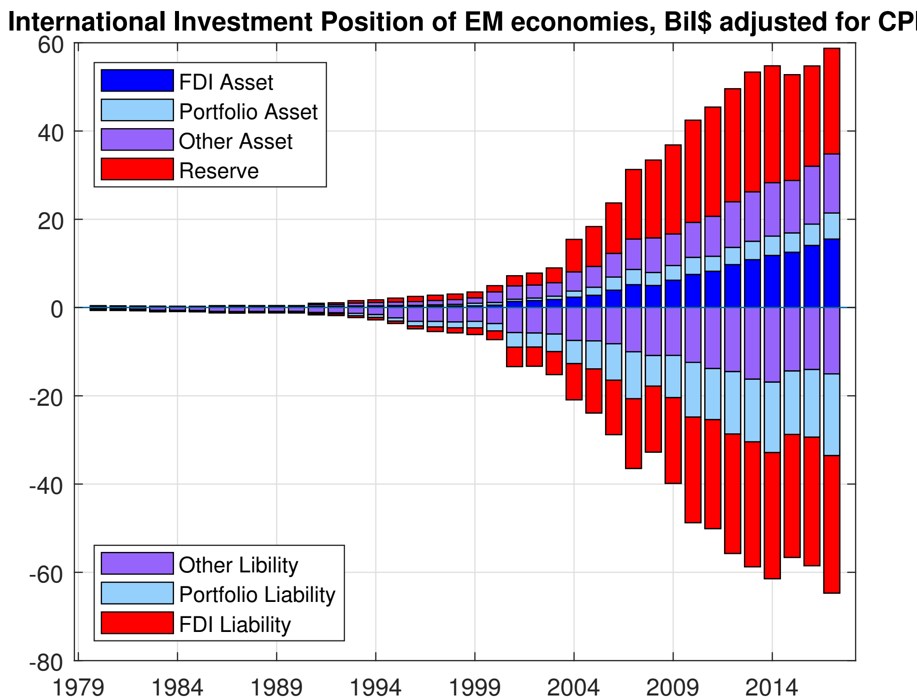


Note: This figure plots the average short-term debt as a percentage of foreign reserves for EM economies from 1980 to 2017.
Source: World Bank WDI

As reserves have grown to become a large proportion of the assets of EM economies, FDI has become their most significant form of liability. Figure 3 plots the components of the gross liability and gross asset positions of EM economies in recent decades. The positive y-axis plots the decomposition of the gross asset position. It has been growing in size since the late 1990s. Meanwhile, reserve assets have taken up an increasing share of the gross asset position, suggesting that EM economies mainly save through reserves. The negative y-axis plots the decomposition of the total liability position, of which FDI liability is the major component.

FDI has in conjunction with the size of the liability of EM economies and has constituted half of their external liability in recent years. This suggests that FDI inflow has been the main form of capital inflow into EM economies. This graph indicates that as these economies gradually build up their balance sheets, both reserves and FDI have become significant parts. When analyzing the behavior of capital inflows and outflows of EM economies, it is important to closely examine to the most significant asset (reserves) and the most significant liability (FDI).

Figure 3: Asset and liability decomposition of emerging market economies



Note: This figure plots the gross asset and gross liability decomposition for EM economies. Source: IMF International Financial Statistics (IFS)

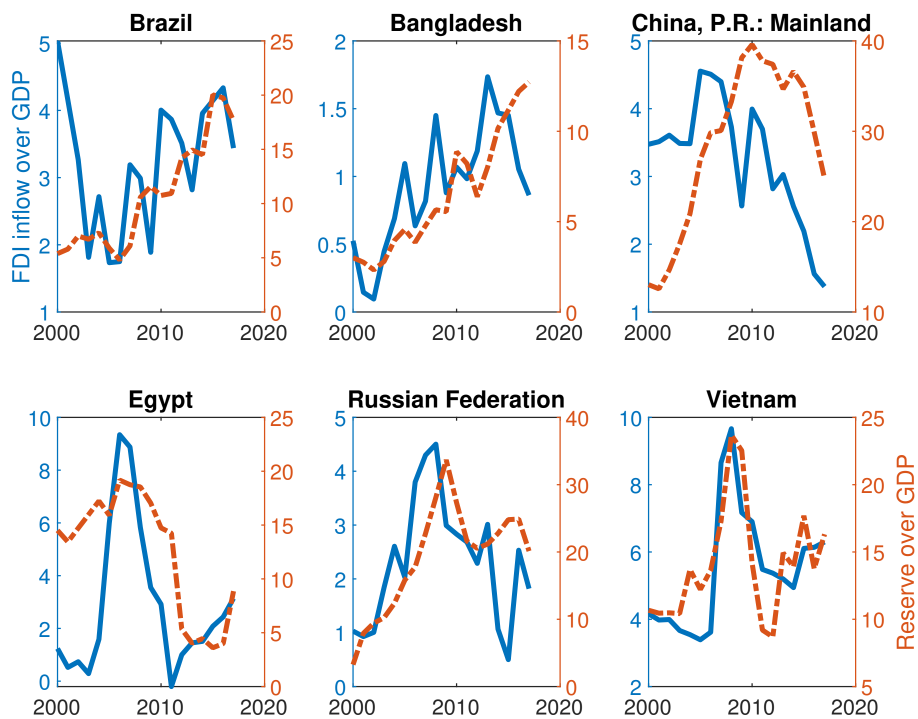
3.2 Correlation between foreign exchange reserves and FDI

Having examined the trends in the RX and FDI series in recent decades, I can put them in a single analysis and explore the interactions between them. A positive correlation between reserves and FDI can be established through different specifications and definitions of variables. This relationship is robust across all analyses. The five different specifications showing the positive correlation between FDI and reserves are summarized below.

3.2.1 Co-movement of reserves and FDI inflow in major EM economies

Figure 4 plots the co-movement of the reserves-to-GDP ratio[3] and FDI-inflow-to-GDP ratio for six major EM economies. These economies have high levels of the reserves-to-GDP ratio, ranging from 20% to 50% in recent years. The inflow of FDI in each year is also substantial, ranging from 0.5% to 10% of GDP. Within each economy, the periods with a high reserves-to-GDP ratio are also the periods with a high FDI-inflow-to-GDP ratio.

Figure 4: Co-movement between FDI inflow and reserves



Note: This figure plots the reserve-to-GDP ratio and FDI-inflow-to-GDP ratio for six major EM economies.
Source: IMF IFS and World Bank WDI

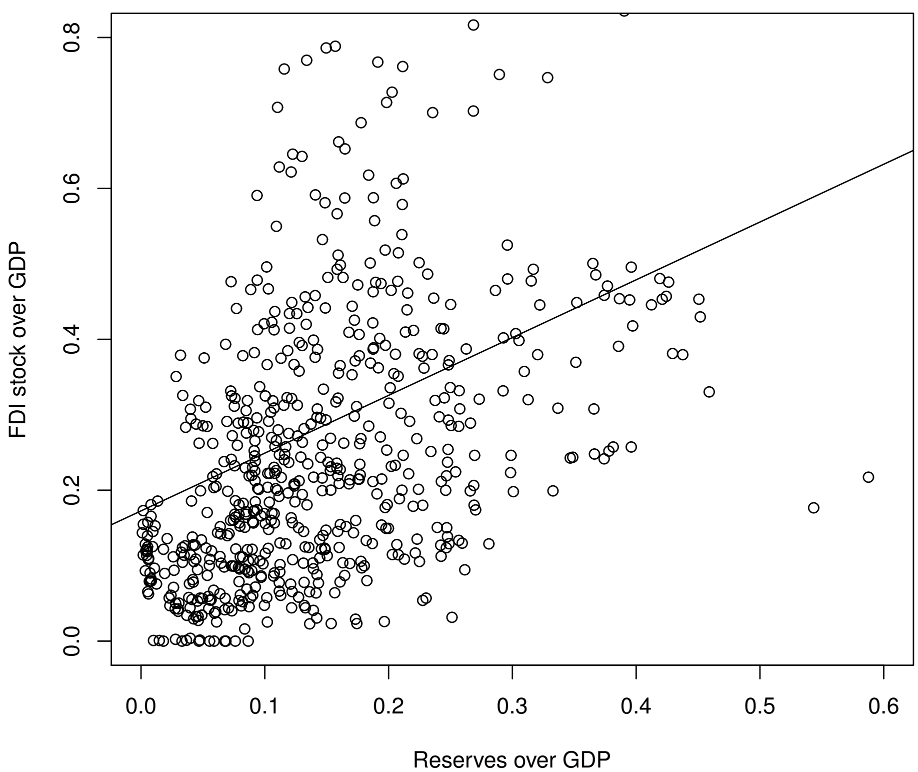
3.2.2 Cross-economy and cross-time analysis of the FDI-stock-to-GDP ratio and reserve-to-GDP ratio

I can apply this analysis to a broader set of economies by examining the correlation between the reserves-to-GDP ratio and FDI-stock-to-GDP ratio across economies and years. Figure 5

[3]Unless otherwise stated, all the empirical analyses take the reserves-to-GDP ratio to be the lagged reserves divided by current GDP.

plots the FDI-stock-to-GDP ratio on the y-axis and the reserves-to-GDP-ratio on the x-axis for all EM economies in the sample. Each dot represents a country-year combination. The figure suggests a robust positive correlation between the two variables; across economies and over time, higher savings in foreign reserves correspond to higher 'borrowing' in FDI.

Figure 5: Positive correlation between FDI stock and reserves



Note: This scatter plot shows the reserves-to-GDP ratio and FDI-stock-to-GDP ratio for 33 EM economies for the period 1980-2017. Each dot represents a economy-year combination
Source: IMF IFS and World Bank WDI

There are analyses in other dimensions to show the positive correlation between to two variables, for example, the economies with larger changes in reserves asset position correspond to the ones with larger changes in FDI liability position. They are included in the Appendix A.2 for further reference.

3.2.3 Strong positive correlation between RX/GDP and FDI/GDP

This positive correlation between FDI and reserves can be further illustrated with a reduced-form regression by specifying the following model:

$$\frac{FDI_{it}}{GDP_{it}} = \beta_1 + \beta_2 \frac{RX_{it-1}}{GDP_{it}} + a_t + c_i + u_{it}$$

Table 1 reports the results of regressing the FDI-stock-to-GDP ratio on the one-period-lagged reserves-to-GDP ratio. The four specifications correspond to no fixed effects, time fixed effects, economy fixed effects and both fixed effects. The table suggests that across all specifications, the FDI-stock-to-GDP ratio and reserves-to-GDP ratio have strong positive correlations. A 1 unit increase in the reserves-to-GDP ratio correlates significantly with a 0.4-1 unit increase in the FDI-stock-to-GDP ratio, depending on the specification.

Table 2 reports the corresponding analysis by taking the FDI-inflow-to-GDP ratio as the dependent variable in the empirical specification. A 1 unit increase in the reserves-to-GDP ratio correlates significantly with a 0.02-0.076 unit increase in the FDI-inflow-to-GDP ratio. These observations will also be discussed later in the model calibration part of the paper.

As robustness checks, other factors that are commonly considered to be determinants of FDI liabilities are included in the regression equation. The full set of results is reported in the Appendix A.3. The results suggest that this positive correlation is robust to the inclusion other control variables. I also include one empirical specification in which the dependent variable is changed to the FDI-stock-to-total-liability ratio. The positive and significant correlation suggests that a high level of the reserves-to-GDP ratio also correlates with a high FDI liability as a percentage of total liability, suggesting that a larger fraction of the 'borrowing' of the economy is in the form of FDI.

Table 1: Regression of FDI stock/GDP on Reserves/GDP

	Dependent variable: FDI stock/GDP			
	(1)	(2)	(3)	(4)
$\text{Res}_{it-1}/GDP_{it}$	0.552***	0.369***	1.097***	0.417***
	(0.054)	(0.033)	(0.100)	(0.056)
Time fixed effect	*No*	*Yes*	*No*	*Yes*
Country fixed effect	*No*	*No*	*Yes*	*Yes*
Observations	618	618	618	618
R^2-Standard	0.144	0.292	0.695	0.834

Note: Standard errors are clustered at the economy and time levels **p<0.05; ***p<0.01

All empirical investigations consistently suggest positive co-movement between the level of FDI stock/inflow and the level of reserves. This positive correlation is the feature that will be

Table 2: Regression of FDI inflow/GDP on Reserves/GDP

	Dependent variable: FDI inflow/GDP			
	(1)	(2)	(3)	(4)
$\text{Res}_{it-1}/GDP_{it}$	0.076***	0.054***	0.043***	0.020**
	(0.017)	(0.012)	(0.013)	(0.009)
Time fixed effect	No	Yes	No	Yes
Country fixed effect	No	No	Yes	Yes
Observations	998	998	998	998
R^2-Standard	0.101	0.221	0.384	0.487

Note: Standard errors are clustered at the country and time levels **p<0.05; ***p<0.01

built into the model to explain the high level of reserves held by EM economies.

4 The model

4.1 Households

The economy is assumed to be populated by a large representative family with a continuum of members. Consumption is identical across family members. Household preferences are defined over per capita consumption, c_t, and are described by the utility function

$$E_0\{\sum_{t=0}^{\infty}\beta^t U(c_t)\} \tag{1}$$

where $\beta \in (0,1)$ represents a subjective discount factor, and U is a period utility index assumed to be strictly increasing and strictly concave.

Households have access to a bond issued by central bank, b_t^H, which pays the gross interest rate R_{t-1}^H when held from $t-1$ to t. The household can hold positive or negative quantities of this bond. When the quantity is positive, the household is lending to the central bank. When the quantity is negative, the household is borrowing from the central bank. The household also receives dividends from the ownership of producing firms, denoted Π_t. The household's period-by-period budget constraint is given by:

$$c_t + b_t^H = R_{t-1}^H b_{t-1}^H + \Pi_t \tag{2}$$

Households take R_t^H and Π_t as given. In the model, households do not have access to the international financial market, and their only access to borrowing and saving is through b_t^H.

The household chooses processes for c_t and b_t^H to maximize the utility function (1) subject to (2). The Lagrangian associated with this maximization problem is

$$L = E_0(\sum_{t=0}^{\infty}\beta^t\{U(c_t)) + \lambda_t^{1d}(R_{t-1}^H b_{t-1}^H + \Pi_t - c_t - b_t^H)\}$$

where λ_t^{1d} represent the Lagrangian multiplier of equation (2). The first-order conditions with respect to c_t and b_t^H are,respectively, given by

$$U'(c_t) - \lambda_t^{1d} = 0 \tag{3}$$

$$\lambda_t^{1d} = \beta E_t\{\lambda_{t+1}^{1d} R_t^H\} \tag{4}$$

4.2 Firms

The economy is also populated by a representative firm that makes investment and production decisions. The production technology is given by

$$y_t = A_t k_t^{\alpha} \tag{5}$$

where A_t is an exogenous technology process, k_t denotes physical capital and the parameter $\alpha \in (0,1)$. Capital depreciates at rate δ and firms will invest to build capital for the next period, so that

$$k_{t+1} = (1-\delta)k_t + i_t \tag{6}$$

where i_t denotes investment.

Firms need to decide whether to finance the investment through equity, which is FDI i_t^F, or debt, which is domestic investment i_t^H. Total investment is a function of both i_t^F and i_t^H. The investment aggregator $F(i_t^H, i_t^F)$ is increasing and concave in both elements.

$$i_t = F(i_t^H, i_t^F) \tag{7}$$

If the firm uses domestic investment, it needs to borrow from the central bank at an interest rate R_t^i and will need to repay the principal and interest in the next period. If the firm uses foreign investment i_t^F, it correspondingly needs to sell a new share $\frac{i_t^F}{k_{t+1}}$ to foreign investors and repay a share of the profit to foreign investors in the next period. The total share of the firm x_t owned by foreign investors also depreciates at rate δ, the depreciation rate of capital. The evolution of the shares held by foreign investors is thus given by

$$x_t = (1-\delta)x_{t-1} + \frac{i_t^F}{k_{t+1}} \tag{8}$$

To capture the advantage of using FDI i_t^F over domestic investment i_t^H, the productivity A_t is modeled as an increasing function of the existing FDI inflow from last period i_{t-1}^F

$$A_t = F(i_{t-1}^F) = A_{t,3} + A^1(\frac{i_{t-1}^F}{i_{ss}^F})^{A_{t,2}} \tag{9}$$

where A_1 is a parameter and A_t^2 and A_t^3 are two exogenous processes. i_{ss}^F refers to the steady-state level of foreign investment. The firm also pays a proportional tax on its foreign investment i_t^F in each period at rate τ_t and receives a lump-sum transfer from the government T_t

Under these assumptions, the profit of the firm after paying dividends to the foreign investors in period t is given by

$$\Pi_t = (1 - x_{t-1})(A_t k_t^\alpha) - R_{t-1}^i i_{t-1}^H + T_t - \tau_t i_t^F \tag{10}$$

where the firm pays dividends to the foreign investors $x_{t-1}A_t k_t^\alpha$, repays the debt $R_{t-1}^i i_{t-1}^H$ from borrowing from the central bank in the previous period $t-1$, is subject to a tax/transfer T_t from the government and pays a proportional tax for inducing foreign investment $\tau_t i_t^F$.

The firm thus chooses processes for k_{t+1}, i_t, x_t, i_t^H, i_t^F to maximize the present discounted value of dividends, subject to equations (6)$-$(9), replaces A_t with equation (9), and takes τ_t, T_t, R_t^i as given.

$$max_{\{i_t,k_{t+1},x_t,i_t^H,i_t^F\}} E_0 \sum_{t=0}^{\infty} M_{0,t}\Big((1-x_{t-1})A_t k_t^\alpha - R_{t-1}^i i_{t-1}^H + T_t - \tau_t i_t^F\Big)$$

where $M_{0,t}$ is the stochastic discount factor between time 0 and t for firms. Since the profit is rebated to the household, $M_{0,t} \equiv \beta^t \frac{\lambda_t^{1d}}{\lambda_0^{1d}}$. Lagrangian is given by

$$\begin{aligned} L = \sum_{t=0}^{\infty} E_0\{M_{0,t}\Big(&(1-x_{t-1})A_t k_t^\alpha - R_{t-1}^i i_{t-1}^H + T_t - \tau_t i_t^F + \\ &\lambda_t^{2d}((1-\delta)k_t + i_t - k_{t+1}) + \lambda_t^{3d}(F(i_t^H, i_t^F) - i_t) + \lambda_t^{4d}((1-\delta)x_{t-1} + \frac{i_t^F}{k_{t+1}} - x_t)\Big)\} \end{aligned}$$

where $\lambda_t^{2d} - \lambda_t^{4d}$ corresponds to the Lagrangian multipliers of equations (6) - (8), with equation (9) substituted into the objective function. The first-order conditions with respect to i_t, k_{t+1}, x_t, i_t^H and i_t^F are

$$M_{0,t}(\lambda_t^{2d} - \lambda_t^{3d}) = 0 \tag{11}$$

$$E_t\{M_{0,t+1}(1-x_t)A_{t+1}k_{t+1}^{\alpha-1}\alpha + M_{0,t+1}\lambda_{t+1}^{2d}(1-\delta) - M_{0,t}\lambda_t^{2d} - M_{0,t}\lambda_t^{4d}\frac{i_t^F}{k_{t+1}^2}\} = 0 \tag{12}$$

$$E_t\{-M_{0,t+1}A_{t+1}k_{t+1}^{\alpha} + M_{0,t+1}(1-\delta)\lambda_{t+1}^{4d}\} = \lambda_t^{4d}M_{0,t} \tag{13}$$

$$E_t\{M_{0,t+1}R_t^i\} = M_{0,t}\lambda_t^{3d}F_{i_t^H} \tag{14}$$

$$E_t\{M_{0,t+1}(1-x_t)\frac{\partial A_{t+1}}{\partial i_t^F}k_{t+1}^{\alpha}\} + M_{0,t}\lambda_t^{3d}F_{i_t^F} + M_{0,t}\lambda_t^{4d}\frac{1}{k_{t+1}} - M_{0,t}\tau_t = 0 \tag{15}$$

4.3 Central bank and government

The central bank makes decisions on the interest rate of lending/borrowing from the households R_t^H and the interest rate of lending to firms R_t^i. The central bank's budget constraint is given by the following equation

$$\underbrace{b_t^H - b_{t-1}^H R_{t-1}^H}_{\text{Household}} - \underbrace{(i_t^H - i_{t-1}^H R_{t-1}^i)}_{\text{Firms}} = \underbrace{rx_t - rx_{t-1}R_{t-1}^r}_{\text{Reserves}} - \underbrace{(b_t^F - b_{t-1}^F R_{t-1}^f)}_{\text{External Debt}} \tag{16}$$

The interest rate on reserves R_t^r and the interest rate on foreign debt R_t^f follow two exogenous processes.

The central bank can choose to finance the accumulation of foreign reserves rx_t through foreign debt b_t^F or by suppressing domestic consumption by borrowing from households b_t^H. This is the cost of holding foreign reserves for EM economies in the model. The economy will experience temporary loss in consumption or borrow from the rest of the world at a higher interest rate than the safe interest rate paid on holding foreign reserves.

The government collects a proportional tax from the firms and then rebates it back to the firms as a lump-sum transfer

$$\tau_t i_t^F = T_t \tag{17}$$

4.4 Foreign investors

Foreign investors are the shareholders in domestic firms because they invest in the firms through FDI inflow i_t^F and obtain a share of the firm's output in the subsequent periods. An investor decides in every period how much to invest in the firms. Foreign investors do not have perfect information on the economic conditions of the recipient EM economies. According to the investors' beliefs, the level of reserves rx_t affects the volatility of the return on their investment.[4]. This is the key assumption in the model: the expected volatility of the return on FDI is a decreasing function of the level of reserves. Some empirical evidence on this assumption will be provided in the next section.

Let us assume that foreign investors face two choices of assets in each period. One is the risky asset FDI i_t^F, which foreign investors believe will pay a stochastic return, and the volatility of which depends on the level of reserves. The other is a global safe asset that always

[4]A simple micro-foundation of this belief is provided in the Appendix

pays a constant interest rate. Then, this is an infinite-horizon portfolio allocation problem between risky and riskless assets.

Allow me to deviate from the main setup of the investors' problem by assuming for the present that the FDI asset is a one-period risky asset for the foreign investors to characterize the problem and provide the explicit solution of the investors' demand for FDI.

Suppose that a foreign investor at the beginning of every period has wealth W_t; he or she needs to decide how much to consume and how much to invest in a risky asset, which is FDI in EM economies, or a risk-free asset. Let R_1 denote the gross return on the risk-free asset. Let G_t^f denote the gross risky return on the FDI asset. Investors expect lnG_t^f to follow a normal distribution $N(d_t, \sigma_t^2)$. The utility function of the investors follows a constant relative risk aversion utility function with γ being the risk-aversion parameter. a_t represents the share of wealth that investors invest in the risky FDI asset. The budget constraint and maximization function of the foreign investors are thus as follows:

$$\sum \beta^t U(C_t^i)$$

subject to

$$W_{t+1} = a_t(W_t - C_t)G_{t+1}^f + (1 - a_t)(W_t - C_t)R_1$$

Here, I follow the same setup as the Merton-Samuelson discrete-time portfolio allocation model. I obtain the solution that C_t^i is a constant fraction of W_t, bW_t. a_t is an explicit function of the other variables in the model:

$$a_t = \frac{E_t G_{t+1}^f - R_1 + \frac{\sigma_{t+1}}{2}}{\gamma \sigma_{t+1}}$$

The share of wealth that the investors will invest in the risky FDI asset depends on two key factors. The first is the spread between the expected return and the safe asset. The second is the volatility of the demand for the FDI investment under the assumption that FDI is a one-period asset:

$$i_t^F = a_t(W_t - C_t) = (1 - b)W_t \frac{E_t G_{t+1}^f - R_1 + \frac{\sigma_{t+1}}{2}}{\gamma \sigma_{t+1}}$$

However, it is unrealistic to assume that the FDI asset only pays a one-period return. To keep the model tractable and incorporate the features of FDI, I take the functional form from the single-period payment case and apply it to the multi-period payment of FDI, which yields the following investment decision for the foreign investors:

$$i_t^F = W_t E_t(\frac{G_{t+1}^f}{\gamma} r x_t^{\zeta}) \tag{18}$$

where σ_{t+1} explicitly takes the form of $rx^{-\zeta}$, which is a decreasing function of the level of reserves. W_t represents the wealth of the foreign investors. G^f_{t+1} represents the return on the asset. γ is the risk aversion of the investors in their power utility function

The share owned by the investors for the newly invested i^F_t in the firm is defined as $\frac{i^F_t}{k_{t+1}}$. This share will remain in the recipient firm. Investors can obtain a series of returns for every period in the future, but the share will depreciate at the depreciation rate of capital, δ. The expected payoff is the share owned $\frac{i^F_t}{k_{t+1}}$ times the output of the firm $A_{t+1}k^\alpha_{t+1}$ from the next period, and the resale value takes into account the devaluation in the next period $p^i_{t+1}(1-\delta)$ over the investment made in this period i^F_t. Thus, the expected return on FDI asset G^f_{t+1} is characterized as follows:

$$E_t(G^f_{t+1}) = \frac{E_t(\frac{i^F_t}{k_{t+1}}(A_{t+1}k^\alpha_{t+1} + p^i_{t+1}(1-\delta)))}{i^F_t} \tag{19}$$

The price for the firm is the production in next period plus the discounted price of next period.

$$p^i_t = E_t(A_{t+1}k^\alpha_{t+1} + \beta^i p^i_{t+1}) \tag{20}$$

where β^i represents the discount factor for foreign investors and is endogenously given. The evolution of the wealth of investors is given by

$$W_{t+1} = (1-b)W_t - i^F_t + x_{t-1}A_t k^\alpha_t \tag{21}$$

4.5 What is FDI?

The setup in the model of modeling FDI liability is obviously a reduced form of modeling of the real activities related to FDI. However, I argue that this modeling captures the critical features of FDI that are of interest in this paper and its relationship linking foreign and domestic entities. I also attempt to evaluate the effect of this simplification on the main mechanism that I capture in the model.

The model simplifies the forms of private inflow into domestic firms. FDI is, in reality, defined as foreign residents owning more than 10% (voting power) of domestic firms, both directly and indirectly. Firms can take on foreign investment through portfolio investment in the form of an equity acquisition of less than 10% and through private debt flow. In the model, FDI is modeled as a foreign investment in the domestic firm via equity acquisition. The private inflow is also simplified to consist only of FDI inflow, not other equity inflows or debt instruments. This simplification is a fair representation of private inflows into EM economies in recent years. Most of the private investment in EM economies is through a direct investment relationship. I use the data from IFS to demonstrate that FDI is indeed the main form of private capital inflow into firms. IFS has decomposed the net international liability position of the economies into Direct investment, Portfolio investment and Other investment. Under Portfolio investment and Other investment, there are three subcategories of private corporations' liabilities. I calculate the share of the direct investment over the sum of direct investment and these three other corporate liability categories. The results suggest that FDI liability has increased to approximately 85% of total liabilities in recent years. This justifies my modeling choice of only including FDI liabilities for the firms.

The model also differs from the actual accounting value of FDI stocks and inflows. Based on IMF standard BPM6 and OECD standard BMD4, the recommendation for reporting FDI statistics, both stocks and flows, is that they be based on market value. However, it is challenging to produce market values of FDI positions because the equity of many direct investment enterprises is not listed. Often, the only information available to compilers is the book value available on either the books of the direct investor or the target enterprise. Thus, if market values are unavailable, FDI data at book value should be adjusted or imputed to estimate the market value. The main purpose of using market value is to enable the comparison of different forms of accounting items for the economy/firms. Due to the complexity of market value accounting and the irrelevancy of using market value to the main mechanism of the model, I do not use the market value of the firms to represent FDI inflows and stocks.

The FDI inflow into the model economy is defined as i_t^F, or the inflow of foreign investment, which is in the unit of domestic and foreign consumption goods. Since the units of reserves and foreign debts in terms of consumption goods are also one, one can compare the current account component of the economy, including the reserve flow, the debt flow and the FDI flow in the model.

The FDI stock in the model is defined as the claim on the firm's capital instead of the claim on the market value of the firm. This is to simplify the net investment position of domestic firms. Given the definition of FDI inflow in the model, the claim on domestic capital enables us to decompose the change in investment position from the change in foreign-owned FDI stock into the new inflow i_t^F and the depreciation of the claim $-\delta x_{t-1} k_{t+1}$, such that one would have $(x_t - x_{t-1})k_{t+1} = i_t - \delta x_{t-1} k_{t+1}$, which is precisely equation (8).

These deviations from the standard accounting of FDI do not jeopardize our main channel and the correlation analysis between different forms of capital flows.

These deviations from the standard accounting for FDI do not jeopardize our main channel or the correlation analysis between different forms of capital flows.

There are several features of FDI that the model captures and are critical for the mechanism in the model. The first is that the inflow of FDI is influenced by the reserve level. This is to capture the advantages of foreign investment over domestic investment. The second feature is the equity payment of the FDI investment. This is to capture the riskiness of FDI, namely, that the return on FDI depends on the return on capital and the unknown production within the economy. The feature of the risky return contributes to explaining why, in reality, the investors care about the stability of the country and why, in the model, the reserve serves as a signaling device. The third feature captured by the model is that FDI pays multi-period returns, which ensures that there will be a reasonable measure of the FDI stock in the economy. There are obviously other features pf FDI and reserves in reality that are not captured by the model. However, they are not central to our analysis, and adding them would complicate the model without offering further insights.

4.6 Competitive equilibrium

A stationary competitive equilibrium is a set of stationary processes c_t, y_t, b_t^H, Π_t, k_{t+1}, x_t, i_t^H, i_t^F, i_t, λ_t^{1d}, λ_t^{2d}, λ_t^{3d}, λ_t^{4d}, A_t, G_{t+1}^f, p_t^i, W_t, R_t^H, R_t^i, rx_t, b_t^F, and T_t that satisfies equations (2)-(17), given a central bank policy on R_t^i and R_t^H, τ_t, exogenous stochastic processes on technology A_t^2, A_t^3 and interest rate R_t^r, R_t^f.

4.7 The social planner

Let us solve the benevolent social planners' problem in choosing the stochastic processes c_t, rx_t, k_{t+1}, i_t, b_t^F, x_t, i_t^H, and i_t^F that maximize

$$\sum_{t=0}^{\infty} E_0(\beta^t U(c_t))$$

subject to the constraints

$$i_t^H + c_t + rx_t - rx_{t-1}R_{t-1}^r = (1 - x_{t-1})A_t(i_{t-1}^F)k_t^\alpha + b_t^F - b_{t-1}^F R_{t-1}^f \tag{22}$$

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{23}$$

$$i_t = F(i_t^H, i_t^F) \tag{24}$$

$$x_t = (1 - \delta)x_{t-1} + \frac{i_t^F}{k_{t+1}} \tag{25}$$

$$i_t^F = W_t \frac{E_t(G_{t+1}^f)}{\gamma} rx_t^\zeta \tag{26}$$

Equation (22) is the aggregate economy-wide resource constraint. The output of the economy, after the dividend payment to foreign investors, is used to consume c_t, invest i_t^H, accumulate/decumulate foreign reserves rx_t and accumulate/decumulate external debt b_t^F. It is the combination of household budget constraint equation (2), central bank budget constraint equation (16), government budget constraint (17) and the definition of profit Π_t equation (10).

If I let λ_t^1, λ_t^2, λ_t^3, λ_t^4 and λ_t^5 correspond to the Lagrangian multipliers of the above five constraints respectively, the Lagrangian associated with the maximization problem is given by

$$L = \sum_{t=0}^{\infty} \beta^t E_0\{U(c_t) + \lambda_t^1\big((1-x_{t-1})A_t k_t^{\alpha} + b_t^F - b_{t-1}^F R_{t-1}^f - i_t^H - c_t - rx_t + rx_{t-1}R_{t-1}^r\big)$$
$$+ \lambda_t^2\big((1-\delta)k_t + i_t - k_{t+1}\big) + \lambda_t^3\big(F(i_t^H, i_t^F) - i_t\big) + \lambda_t^4\big((1-\delta)x_{t-1} + \frac{i_t^F}{k_{t+1}} - x_t\big)$$
$$+ \lambda_t^5\big(W_t \frac{E_t(G_{t+1}^f)}{\gamma} rx_t^{\zeta} - i_t^F\big)\}$$

The first-order conditions with respect to c_t, rx_t, b_t^F, i_t k_{t+1}, x_t, i_t^H, i_t^F in that order is

$$U'(c_t) = \lambda_t^1 \tag{27}$$

$$E_t(\beta\lambda_{t+1}^1 R_t^r) = \lambda_t^1 - \lambda_t^5 \frac{\partial i_t^F}{\partial rx_t} \tag{28}$$

$$\lambda_t^1 = E_t(\beta\lambda_{t+1}^1 R_t^f) \tag{29}$$

$$\lambda_t^2 - \lambda_t^3 = 0 \tag{30}$$

$$E_t(\beta\lambda_{t+1}(1-x_t)A_{t+1}\alpha k_{t+1}^{\alpha-1} + \beta(1-\delta)\lambda_{t+1}^2) = \lambda_t^2 + \lambda_4 \frac{i_t^F}{k_{t+1}^2} \tag{31}$$

$$E_t(-\beta\lambda_{t+1}^1 A_{t+1}k_{t+1}^{\alpha} + \beta\lambda_{t+1}^4(1-\delta)) = \lambda_t^4 \tag{32}$$

$$-\lambda_t^1 + \lambda_t^3 F_{i_t^H} = 0 \tag{33}$$

$$E_t(\beta\lambda_{t+1}^1(1-x_t)\frac{\partial A_{t+1}}{\partial i_t^F} k_{t+1}^{\alpha}) + \lambda_t^3 F_{i_t^F} + \lambda_t^4 \frac{1}{k_{t+1}} - \lambda_t^5 = 0 \tag{34}$$

The Ramsey equilibrium is thus defined as a series of stochastic processes c_t, rx_t, k_{t+1}, i_t, b_t^F, x_t, i_t^H, i_t^F, λ_t^1, λ_t^2, λ_t^3, λ_t^4, λ_t^5, W_t, G_{t+1}^f, and p_t^i that satisfies condition equations (22) - (34) and equations (19) - (21), given the same exogenous stochastic processes on technology A_t^2, A_t^3 and interest rate R_t^r, R_t^f as in the competitive equilibrium, which will be discussed later in terms of the specific functional form.

4.8 Social planners and the competitive equilibrium

I argue that by choosing the optimal policy variables, the central bank and government can replicate the social planner's optimal solution under competitive equilibrium.

Suppose that we have the solution from the social planner's problem. The variables λ_t^1-λ_t^5, c_t, rx_t,k_{t+1}, i_t, b_t^F, x_t, i_t^H, and i_t^F are already pinned down by the social planner's equilibrium conditions. Now, we need to pin down the other corresponding variables by letting them satisfy all the competitive equilibrium conditions. Then, I argue that by satisfying equation (22) to equation (34) and equations (19) to (21), the competitive equilibrium conditions (2)-(17) are also satisfied.

Let us first consider the firm's problem in the competitive equilibrium. Given the definition of $M_{0,t} \equiv \frac{\beta^t \lambda_t^{1d}}{\lambda_0^{1d}}$, if I let $\lambda_t^1 = \lambda_t^{1d}$, $\lambda_t^{2d} = \frac{\beta^t \lambda_t^2}{M_{0,t}\lambda_0^1}$, $\lambda_t^{3d} = \frac{\beta^t \lambda_t^3}{M_{0,t}\lambda_0^1}$, $\lambda_t^{4d} = \frac{\beta^t \lambda_t^4}{M_{0,t}\lambda_0^1}$, $\tau_t = \frac{\lambda_t^5}{M_{0,t}} \frac{\beta^t}{\lambda_0^1}$, $R_t^i = \frac{\lambda_t^1}{\beta E_t(\lambda_{t+1}^1)}$, this replicates the first-order conditions in the social planner's problem. Specifically, the equations for the firm's problem under competitive equilibrium (6) (7) (8) and (11) to (15), given the value of the variables discussed above, are exactly equal to equations (23) (24) (25) and equations (30) to (34). The former sets of equations hold as long as the latter sets hold. Equation (5) is satisfied automatically by defining the output level. Equation (10) is satisfied automatically by defining the profit level. Equation (9) is satisfied automatically by defining the technology level.

Let us then match the household problem under the competitive problem and the social planner's equilibrium. Equation (3) corresponds to equation (27), as is satisfied by the equivalence of $\lambda_t^{1d} = \lambda_t^1$. Equation (4) pins down the level of R_t^H. $R_t^H = \frac{\lambda_t^1}{\beta E_t(\lambda_{t+1}^1)}$. Note that this is the same as the interest rate of borrowing from abroad R_t^f by equation (29)

In the foreign investors' problem, (18) to (21) are also included in the social planner's problem, so they are automatically satisfied.

The only equations left in the competitive equilibrium are equations (2), (16) and (17). Since everything else is pinned down, (2) pins down the level of domestic borrowing/saving b_t^H. Then, (16) is automatically satisfied by subtracting (2) from (22). Equation (17) pins down the level of T_t

This completes the proof that competitive equilibrium. Equations (2)-(17) are also satisfied once we have the social planner's optimal solution and the corresponding optimal interest rate and optimal tax rate on FDI.

5 Model parametrization

5.1 Households and preferences

I use the following standard functional forms for utility

$$U = log(c_t)$$

The household discount factor β is pinned down by the interest rate on foreign debt to ensure the existence of steady state.

5.2 Foreign investors

ζ is the key parameter in the analysis of foreign investors and is central to the mechanism of the model. As first shown in equation (18), ζ measures the reserves' ability to reduce the volatility of the expected return on FDI and thus increase FDI inflow. Therefore, I calibrate ζ by directly measuring its impact on the volatility of stock market returns from the data. The procedures for doing so will be explained in section 6.1. It is set to 0.1 in the baseline analysis. I also compare it to the result from directly assessing reserves' impact on increasing FDI inflow from the data. These two empirical measures are compatible with one another, although the analysis with FDI inflow provides a wider region for pinning down the parameter. The detailed discussion on this parameter is provided in both the empirical findings discussed in section 3.2 and the analysis on the effect of the critical parameters in section 6

β^i is the discount factor for foreign investors and is calibrated using the empirical fact that the return on FDI is on average approximately 10% as documented by Bosworth, Collins and Chodorow-Reich (2007)

γ is the risk-aversion parameter for foreign investors and set to 4, which is a standard value in the literature

b is the fraction of wealth consumed by investors. It is pinned down by the steady-state level of the consumption-to-wealth ratio for US households, which is 0.7

5.3 Firms and production

The aggregation function for total investment (i_t) composed of foreign investment (i_t^F) and domestic investment (i_t^H) is modeled as taking a constant elasticity of substitution form.

$$i_t = H(\phi {i_t^F}^\rho + (1-\phi){i_t^H}^\rho)^{\frac{1}{\rho}}$$

where H is a constant and ϕ measures the share of domestic and foreign investment in total investment. ρ measures the elasticity of substitution between domestic and foreign investment.

ρ is set to 0.1 in the benchmark analysis, and I will discuss the model's robustness to different values of ρ. In the benchmark case, the elasticity is $1/(1-0.1) = 1.11$, which is slightly higher than in the Cobb-Douglas case, in which $\rho \to 0$ and the elasticity is 1. However, it will be shown that the level of ρ does not affect our result of interest.

ϕ and $1-\phi$ are calibrated to match the empirical observation that $\frac{i_t^F}{i_t} = 0.15$, which is calculated using the average FDI inflow over the fixed capital formation for the EM economies in the sample.

H is chosen to match the steady-state domestic investment share in total investment and set to 0.85, $\frac{i_t^H}{i_t} = 0.85$. Thus, total investment in the steady state is equivalent to the summation of domestic and foreign investment. The price of total investment in terms of consumption/investment goods is exactly 1 in the steady state.

In the production function, $y = Ak^{\alpha}$, the Cobb-Douglas parameter α is set to 0.3, which is a common value in the literature.

5.4 Technology process and shocks

The technology process takes the following functional form

$$A_t = A_{t,3} + A_1(\frac{i_t^F}{i_{ss}^F})^{A_{t,2}} \tag{35}$$

The technology level A increases when there is an increase in FDI inflow i_t^F. $A_{t,3}$ is the level of technology when foreign investment (FDI inflow) is 0, in which case total investment consists solely of domestic capital. $A_{t,2}$ measures the ability of FDI inflow to increase technological development. A_1 is a constant to adjust for the share of the technology level that is affected by FDI inflow. A_t^3 and A_t^2 follow an exogenous process subject to shocks in the economy

$$lnA_{t+1,2} - lnA_2 = \rho_A(lnA_{t,2} - lnA_2) + \epsilon_{t+1}^{A_2} \tag{36}$$

$$lnA_{t+1,3} - lnA_3 = \rho_A(lnA_{t,3} - lnA_3) + \epsilon_{t+1}^{A_3} \tag{37}$$

where $\epsilon_{t+1}^{A_2} \sim N(0, \sigma_{A_2})$, $\epsilon_{t+1}^{A_3} \sim N(0, \sigma_{A_3})$
A_1, A_2, and A_3 are calibrated using the following three conditions.

- A_3 is set at 57.9% of the level of A, which matches the average technology level in 1980 relative to the technology level in 2017 for 33 EM countries. The technology level in 1980 is assumed to represent the condition when bilateral FDI flows are 0.

- In steady state, the investment-to-output ratio is the average investment-to-output ratio for the 33 EM countries in the sample, which is 0.24.

- The standard percentage deviation from the trend of the reserves-to-GDP ratio in the data is 14.32 %

The persistence of both shocks is taken to be the same, $\rho_A = 0.9$. This is taken from Eichenbaum (1991), where different values of the quarterly persistence level between 0.863 and 0.986 are considered.

The standard deviations σ_{A_2} and σ_{A_3} are taken to be the same to match the standard deviation of technology A over the mean of technology in the data, which is a 5.3% deviation from the trend. The technology level is calculated using the residual from capital and labor input in EM countries. I follow the method of Gourinchas and Jeanne (2013). The precise steps are explained in detail in Appendix D

The other two shocks in the economy come from exogenous processes R_t^r and R_t^f, the return on reserve assets and the borrowing cost of external debt, respectively. I assume that both of them follow an AR(1) process with persistence levels of ρ_{R^r} and ρ_{R^f}, respectively:

$$R_{t+1}^r - R^r = \rho_{R^r}(R_t^r - R^r) + \epsilon_{t+1}^r \tag{38}$$

$$R_{t+1}^f - R^f = \rho_{R^f}(R_t^f - R^f) + \epsilon_{t+1}^f \tag{39}$$

The level of R^r is calibrated using the US Federal Funds rate, the level of R_f is calibrated using the EMBI sovereign spread index, and their corresponding standard deviations are σ_{R^f} and σ_{R^r}.

The persistence levels of the shocks ρ_{R^r} and ρ_{R^f} are taken from Neumeyer and Perri (2005), where the persistence level of world interest rate shock is estimated to be 0.81, and that of emerging economies is 0.78. The full set of parameters and their corresponding values are summarized in Table 3 below.

Table 3: Parameter values after calibration

Parameter	Description	Value
Household		
β	Household discount factor	0.9434
Firm		
α	Cobb-Douglas parameter	0.3
ϕ	foreign investment share in total investment	0.0925
ρ	elasticity of substation between foreign and domestic investment	0.1
H	scale parameter of investment	1.3647
δ	capital depreciation rate	0.1
Foreign investors		
ζ	reserves ability in attracting FDI inflow	0.1
γ	risk aversion parameter or investors	4
b	investors' share of consumption in total wealth	0.7
β^i	investors' discount factor	0.4512
Exogenous		
R_r	steady state return on reserve assets	1.015
R_f	steady state borrowing cost of external debt	1.06
A_1	parameter in A	0.1941
A_2	steady state of $A_{t,2}$, FDI technology	0.52
A_3	steady state of $A_{t,3}$	0.27

ρ_A	persistence level of the shock process	0.9
ρ_{r1}	persistence level of the shock process	0.81
ρ_{r2}	persistence level of the shock process	0.78
σ_{R_r}	sd of R_r	0.0063
σ_{R_f}	sd of R_f	0.0259
σ_A	sd of A_2 and A_3	2.15%

Note: The table summarizes the parameter values taken in the model

6 Equilibrium level of foreign reserves and key parameters

Given the parameter values discussed above, the model predicts that the reserves-to-GDP ratio is 23% in steady-state value.

This prediction is much larger than those in the previous literature. It is closer to the average in recent years, as I show in the graphs in the empirical section 3.2, and closer to the average level of the reserves-to-GDP ratio for the East Asian economies. This suggests that by incorporating the incentives for attracting FDI inflow for reserve holding, I can explain the larger amount of reserves held by EM economies.

However, this result is sensitive to the parameter values I assign to the two main mechanisms in the model: the parameter that measures reserves' ability to increase FDI, ζ, and the parameter that represents the level of FDI inflow to increase the technology level, A_2. I next discuss in detail some empirical evidence on these parameters, how I calibrate them and what the economic meanings of the values of the these parameters are.

6.1 The value of ζ

Recall that ζ governs the inflow of FDI with respect to the level of reserves:

$$i_t^F = W_t \frac{E(G_{t+1}^f)}{\gamma} rx_t^\zeta \tag{40}$$

This equation is a reduced-form equation taken from a simpler case when letting FDI be a one-period asset for foreign investors.

$$i_t^F = a_t(W_t - C_t) = (1-b)W_t \frac{E_t G_{t+1}^f - R_1 + \frac{\sigma_{t+1}}{2}}{\gamma \sigma_{t+1}}$$

where I assume that rx_t reduces the volatility of the return σ_{t+1} by letting $\sigma_{t+1} = rx^{-\zeta}$.

I first show that there is indeed a negative correlation between the volatility of returns and the level of reserves in the data, and I then measure how large the parameter ζ is.

I use stock market volatility as a proxy for the volatility of the expected return on risky FDI assets for foreign investors. There are two reasons that I select this proxy. The first is that there is no good measure of the precise return on FDI from investing in each of the EM economies. The stock market return is relatively more accessible and comparable across economies. The MSCI country index has a good coverage of 30 economies out of the 33 economies in my sample. The similar standard in picking stocks to ensure a fair representation of the underlying local equity markets make it more comparable across economies. The second reason is that since engaging in FDI aims to buy a share of a firm, it is reasonable to assume that it pays the same return as other portfolio investment, which is represented in general by the economies' stock market returns.

The stock market return index, MSCI country index,[5] covers all the EM economies in my sample except Iran, Indonesia, and Saudi Arabia. I take the monthly data from 1980-present and calculate each month's return based on the index. The volatility of the return in each year is calculated as the past three years of volatility and then annualized.

I plot the volatility against the reserves-to-GDP ratio across economies and over time in figure 6. As the figure shows, high return volatility is correlated with a lower level of the reserves-to-GDP ratio, and this negative relationship is very significant. The negative relationship is also robust to grouping the observations by economy or by date, as shown in Appendix C.

To pin down the level of ζ in $\sigma_{t+1} = rx_t^{-\zeta}$, I run the following empirical regression.

$$log(\sigma_{it}) = \beta + \beta_1 log(rx_{it-1}) + \beta_2 log(gdp_{it-1}) + a_i + b_t + \epsilon_{it} \tag{41}$$

where a_i refer to time fixed effects and b_t refers to economy fixed effects

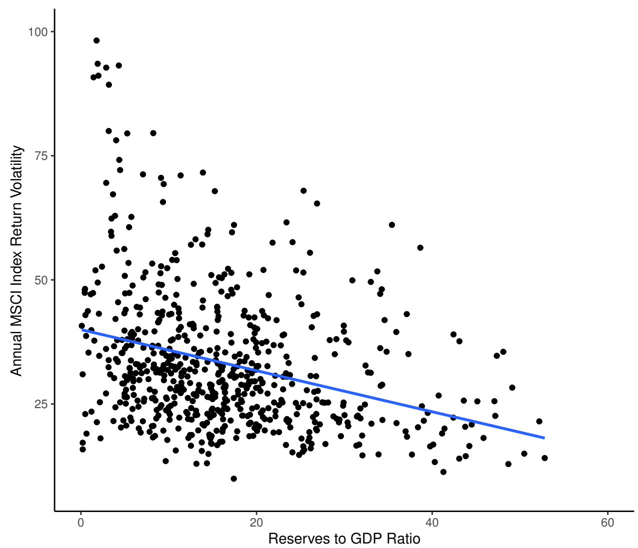
The result of this empirical analysis on parameter ζ is presented in table 4. Because I assume that $\sigma_{t+1} = rx_t^{-\zeta}$, the parameter in front of the variable $log(rx)$ is the empirical measure of ζ. The number ranges from -0.08 to -0.136. I employ -0.1 in the baseline analysis in the model, which means that $\zeta = 0.1$

I also compare this number with the empirical analysis on FDI inflow and reserve levels to determine whether the level of ζ also conforms with the empirical evidence corresponding to equation 40.

The empirical regression I present in table 2 suggests that a 1% increase in the reserves-to-GDP ratio corresponds to a 0.02%-0.076% increase in the FDI-inflow-to-GDP ratio. In essence, if we take GDP as constant across all cases and recognize that the reserve level is five times that of FDI inflow, this indicates that a 1% increase in the reserve level corresponds to a 0.1%--0.35% increase in the FDI inflow level. I further run a similar regression of equation 41 by regressing $log(FDI\ inflow)$ on $log(rx)$, which yields an estimate of ζ in the range $0.16 - 0.28$.

[5] The data are obtained from Factset

Figure 6: Return volatility and reserve-over-GDP ratio



Note: This figure plots the stock market return volatility again the reserves-to-GDP ratio for the EM economies in the sample.
Source: World Bank WDI, Factset MSCI

Table 4: Correlation between stock market volatility and reserve level

	Dependent variable: Stock market volatility (3-year moving window)			
	(1)	(2)	(3)	(4)
Log(rx)	-0.136^{***}	-0.085^{***}	-0.098^{***}	-0.087^{***}
	(0.051)	(0.029)	(0.037)	(0.027)
Log(gdp)	0.094	-0.182^{***}	0.112^{**}	-0.213^{**}
	(0.063)	(0.042)	(0.055)	(0.085)
Time fixed effect	No	No	Yes	Yes
Country fixed effect	No	Yes	No	Yes
Observations	643	643	643	643
R^2	0.107	0.467	0.418	0.694
Note:				*p<0.1; **p<0.05; ***p<0.01

This evidence suggests that $\zeta = 0.1$ is a reasonable and conservative measure for how reserves reduce the volatility of the return and provide a lower bound in the data on how reserves increase FDI inflow. I employ $\zeta = 0.1$ in the benchmark analysis if not otherwise stated and relax the number in the robustness checks in Appendix E

6.2 The level of $\mathbf{A}_2$

The change in the level of parameter A_2 also changes the steady-state level of the reserves-to-GDP ratio. As in equations (9) and (36), A_2 is the steady state of $A_{t,2}$, which measures the ability of FDI inflow to increase the technology level. Since it is not feasible to obtain a direct measure of this effect, I calibrate it to match the percentage deviation from the trend of the reserves-to-GDP ratio in the model with the data. The interpretation of A_2, combined with the level of A_3 as a fraction of A, is the percentage increase in FDI inflow for increasing the technology level. In steady state, A_3 is calibrated to be 0.58, and A_2 is calibrated to be 0.4. This approach gives the interpretation that a 1% increase in FDI inflow over GDP yields a 0.22% increase in the technology level.

This value of A_2 is also a reasonable and conservative estimation based on the empirical evidence in this study. To provide an empirical justification for the level of A_2, I calculate the technology process of the 33 EM economies in the sample by calculating the Solow residuals based on the information on GDP, employment, and capital level. The detailed steps of calculation are described in Appendix D. Then I run the following regression between productivity level A across economies and the FDI inflow across economies.

$$lnA_{it} = \beta lnFDIinflow_{it} + a_i + \epsilon_{it}$$

To account for the reverse causality that investors will invest more when the productivity level is higher, I use the risk-free rates taken from the Kenneth R. French Library to instrument for the level of FDI inflow. The intuition is that when using the risk-free rates to instrument for FDI inflows, we are utilizing the variations in FDI inflows due to the changes in the risk-free rates. When risk-free rates are higher, the investors have lower incentives to invest in risky FDI assets. However when the interest rates are lower, investors seek yields and thus invest more in the risky FDI assets, leading to more FDI inflows for EM economies. On the other hand, the change in the risk-free rates should not affect the variations in technology levels. I further add economy fixed effects to the regression to account for potential difference across economies.

Table 5 reports the result of the IV regression using three different periods. The earliest record for FDI inflow for the 33 EM economies in the sample is from 1968, which is presented in column 1. I also check the periods corresponding to the main period of empirical analysis in section 3, from 1980 to 2017, in column 2. Then I check the periods using the same length as the calibration to ζ, from 1987 to 2017, in column 3. The three regressions by including different periods suggest that β is between 0.21-0.32. The interpretation of β is the percentage increase in A when there is a 1% increase in FDI inflow. The regression results suggest that that number used in the baseline model 0.22 is a conservative and reasonable estimation

Table 5: Correlation between technology level and FDI inflow

	1968-2017	1980-2017	1987-2017
	lnA	lnA	lnA
lnfdi	0.217***	0.230***	0.319***
	(0.0128)	(0.0137)	(0.0223)
_cons	7.839***	7.703***	6.903***
	(0.106)	(0.116)	(0.194)
N	1079	1002	867

Note: Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

also based on empirical evidence. The robustness checks for the equilibrium and dynamic movements of the model by varying A_2 are discussed in the Appendix E

The two key parameters also suggest an explanation for the heterogeneity in reserve holding behavior across EM economies. The crucial impacts that these parameters have on the steady-state level of the reserves-to-GDP ratio suggest that exploring the exact levels of these parameters can be used to explain the heterogeneity in the levels of reserves across economies. The average level of Latin American economies' reserves-to-GDP ratio is approximately 10%, which can be explained by motives such as precautionary savings. However, the average level of the reserves-to-GDP ratio for Asian economies is over 20%, which requires a high probability of crisis to justify under the precautionary saving motive. However, using the possible incentive of attracting FDI inflow by holding reserves can account for the gap between the optimal level of reserves in past theories and the data. To better estimate the magnitude of this channel and its relative significance compared to other reserve holding incentives, we need to obtain more precise measures from the data for different economies of interest. This will be an essential aspect of future research on this channel if more micro-level data become available.

7 Technology shock

After discussing the equilibrium level of the reserves-to-GDP ratio, we can examine the dynamics of the variables of interest after technology shocks.

There are two kinds of technology shocks in the model based on the technology process specified in the model.

$$A_t = A_{t,3} + A^1 (i_{t-1}^F)^{A_{t,2}}$$

where A_t^3 and A_t^2 follow two exogenous processes:

$$lnA_{t+1,2} - lnA_2 = \rho_A(lnA_{t,2} - lnA_2) + \epsilon_{t+1}^{A_2}$$

$$lnA_{t+1,3} - lnA_3 = \rho_A(lnA_{t,3} - lnA_3) + \epsilon_{t+1}^{A_3}$$

We call the shock to $A_{t,2}$ the FDI technology shock and the shock to $A_{t,3}$ the classical technology shock. An increase in $A_{t,2}$ means that an increase in FDI inflow is more capable of increasing the technology level of the economy. This can be regarded as a better absorption of the advanced technology of the parent economy through better training or the presence of more high-skilled workers. An increase in $A_{t,3}$ is similar to standard technology shocks, where productivity is higher, which has nothing to do with the level of FDI inflow. It is reasonable to assume that both kinds of technology shock vary along the business cycle. FDI inflow sometimes contributes more to technology and sometimes contributes less. The classic technology shock also experiences booms and busts.

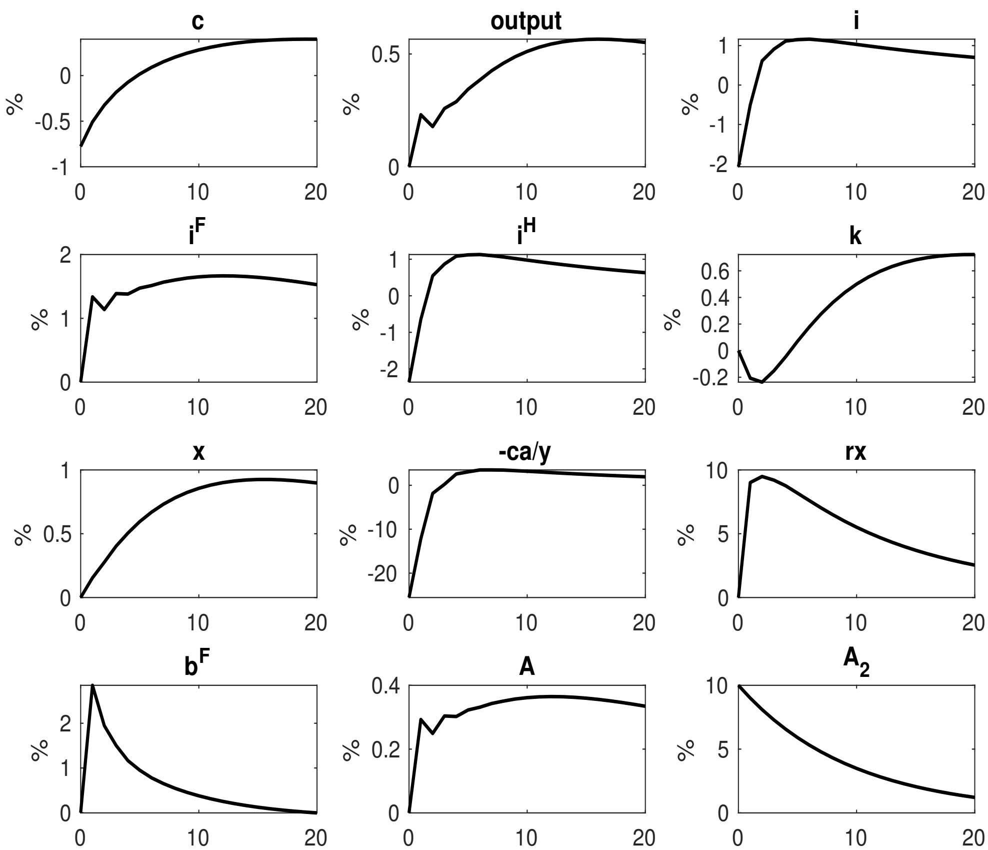
7.1 The FDI technology shock A_2

Figure 7 plots a 10% increase in the FDI technology $A_{t,2}$. We can see that $A_{t,2}$ increases by 10% following the shock and gradually returns to the steady state. The increase in $A_{t,2}$ increases in the marginal benefit of FDI inflow and thus the marginal benefit of holding reserves. We hence observe an increase in reserves after the shock. The FDI inflow correspondingly increases, so we observe an increase in i_t^F. The increase in reserves is financed by an increase in the foreign debt level b_t^F, a temporary decrease in the consumption level c, and a temporary decrease in domestic investment i_t^H. These are the costs of holding reserves. The simultaneous increase in the marginal cost and marginal benefit of holding reserves generates the co-movement between external debt and the reserve level. This explains why the economy simultaneously borrows and saves.

7.2 The classic technology shock A_3

Figure 8 plots a 10% increase in the classic technology $A_{t,3}$. We can see that $A_{t,3}$ increases by 10% following the shock and gradually returns to the steady state. The increase in the classic technology shock increases the expected return on capital. One key reason that foreign investors invest in the domestic economy is the expected return. Following the increase in the classic technology shock, foreign investors become more interested in investing in the economy for a given reserve level. The increase in the capital stock decreases the marginal productivity of capital and thus reduces the marginal benefit of holding reserves to attract one additional unit of capital. We can thus observe an initial decrease in the reserve level. The economy also allocates resources to domestic investment to make use of high technology. We can see that

Figure 7: 10% increase in A_2

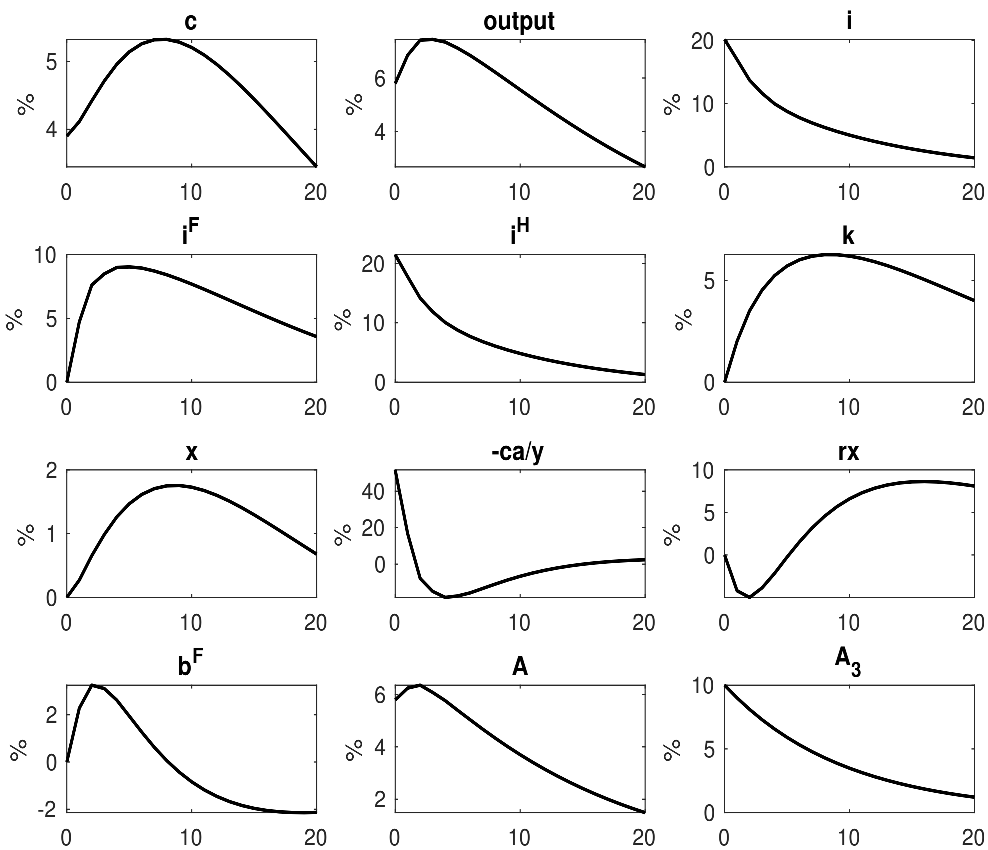


Note: This graph plots the impulse responses for a 10 % FDI technology shock on $A_{t,2}$

there is an increase in i_t^H, and the economy is borrowing more from the rest of the world b_t^F. These behaviors are the same as in the classic RBC model, which predicts that a positive technology shock leads to more investment, less saving and more borrowing. The FDI inflow is higher following the shock because of the high expected return. As the shock gradually vanishes, we observe that the economy begins to accumulate more reserves to slow the process of FDI inflow converging back to the steady state.

The difference between the increases in A_3 and A_2 is the change in the marginal benefit of holding reserves, which can be decomposed into two parts: the marginal benefit of investment and the marginal increase in investment due to a unit increase in the reserve level. An increase in the FDI technology shock does not significantly affect the level of technology but increases the marginal benefit of foreign investment. In contrast, an increase in classic technology shock substantially affects the level of technology and causes an increase in foreign investment. This increase in foreign investment significantly lowers the marginal benefit of the investment. The

Figure 8: 10% increase in A_3



Note: This graph plots the impulse responses for a 10 % classic technology shock on $A_{t,3}$

different directions of the movements in the marginal benefit of investment cause reserve levels to move differently following the two technology shocks.

$$E_t\Big(\underbrace{(\beta\lambda^1_{t+1}(1-x_t)\frac{\partial A_{t+1}}{\partial i^F_t}k^{\alpha}_{t+1} + \lambda^3_t F_{i^F_t} + \lambda^4_t \frac{1}{k_{t+1}})}_{\text{marginal benefit of investment}} \qquad \underbrace{\frac{\partial i^F_t}{\partial rx_t}}_{\text{change in investment}} \qquad \Big) = \underbrace{\beta E_t \lambda_{t+1}(R^f_t - R^r_t)}_{\text{marginal external cost}} \tag{42}$$

8 Allocation puzzle

The allocation puzzle refers to the empirical observation that there is a negative correlation between economies' net inflows of capital and productivity growth and was initially highlighted by Gourinchas and Jeanne (2013). It is considered a puzzle since one would expect that according to the classical real business cycle model, capital would flow to economies with higher technology growth. This means that if we compare economies in recent decades, we would expect those with higher average productivity growth to be those with higher capital inflow. However, empirically, as I show below, the pattern is the opposite: economies with higher average productivity growth are those with lower capital inflow. The model discussed in this paper can generate the pattern described in the puzzle. The short explanation is that following technology shocks, there is public capital outflow (reserves) intended to attract private capital inflow (FDI inflow). On net, we observe a negative correlation between net capital inflow and technology growth.

I first provide empirical evidence of this phenomenon by replicating the result of Gourinchas and Jeanne (2013). I then present the empirical counterpart in this paper by showing that the puzzle is resolved if we only consider the empirical FDI flow. However, the foreign reserve outflow is also higher in economies with higher technology growth. The last step is to simulate the model and demonstrate that it generates the same pattern as we observe in the data, and the explanation for the puzzle is precisely the mechanism proposed in the model.

8.1 Empirical observations

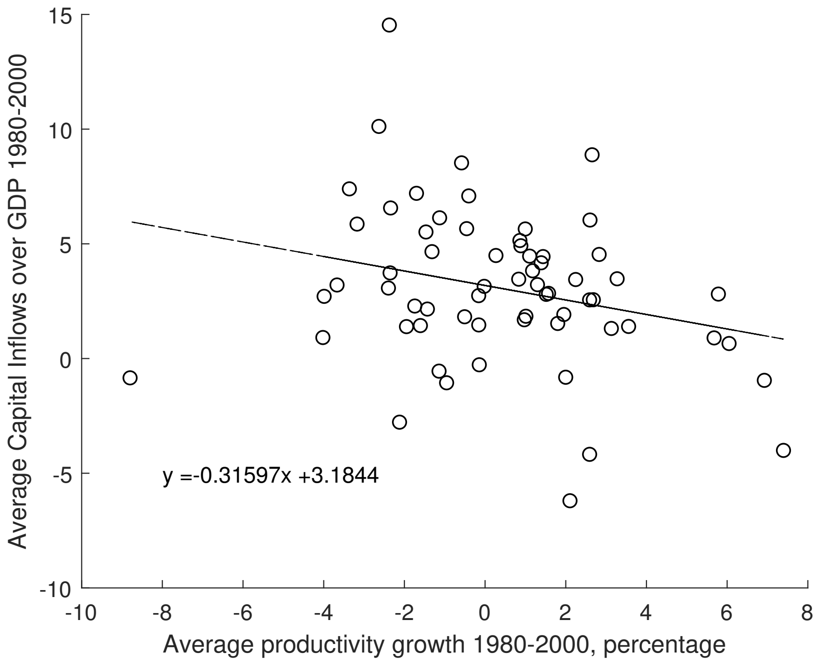
The empirical observation of the allocation puzzle is summarized in Figure 9. I plot the average productivity growth over the period 1980-2000 for 66 economies on the x-axis, and the average capital inflow is represented by the negative of the current-account-to-GDP ratio $\frac{CA_t}{GDP_t}$ on the y-axis. The details of the data construction can be seen in Appendix D. The graph suggests that among the 66 developing economies in the sample, economies with higher average productivity growth over the period 1980-2000 experience less net capital inflow on average.

Gourinchas and Jeanne (2013) further divides the net capital inflow into private flows and public flows and concludes that there is no puzzle if we only consider private flows. Economies with higher productivity growth are indeed associated with higher net private capital inflow. However, public flows show that economies with higher productivity growth are associated with less net public inflow.

When describing the motivation for this study, I noted that the liabilities of emerging economies are mainly composed of FDI liability (private capital inflow) and that their assets are mainly composed of RX (public capital outflow). When FDI inflow and reserve assets are plotted against productivity growth, we observe the pattern that generates the allocation puzzle. The private FDI inflow correlates positively with productivity growth, as does the public reserve outflow.

Figure 10 shows the positive correlation between FDI inflow and productivity growth. On the x-axis, I have the same productivity growth for the 66 countries shown in Figure 9. On the y-axis, I plot the change in FDI between 1980 and 2000 by summing all the FDI inflows

Figure 9: Allocation puzzle, net capital inflow and productivity growth



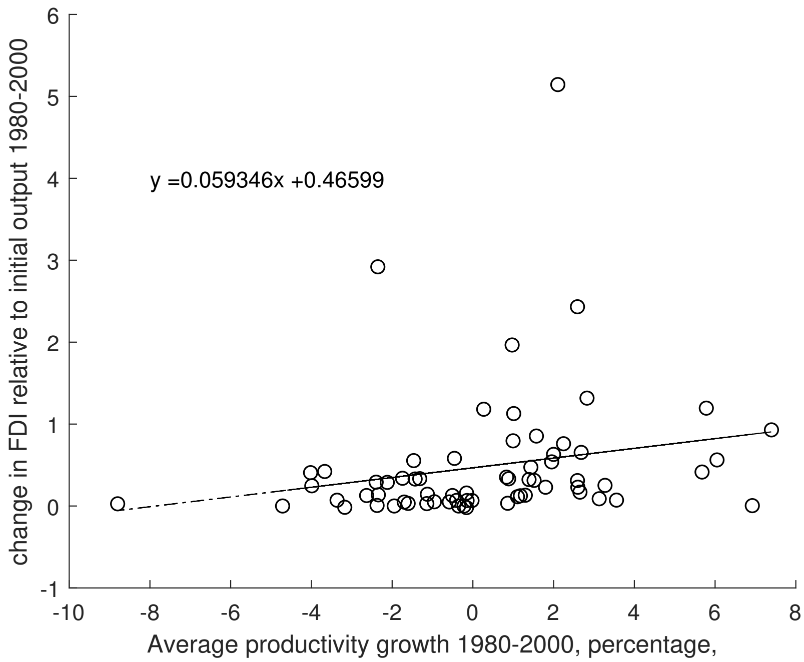
Note: Author's calculation. Due to the lack of current account data for some economies, there are 59 observations in this figure. The calculations of the technology growth and net capital inflow are described in Appendix D.
Source: Penn World Table and World Bank WDI.

during the 20-year period and scale each year's FDI inflow by the investment price level in that year. The details of the data are described in Appendix D. Figure 10 suggests that when considering private FDI flows, countries with higher technology growth indeed attract higher FDI inflow.

Figure 11 shows the positive correlation between the increase in reserve position and productivity growth. I have the same x-axis as in the previous two figures. The y-axis plots the change in reserve position over the period 1980-2000. The construction of the reserve outflow is analogous to that of the FDI inflow. Figure 11 suggests that when considering public reserve outflow, economies with higher technology growth also experience larger reserve outflows.

To summarize, the empirical evidence suggests a negative correlation between capital inflow and technology growth. However, if I decompose the inflow into public and private, specifically reserve outflow and FDI inflow, there is a positive correlation between FDI inflow and technology growth and a positive correlation between reserve outflow and technology

Figure 10: FDI inflows and productivity growth



Note: Author's calculation. There are 66 observations in the figure. The calculations of the technology growth and FDI inflow are described in Appendix D
Source: Penn World Table and IMF International Financial Statistics

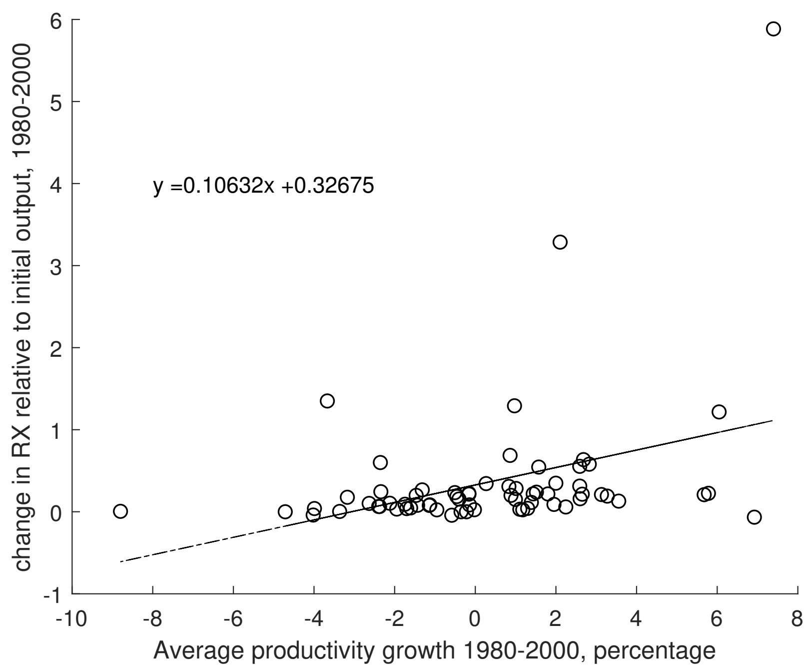
growth. This causes a negative correlation between net inflow and technology growth.

8.2 Model simulations

The model can generate the same correlations we see in the empirical observations. The model indicates that higher productivity growth correlates with less net capital inflow, larger FDI inflow, and larger reserve accumulation. To mimic the same technology growth process we observe in the data, I use the technology growth inferred from the data to feed into the shock processes in the model.

There are four shock processes in the model, namely, equations (36) to (39). Since the technology shocks in the model affect the components of A_t, $A_{t,2}$ and $A_{t,3}$, I do not have direct observations of the processes of $A_{t,2}$ and $A_{t,3}$ in the data. The only observation I have from the data is on the aggregate technology growth A_t. Thus, I feed into the simulation process the same technology growth rate as in the data to both $A_{t,2}$ and $A_{t,3}$. If the observed growth rate of technology in a specific year is 3%, then I let $A_{t,2}$ and $A_{t,2}$ experience the same percentage shock in that year. The interest rate shocks on R_t^r and R_t^f are shut down in the

Figure 11: Reserve outflows and productivity growth



Note: Author's calculation. There are 66 observations in the figure. The calculation of technology growth and the change in reserves assets are described in the Appendix D
Source: Penn World Table and IMF International Financial Statistics

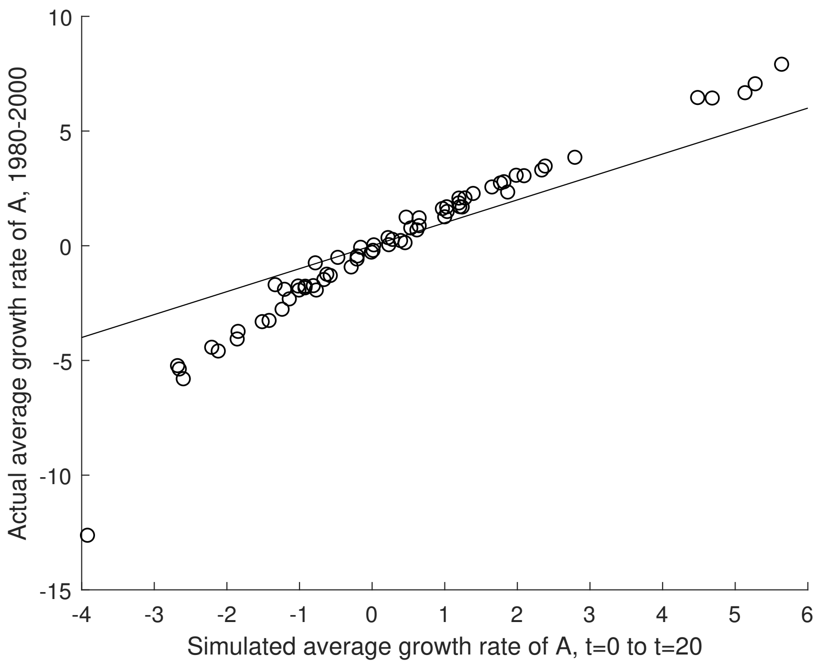
simulation. The simulation is performed for the technological progress of all 66 countries used in the empirical analysis.

I begin the simulation from the steady state, treating that as t=0. The persistence parameter ρ_A is changed to 1 so I can more easily match the actual technology growth. The actual data I have are a series of technology growth observations from 1980 to 2000 for 66 economies, where each number represents the growth rate between two consecutive years, e.g., g_t^{data}. Each period's technology shock is calculated from the actual series by

$$\epsilon_t^A = \prod_{i=1}^{t} g_i^{data} - \prod_{i=1}^{t-1} g_i^{data}$$

Under this construction, the percentage change in A_t^2 and A_t^3 from the steady state in the model is the same as the percentage change in A from the initial period in the data. Although this setup does not necessarily generate the same average productivity growth in the model as in the data, the two are quite similar. In Figure 12, I have the simulated average productivity growth for 20 periods plotted on the x-axis and the actual average productivity growth from the data plotted on the y-axis. The simulated average technology growth is calculated by using the last period's technology level A^i_{20} and taking the geometric average. The dots almost align on a 45-degree line, suggesting that the simulated productivity growth is close to the actual productivity growth.

Figure 12: Model simulated productivity growth and actual productivity growth

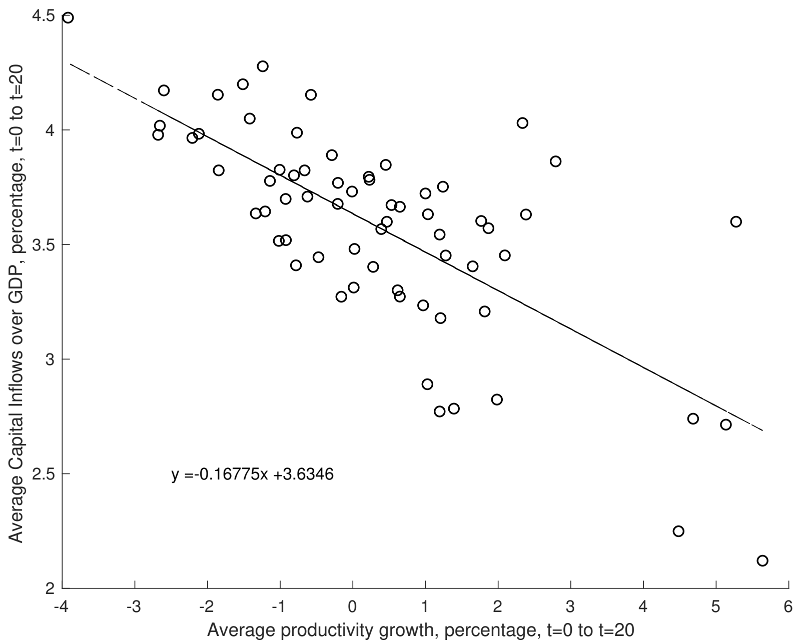


Note: The x-axis depicts the simulated technology growth. The y-axis depicts the actual technology growth. The straight line is a 45-degree line, suggesting $y = x$.

Figure 13 depicts the model simulated version of the allocation puzzle. The y-axis plots the average net capital inflow over the initial output. The x-axis plots the average technology growth. There is a clear negative correlation between technology growth and average capital inflow. After I simulate the model with the shocks described above, I calculate the average current-account-to-GDP ratio (ca/y) as the average over the 20 periods and take the negative of it as the net capital inflow. The average productivity growth is the same as discussed above.

Note that the negative correlation in Figure 13 is the combination of the effect of the FDI technology shock, $A_{t,2}$, and the classic technology shock $A_{t,3}$. The FDI technology $A_{t,2}$ shock increases the marginal benefit of FDI inflow, since now one unit of FDI inflow can contribute more to the technology level. The increase in the marginal benefit of FDI inflow needs to

Figure 13: Model simulation of the allocation puzzle

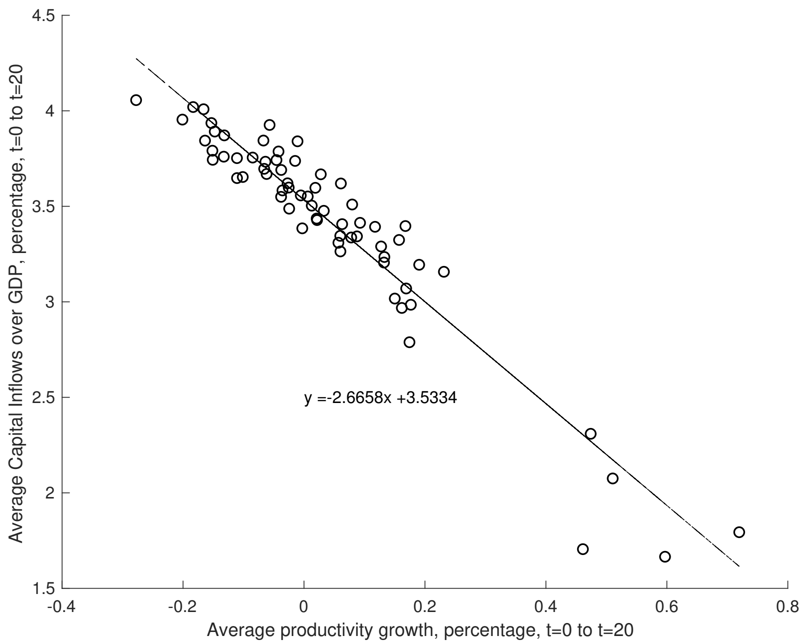


Note: Model simulation of the allocation puzzle.

be accompanied by a decrease in the marginal increase in FDI inflow per unit increase in reserves to equalize equation (42). The decrease in the marginal increase in FDI inflow can be achieved by an increased level of reserves. The increase in FDI inflow cannot compensate for the increase in reserve level, and we see a strong negative correlation between technology growth and capital inflow under $A_{t,2}$, which is shown in Figure 14

The classic technology shock $A_{t,3}$ generates a consistent result with what we would expect from a neoclassic growth model. Higher technology growth corresponds to larger capital inflows, which is shown in Figure 15. The graph is plotted by only feeding in the model the classic technology shock $A_{t,3}$ and shutting down the FDI technology shock $A_{t,2}$. The classic technology shock induces a sizable increase in foreign investment in the first place but not the marginal benefit of foreign investment. This significant increase in FDI inflow reduces the marginal benefit of FDI inflow. Thus, it needs to be compensated by a reduction in reserve levels, that is, to make the marginal increase in FDI inflow larger for a unit increase in the reserve level, to make equation (42) equalized. The increase in the FDI inflow and the reduction in the reserve level together contribute to the positive correlation of technology growth and capital inflow.

Figure 14: Model simulation with only FDI technology shock $A_{t,2}$



Note: Model simulation of the allocation puzzle, with only the FDI technology shock.

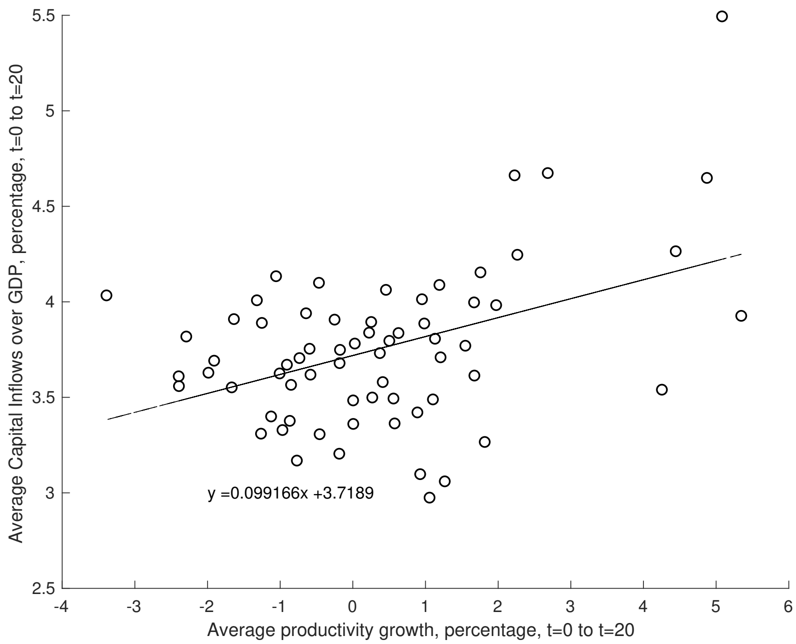
Overall, with both shocks in $A_{t,2}$ and $A_{t,3}$, the combination suggests that the $A_{t,2}$ overturns the positive correlation arising from the $A_{t,3}$ shock and gives us the negative correlation of the allocation puzzle with both shocks in the model.

Figure 16 shows the model-simulated version of the positive correlation between technology growth and the change in reserve position. The change in reserve position is taken as the deviation of the last period's reserve from the steady state. When productivity growth is high, there is also more substantial reserve outflow.

Figure 17 shows the model-simulated version of the positive correlation between technology growth and FDI inflow. The FDI inflow is calculated as the total FDI inflow for all periods relative to the initial output. When productivity growth is high, there is a larger FDI inflow.

Overall, the model matches the observations in the data. The explanation for the allocation puzzle based on the model is precisely the mechanism of the model, in which the public capital outflow is used to incentivize private capital inflow.

Figure 15: Model simulation with only FDI technology shock $A_{t,3}$



Note: Model simulation of the allocation puzzle, with only the classic technology shock.

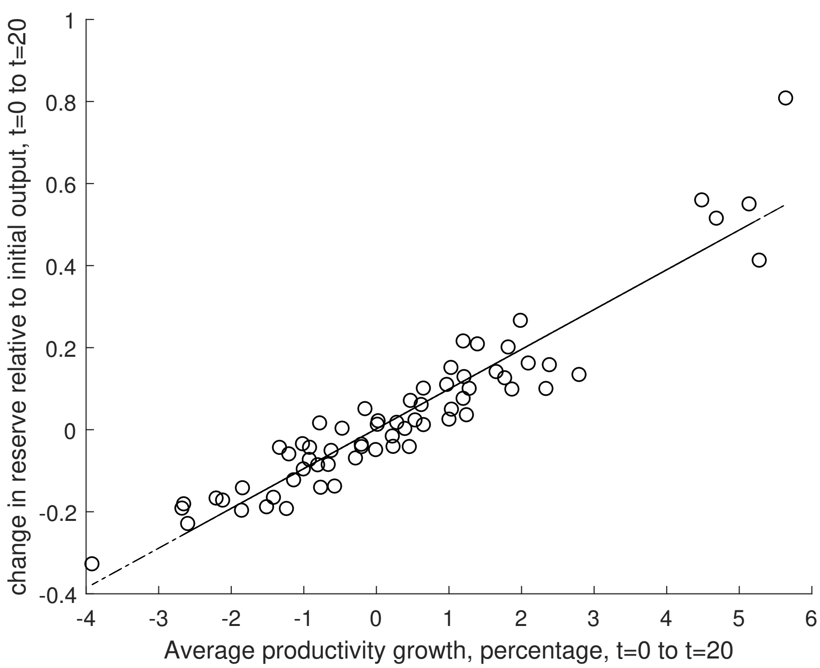
9 Reserves correlation with other variables

In this section, I compare the model's unconditional moments to the data. To do so, I conduct a three-million-period simulation of the model by drawing a sequence of $\epsilon_t^{R^r}$, $\epsilon_t^{R^f}$, $\epsilon_t^{A_2}$, and $\epsilon_t^{A_3}$ and feed them into the policy functions to obtain the time-series for all other variables.

Table 6 shows the moments and correlations of reserves and other macro variables, providing a comparison between the data and model. I report the comparison of the data to the model both under all shocks and one shock at a time.

In general, I can reproduce the correlations between reserves and other variables. The model predicts a similar level of volatility of the current-account-to-GDP ratio as observed in the data. The correlations between the reserves-to-GDP ratio and FDI-inflow-to-GDP ratio are close to the actual moments from the data. The correlation between the reserves-to-GDP ratio and FDI-stock-to-GDP is more substantial than the data, probability due to the lack of adjustment cost for both variables. The correlation between the reserves-to-GDP ratio and GDP is positive, as in the data. The correlation between the reserves-to-GDP ratio and the external-debt-to-GDP ratio for the full model is slightly negative compared to what we see in the data. However, we do observe positive comovement between the two series following

Figure 16: Model simulation of reserve outflows and productivity growth



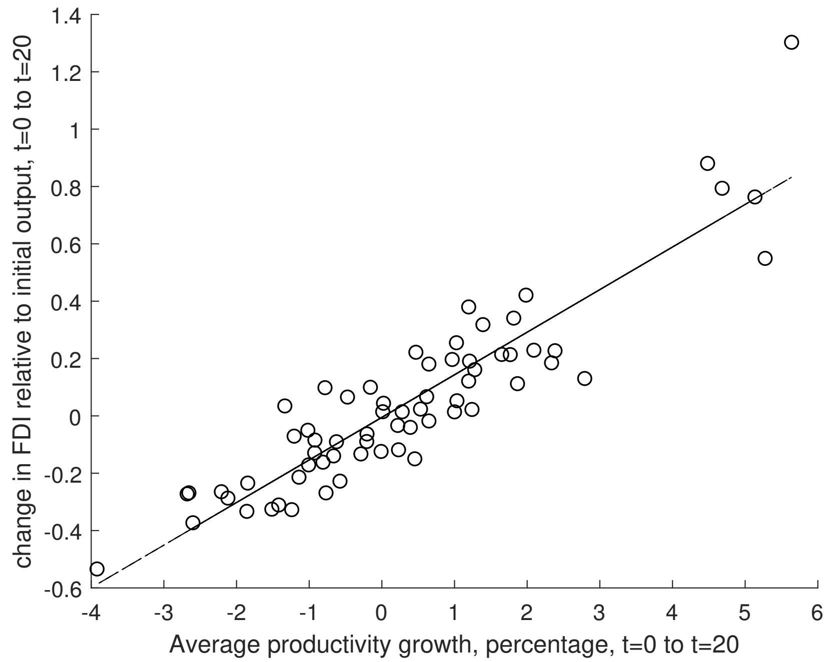
Note: Model simulation of the allocation puzzle between reserves and technology growth.

specific shocks (A_2, R_t^f, R_t^r). The negative correlation is dominated by the classical technology shock A_3, due to the restriction that the sizes of the two shocks be equal.

Comparing the model's performance under specific shocks, we can see that the model under the classical technology shock indicates that the reserves-to-GDP ratio has negative correlations with the FDI-inflow-to GDP ratio, the output level and the external-debt-to-GDP level. These are of the opposite directions from the observations in the data.

The model predicts a higher level of the reserves-to-GDP ratio for EMs than observed in the data, suggesting that countries are under-accumulating reserves on average. However, the predicted level of the reserves-to-GDP ratio is closer to the level of Asian economies, which are the economies that are accused of over accumulating reserves to an extent that is difficult to justify using classical incentives. The result from the model shows that, by allowing for the incentive of attracting FDI inflows into the country, one can generate a much larger level of the equilibrium reserves-to-GDP ratio.

Figure 17: Model simulation of FDI inflows and productivity growth



Note: Model simulation of the allocation puzzle between FDI inflow and technology growth.

Table 6: Business Cycle Moments

Variable	Data	Model	Model (A2)	Model (A3)	Model (R1)	Model (R2)
Targeted						
$\sigma_{rx/y}$	14.37%	14.63%	5.64%	7.48%	13.09%	17.3%
Non-Targeted						
$\mu_{rx/y}$	13%	23%	23%	23%	23%	23%
$\sigma_{ca/y}$	80%	89.34%	6.37%	14.75%	20.38%	76.49%
$\rho(rx/y, i_t^F/y)$	0.52	0.53	0.61	-0.28	0.73	-0.38
$\rho(rx/y, k_t^F/y)$	0.43	0.89	0.52	0.41	0.39	0.35
$\rho(rx/y, y)$	0.10	0.57	0.61	-0.24	0.54	0.72
$\rho(rx/y, , bf/y)$	0.35	-0.11	0.34	-0.15	0.59	0.89

10 Conclusion

Why do we observe large accumulations of reserves? This paper answers this question from a novel perspective: public capital outflow is intended to attract more private capital inflow.

This explanation also speaks to the co-movements of capital flows observed in the data on EM economies.

It is important to emphasize that I abstract from some important features of foreign exchange reserves as analyzed in the literature, including precautionary savings and reducing the probability of a crisis. Including these features would unambiguously lead to an increase in the optimal level of reserves. However, the aim of this paper is not to provide policy guidance regarding the optimal level of reserves. It should instead be used to understand the unexplored incentives for reserve-holding behavior. If there were policy coordination between investor economies and the recipient economies on other signals instead of foreign exchange reserves, the cost of attracting private investment will be significantly reduced for EM economies. Such policy discussions regarding optimal reserve-holding behavior could be further pursued by multilateral institutions such as the IMF and central banks around the world as an important area for future research.

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A Empirical analysis

A.1 Sample of countries

Emerging Market Economies

In the empirical analysis, the sample of EM economies is taken by using the union of the definitions from MSCI, IMF and Brics+Next eleven. The three definitions include the 33 economies listed below:

Argentina, Bangladesh, Brazil, Bulgaria, China, Chile, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Iran, Malaysia, Mexico, Nigeria, Pakistan, Peru, Philippines, Poland, Qatar, Romania, Russian Federation, Saudi Arabia, South Africa, South Korea, Thailand, Turkey, United Arab Emirates, Ukraine, Vietnam, Venezuela.

Advanced Economies

The sample of advanced economies follows the definition of IMF, covering 47 economies:

Andorra, Austria, Belgium, Cyprus, Czech Republic, Denmark, Estonia, Faroe Islands, Finland, France, Germany, Greece, Guernsey, Holy See, Iceland, Ireland, Italy, Jersey, Latvia, Liechtenstein, Lithuania, Luxembourg, Malta, Monaco, Netherlands, Norway, Portugal, San Marino, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom, Hong Kong, Israel, Japan, Macau, Singapore, South Korea, Taiwan, Bermuda, Canada, Puerto Rico, United States, Australia, New Zealand

A.2 Other empirical evidences of co-movements

This section describe further empirical observations of the co-movement between reserve levels and FDI levels, supplementing section 3

A.2.1 Yearly snapshot of FDI-stock-to-GDP ratio and reserves-to-GDP ratio

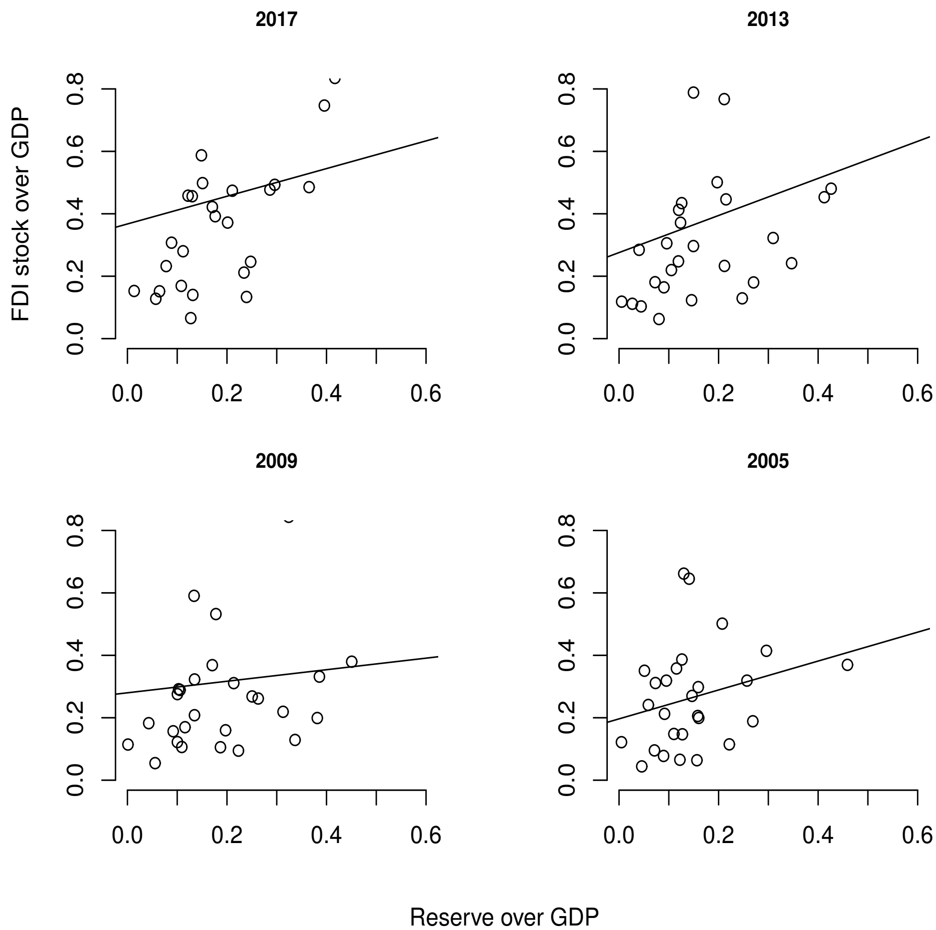
Figure 18 is composed of multiple yearly snapshots of Figure 5. I plot, within each year, the FDI-stock-to-GDP ratio against the reserves-to-GDP ratio for all the economies in the sample[6] for the years 2017, 2013, 2009, and 2005. The positive correlation still holds when I restrict the economies to these individual years. Economies with a high reserves-to-GDP ratio are also economies with a high FDI-stock-to-GDP ratio.

A.2.2 The change in the reserves-to-GDP ratio and FDI-stock-to-GDP ratio

Figure 19 plots the percentage change in the reserves-to-GDP ratio and FDI-stock-to-GDP ratio for the period 2009-2017. Economies that have a higher growth rate of the reserves-to-GDP ratio also have a higher growth rate of the FDI-stock-to-GDP ratio.

[6] Depending on data availability, the years have slightly different samples.

Figure 18: FDI stock/GDP and Reserve/GDP, yearly snapshot



Note: This scatter plot shows the reserves-to-GDP ratio and FDI-stock-to-GDP ratio for 33 major EM economies in four years.
Source: IMF IFS and World Bank WDI

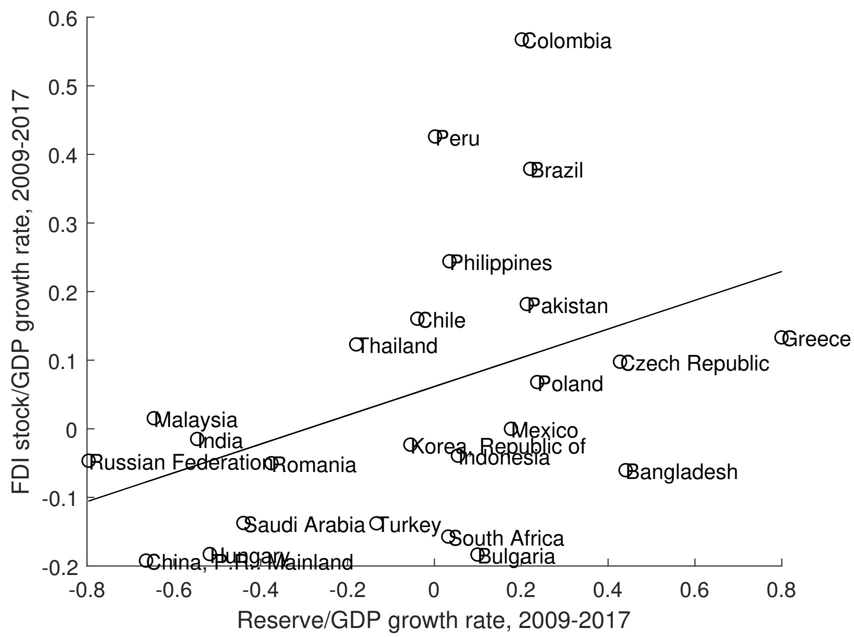
A.3 Regression with control variables

The empirical model with other control variables is specified as follows

$$\frac{FDI_{it}}{GDP_{it}} = \beta_1 + \beta_2 \frac{Res_{it-1}}{GDP_{it}} + Z_{it}\Gamma_{it} + a_i + c_t + u_{it}$$

FDI_{it} refers to the FDI variable of interest; it can be FDI stock or FDI inflow as in section 3.
Res_{it-1} is the amount of RX lagged by one period.
Z_{it} are the control variables employed in the literature that are expected to affect the level of FDI. I include GDP growth, the interest rate, the depreciation rate, financial market

Figure 19: Change in reserves and FDI from 2009 to 2017



Note: This scatter plot shows the change in reserve stock and the change in FDI stock from 2009 to 2017 for major EM economies
Source: IMF IFS and World Bank WDI

accessibility, financial market efficiency, trade openness and the external debt level.

GDP growth is directly taken from the World Bank WDI. The interest rate is operationalized as the deposit rate for economies from the World Bank WDI. Trade openness is operationalized as the gross imports and gross exports over the GDP level of the economy by using the corresponding series from World Bank WDI. External debt is calculated as the external-debt-to-GDP ratio by using the corresponding series from World Bank WDI.

The depreciation rate is calculated using the local currency and dollar exchange rate from the IMF IFS. Financial market accessibility and financial market efficiency are taken from the IMF Financial Development Index Database (FDID). Financial market accessibility compiles data on large companies' percent of market capitalization and total number of issuers of debt per 10,000 adults. Financial market efficiency compiles data on the stock market turnover ratio for each economy.

c_t is a time fixed effect, which controls for the world interest rate, investor appetites, risk-taking features, etc.

a_i is the economy fixed effect, which controls for the geographic features and economy-specific characteristics.

Table 7 uses the FDI-stock-to-GDP ratio as the dependent variable. Table 8 employs the FDI-flow-to-GDP ratio as the dependent variable. Table 9 employs the FDI-liability-to-total-liability ratio as the dependent variable

Table 7: Regression of the FDI-stock-to-GDP ratio on reserves and other controls

	Dependent variable: FDI stock/GDP			
	(1)	(2)	(3)	(4)
$\text{Res}_{it-1}/GDP_{it}$	0.556***	0.507***	0.496***	0.405***
	(0.042)	(0.071)	(0.070)	(0.047)
GDP Growth_{it}	−0.001	−0.001*	0.0001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)
Deposit rate_{it}	−0.010	−0.009***	−0.010***	−0.010***
	(0.002)	(0.001)	(0.001)	(0.001)
Devaluation rate_{it}	0.068	0.107***	0.066*	0.074*
	(0.049)	(0.037)	(0.039)	(0.043)
Financial $\text{Accessibility}_{it}$	−0.358***	−0.431***	−0.392***	−0.395***
	(0.043)	(0.047)	(0.073)	(0.043)
Financial openness_{it}	−0.197***	−0.219***	−0.081	−0.099***
	(0.032)	(0.068)	(0.084)	(0.030)
Trade/GDP_{it}	−0.002	−0.002***	−0.0001	−0.001***
	(0.0003)	(0.0003)	(0.0005)	(0.0002)
Debt/GDP_{it}	0.466	0.392***	0.217	0.112
	(0.237)	(0.149)	(0.169)	(0.230)
Constant	0.413***			
Time fixed effect	*No*	*Yes*	*No*	*Yes*
Country fixed effect	*No*	*No*	*Yes*	*Yes*
Observations	277	277	277	277

Note: Data come from World Bank WDI, IMF IFS and IMF FDID

Table 7 suggests that after controlling for the commonly used control variables for FDI, reserves still have a positive and significant impact on the level of the FDI-stock-to-GDP ratio. Financial market accessibility and openness have a negative impact on the level of FDI, suggesting that economies with more developed financial markets might borrow more from the stock market and debt market. This is in line with our defined characteristics for EM economies, in that they have less developed financial markets and need to finance through FDI to fund their private firms. These characteristics give reserves the opportunity to play a signaling role.

Table 8: Regression of the FDI-inflow-to-GDP ratio on reserves and other controls

	Dependent variable: FDI inflow/GDP			
	(1)	(2)	(3)	(4)
$\text{Res}_{it-1}/GDP_{it}$	0.100***	0.090***	0.015	0.006
	(0.015)	(0.011)	(0.010)	(0.015)
GDP Growth_{it}	0.0001	0.00001	0.0002	0.00004
	(0.0004)	(0.0003)	(0.0003)	(0.0003)
Deposit rate_{it}	−0.0004	−0.0004	−0.0002	−0.0004
	(0.0004)	(0.0002)	(0.0003)	(0.0003)
Devaluation rate_{it}	0.001	0.008	−0.004	0.004
	(0.012)	(0.009)	(0.009)	(0.011)
Financial $\text{Accessibility}_{it}$	−0.034***	−0.042***	−0.004	−0.026***
	(0.007)	(0.013)	(0.012)	(0.007)
Financial openness_{it}	−0.007***	−0.009**	0.016***	0.008*
	(0.005)	(0.005)	(0.005)	(0.005)
Trade/GDP_{it}	−0.0001	−0.0001***	0.0001	0.0001***
	(0.00004)	(0.00004)	(0.00004)	(0.00004)
Debt/GDP_{it}	0.284	0.275***	0.249***	0.242***
	(0.079)	(0.069)	(0.069)	(0.081)
Constant	0.028*			
Time fixed effect	*No*	*Yes*	*No*	*Yes*
Country fixed effect	*No*	*No*	*Yes*	*Yes*
Observations	372	372	372	372

Note: See note to Table 7

Table 8 suggests that when we account for the control variables, the effect of the reserves-to-GDP ratio has a weaker impact on the FDI-inflow-to-GDP ratio, but the positive correlation persists. This could be because the control variables are highly correlated with the level of reserves and because both reserves and FDI inflow are somewhat driven by the control variables.

Table 9: Regression of the FDI-stock-to-total-liability ratio on reserves and other controls

	Dependent variable: FDI stock/Total Liability			
	(1)	(2)	(3)	(4)
$\text{Res}_{it-1}/GDP_{it}$	0.509***	0.427***	0.657***	0.396***
	(0.060)	(0.076)	(0.092)	(0.070)
GDP Growth_{it}	0.001	0.002***	0.0001	0.001**
	(0.001)	(0.001)	(0.001)	(0.001)
Deposit rate_{it}	−0.008	−0.004***	−0.015***	−0.010***
	(0.002)	(0.001)	(0.002)	(0.002)
Devaluation rate_{it}	−0.016	0.002	0.028	−0.023
	(0.056)	(0.037)	(0.029)	(0.063)
Financial $\text{Accessibility}_{it}$	−0.039***	−0.153***	−0.210***	−0.323***
	(0.041)	(0.049)	(0.064)	(0.044)
Financial openness_{it}	−0.088***	−0.106	0.016	0.018
	(0.015)	(0.068)	(0.060)	(0.021)
Trade/GDP_{it}	−0.001	−0.001***	0.001***	−0.0003
	(0.0002)	(0.0004)	(0.0004)	(0.0002)
Debt/GDP_{it}	0.058	−0.047	0.214***	0.114
	(0.098)	(0.104)	(0.080)	(0.098)
Constant	0.423***			
Time fixed effect	*No*	*Yes*	*No*	*Yes*
Country fixed effect	*No*	*No*	*Yes*	*Yes*
Observations	277	277	277	277

Note: See note to Table 7

Table 9 suggests that the reserves-to-GDP ratio has a positive and significant impact on the FDI-liability-to-total-liability ratio. The higher the level of reserves-to-GDP ratio is, the larger the fraction of external liability is in the form of FDI. The financial market variables have a negative impact on this ratio, also suggesting that the more open and efficient an economy's financial market is, the more likely the economy is to self-finance through debt and the stock market. The deposit rate has a significant and negative impact on this ratio and could be preliminary evidence of a possible carry trade in economies with high deposit rates.

B Micro-foundations of investor behavior

B.1 Investment decision rule of foreign investors

In the main formulation of the model, I let b represent the constant share of wealth that is consumed by foreign investors and, without loss of generality, model the gross safe asset interest rate R_1 as 1. In the multi-period case, I model investors in an overlapping generations setup to avoid accounting for multiple shareholders of the firm and mimicking the results discussed above. Suppose that each investor lives for two periods, young and old. The young investor in period t is endowed with $(1-b)W_t$. He or she buys the share x_{t-1} from the previous generation using $(1-\delta)p_t^x$. He or she also obtains the proceeds that the previous generation had from selling the asset. He or she then makes an investment decision regarding the FDI asset in period t i_t^Fbased on the return that the next generation will obtain. He or she also receives a dividend payment from the domestic firms $x_{t-1}A_{t\,t}^{k\alpha}$. At the end of the period, he or she becomes old, consumes $(1-b)W_{t+1}$ of the total wealth, endows the rest of his or her wealth and sells his or her shares in firm to the next young generation. In this way, I derive an investor wealth process that is similar to that in the single-period return case:

$$
\begin{aligned}
W_{t+1} = (1-b)W_t - i_t^F + x_{t-1}A_t^\alpha k_t^\alpha - (1-\delta_x)x_{t-1}p_t^x + (1-\delta_x)x_{t-1}p_t^x \\
= (1-b)W_t - i_t^F + x_{t-1}A_t^\alpha k_t^\alpha
\end{aligned}
$$

B.2 Reserves and volatility of return

In the previous section and throughout the model, I assume that the volatility of returns is affected by the level of reserves. This correlation could operate through multiple channels. The reserves could serve as insurance held by the central bank whereby when firms seek to exit the country, the central bank can guarantee the repayment of investments made in them. The reserves can also serve as insurance for investors through the traditional role of maintaining exchange rate stability if the profit of the firms is denominated in local currency but the dividend is paid in USD. Although there is a large empirical literature on the ability of reserves to stabilize the exchange rate, as discussed in the literature review, there is no theoretical model of this mechanism.

I do not intend to limit the analysis to a specific channel, and I would argue that the reality is a mixture of these possible channels. However, I provide here a simple micro-foundation for the exchange rate channel.

Apart from the players described in the main model, suppose that all decisions are made for period t and that there is a certain amount of bonds held by traders. This means that in addition to holding reserve assets, the central bank is borrowing in domestic currency from the rest of the world, which pays interest rate R. The traders can decide whether to hold the bond or exchange it for dollars, depending on their expectation of future devaluation. The payoff is as follows

	Devaluation	No devaluation
Dollar	1-g(rx)	0
Bond	r	r

If the central bank devalues, the return on devaluation is equal to a function of reserve level. $g(rx)$ is an increasing function of the reserve level. The intuition is that when the reserve level is high, the devaluation rate will be lower since more reserves would stabilize more movement. The return in the no-devaluation case is 0 for dollar holders. The returns for bond holders are r, which is the predetermined interest rate in the two cases considered here. The central bank's decision to devalue the currency depends on its loss of reserves, L/rx. A measures the total amount withdrawn by traders. The net value of maintaining a fixed exchange rate is given by θ-$\frac{L}{rx}$, where θ is exogenous and refers to the value of a peg without any reserve loss. The central bank will devalue if and only if

$$\theta < \frac{L}{rx}$$

In the first stage of the game, nature selects θ from an improper uniform distribution over the entire real line. Then, each trader observes an idiosyncratic signal about θ, which is x_i and $x_i \sim N(\theta, \sigma^{-1})$. The cumulative distribution function of the private signal is given by $\Phi(\sqrt{\sigma}(x - \theta))$, which also equals the fraction of traders who observe a signal $x_i \leq x$. In stage 2, the traders chose whether to exchange their bonds for dollars with the central bank. The traders submit bid $a_i(x_i, r)$ to the central bank if they wish to exchange their bonds for dollars. The amount of reserve loss is equal to the total demand for dollars:

$$L(\theta, r) = \int a(x, r)\sqrt{\sigma}\phi(\sqrt{\sigma}(x - \theta))$$

Suppose that the solution is a threshold rule for both central banks and traders. Traders demand dollars whenever their private signal satisfies $x_i \leq x^*(r)$, and the central bank devalues if $\theta \leq \theta^*(r)$. The expected payoff when observing $x^*(r)$ should be equalized across the two cases; let $p(x^*(r), r)$ refer to the expected probability of devaluation:

$$p(x^*(r), r)(1 - g(rx)) = r = p(\theta \leq \theta^* | x^*) = \Phi(\sqrt{\sigma}(\theta^*(r) - x^*(r)))$$

$$\theta^*(r) = \frac{L(\theta^*(r), r)}{rx} = \frac{\Phi(\sqrt{\sigma}(x^*(r) - \theta^*(r))}{rx}$$

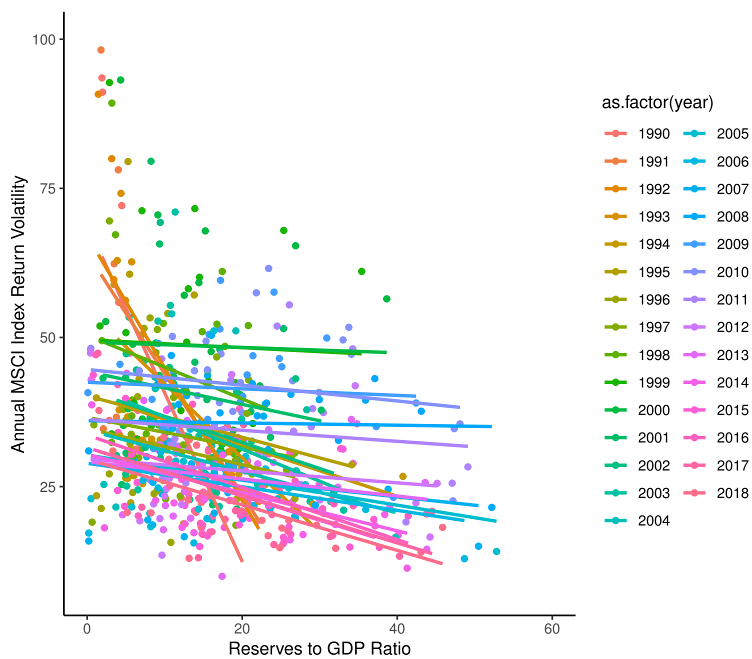
The levels of $\theta(r)$ and $x^*(r)$ for the corresponding r can be solved using the above equations. A higher level of reserves means a lower level of $\theta^*(r)$ for any r, which reduces the probability of devaluation, leading to the conclusion that reserves decrease the volatility of the exchange rate. On the equilibrium path, the central bank does not devalue with a deterministic θ but knowns that there is a possibility that θ will be subject to a shock and drawn from the distribution noted above.

This provides one micro-foundation for the level of reserves affecting the volatility of the return on assets through exchange rate stability.

C Return volatility and reserve levels by economy and by date

Figure 20 plots the same observations as presented in figure 6 but separates the observations by year. Within any given year, an economy with a larger return volatility has to a lower reserves-to-GDP ratio. The negative correlation persists even if we consider a single year.

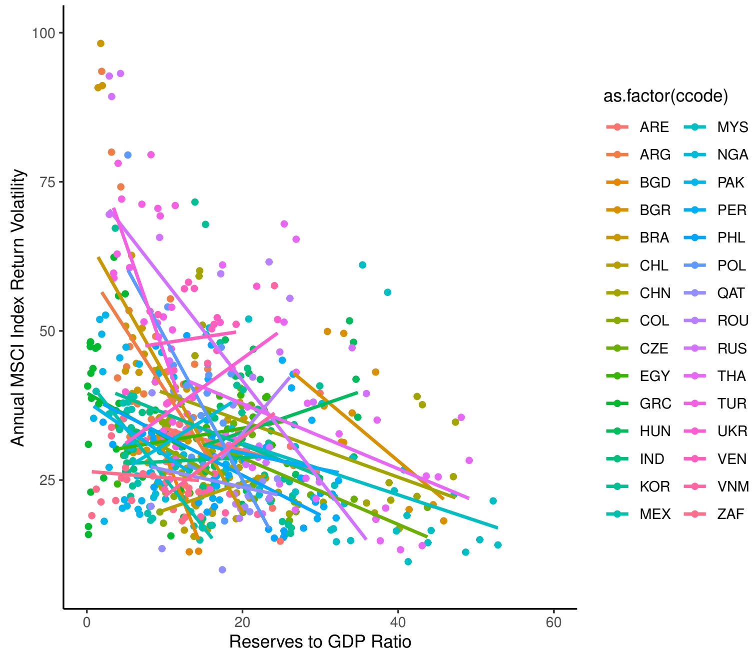
Figure 20: Return volatility and the reserves-to-GDP ratio by date



Note: This figure plots the stock market return volatility again the reserves-to-GDP ratio for the EM economies in the sample.
Source: World Bank WDI, Factset MSCI

Figure 21 plots the same observations as presented in figure 6 but separates the observations by country. Of the 30 EM economies in my sample, most show negative correlations. Overall, this negative correlation is quite robust across economies and over time.

Figure 21: Return volatility and the reserves-to-GDP ratio by economy



Note: This figure plots the stock market return volatility against the reserves-to-GDP ratio for the EM economies in the sample.
Source: World Bank WDI, Factset MSCI

D Data construction for allocation puzzle

The construction of the data used in the allocation puzzle follows the method used by Gourinchas and Jeanne (2013). The raw data come from the latest version of the Penn World Table Version 9.1, termed PWT hereafter, and the IMF's International Financial Statistics, termed the IFS hereafter

Figure 9 suggests a negative correlation between the average net capital inflow and average technology growth. The data are constructed as follows:

1. Technology growth is measured using the Cobb-Douglas production function $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$; α is set to 0.3. The country-specific output Y_t and L_t are directly taken from the PWT.

2. K_t in the above equation is measured by constructing the times-series of real investment $K_t = (1-\delta)K_{t-1} + I_t$. Real investment I_t in each period is calculated using the series Real domestic absorption and Real consumption in the PWT.

$$I_t = \text{Real domestic absorption - Real consumption}$$

The initial level of capital K_0 is constructed using the standard accounting method, where $K_0 = \frac{I_0}{\delta+\bar{g}}$. I_0 is the first observation of capital, and $\delta = 0.06$ is the depreciation rate employed in the paper. $\bar{g}$ is the geometric average growth rate of investment for the first ten observations of investment data. Then, I can obtain the series of capital levels K_t in recursive order.

3. Average productivity growth is measured using the series of productivity A calculated above and taking the geometric average.

4. Average capital inflow is calculated as the arithmetic average of the current account balance -$\frac{CA_t}{GDP_t}$ for each year because a negative current account balance indicates capital inflow. The current account-to-GDP ratio is taken from IFS.

Figure 10 suggests that there is a positive correlation between FDI inflow and productivity growth.

- FDI inflow is calculated as the accumulation of all FDI inflow from 1980 to 2000. FDI inflow data are taken from the IFS balance of payment data at the country level.

- I adjust the inflow data from nominal USD to real USD by adjusting for the investment price level pl_i in the PWT. Then, I sum up all the real FDI inflows between 1980 and 2000 and divide it by the real GDP level in the initial year, 1980:

$$\frac{\sum_{1980}^{2000} \text{Real FDI inflow}_t}{\text{Real GDP}_{1980}}$$

Figure 11 suggests a positive correlation between RX assets (capital outflow) and productivity growth; countries with higher productivity growth also accumulate higher reserve assets, which is the capital outflow, which contributes to explaining the allocation puzzle. The construction of RX change is the same as the construction of FDI change, with the RX data coming from the IFS balance of payment data.

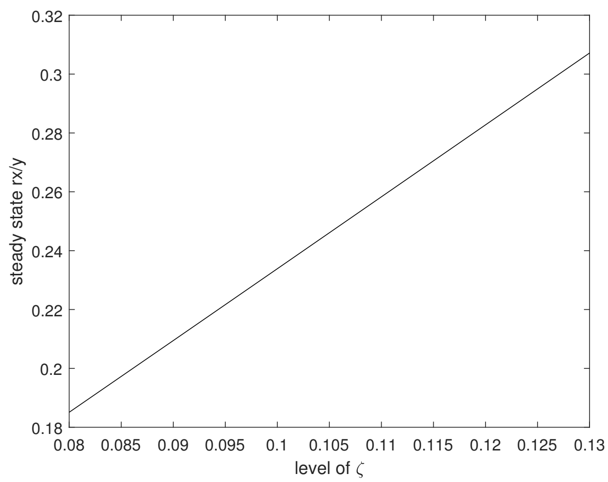
E Robustness check

E.1 The parameter ζ

Parameter ζ measures the ability of reserves to attract FDI inflow and affect the equilibrium level of the reserves-to-GDP ratio. From the empirical analysis, I have pinned down the range of ζ as $0.08 - 0.13$, as discussed in section 6. In the main analysis, I set the level of ζ at 0.1, within the range of the empirical observations. Now, I can vary the value of ζ and examine how the main analysis changes with the change in ζ.

The application here is analogous to that for varying the parameter ρ described above, in that I change the value of ζ but fix the other parameters that I use in the main analysis to obtain comparative results on the equilibrium level of the reserves-to-GDP ratio

Figure 22: Change in the level of ζ



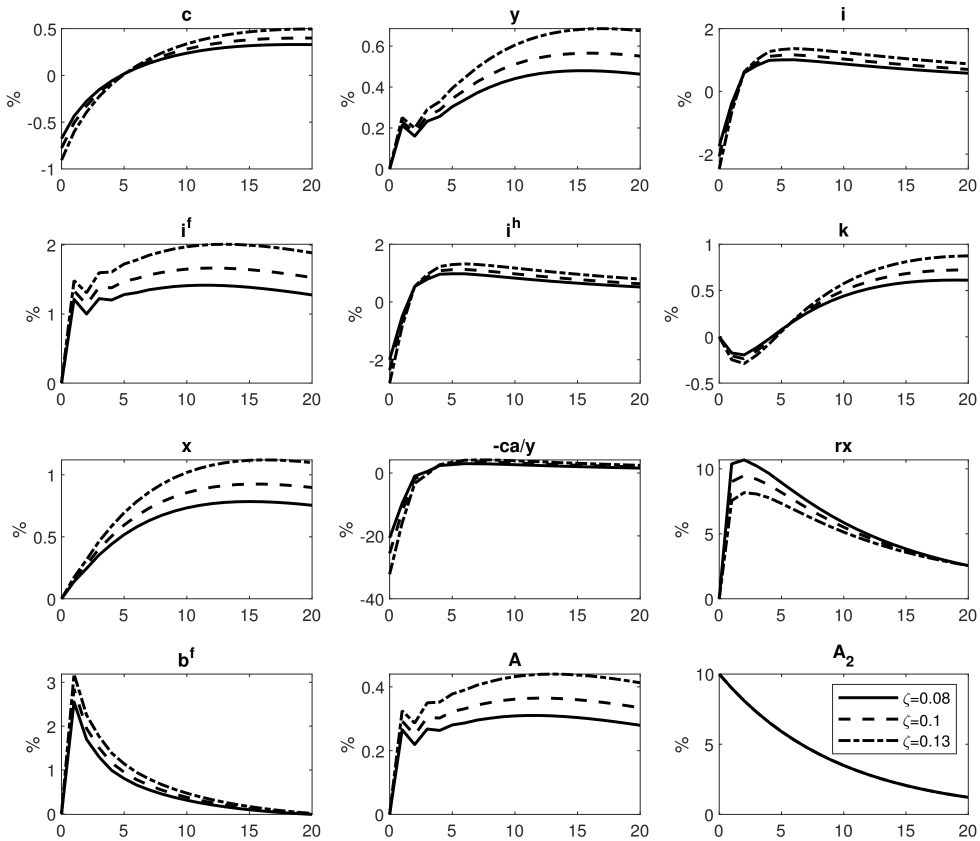
Note: This graph plots the equilibrium level of the reserves-to-GDP ratio for different values of ζ

Figure 22 suggests that the higher ζ is, the larger amount of reserves held by central banks in equilibrium. The interpretation is that a central bank would hold more reserves if one unit of reserves can attract more FDI inflow. One might expect that if the ability of reserves to attract FDI inflow is very high, the central bank would not need to hold a high level of reserves to attract a given amount of FDI inflow. However, the equilibrium level of variables is pinned down by the optimal conditions, in which the marginal cost of holding reserves and the marginal benefit of holding reserves should be equalized. Since for any level of ζ, the marginal benefit of holding reserves is decreasing in the level of reserves, a large value of ζ corresponds to a high level of reserves provided that the cost is the same. This is because when the reserve level is very high, increasing reserves by one unit does not increase the benefit of holding reserves.

The different levels of ζ also produce different responses to shocks. The impulse response for different values of ζ for a given FDI technology shock A_2 is shown in Figure 23, where ζ varies from 0.03 to 0.13. The direction and shape of the reactions of all variables to shocks are the same, but the magnitude differs. When ζ is small, the magnitude of the increase in the reserve level is large. This is because, for a given increase in the reserve level, a smaller ζ means less ability to attract FDI inflow. A large increase in reserves is need to raise the FDI inflow to the desired level. The increase in FDI inflow i^f under different specifications of

ζ also differs. Specifically, a smaller ζ, which indicates a smaller impact of reserves on FDI inflow, is accompanied by both a larger increase in reserves and a smaller increase in FDI inflow. The increase in FDI inflow increases in ζ.

Figure 23: IRF for different values of ζ

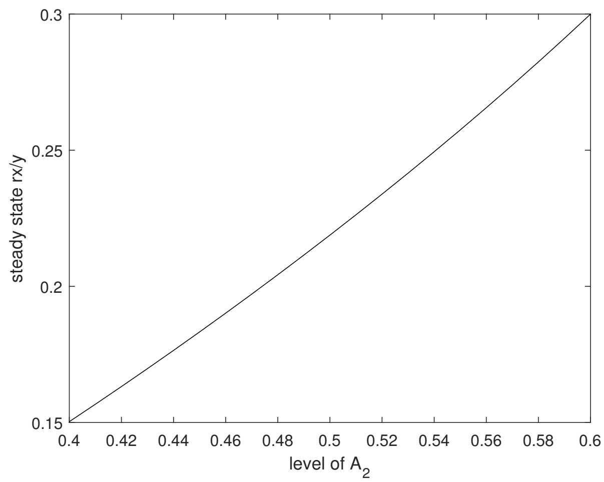


Note: This graph plots the impulse responses for a 10 % FDI technology shock on $A_{t,2}$ for different values of ζ

E.2 The parameter A_2

The parameter A_2 measures the ability of an increase in FDI inflow to increase productivity. I conduct similar exercises involving fixing other parameters in the model when varying the level of A_2 to compare the steady-state levels of the reserves-to-GDP ratio. The effect of a change in A_2 on the equilibrium level of the reserves-to-GDP ratio is similar to that of a change in parameter ζ. As suggested in Figure 24, a larger value of A_2 yields a higher equilibrium reserves-to-GDP ratio. The interpretation is the same as discussed in the previous section. The impulse responses that correspond to different values of A_2 are shown in Figure

Figure 24: Change in the level of A_2



Note: This graph plots the equilibrium level of the reserves-to-GDP ratio for different values of A_2

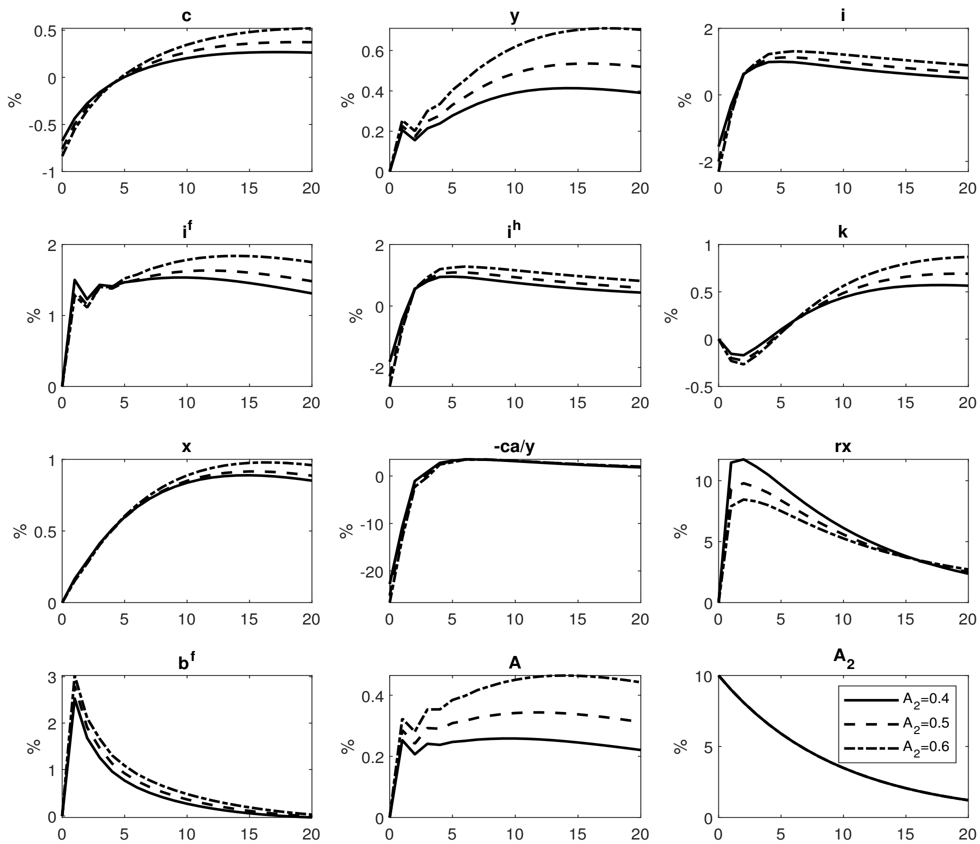
25. A higher level of A_2 corresponds to a smaller increase in reserves and a larger increase in the FDI inflow.

E.3 The elasticity of substitution, ρ

The parameter ρ, which governs the elasticity of substitution between domestic investment and foreign investment, seems to be an essential parameter for determining the level of foreign investment and thus the equilibrium level of reserves. However, it has little effect on the equilibrium level of the reserves-to-GDP ratio.

If I perform a similar exercise that involves holding the other parameters fixed from the baseline analysis and only varying the level of ρ from 0 to 0.5, meaning that the elasticity of substitution ranges from 1 to 2, the equilibrium level of the reserves-to-GDP ratio is always approximately 23%. The equilibrium level of foreign investment as a percentage of total investment decreases as the elasticity of substitution increases. However, the decrease in i_t^F increases the marginal benefit of holding one more unit of i_t^F on the level of technology. Since the cost of holding reserves has not changed, the central bank would always want to hold reserves at the level that balances the costs and benefits of doing so. Therefore, we observe that the equilibrium level of reserves is does not appear to be significantly sensitive to a change in the elasticity of substitution.

Figure 25: IRF for different values of A_2



Note: This graph plots the impulse responses for a 10 % FDI technology shock on $A_{t,2}$ for different values of A_2, the steady-state value of $A_{t,2}$