Macroeconomic forecasting with outlier-robust VAR

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September 22, 2025

Abstract

The vector autoregressive (VAR) model is one of the key models for macroeconomic forecasting and structural analysis. However, the COVID-19 pandemic has posed significant challenges to estimating and forecasting with VAR models in the post-pandemic era, highlighting the need for outlier-robust models. This study proposes an outlier-robust VAR model that decomposes the reduced-form error into a regular component and an outlier component. The VAR coefficients are estimated by imposing an L_1 penalty on the outlier component, yielding an outlier-robust estimator that is equivalent to the multivariate Huber M-estimator. The asymptotic normality of the proposed estimator is then established. To assess its practical relevance in improving the accuracy of forecasts for the post-COVID period, we estimate a five-variable VAR model using monthly data from March 1959 to June 2024 obtained from the FRED-MD database. The results show that the out-of-sample forecasts for the post-COVID period obtained from the outlier-robust VAR are generally more accurate than those obtained from the standard homoskedastic VAR and the outlier-augmented stochastic-volatility Bayesian VAR proposed by Carriero, Clark, Marcellino, and Mertens (2024).

1 Introduction

Since the seminal work of Sims (1980), the VAR model has become a cornerstone of macroeconomic forecasting and structural analysis. The COVID-19 pandemic, however, has introduced new challenges for estimation and inference with VAR models, creating a demand for approaches that are robust to extreme observations and outliers. Several strategies have been proposed to address this issue. For example, Ng (2021) employ pandemic-related indicators as controls to pre-clean the data prior to estimation. Schorfheide and Song (2024) exclude several months of pandemic-affected data when generating forecasts with VARs, rather than modeling outliers explicitly. Lenza and Primiceri (2022) argue that this ad hoc strategy of dropping extreme observations may be acceptable for parameter estimation but is inappropriate for forecasting, as it risks underestimating uncertainty. More recently, Carriero, Clark, Marcellino, and Mertens (2024) propose stochastic-volatility Bayesian VAR (SV-BVAR) models augmented with outlier terms to capture both persistent and transitory changes in volatility arising from COVID-19.[1]

Two main approaches are available for addressing outliers in regression models. The first approach involves detecting outliers in the data and then either removing them or replacing them with alternative values. Then, the regression model is estimated using the modified sample. Early contributions to the outlier-detection literature include Box and Tiao (1968) and Fox (2018). Subsequent developments extend these methods to univariate and multivariate time series, as in Pankratz (1993) and Tsay, Pena, and Pankratz (2000). A key limitation of these methods, however, is that they can fail when multiple outliers mask one another.

The second approach is to employ estimators that are relatively robust to the influence of extreme observations, even in the presence of masking. Since the seminal contributions of Huber (1964, 1973) and Hampel (1971, 1974), numerous robust estimators have been developed to mitigate the impact of outliers on parameter estimation. Notable examples include robust estimators for coefficients in univariate autoregressive moving-average (ARMA) regression models (Denby and Martin, 1979; Connor, Martin, and Atlas, 1994; Sejling, Madsen, Holst, Holst, and Englund, 1994; Politis, 2009; Muler and Yohai, 2009). A comprehensive review of robust procedures for ARMA models is provided by Maronna, Martin, Yohai, and Salibian-Barrera (2006).

Obtaining an outlier-robust estimator of VAR coefficients is crucial for macroeconomic forecasting, since VAR forecasts are generated by iteratively multiplying the estimated coefficients. Consequently, if the coefficient estimates are distorted by extreme observations,

[1] Alvarez and Odendahl (2022) apply this approach to reduced-form BVARs.

the resulting forecasts will also be affected, particularly at longer horizons. This study illustrates this point by comparing forecasts made in April 2020 using outlier-robust estimates with those using standard ordinary least squares (OLS) estimates.

Robust estimators for multivariate regression models are often developed by extending robust methods originally proposed for univariate regression models. For example, Ben, Martinez, and Yohai (1999) generalize the RA-based robust estimator for univariate ARMA models of Bustos and Yohai (1986) to VAR models. Croux and Joossens (2008) apply the multivariate least trimmed squares (MLTS) method of Agulló, Croux, and Van Aelst (2008) to VAR models. Similarly, Muler and Yohai (2013) extend the robust estimator for univariate ARMA coefficients proposed by Muler and Yohai (2009) to the VAR framework. More recently, Wang and Tsay (2023) develop a robust estimation procedure for high-dimensional VAR models, encompassing sparse, reduced-rank, banded, and network VAR specifications. A comprehensive review of robust estimation methods for multivariate and high-dimensional models is provided in Pena and Yohai (2023).

In robust estimation, outliers are typically treated as nuisance parameters, and their identification is not regarded as essential. In macroeconomics, however, identifying outliers can be particularly informative, as they often correspond to major historical or economic events. For example, outliers in a macroeconomic VAR model may be associated with episodes of heightened economic uncertainty, such as recessions or pandemics. In this regard, this study proposes a VAR framework that allows for the joint estimation of VAR coefficients and outliers.

This study assumes that the reduced-form error of the VAR model consists of two components: a regular component and an outlier component. The VAR coefficients and the outlier component can be jointly estimated using penalized least squares (PLS). Specifically, the estimators are obtained as the minimizers of an objective function defined as the sum of squared regular errors and the L_1 norm of the outlier multiplied by a regularization parameter λ.

The contributions of this paper are as follows. First, it is shown that the outlier-robust estimator of the VAR coefficients is equivalent to the multivariate Huber M-estimator, which can be viewed as a multivariate extension of Gannaz (2007). This equivalence allows the asymptotic normality of the proposed estimator to be established directly from existing results on the asymptotic properties of multivariate M-estimators. Second, it is shown that the estimated outlier component corresponds to the reduced-form error whenever it exceeds λ in absolute value, implying that the regularized error can interpreted as the reduced-form error thresholded at $\pm\lambda$.

For the empirical application, we construct a five-variable VAR model with 12 lags using monthly macroeconomic data from the FRED-MD database, following the specifications of Lenza and Primiceri (2022) and Carriero, Clark, Marcellino, and Mertens (2024). The dataset spans March 1959 to June 2024, and it is drawn from the July 2024 vintage of the FRED-MD database. The model includes industrial production, real consumption, unemployment rate, non-farm payrolls, and price index. All variables are expressed in growth rates except for the unemployment rate. The interest rate is excluded to avoid complications arising from the zero lower bound (ZLB) on nominal interest rates, as discussed by Lenza and Primiceri (2022). The empirical results demonstrate that employing the outlier-robust estimator of the VAR coefficients improves forecast accuracy in the post-pandemic period.

The remainder of the paper is organized as follows. Section 2 introduces the regression model and derives the estimators for the VAR coefficients and outliers. Section 3 outlines the iterative estimation algorithm. Section 4 establishes the asymptotic normality of the outlier-robust VAR estimator. Section 5 describes the data and presents the empirical results. Section 6 concludes.

2 Model

Let y_t be an $N \times 1$ vector for a fixed N. The reduced-form VAR(p) model for y_t is given by

$$y_t = c + \pi_1 y_{t-1} + ... + \pi_p y_{t-p} + u_t = \Pi x_t + u_t, \quad t = p+1, ..., T \tag{1}$$

where $\Pi = [c, \pi_1, ..., \pi_p]$ is an $N \times (Np+1)$ coefficient matrix. The n^{th} row of Π is denoted as $\Pi_{n\cdot}, n = 1, ..., N$. $x_t = [1, y'_{t-1}, ..., y'_{t-p}]'$ is an $(Np+1) \times 1$ vector, and u_t is an $N \times 1$ vector of reduced-from errors with outliers. It is assumed that u_t is the sum of two components.

$$u_t = e_t + v_t \tag{2}$$

In Equation (2), e_t represents the regular component (regularized error), and v_t represents the outlier component. This specification implies that v_t is classified as an innovative outlier, as the autoregressive structure of the model implies that v_t affects not only y_t but also the subsequent observations, $y_{t+1}, ..., y_{t+p-1}$. Therefore, the definition of an outlier in this study excludes the case in which an outlier only affects y_t without propagating to subsequent observations, which is classified as an additive outlier in the literature. An alternative approach to modeling outliers is to assume that the reduced-form error is a product of a regular component and an outlier component (e.g., $u_t = V_t e_t$ where V_t is an $N \times N$ matrix that represents the outlier component), a formulation often employed in the Bayesian VAR literature.[2]

The advantage of adopting Equation (2) is that it enables both the robust estimation of Π and the estimation of $\{v_t\}_{t=1}^T$ by a penalized regression framework, as demonstrated in Gannaz (2007) and She and Owen (2011) in the context of univariate regression models. In what follows, we show that this outlier-robust estimator of Π is equivalent to the multivariate M-estimator of Π.

The following notation is used throughout the paper. A random vector v is in L_q if $\mathbb{E}(|v|^q) < \infty$. Let $||\cdot||_q$ denote the L_q norm, $q \geq 0$. Then, the L_q norm of v is given by $||v||_q = [E(|V|^q)]^{\frac{1}{q}}$. For notational simplicity, $|\cdot| = ||\cdot||_1$ is used for the L_1 norm and $||\cdot|| = ||\cdot||_2$ is used for the L_2 norm. $sgn(\cdot)$ denotes the sign function and $f^+ = max(f, 0)$. I_p denotes a $p \times p$ identity matrix.

2.1 OLS Estimation

The standard OLS estimator of Π, denoted by $\widetilde{\Pi}$, is given by

$$\widetilde{\Pi} = \underset{\Pi}{\operatorname{argmin}} \sum_{t=p+1}^{T} u_t' u_t = \underset{\Pi}{\operatorname{argmin}} \sum_{t=p+1}^{T} (y_t - \Pi x_t)'(y_t - \Pi x_t) \tag{3}$$

$$= (\sum_{t=p+1}^{T} y_t x_t')(\sum_{t=p+1}^{T} x_t x_t')^{-1} \tag{4}$$

Equation (4) implies that $\widetilde{\Pi}$ is essentially a row-wise OLS estimator. That is, the n^{th} row of Π, denoted by $\Pi_{n\cdot}$, is obtained from the OLS regression of y_{nt} on x_t.

2.2 L_1-Penalized Estimation

Let $\theta = [\Pi, v_1, ..., v_T]$ be a collection of the VAR coefficients and the outliers in the reduced-form errors. Then, the L_1-penalized least squares estimator of θ satisfies

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}}\, L_{pen}(\theta) \tag{5}$$

where $L_{pen}(\theta)$ denotes a penalized loss function. It is defined as

[2] For example, the outlier-augmented SV-BVAR models of Carriero, Clark, Marcellino, and Mertens (2024) assume that the reduced-form errors are the product of normally distributed structural shocks and outliers that follow either a uniform distribution or a t-distribution. Alvarez and Odendahl (2022) take a similar approach by assuming that the reduced-form errors are the product of normally distributed errors and outliers.

$$L_{pen}(\theta) = \sum_{t=p+1}^{T} \frac{1}{2} e_t' e_t + \lambda ||v_t||_1 \tag{6}$$

$$= \frac{1}{2} \sum_{t=p+1}^{T} (y_t - \Pi x_t - v_t)'(y - \Pi x_t - v_t) + \lambda \sum_{t=p+1}^{T} \sum_{n=1}^{N} |v_{nt}| \tag{7}$$

where λ is a threshold parameter that will be discussed later.

Let $\hat{\theta} = [\hat{\Pi}, \hat{v}_1, ..., \hat{v}_T]$ be the minimizer of $L_{pen}(\theta)$. Proposition 1 establishes that $\hat{\Pi}$ is equivalent to the multivariate Huber M-estimator of Π. It also shows that the estimated outliers, $\{\hat{v}_t\}_{t=1}^{T}$, correspond to the regression residuals whose absolute values exceed λ.

Proposition 1. *Let $\hat{\theta} = [\hat{\Pi}, \hat{v}_1, ..., \hat{v}_T]$ be a solution to Equation (5). Then, $\hat{\Pi}$ is identical to the multivariate Huber M-estimator of Π that minimizes*

$$L_{Huber}(\Pi) = \sum_{t=p+1}^{T} \varrho(y_t - \Pi x_t) \tag{8}$$

where $\varrho : \mathbb{R}^N \to \mathbb{R}$ is defined as a sum of univariate Huber loss functions with a common threshold parameter λ. That is,

$$\varrho(y_t - \Pi x_t) = \sum_{n=1}^{N} \rho_\lambda(y_{nt} - \Pi_{n\cdot} x_t) \tag{9}$$

where $\Pi_{n\cdot}$ is the n^{th} row of Π and $\rho_\lambda(z) : \mathbb{R} \to \mathbb{R}$ is a univariate Huber loss function with a threshold parameter λ.

$$\rho_\lambda(z) = \begin{cases} z^2/2 & \text{if } |z| \le \lambda \\ \lambda|z| - \lambda^2/2 & \text{if } |z| > \lambda \end{cases} \tag{10}$$

The estimated outlier, $\hat{v}_{nt}$, is given by

$$\hat{v}_{nt} = sgn(y_{nt} - \hat{\Pi}_{n\cdot} x_t)(|y_{nt} - \hat{\Pi}_{n\cdot} x_t| - \lambda)^+, \quad n = 1, ..., N, \quad t = p+1, ..., T \tag{11}$$

That is, the estimated outlier is the regression residual that exceeds λ in absolute value.

Proof. Suppose that Π is given. Then, the optimal v_{nt} that minimizes $L_{pen}(v_{nt}; \Pi)$ is

$$\hat{v}_{nt} = sgn(y_{nt} - \Pi_{n\cdot} x_t)(|y_{nt} - \Pi_{n\cdot} x_t| - \lambda)^+ \quad n = 1, ..., N, \quad t = p+1, ..., T \tag{12}$$

Then, the optimal Π should minimize the loss function given $v_{nt} = \hat{v}_{nt}$, denoted as $L_{pen}(\Pi; \hat{v}_1, ..., \hat{v}_T)$.

This loss function can be expressed as

$$L_{pen}(\Pi;\hat{v}_1,...,\hat{v}_T) = \frac{1}{2}\sum_{t=p+1}^{T}(y_t-\Pi x_t-\hat{v}_t)'(y-\Pi x_t-\hat{v}_t)+\lambda\sum_{t=p+1}^{T}\sum_{n=1}^{N}|\hat{v}_{nt}| \tag{13}$$

$$=\frac{1}{2}\sum_{t=p+1}^{T}(y_t-\Pi x_t)'(y_t-\Pi x_t)-\sum_{t=1}^{T}(y_t-\Pi x_t)'\hat{v}_t+\sum_{t=p+1}^{T}(\frac{1}{2}\hat{v}_t'\hat{v}_t+\lambda\sum_{n=1}^{N}|\hat{v}_{nt}|) \tag{14}$$

$$=\frac{1}{2}\sum_{t=p+1}^{T}\sum_{n=1}^{N}(y_{nt}-\Pi_{n\cdot}x_t)^2-\sum_{t=1}^{T}\sum_{n=1}^{N}(y_{nt}-\Pi_{n\cdot}x_t)\hat{v}_{nt}+\sum_{t=p+1}^{T}\sum_{n=1}^{N}(\frac{1}{2}\hat{v}_{nt}^2+\lambda|\hat{v}_{nt}|) \tag{15}$$

$$=\sum_{t=p+1}^{T}\sum_{n=1}^{N}\rho_{pen}(\Pi_{n\cdot};\hat{v}_{nt}) \tag{16}$$

where

$$\rho_{pen}(\Pi_{n\cdot};\hat{v}_{nt})=\frac{1}{2}(y_{nt}-\Pi_{n\cdot}x_t)^2-(y_{nt}-\Pi_{n\cdot}x_t)\hat{v}_{nt}+(\frac{1}{2}\hat{v}_{nt}^2+\lambda|\hat{v}_{nt}|) \tag{17}$$

Suppose $|y_{nt}-\Pi_{n\cdot}x_t|>\lambda$. Then, Equation (11) implies that

$$\rho_{pen}(\Pi_{n\cdot};\hat{v}_{nt})=\frac{1}{2}\lambda^2+(\lambda|y_{nt}-\Pi_{n\cdot}x_t|-\lambda^2)=\lambda|y_{nt}-\Pi_{n\cdot}x_t|-\frac{\lambda^2}{2} \tag{18}$$

If $|y_{nt}-\Pi_{n\cdot}x_t|\le\lambda$, then $\hat{v}_{nt}=0$. Therefore, we have

$$\rho_{pen}(\Pi_{n\cdot};\hat{v}_{nt})=\frac{1}{2}(y_{nt}-\Pi_{n\cdot}x_t)^2 \tag{19}$$

Notice that $\rho_{pen}(\Pi_{n\cdot};\hat{v}_{nt})=\rho_\lambda(y_{nt}-\Pi_{n\cdot}x_t)$. Therefore,

$$L_{pen}(\Pi;\hat{v}_1,...,\hat{v}_T)=\sum_{t=p+1}^{T}\sum_{n=1}^{N}\rho_{pen}(\Pi_{n\cdot};\hat{v}_{nt}) \tag{20}$$

$$=\sum_{t=p+1}^{T}\sum_{n=1}^{N}\rho_\lambda(y_{nt}-\Pi_{n\cdot}x_t) \tag{21}$$

$$=\sum_{t=p+1}^{T}\varrho(y_t-\Pi x_t) \tag{22}$$

$$=L_{Huber}(\Pi) \tag{23}$$

where Equation (22) uses the definition of ϱ in Equation(9). Therefore, the estimator of Π obtained from the L_1-penalized least squares is identical to the multivariate Huber M-estimator. Lastly, $\hat{v}_{nt}$ can be obtained by replacing $\Pi_{n\cdot}$ in Equation 12 with $\hat{\Pi}_{n\cdot}$. □

Given the estimates of the outliers, the regular component of the residuals, referred to as the regularized residuals, can also be obtained. Corollary 1 shows that the regularized residuals correspond to the residuals thresholded at λ or $-\lambda$, depending on their signs.

Corollary 1. *The regularized residual, denoted as $\hat{e}_{nt}$, is given by*

$$\hat{e}_{nt} = y_{nt} - \hat{\Pi}_{n\cdot}x_t - \hat{v}_{nt} = sgn(y_{nt} - \hat{\Pi}_{n\cdot}x_t)\min(|y_{nt} - \hat{\Pi}_{n\cdot}x_t|, \lambda), \quad \forall n, t \tag{24}$$

Imposing a common threshold λ can be problematic when the variables in the VAR model differ in scale. For example, if one variable has a much larger scale than other variables, then outliers are likely to be identified almost exclusively from that one variable. To mitigate this issue, it is essential to normalize all variables in the VAR model prior to estimation.

Alternatively, the objective function in Equation (7) can be modified to incorporate variable-specific thresholds. Consider the modified objective function

$$L_{modified}(\theta) = \frac{1}{2}\sum_{t=p+1}^{T}(y_t - \Pi x_t - v_t)'(y - \Pi x_t - v_t) + \sum_{t=p+1}^{T}\sum_{n=1}^{N}\lambda_n|v_{nt}| \tag{25}$$

Here, the threshold parameters, $\{\lambda_n\}_{n=1}^{N}$, are allowed to vary across variables. By following the steps in the proof of Proposition 1 and adapting them accordingly, it can be shown that the solution to Equation (25) is likewise equivalent to the multivariate M-estimator of Π where the objective function is defined as the sum of univariate Huber loss functions with variable-specific thresholds.

Proposition 2. *Let $\hat{\theta} = [\hat{\Pi}, \hat{v}_1, ..., \hat{v}_T]$ be the solution to (25). Then, $\hat{\Pi}$ is identical to the multivariate M-estimator of Π that minimizes*

$$L_{Huber}(\Pi) = \sum_{t=p+1}^{T}\varrho(y_t - \Pi x_t) \tag{26}$$

where $\varrho : \mathbb{R}^N \to \mathbb{R}$ is a sum of univariate Huber loss functions with variable-specific thresholds, denoted by $\{\lambda_n\}_{n=1}^{N}$. That is,

$$\varrho(z) = \sum_{n=1}^{N}\rho_{\lambda_n}(y_{nt} - \Pi_{n\cdot}x_t) \tag{27}$$

where $\rho_{\lambda_n}(z) : \mathbb{R} \to \mathbb{R}$ is the univariate Huber loss function with a variable-specific threshold

λ_n. *That is,*

$$\rho_{\lambda_n}(z) = \begin{cases} z^2/2 & \text{if } |z| \leq \lambda_n \\ \lambda|z| - \lambda^2/2 & \text{if } |z| > \lambda_n \end{cases} \tag{28}$$

Also, the estimated outlier is given by

$$\hat{v}_{nt} = sgn(y_{nt} - \hat{\Pi}_{n\cdot}x_t)(|y_{nt} - \hat{\Pi}_{n\cdot}x_t| - \lambda_n)^+, \quad \forall n, t \tag{29}$$

Corollary 2. *The regularized residual, denoted as $\hat{e}_{nt}$, is given by*

$$\hat{e}_{nt} = y_{nt} - \hat{\Pi}_{n\cdot}x_t - \hat{v}_{nt} = sgn(y_{nt} - \hat{\Pi}_{n\cdot}x_t)\min(|y_{nt} - \hat{\Pi}_{n\cdot}x_t|, \lambda_n), \quad \forall n, t \tag{30}$$

The only difference from Proposition 1 is that Huber loss functions now depend on variable-specific thresholds rather than a common threshold. This formulation has the advantage of allowing the number of outliers in each variable to be controlled through the adjustment of its corresponding threshold. For the remainder of this paper, we focus on the penalized regression model with variable-specific thresholds.

3 Estimation

This section describes the procedure for estimating Π. In particular, we focus on the steps for the M-estimation of Π because the L_1-penalized estimator of Π has been shown to be equivalent to the multivariate M-estimator of Π. Although M-estimation requires an iterative procedure, its computational cost is modest because the objective function in Equation (27) permits row-wise estimation similar to the OLS estimation.

3.1 M-estimator as a Weighted Least Squares Estimator

Consider the M-estimator of Π derived from the objective function (26), which imposes variable-specific thresholds. Equation (27) implies that $\varrho_{\lambda_n}(y_t - \Pi x_t)$ can be expressed as the sum of N univariate Huber loss functions, $\rho_{\lambda_n}(y_{nt} - \Pi_{n\cdot}x_t)$ where $y_{nt} - \Pi_{n\cdot}x_t$ is the residual of the regression of y_{nt} on x_t. Consequently, the M-estimator of Π can be obtained from variable-wise regressions. The M-estimator of the n^{th} row of Π satisfies

$$\hat{\Pi}_{n\cdot} = \underset{\Pi_{n\cdot}}{\operatorname{argmin}} \sum_{t=p+1}^{T} \rho_{\lambda_n}(y_{nt} - \Pi_{n\cdot}x_t), \quad n = 1, ..., N \tag{31}$$

Then, $\hat{\Pi}_{n\cdot}$ satisfies

$$\sum_{t=p+1}^{T} \psi_{\lambda_n}(y_{nt} - \Pi_{n\cdot} x_t) x_t' = 0 \tag{32}$$

where

$$\psi_{\lambda_n}(u) = \frac{\partial \rho_{\lambda_n}(u)}{\partial u} = \begin{cases} u & \text{if } |u| \leq \lambda_n \\ \lambda_n sgn(u) & \text{if } |u| > \lambda_n \end{cases} \tag{33}$$

is the first derivative of ρ_{λ_n}. Define the weight function w_{nt} as

$$w_{nt} = \frac{\psi_{\lambda_n}(y_{nt} - \Pi_{n\cdot} x_t)}{y_{nt} - \Pi_{n\cdot} x_t}, \quad n = 1, ..., N, \quad t = p+1, ..., T \tag{34}$$

Then, Equation (31) can be written as

$$\sum_{t=p+1}^{T} (y_{nt} - \Pi_{n\cdot} x_t) w_{nt} x_t' = 0 \tag{35}$$

and therefore $\hat{\Pi}_{n\cdot}$ is defined as

$$\hat{\Pi}_{n\cdot} = (\sum_{t=p+1}^{T} y_{nt} w_{nt} x_t')(\sum_{t=p+1}^{T} x_t w_{nt} x_t')^{-1} \tag{36}$$

After estimating $\hat{\Pi}_{n\cdot}, n = 1, ..., N$, $\hat{\Pi}$ can be obtained as $\hat{\Pi} = [\hat{\Pi}_{1\cdot}', \hat{\Pi}_{2\cdot}', ..., \hat{\Pi}_{N\cdot}']'$. Equation (36) implies that $\hat{\Pi}_{n\cdot}$ is a weighted least squares estimator, with weights depending on $\hat{\Pi}_{n\cdot}$ itself. Consequently, an iterative procedure is required to obtain $\hat{\Pi}_{n\cdot}$. The iterative estimation algorithm is presented in Section 3.2.

3.2 Algorithm for Iteratively Re-weighted Least Squares

Let $\hat{\Pi}_{n\cdot}^{(j)}$ denote the weighted least squares estimator of $\Pi_{n\cdot}$ after the j^{th} iteration. The threshold parameters $\{\lambda_n\}_{n=1}^{N}$ are known and fixed throughout the iteration. The choice of threshold parameters is discussed in Section 3.3.

To algorithm is initialized by using the OLS estimator of Π is used as an initial value. That is, $\hat{\Pi}^{(0)} = \widetilde{\Pi}$.[3] Then, the weight assigned to variable n at time t in the first iteration is given by

$$w_{nt}^{(1)} = \frac{\psi_{\lambda_n}(y_{nt} - \hat{\Pi}_{n\cdot}^{(0)} x_t)}{y_{nt} - \hat{\Pi}_{n\cdot}^{(0)} x_t}, \quad n = 1, ..., N, t = p+1, ..., T \tag{37}$$

Then, the weighted least squares estimator of $\Pi_{n\cdot}$ in the first iteration, denoted by $\hat{\Pi}_{n\cdot}^{(1)}$, is

[3]This is equivalent to setting the initial weights to be identical across variables.

given by

$$\hat{\Pi}_{n\cdot}^{(1)} = (\sum_{t=p+1}^{T} y_{nt} w_{nt}^{(1)} x_t')(\sum_{t=p+1}^{T} x_t w_{nt}^{(1)} x_t')^{-1}, \quad n = 1, ..., N, t = p+1, ..., T \tag{38}$$

In general, let $\hat{\Pi}_{n\cdot}^{(j)}$ denote the estimator for $\Pi_{n\cdot}$ after the j^{th} iteration. Then, the weight assigned to variable n at time t in the $(j+1)^{th}$ iteration is given by

$$w_{nt}^{(j+1)} = \frac{\psi_{\lambda_n}(y_{nt} - \hat{\Pi}_{n\cdot}^{(j)} x_t)}{y_{nt} - \hat{\Pi}_{n\cdot}^{(j)} x_t} \tag{39}$$

Then, the estimator for $\Pi_{n\cdot}$ after the $(j+1)^{th}$ iteration is given by

$$\hat{\Pi}_{n\cdot}^{(j+1)} = (\sum_{t=p+1}^{T} y_{nt} w_{nt}^{(j+1)} x_t')(\sum_{t=p+1}^{T} x_t w_{nt}^{(j+1)} x_t')^{-1} \tag{40}$$

The iteration repeats until $\frac{||\hat{\Pi}_{n\cdot}^{(j+1)} - \hat{\Pi}_{n\cdot}^{(j)}||_2}{||\hat{\Pi}_{n\cdot}^{(j)}||_2}$ converges. Once the convergence is achieved, the outlier-robust estimator of $\Pi_{n\cdot}$, denoted by $\hat{\Pi}_{n\cdot}$, is obtained. Repeating this procedure for all n yields the robust estimator of Π: $\hat{\Pi} = [\hat{\Pi}_{1\cdot}', \hat{\Pi}_{2\cdot}', ..., \hat{\Pi}_{N\cdot}']'$.

3.3 Choice of Threshold Parameters

The definitions of the estimated outliers in Equation (11) and Equation (29) imply that the threshold parameters determine the degree of sparsity in the identified outliers. Since the objective is to keep the number of identified outliers small compared to the sample size, the choice of threshold parameters is important. In the existing literature, thresholds are typically defined as a constant multiple of a robust measure of scale. For example, Huber (1981) proposes $\lambda_n = \frac{1}{0.6745} MAD(u_{n,t})$ where $MAD(u_{n,t})$ is the median absolute deviation of $u_{n,t}$, which is an unbiased estimator of σ if $u_{n,t} \stackrel{iid}{\sim} N(0, \sigma^2)$. Following this approach, this study defines λ_n as a constant multiple of the inter-quartile range (IQR) of $u_{n,t}$, which is also used as a robust measure of scale in the macroeconomic literature.[4] That is,

$$\lambda_n = c_n IQR(u_{n,t}), \quad n = 1, ..., N \tag{41}$$

where c_n is a variable-specific constant selected by cross-validation.

[4]If $u_{n,t} \stackrel{iid}{\sim} N(0, \sigma^2)$, then $IQR(u_{n,t}) = 2MAD(u_{n,t})$.

4 Asymptotic Normality of the Outlier-Robust Estimator

In this section, we establish the asymptotic normality of the proposed estimator. The following assumptions are imposed to derive the central limit theorem. Then, the asymptotic normality of $\hat{\Pi}$ can be derived from the results of Bai, Rao, and Wu (1992) on the asymptotic properties of M-estimators in multivariate linear regressions.[5] With this in mind, we state the following assumptions.

Assumption 1. $\{u_t\}_{t=1}^{T}$ *are i.i.d.* $N \times 1$ *vectors with bounded variance.*

Assumption 2. *Define* $\Psi(u)$ *as an* $N \times 1$ *vector of the first derivative of* $\varrho(u)$ *with respect to* u*. Then,* $||\Psi(u_t + u) - \Psi(u_t)||$ *is continuous at* $u = 0_{N\times 1}$*.*

Assumption 3. *Define* $\Phi(u) = \mathbb{E}[\Psi(u_t + u)]$ *where* u *is an* $N \times 1$ *vector. Let* $\dot{\Phi}(u)$ *denote an* $N \times N$ *matrix of the first derivative of* $\Phi(u)$ *with respect to* u*. Then,* $\dot{\Phi}(0_{N\times 1})$ *is a positive definite matrix.*

Assumption 4. $\mathbb{E}[\Psi(u_t)] = 0_{N\times 1}$ *and* $\mathbb{E}[\Psi(u_t)\Psi(u_t)']$ *is a positive definite matrix.*

Assumption 5. *Define* $S_x = \sum_{t=p+1}^{T} x_t x_t'$*. Then,* $\max_{p+1\leq t\leq T} tr[x_t' S_x^{-1} x_t] \to 0$ *as* $T \to \infty$*.*

Assumption 1 states that u_t is independent and identically distributed (i.i.d.) across t and has finite variance, which is often assumed in the VAR literature. Assumptions 2-4 correspond to conditions (M3)-(M5) in Bai, Rao, and Wu (1992). Assumption 5 implies condition (M6) in Bai, Rao, and Wu (1992), which is the Lindeberg-Feller-type condition.[6] The following theorem establishes the asymptotic normality of $\hat{\Pi}$ under these assumptions.

Theorem 1. *Let* $X_t = x_t \otimes I_N$ *be an* $N(Np + 1) \times N$ *matrix and define*

$$K = \sum_{t=p+1}^{T} X_t \dot{\Phi}(0_{N\times 1}) X_t' \qquad \text{and} \qquad S = \sum_{t=p+1}^{T} X_t \Psi(u_t)\Psi(u_t) X_t' \tag{42}$$

where $\dot{\Phi}_\lambda(0)$ *is the derivative of* $\Phi_\lambda(u)$ *with respect to* u *at* $u = 0_{N\times 1}$*. Then,* $\hat{\Pi}$ *is asymptotically normally distributed.*

$$S^{-1/2} K(vec(\hat{\Pi}) - vec(\Pi)) \xrightarrow{d} N(0, I_{N(Np+1)}) \tag{43}$$

[5] As noted by Maronna, Martin, Yohai, and Salibian-Barrera (2006), any robust regression framework based on minimizing a function of residuals can be applied to the autoregressive models.

[6] The definition of ϱ in this study satisfies condition (M1) and (M2) in Bai, Rao, and Wu (1992), so those conditions are omitted.

Proof. From $y_t = \Pi x_t + u_t$,

$$y_t = vec(y_t) = vec(\Pi x_t + u_t) = (x_t' \otimes I_N)vec(\Pi) + vec(u_t) = X_t' vec(\Pi) + u_t \tag{44}$$

We only need to prove that Assumption 5 implies condition (M6) in Bai, Rao, and Wu (1992). Let S_X be

$$S_X = \sum_{t=p+1}^{T} X_t X_t' = \sum_{t=p+1}^{T} (x_t \otimes I_N)(x_t \otimes I_N)' = \sum_{t=p+1}^{T} (x_t x_t') \otimes I_N = (\sum_{t=p+1}^{T} x_t x_t') \otimes I_N \tag{45}$$

Then,

$$\begin{aligned} X_t S_X^{-1} X_t &= (x_t' \otimes I_N)((\sum_{t=p+1}^{T} x_t x_t') \otimes I_N)^{-1}(x_t \otimes I_N) && (46) \\ &= (x_t' \otimes I_N)((\sum_{t=p+1}^{T} x_t x_t')^{-1} \otimes I_N)(x_t \otimes I_N) && (47) \\ &= (x_t'(\sum_{t=p+1}^{T} x_t x_t')^{-1} x_t) \otimes I_N && (48) \\ &= (x_t' S_x^{-1} x_t) \otimes I_N && (49) \end{aligned}$$

Then,

$$tr(X_t S_X^{-1} X_t) = tr((x_t' S_x^{-1} x_t) \otimes I_N) = N tr(x_t' S_x^{-1} x_t) \tag{50}$$

Therefore, for fixed N, Assumption 5 implies that $\max_{p+1 \leq t \leq T} tr(X_t S_X^{-1} X_t) \to 0$ as $T \to \infty$. Since conditions (M1)-(M6) in Bai, Rao, and Wu (1992) are satisfied, Equation (43) follows from Theorem 2.4 in Bai, Rao, and Wu (1992). □

5 Forecasting Macroeconomic Variables in the Post-COVID Era

In this section, we forecast a set of macroeconomic variables in the post-pandemic period. Similar empirical exercises have been conducted by Lenza and Primiceri (2022) and Carriero, Clark, Marcellino, and Mertens (2024) using monthly macroeconomic and financial variables drawn from the FRED-MD database. Lenza and Primiceri (2022) estimate a 7-variable BVAR with 13 lags using data from December 1988 to May 2021, and Carriero, Clark, Marcellino, and Mertens (2024) estimate a 16-variable outlier-augmented SV-BVAR with 12 lags using data from March 1959 to March 2021.

However, both studies rely on data ending in early to mid-2021, leaving too few post-

COVID observations to allow for a meaningful evaluation of forecast accuracy. By contrast, this study uses monthly data from March 1959 to June 2024, obtained from the July 2024 vintage of the FRED-MD database, to compare the forecast performance of alternative VAR models in the post-pandemic period.

5.1 Data and potential outliers

This study estimates a 5-variable VAR with 12 lags using monthly macroeconomic variables from the FRED-MD database. The variables and their transformations into logs or log-differences are provided in Table 1. The transformations follow Carriero, Clark, Marcellino, and Mertens (2024), except that their analysis uses annualized growth rates. Following Lenza and Primiceri (2022), the interest rate is excluded to avoid complications arising from the zero lower bound (ZLB) on nominal interest rates. The federal funds rate was constrained by the ZLB from late 2008 to 2016 and again from March 2020 to March 2022, covering a substantial portion of the post-COVID sample. Including the federal funds rate would therefore risk distorting forecasts in the post-COVID period. [7]

Table 1: List of variables and transformations

Variable	FRED-MD code	Transformation
Industrial production	INDPRO	$\Delta log(x_t) \cdot 100$
Real consumption	DPCERA3M086SBEA	$\Delta log(x_t) \cdot 100$
Unemployment rate	UNRATE	
Non-farm payrolls	PAYEMS	$\Delta log(x_t) \cdot 100$
PCE price index	PCEPI	$\Delta log(x_t) \cdot 100$

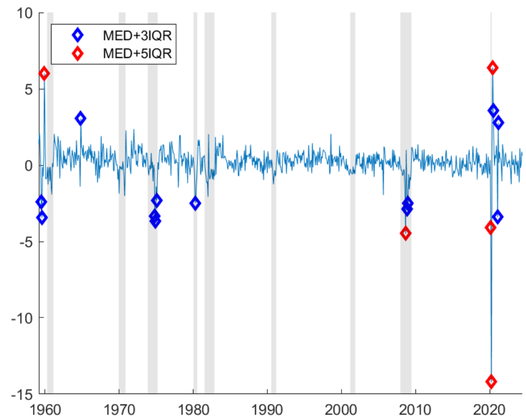
Note: Monthly observations from March 1959 to June 2024 are obtained from the July 2024 vintage of FRED-MD database.

Figure 1 presents the transformed data, with potential outliers highlighted in each series. In each plot, blue dots denote observations more than 3 IQRs from the sample median, and red dots denote observations more than 5 IQRs from the sample median. Shaded regions correspond to NBER recession periods.[8] Table 2 lists the potential outlier dates. Observations on these dates are more than 3 IQRs from the sample median, as indicated by the blue dots in Figure 1. Dates shown in bold correspond to observations more than 5 IQRs from the median, as indicated by the red dots in Figure 1.

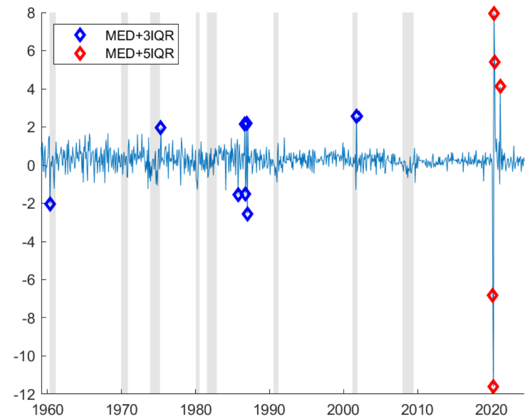
[7] To circumvent this issue, Carriero, Clark, Marcellino, and Mertens (2024) only include longer-term interest rates in their model.

[8] The choice of 5 IQRs follows Carriero, Clark, Marcellino, and Mertens (2024), who treat such observations as missing in their SV-outmiss model.

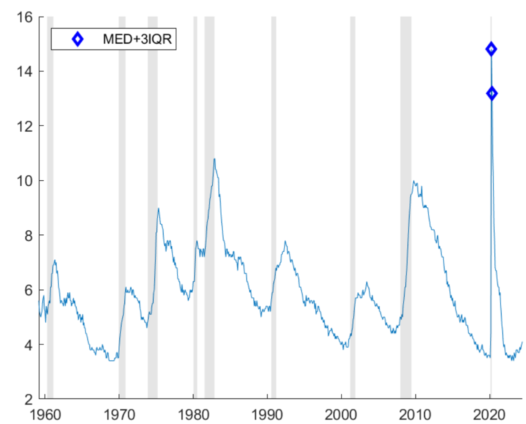
Figure 1: Data series and potential outliers



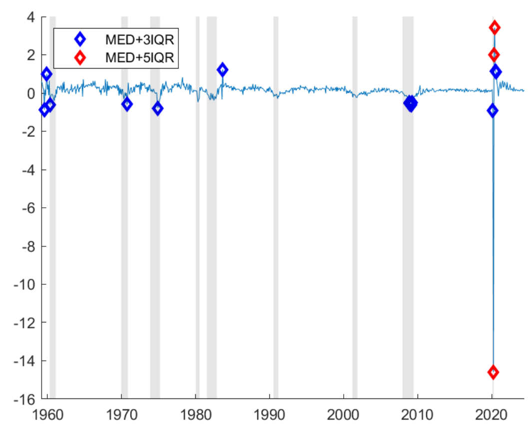
(a) IP growth rate



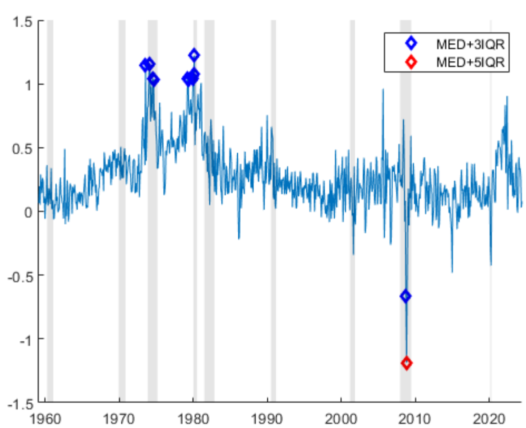
(b) Consumption growth rate



(c) Unemployment rate



(d) Payroll growth rate



(e) Inflation rate (PCE index)

Table 2: List of potential outliers

Variable	Dates
IP growth rate	1959:07, 1959:08, **1959:12**, 1964:11, 1974:11, 1974:12, 1980:05 **2008:09**, 2008:12, 2009:01, **2020:03, 2020:04, 2020:06**, 2020:07 2021:02, 2021:03
Consumption growth rate	1960:05, 1975:05, 1985:10, 1986:09, 1986:10, 1986:12, 1987:01 2001:10, **2020:03, 2020:04, 2020:05, 2020:06, 2021:03**
Unemployment rate	2020:04, 2020:05
Payroll growth rate	1959:08, 1959:12, 1960:05, 1970:10, 1974:12, 1983:09 2008:11, 2008:12, 2009:01, 2009:02, 2009:03, 2009:04 2020:03, **2020:04, 2020:05, 2020:06**, 2020:07, 2020:08
Inflation rate	1973:08, 1974:02, 1974:03, 1974:08, 1974:09, 1979:04, 1979:05 1980:01, 1980:02, 1980:03, 2008:10, **2008:11**

Note: A list of observations that are more than 3 IQRs from the sample median. The observations in bold are more than 5 IQRs from the sample median.

As expected, the initial months of COVID-19 are identified as potential outliers for all variables except the inflation rate. Although the inflation rate declined during the early stages of the pandemic, the magnitude was considerably smaller than during the global financial crisis (GFC). The most severe months of the GFC are also classified as potential outliers in the growth rate of industrial production, the growth rate of non-farm payrolls, and the inflation rate. While most potential outliers coincide with recessions, some instances, such as those in consumption growth during the mid-1980s, are not associated with recessions. A similar pattern is observed for the inflation rate, with clusters of potential outliers in 1973–1974 and 1979–1980, attributed to the 1973–1974 oil crisis and the resurgence of inflation in the late 1970s.

Figure 1c shows that no potential outliers are detected in the unemployment rate when the threshold is set too high, which is counterintuitive. In contrast, setting the threshold too low leads to the identification of too many potential outliers, particularly in the growth rates of industrial production and non-farm payrolls, as indicated by the blue dots in Figure 1a and 1d. These findings suggest that the optimal threshold may vary across variables, underscoring the importance of selecting variable-specific thresholds through cross-validation.

It is important to emphasize that these are *potential* outliers, and thus they are not dropped or treated as missing, as is common in the literature. Instead, all observations are retained in the estimation process, as they continue to contain useful information. The identification and treatment of outliers are discussed in the next section.

5.2 Identification of Outliers in Data from March 1959 to June 2024

The VAR model is estimated using the penalized least squares (PLS) method over the full sample from March 1959 to June 2024. Then, outliers are identified based on the variable-specific threshold

$$\lambda_n = c_n IQR(u_{n,t}), \quad n = 1, ..., 5 \tag{51}$$

where $u_{n,t}$ denotes the OLS residual of the regression of $y_{n,t}$ on x_t. c_n is selected by leave-one-out cross-validation from the candidate set, $\{3, 3.5, 4, 4.5, 5\}$.[9] It should be noted that the outliers identified through this regression-based approach need not coincide exactly with the potential outliers shown in Figure 1.

Table 3: Probability of outliers (March 1959 - June 2024)

	Probability (%)		
Variable/Model	Outlier-Robust VAR	SVO (CCCM, 2022)	SVO-t (CCCM, 2022)
IP growth rate	1.17	0.41 [0.08 1.21]	0.19 [0.01 1.61]
Consumption growth rate	0.91	0.53 [0.09 1.49]	0.13 [0.01 0.62]
Unemployment rate	0.52	0.58 [0.19 1.33]	0.27 [0.03 0.85]
Payroll growth rate	1.42	1.32 [0.48 2.66]	0.48 [0.01 1.72]
Inflation rate	0.65	0.38 [0.07 1.09]	0.11 [0.02 0.67]

Note: The probabilities of outliers estimated from each model are reported. For the SVO and SVO-t models, posterior median probabilities are reported. Numbers in brackets denote the 95% credible intervals (0.025 and 0.975 posterior quantiles).

Table 3 reports the probabilities of outliers for each variable, defined as the percentage of observations identified as outliers. For comparison, the table also presents the posterior median probabilities of outliers estimated using the two outlier-augmented SV-BVAR models of Carriero, Clark, Marcellino, and Mertens (2024): the SV-BVAR model with infrequent volatility outliers (SVO) and the SV-BVAR model with infrequent volatility outliers and fat-tailed errors (SVO-t).[10] A brief description of the differences between the SVO and SVO-t models is provided in Appendix A. The last two columns of Table 3 report the posterior median probabilities of outliers from the SVO and SVO-t models, together with their 95% credible intervals (0.025 and 0.975 posterior quantiles).

The results in the second column indicate that the estimated probabilities of outliers in our outlier-robust VAR model are approximately 1 percent, with all estimates lying within

[9]Although out-of-sample cross-validation is often used in time-series forecasting, Bergmeir, Hyndman, and Koo (2018) show that K-fold cross-validation and leave-one-out cross-validation can also be applied to purely autoregressive models with uncorrelated errors.

[10]These models were estimated using the replication codes available on the authors' website (`https://github.com/elmarmertens/CCMMoutlierVAR-code`), with appropriate modifications.

the 95% credible intervals of the SVO model (and of the SVO-t model, except for the growth rate of real consumption). To facilitate comparison with the potential outlier dates listed in Table 2, Table 4 provides the dates of the identified outliers for each variable.

Table 4: List of identified outliers

Variable	Dates
IP growth rate	1964:11, 1970:10, 1970:12, 1974:11, 1974:12 1982:02, 2008:09, 2020:03, 2020:04
Consumption growth rate	1986:09, 1986:12, 1987:01, 2001:10, 2020:03, 2020:04, 2020:05
Unemployment rate	2020:03, 2020:04, 2020:05, 2020:06
Payroll growth rate	1960:05, 1964:11, 1970:10, 1970:12, 1983:08 1983:09, 2020:03, 2020:04, 2020:05, 2020:06
Inflation rate	1973:08, 2005:09, 2008:10, 2008:11, 2022:07

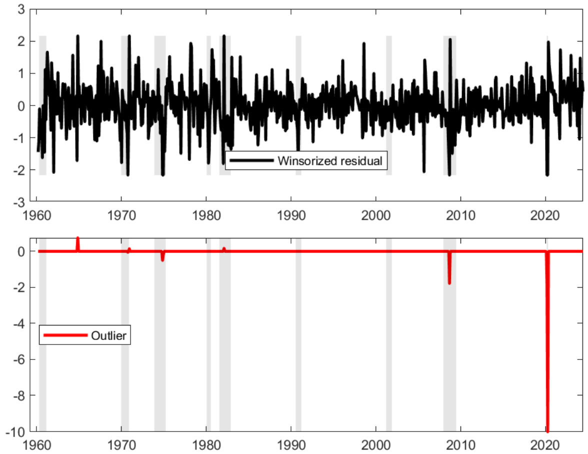
Note: The dates of the identified outliers are reported. Note that these dates are not identical to the potential outliers reported in Table 2.

The initial months of COVID-19 are classified as outliers for all variables except the inflation rate. The outlier in the inflation rate observed in July 2022 is likely to be attributed to the Federal Reserve's rapid tightening of monetary policy, which began in March 2022. The global financial crisis (GFC) is also identified as an outlier, specifically in the growth rate of industrial production (September 2008) and in the inflation rate (November 2008), though the dates differ across variables.

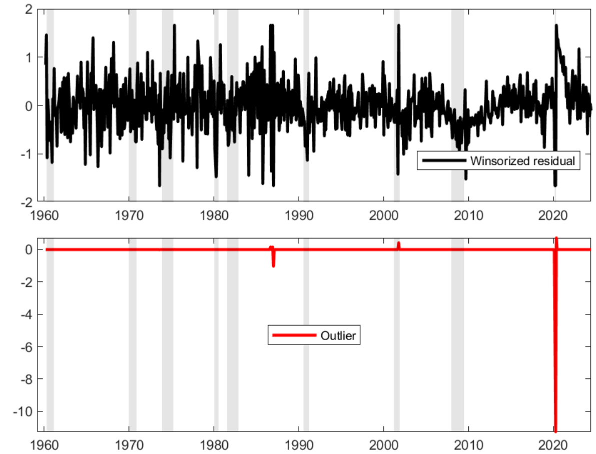
Overall, the number of outliers identified in the outlier-robust VAR model is smaller than the number of potential outliers, with the exception of the unemployment rate. This finding suggests that a two-step procedure of detecting and treating outliers prior to estimation may be overly conservative.

Figure 2 displays the regularized residuals and the estimated outliers. Since no restrictions are imposed on the signs of the outliers, they may be either positive or negative. However, as shown in the figure, most identified outliers are negative. The exception is the unemployment rate, since the unemployment rate rises in economic turmoils. For example, large negative outliers appear in the growth rates of industrial production, consumption, and non-farm payrolls at the onset of COVID-19. In the cases of consumption growth and non-farm payroll growth, these negative outliers are followed by smaller positive outliers, reflecting the sharp rebound in these variables. Negative outliers are also observed in the growth rate of industrial production and in the inflation rate during the GFC. These negative outliers frequently coincide with recession periods, consistent with the notion that recessions are associated with skewness in macroeconomic variables.

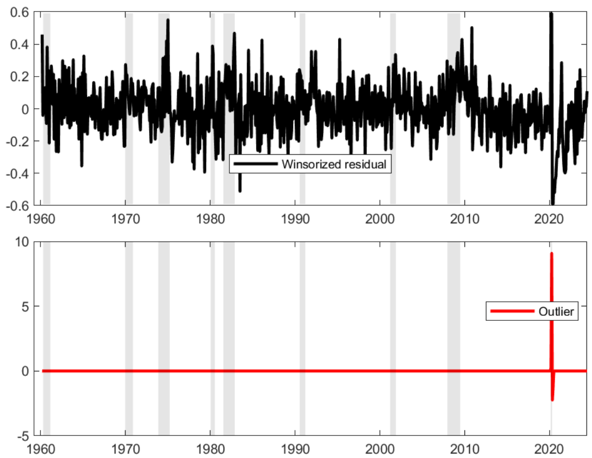
Figure 2: Regularized residuals and estimated outliers



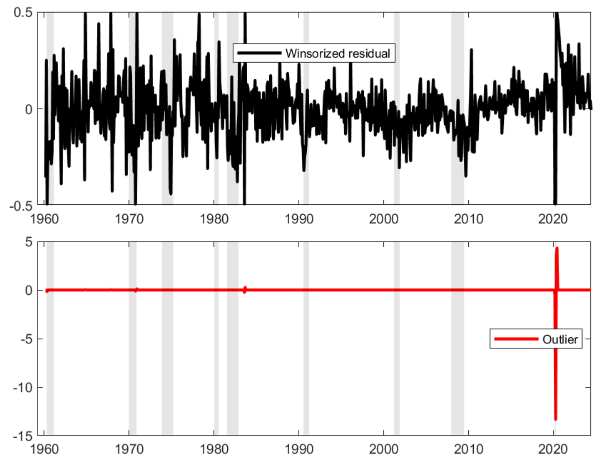
(a) IP growth rate



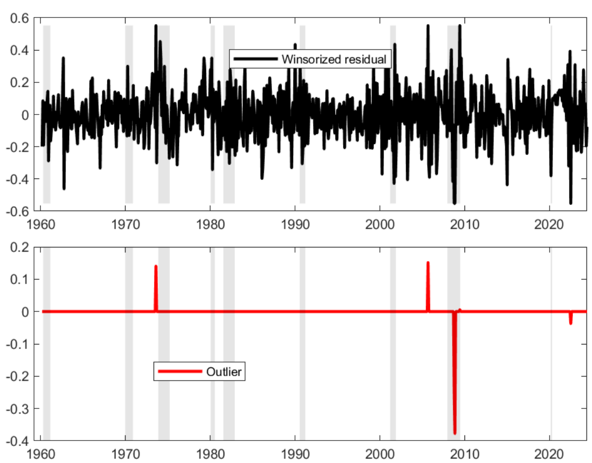
(b) Consumption growth rate



(c) Unemployment rate



(d) Payroll growth rate



(e) Inflation rate

Table 5: In-sample fit of standard VAR and outlier-robust VAR (MAEs)

	Models	
Variable	Standard VAR	Outlier-Robust VAR
IP growth rate	0.56	0.54
Consumption growth rate	0.46	0.43
Unemployment rate	0.20	0.15
Payroll growth rate	0.24	0.15
Inflation rate	0.13	0.12

Note: MAEs computed from the standard VAR and the outlier-robust VAR

Lastly, we compare the in-sample fit of the outlier-robust VAR with that of the standard homoskedastic VAR. Table 5 reports the mean absolute errors (MAEs), a robust measure of fit, for both models. The outlier-robust VAR yields lower MAEs than the standard VAR, indicating an improved in-sample fit. Besides in-sample fit, forecasting performance is a key criterion for evaluating a VAR model. Accordingly, we compare the forecast accuracy of the outlier-robust VAR with that of alternative VAR specifications.

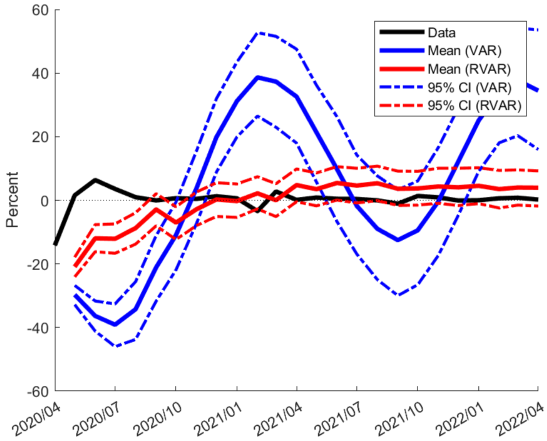
5.3 Comparison of Forecasts Made in April 2020

The economic impact of COVID-19 became apparent in March and April 2020, following the introduction of lockdown measures in mid-March. The subsequent months continued to experience the lingering effects of the pandemic. To assess how these initial disruptions could influence forecasts, we compare April 2020 forecasts from the standard VAR, outlier-robust VAR, and SVO models with actual data spanning March 1959 to April 2020. The forecast horizon extends from 1 to 24 months.

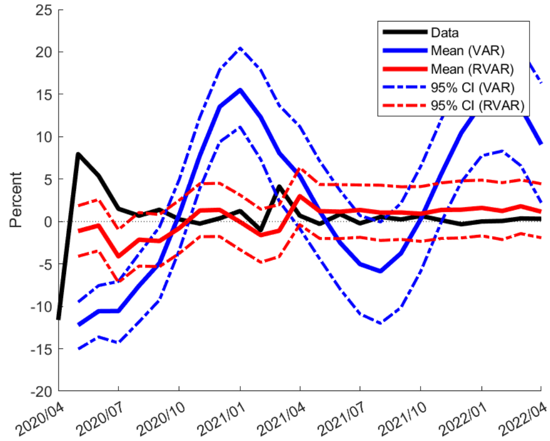
First, we compare the April 2020 forecasts from the standard VAR and the outlier-robust VAR against the actual data. In Figure 3, the black lines represent the actual data from April 2020 to April 2022. The blue lines represent average forecasts from the standard VAR from May 2020 to April 2022, with dashed blue lines indicating their 95% confidence intervals. The red lines represent forecasts from the outlier-robust VAR, with dashed red lines marking the corresponding 95% confidence intervals.

Overall, the outlier-robust VAR model produces forecasts that more closely track the actual data, as reflected in the proximity of the red lines to the black lines relative to the blue lines. The only exception occurs in the unemployment rate forecasts between November 2021 and April 2022 although both models tend to over-predict its trajectory. However, the standard VAR projects a negative unemployment rate in 2022, which is counterintuitive. In the case of the inflation rate, the robust VAR forecasts are slightly below the realized values.

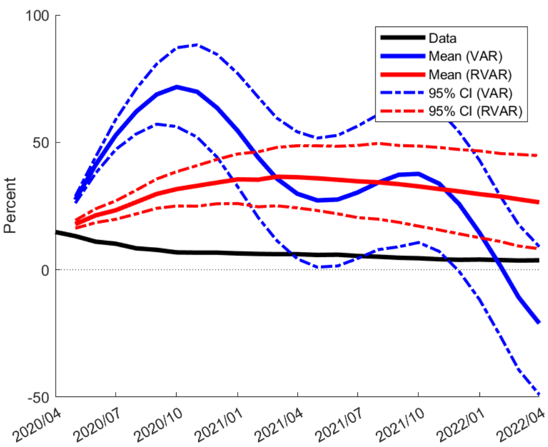
Figure 3: Actual data and forecasts from standard VAR and outlier-robust VAR



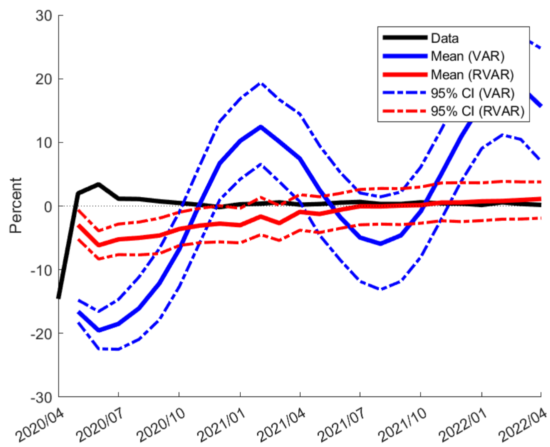
(a) IP growth rate



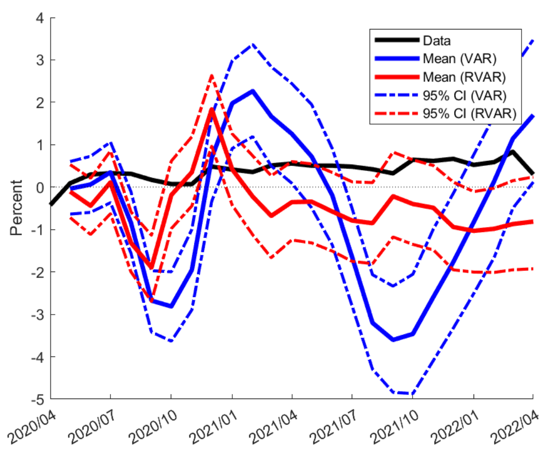
(b) Consumption growth rate



(c) Unemployment rate



(d) Payroll growth rate



(e) Inflation rate

Overall, Figure 3 demonstrates that the forecasts from the standard VAR display excessive variability, whereas the forecasts from the outlier-robust VAR are more stable and closely aligned with the actual data.

We next compare the forecasts from the outlier-robust VAR with those from the SVO model.[11] Figure 4 presents this comparison. The black lines represent the actual data. The blue lines represent the posterior median forecasts from the SVO model, and the dashed blue lines indicate their 95% credible intervals. The red lines represent the average forecasts from the outlier-robust VAR, and the dashed red lines indicate their 95% confidence intervals.[12]

Overall, both the forecasts from the outlier-robust VAR model and the SVO model closely track the observed data. The main exception arises in the forecasts for the inflation rate between September 2020 and January 2021, when the outlier-robust VAR predicts a rise from -2% to 2%, while the SVO forecasts remain stable at -1%. Also, the outlier-robust VAR tends to over-predict the trajectory of the unemployment rate relative to the SVO model. However, these discrepancies are not statistically significant. Both models tend to under-predict the trajectory of payroll growth, particularly during 2020. Finally, the confidence intervals associated with the outlier-robust VAR forecasts are comparable to, or narrower than, those of the SVO forecasts.

In this section, we present only the graphical comparisons of forecasts generated at a specific point in time across different models. The next section evaluates the forecasting accuracy of these models over the post-COVID period.

5.4 Comparison of Forecast Accuracy between March 2020 and June 2023

This section evaluates the forecasts generated between March 2020 and June 2023. A related exercise is conducted by Carriero, Clark, Marcellino, and Mertens (2024) using the SVO and SVO-t models, but their evaluation period is restricted to January 1975–December 2017 due to limited post-pandemic data. Here, we compare the forecasting performance of the standard VAR model, the outlier-robust VAR model, and the SVO model over the March 2020 – June 2023 period. For each forecast origin, the models are re-estimated on expanding samples beginning in March 1959.[13] Forecast horizons range from 1 to 12 months, reflecting the relatively short span of data available since the onset of the pandemic.

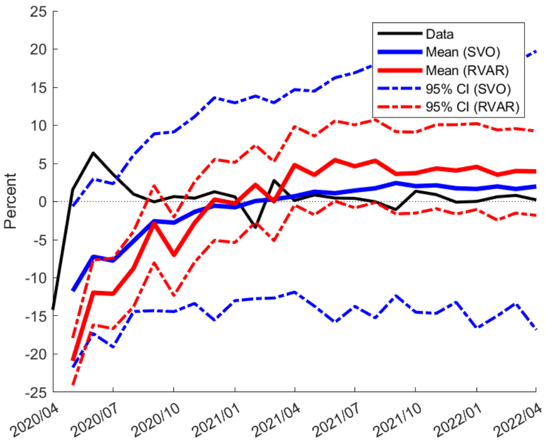
Forecast accuracy is evaluated using two metrics: the root mean square prediction error (RMSPE), the conventional benchmark for forecast accuracy, and the mean absolute

[11]Because the SVO and SVO-t models generate broadly similar forecasts, we report results only for the SVO model. Forecasts from both models are provided in Appendix B.

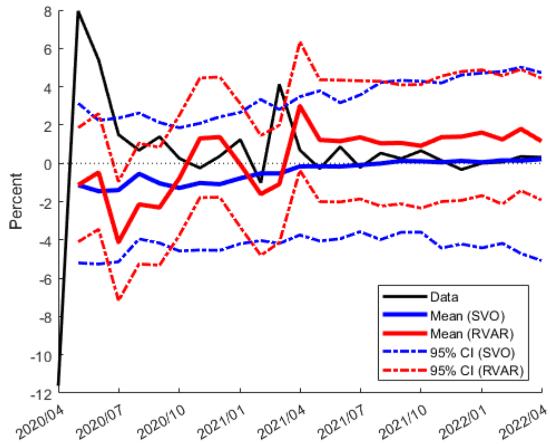
[12]The forecasts from the outlier-robust VAR in Figure 4 are identical to those in Figure 3, but are presented on different scales.

[13]All data are drawn from the July 2024 vintage of FRED-MD, and we abstract from issues related to real-time data availability.

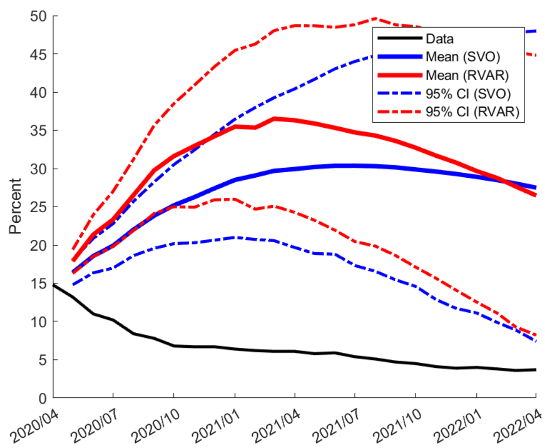
Figure 4: Actual data and forecasts from outlier-robust VAR and SVO



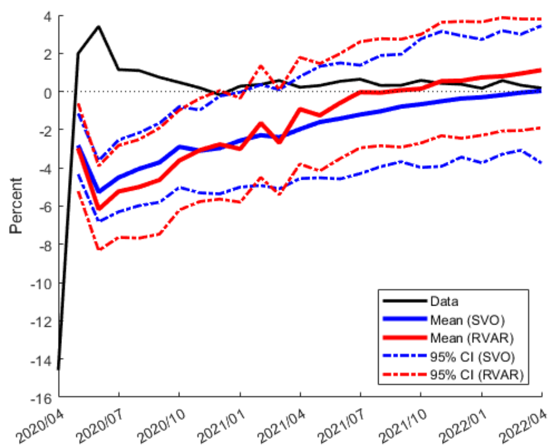
(a) IP growth rate



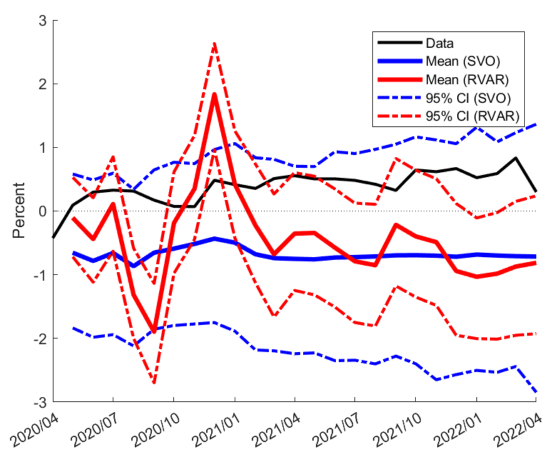
(b) Consumption growth rate



(c) Unemployment rate



(d) Payroll growth rate



(e) Inflation rate

prediction error (MAPE), which is more robust to extreme values than the RMSPE. The h-month-ahead RMSPE for variable n is defined as

$$RMSPE(n,h) = \sqrt{\frac{1}{40}\sum_{t=March2020}^{June2023} (y_{n,t+h} - \hat{y}_{n,t+h|t})^2} \tag{52}$$

and the h-month-ahead MAPE for variable n is defined as

$$MAPE(n,h) = \frac{1}{40}\sum_{t=March2020}^{June2023} |y_{n,t+h} - \hat{y}_{n,t+h|t}| \tag{53}$$

where $y_{n,t+h|t}$ is the h-month-ahead forecast for $y_{n,t+h}$ made at time t.

Table 6: Relative RMSPE and MAPE of outlier-robust VAR and SVO

Relative RMSPE (baseline: standard VAR)								
Model	Outlier-Robust VAR				SVO			
Variables/Horizons (months)	1	4	8	12	1	4	8	12
IP growth rate	1.00	0.99	0.99	1.00	1.34	1.47	1.13	1.05
Consumption growth rate	1.00	0.96	1.00	1.00	1.19	1.09	1.03	1.00
Unemployment rate	1.01	1.03	1.02	0.93	0.61	1.72	3.88	5.58
Payroll growth rate	1.00	0.75	0.92	1.03	1.11	2.35	2.44	2.35
Inflation rate	0.99	0.98	0.99	1.00	0.98	1.18	1.13	1.48
Relative MAPE (baseline: standard VAR)								
Model	Outlier-Robust VAR				SVO			
Variables/Horizons (months)	1	4	8	12	1	4	8	12
IP growth rate	0.95	1.00	0.98	1.00	1.37	1.33	1.08	1.06
Consumption growth rate	1.02	0.94	1.02	0.97	1.20	1.14	1.09	0.94
Unemployment rate	1.00	0.98	0.93	0.83	0.39	1.05	1.87	2.67
Payroll growth rate	0.87	0.60	0.87	1.05	1.03	1.07	1.47	1.75
Inflation rate	0.99	0.98	0.99	1.00	0.94	1.11	1.15	1.37

Note: Comparison of the forecast accuracy of the outlier-robust VAR and the SVO against the standard VAR, which serves as the baseline (denominator). The value below 1 indicates an improvement over the baseline.

Table 6 reports the relative RMSPE and MAPE of the average forecasts from the robust VAR and the SVO, using the standard VAR as the benchmark (denominator), at forecast horizons of 1, 4, 8, and 12 months.[14] A value below unity indicates an improvement in forecast accuracy of the robust VAR or the SVO relative to the standard VAR.

[14]The corresponding levels of RMSPE and MAPE are reported in Appendix C.

The upper panel of Table 6 shows that the RMSPEs of the standard VAR and the outlier-robust VAR are generally close to unity, except for the 12-month-ahead forecast of unemployment and the 4- and 8-month-ahead forecasts of payroll growth. Forecast accuracy improves substantially for payroll growth at the 4-month horizon. The lower panel of Table 6 indicates that the outlier-robust VAR typically improves forecast accuracy when measured by MAFE. In particular, forecasts from the outlier-robust VAR exhibit smaller MAPEs than those from the standard VAR, even though the opposite often holds in terms of RMSPEs. Gains are especially pronounced for payroll growth across horizons.

The forecast performance of the SVO model varies considerably across variables and horizons. Interestingly, its accuracy is generally lower than that of the standard VAR, except for the 1-month-ahead forecasts of unemployment and inflation. To assess whether variable selection drives this result, we replicate the analysis using the original 16-variable VAR of Carriero, Clark, Marcellino, and Mertens (2024), which includes both macroeconomic and financial variables. The results, reported in Appendix D, show that the SVO improves forecasts for financial variables but does not necessarily improve forecasts for macroeconomic variables.

6 Conclusion

This study proposed an outlier-robust VAR model that decomposes the reduced-form error into a regular component and an outlier component. The VAR coefficients are estimated via penalized least squares, with an L_1 penalty imposed on the outlier component. The resulting estimator is shown to be equivalent to the multivariate Huber M-estimator, with outliers corresponding to residuals whose absolute values exceed a threshold parameter. The asymptotic normality of the outlier-robust estimator is then established using the results for multivariate M-estimators. To demonstrate its practical relevance, we apply the method to a VAR with five monthly macroeconomic variables. Out-of-sample forecasts for the post-COVID period indicate that the outlier-robust VAR delivers greater accuracy than both the standard VAR and the outlier-augmented SV-BVAR of Carriero, Clark, Marcellino, and Mertens (2024).

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Appendices

A Outlier-augmented SV-BVAR models in Carriero, Clark, Marcellino, and Mertens (2024)

We consider two outlier-augmented SV-BVAR in Carriero, Clark, Marcellino, and Mertens (2024). The SVO model assumes that the reduced-form VAR residual satisfies

$$u_t = A^{-1}\Lambda_t^{0.5} O_t \epsilon_t, \qquad \epsilon_t \sim N(0, I_n) \tag{54}$$

where A^{-1} is a lower triangular matrix. The vector of the diagonal elements of Λ_t, denoted as λ_t, satisfies

$$\log \lambda_t = \log \lambda_{t-1} + e_t, \qquad e_t \sim N(0, \Phi) \tag{55}$$

O_t is a diagonal matrix whose diagonal element is given by

$$o_{nt} = \begin{cases} 1 & \text{with probability} 1 - p_n \\ U(2, 20) & \text{with probability} p_n \end{cases} \tag{56}$$

where $U(2, 20)$ denotes a uniform distribution with support between 2 and 20.

While the SVO model assumes that the size of outliers follows a uniform distribution in the SVO model, the SVO-t model assumes that

$$u_t = A^{-1}\Lambda_t^{0.5} O_t Q_t \epsilon_t \tag{57}$$

with A^{-1}, $\Lambda_t^{0.5}$, and O_t specified as above. Q_t is a diagonal matrix whose diagonal element, denoted as q_{nt}, has inverse-gamma distribution.

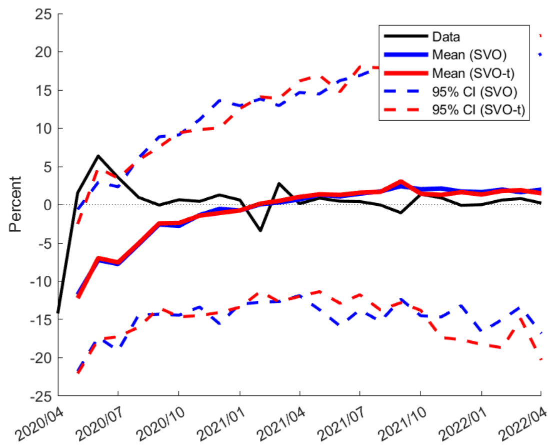
$$q_{nt}^2 \sim IG(\frac{d_n}{2}, \frac{d_n}{2}) \tag{58}$$

Following Carriero, Clark, Marcellino, and Mertens (2024), the prior for the probability of outliers is set to imply an average outlier frequency of once every 4 years (= 1/48) for the SVO model and once every 10 years (= 1/120) for the SVO-t model.[15]

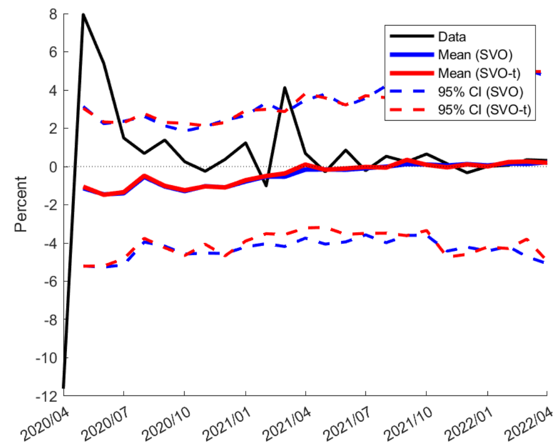
[15] A detailed description of the SVO and SVO-t models are available on page 4 of Carriero, Clark, Marcellino, and Mertens (2024).

B Comparison of Forecasts between the SVO and the SVO-t

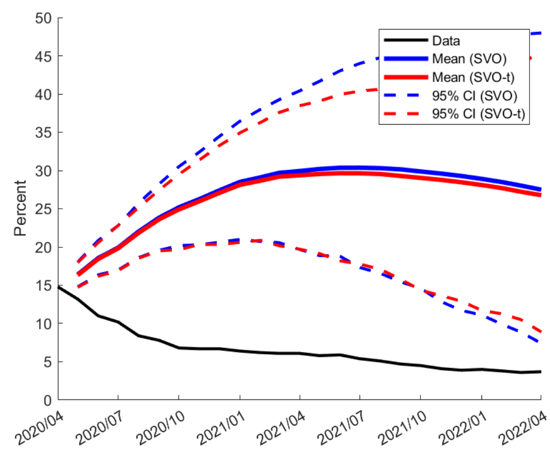
Figure 5: Actual data and forecasts made in April 2020 (SVO and SVO-t)



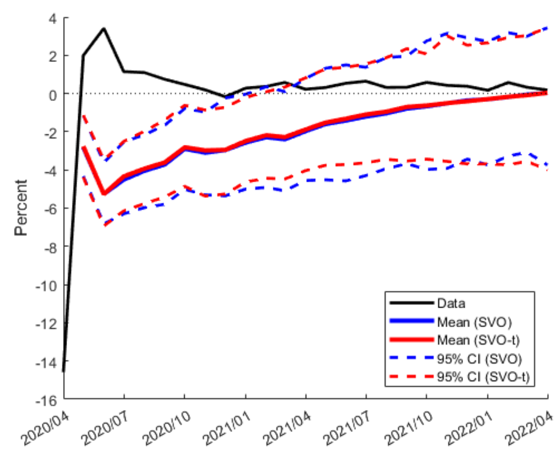
(a) IP growth rate



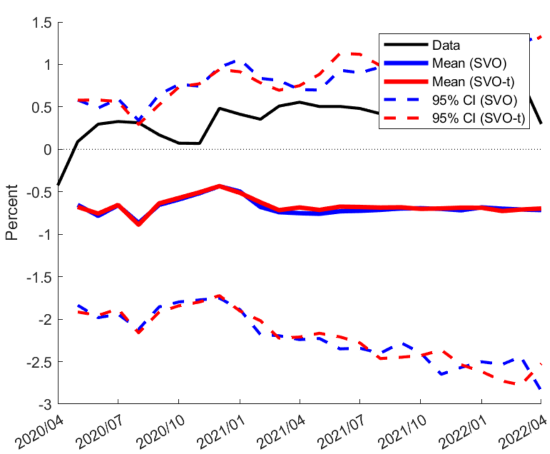
(b) Consumption growth rate



(c) Unemployment rate



(d) Payroll growth rate



(e) Inflation rate

C Forecast Performance of the Standard VAR, the Outlier-Robust VAR, and the SVO

Table 7: RMSPE and MAPE of standard VAR, outlier-robust VAR, and SVO

RMSFE												
Model	Standard VAR				Outlier-Robust VAR				SVO			
Variables/Horizons (months)	1	4	8	12	1	4	8	12	1	4	8	12
IP growth rate	2.70	1.04	0.92	0.71	2.69	1.03	0.92	0.71	3.63	1.54	1.04	0.74
Consumption growth rate	2.50	0.83	0.74	0.69	2.50	0.80	0.74	0.69	2.98	0.90	0.76	0.69
Unemployment rate	2.98	1.72	1.06	0.83	3.02	1.77	1.08	0.77	1.83	2.97	4.12	4.65
Payroll growth rate	2.43	0.41	0.22	0.18	2.42	0.31	0.20	0.19	2.69	0.97	0.54	0.43
Inflation rate	0.28	0.27	0.28	0.25	0.28	0.26	0.27	0.25	0.27	0.31	0.31	0.37
MAFE												
Model	Standard VAR				Outlier-Robust VAR				SVO			
Variables/Horizons (months)	1	4	8	12	1	4	8	12	1	4	8	12
IP growth rate	1.27	0.65	0.60	0.50	1.21	0.65	0.60	0.50	1.74	0.87	0.66	0.53
Consumption growth rate	1.05	0.47	0.40	0.37	1.07	0.45	0.41	0.35	1.26	0.54	0.44	0.34
Unemployment rate	1.85	1.21	0.87	0.76	1.84	1.19	0.81	0.63	0.72	1.27	1.63	2.02
Payroll growth rate	0.86	0.33	0.16	0.12	0.75	0.2	0.14	0.13	0.89	0.36	0.24	0.21
Inflation rate	0.22	0.22	0.23	0.21	0.22	0.21	0.23	0.21	0.21	0.24	0.26	0.28

D Result from 16-variable VAR

The following is the list of variables used for the 16-variable VAR in Carriero, Clark, Marcellino, and Mertens (2024).

Table 8: List of variables and transformations

Variable	FRED-MD code	Transformation
Real income	RPI	$\Delta log(x_t) \cdot 100$
Real consumption	DPCERA3M086SBEA	$\Delta log(x_t) \cdot 100$
Industrial production	INDPRO	$\Delta log(x_t) \cdot 100$
Capacity utilization	CUMFNS	
Unemployment rate	UNRATE	
Non-farm payrolls	PAYEMS	$\Delta log(x_t) \cdot 100$
Hours	CES0600000007	
Hourly earnings	CES0600000008	$\Delta log(x_t) \cdot 100$
PPI (fin. goods)	WPSFD49207	$\Delta log(x_t) \cdot 100$
PCE price index	PCEPI	$\Delta log(x_t) \cdot 100$
Housing starts	HOUST	$log(x_t)$
S&P 500	SP500	$\Delta log(x_t) \cdot 100$
USD/GBP exchange rate	EXUSUKx	$\Delta log(x_t) \cdot 100$
5-Year Treasury yield	GS5	
10-Year Treasury yield	GS10	
Baa spread	BAAFFM	

Note: Data is obtained from the July 2024 vintage of FRED-MD. Monthly observations from March 1959 to June 2024.

Table 9: Probability of outliers (March 1959 - June 2024)

	Probability (%)		
Variable/Model	PVAR	SVO-t (CCCM, 2022)	SVO (CCCM, 2022)
Real income	1.42	0.83 [0.17 3.05]	3.08 [1.73 5.07]
Real consumption	0.65	0.16 [0.01 0.80]	0.44 [0.09 1.25]
Industrial production	1.04	0.15 [0.01 1.20]	0.42 [0.08 1.23]
Capacity utilization	0.52	0.14 [0.01 0.68]	0.39 [0.06 1.22]
Unemployment rate	0.26	0.31 [0.04 1.04]	0.62 [0.18 1.52]
Non-farm payrolls	0.65	0.84 [0.18 2.00]	1.42 [0.42 2.82]
Hours	0.91	0.42 [0.04 1.34]	0.77 [0.17 2.07]
Hourly earnings	0.26	0.12 [0.01 0.68]	0.42 [0.09 1.19]
PPI (fin. goods)	0.78	0.18 [0.01 0.82]	0.47 [0.10 1.46]
PCE price index	0.52	0.12 [0.02 0.63]	0.39 [0.06 1.12]
Housing starts	0.13	0.12 [0.01 0.55]	0.35 [0.07 1.02]
S&P 500	0.39	0.11 [0.01 0.63]	0.47 [0.09 1.30]
USD/GBP exchange rate	0.78	0.62 [0.18 1.50]	0.93 [0.35 1.87]
5-Year Treasury yield	0.39	0.10 [0.01 0.53]	0.36 [0.06 1.06]
10-Year Treasury yield	0.26	0.11 [0.01 0.57]	0.34 [0.07 1.05]
Baa spread	1.30	0.18 [0.01 0.98]	0.80 [0.19 1.78]

Note: The probabilities of outliers estimated from each model are reported. The posterior median probabilities are reported for the SVO and SVO-t models. The numbers in brackets are the 95% confidence intervals (0.025 and 0.975 posterior quantiles).

Again, the probabilities of outliers identified in the robust VAR are within the 95% credible intervals of those estimated from the SVO and the SVO-t.

Table 10: Relative MAPE of Outlier-Robust VAR and SVO

Model	Outlier-Robust VAR				SVO			
Variables/Horizons (months)	1	4	8	12	1	4	8	12
Real income	0.95	0.93	0.92	0.91	0.99	0.92	1.08	1.05
Real consumption	0.82	0.93	0.99	1.00	0.85	1.02	1.03	0.95
Industrial production	0.85	0.99	0.85	1.04	0.93	1.21	0.84	0.72
Capacity utilization	0.98	0.96	0.81	0.94	0.59	1.34	1.17	0.59
Unemployment rate	1.00	0.94	0.72	0.70	0.33	0.76	0.79	0.66
Non-farm payrolls	0.76	0.66	0.64	1.02	0.74	0.62	0.28	0.31
Hours	0.90	0.86	0.81	0.91	0.99	0.89	0.70	0.50
Hourly earnings	0.97	0.99	1.07	0.96	1.70	1.23	1.11	1.12
PPI (fin. goods)	1.01	0.99	1.07	0.96	1.00	1.01	0.90	1.08
PCE price index	0.99	1.05	1.03	1.04	1.18	1.11	1.11	1.19
Housing starts	1.00	0.99	0.97	1.03	0.47	0.53	0.48	0.53
S&P 500	1.01	1.07	1.00	1.00	1.04	0.95	1.06	0.97
USD/GBP exchange rate	0.97	0.99	1.00	1.00	1.05	0.96	1.08	1.06
5-Year Treasury yield	0.99	1.00	1.05	1.08	0.09	0.20	0.37	0.57
10-Year Treasury yield	0.99	1.00	1.04	1.07	0.08	0.17	0.30	0.42
Baa spread	0.98	0.94	1.03	1.00	0.12	0.33	0.84	1.11

The performance of the SVO model varies considerably across variables and forecast horizons. For instance, the SVO outperforms the outlier-robust VAR in forecasting unemployment, non-farm payrolls, hours worked, housing starts, Treasury yields, and the Baa spread, except at the 12-month horizon. By contrast, it performs poorly relative to the outlier-robust VAR when forecasting real income, real consumption, hourly earnings and inflation.

One caveat to this result is that the SVO model applies dimension reduction through shrinkage priors, whereas the outlier-robust VAR includes all lags of all variables. Consequently, the outlier-robust model is more prone to overfitting in high-dimensional VAR settings, which may in turn impair its forecasting accuracy.