

# The Inflation Target and the Equilibrium Real Rate

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## Abstract

Many economists have proposed raising the inflation target to reduce the probability of hitting the zero lower bound. It is both widely assumed and a feature of standard models that raising the inflation target does not impact the equilibrium real rate. I demonstrate that once we allow for heterogeneity among households, raising the inflation target causes the equilibrium real rate to fall in a New Keynesian model. This implies that raising the inflation target will raise the nominal interest rate by less than expected and thus will be less effective in reducing the probability of hitting the zero lower bound. The channel is that a rise in the inflation target lowers the average markup by price rigidities and a fall in the average markup lowers the equilibrium real interest rate by household heterogeneity. Raising the inflation target from 2% to 4% lowers the equilibrium real rate by 0.38 percentage points in my baseline calibration of a New Keynesian model with life cycle features. I also analyse the optimal inflation level and provide supportive empirical evidence for my channel.

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# 1 Introduction

Many economists have proposed raising the inflation target to reduce the probability of hitting the effective lower bound.<sup>1</sup> Nearly all developed countries were constrained by the effective lower bound during the financial crisis. Moreover, it is widely believed that average real interest rates have fallen.<sup>2</sup> This implies that average nominal interest rates will be lower going forward. Consequently, there has been a re-evaluation of the risk that central banks will hit the effective lower bound. Hitting the bound is bad for economic outcomes because central banks have less room to lower nominal interest rates and stimulate the economy during bad times. Therefore many economists (including [Blanchard et al. \(2010\)](#), [Ball \(2014\)](#), [Krugman \(2014\)](#)) have proposed raising the inflation target from the standard objective of 2% to 4% because this will raise average nominal interest rates and thus reduce the probability of hitting the zero lower bound.

It is widely assumed that raising the inflation target will not affect the equilibrium real rate. The equilibrium real (nominal) rate is the real (nominal) interest rate on short-term safe assets when there are no shocks. Standard Macroeconomic models very commonly assume either flexible prices or a representative agent. With either of these assumptions, the equilibrium real rate is unaffected by changing average inflation. This is also a historic concept introduced by [Fisher \(1907\)](#) and is often taken for granted within policy discussions. Thus, it is widely believed that raising the inflation target by 2p.p. will have no impact upon the equilibrium real rate and will therefore raise the equilibrium nominal rate by a corresponding 2p.p.<sup>3</sup>

My primary contribution is to demonstrate a new channel by which household heterogeneity implies that raising the inflation target will lower the equilibrium real rate. Once we account for household heterogeneity (through either generational features or idiosyncratic risk) within the standard New Keynesian model, we find that raising the inflation target lowers the equilibrium real rate. This implies that a rise in the inflation target will raise the average nominal interest rate by less than expected. Since nominal interest rates will rise by less than expected, raising the inflation target will reduce the probability of avoiding the zero lower bound by less than is commonly believed. The channel by which raising the inflation target lowers the equilibrium real rate is that price rigidities imply that a rise in the inflation target lowers the markup and household heterogeneity implies that a fall in the markup lowers the equilibrium real rate.

The first part of the channel is that raising average inflation lowers the average markup. A firm's markup is just the ratio of its price to its nominal marginal cost. When firms set their prices infrequently, a higher average inflation level has two opposing impacts upon average markups. Firstly, higher inflation means that when a firm doesn't reset its price then its markup falls by relatively more since with higher inflation nominal marginal costs rise relatively more quickly. Secondly, firms observe that their markups fall more quickly and therefore set their markup to be

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<sup>1</sup>The effective lower bound is a lower bound on the nominal interest rate, created by the fact that agents can always get a zero nominal return by holding cash.

<sup>2</sup>Recent estimates by [Negro et al. \(2017\)](#), [Holston et al. \(2017\)](#), [Johannsen and Mertens \(2016\)](#), [Kiley \(2015\)](#), [Laubach and Williams \(2015\)](#), [Lubik and Matthes \(2015\)](#) lie between 0.1% and 1.8%.

<sup>3</sup>For an example of this type of discussion, see [Ball \(2014\)](#).

higher when they do get to reset their prices. It can be shown that with no discounting these two effects cancel out and thus average markups are unchanged by raising average inflation. However, once we allow for discounting, the first effect dominates since firms care more about making profits in the current period and so don't want to set their current markup to be very high when they reset their price. Therefore, a rise in average inflation lowers the average markup.<sup>4</sup>

The novel second part of the channel is that once you allow for household heterogeneity a fall in the markup lowers the equilibrium real rate. Taking the example of heterogeneity through overlapping generations (OLG): A fall in the markup lowers firm profits and thus reduces the value of shares. Households work more when they are young and save for when they are old. If the amount of savings falls then *ceteris paribus* it will lower the consumption of the old relative to the young. Therefore, the price of savings rises. As the price of savings rises, the return on savings (the equilibrium real rate) falls. I do not believe this has been discussed within the literature.

This contrasts to a representative agent New Keynesian model where a fall in the markup has no impact on the equilibrium real rate. A fall in the markup again lowers firm profits and thus reduces the value of shares. However, within a representative agent framework, the agent's consumption path does not depend upon their average savings. Instead, without shocks, they just set their level of consumption to be the same over time and thus the equilibrium real rate is purely determined by the patience of the consumer in the usual way.

I estimate the impact of the channel through a fully calibrated model. I study the effect of raising the inflation target from 2% to 4% within a model with a fully calibrated life cycle model embedded into a model with usual New Keynesian features. Within the basic calibration, I find the equilibrium real rate falls by 0.38p.p. When I reduce the intertemporal elasticity of substitution from 0.5 to 0.1<sup>5</sup>, I find the fall in the equilibrium real rate increases to 0.67p.p.

I compute the inflation target that maximises welfare within my calibrated model. I assess the welfare of the simulated path of the economy under different inflation targets taking into account the zero lower bound and allowing for calibrated shocks. I find that the optimal level of inflation is around 1p.p. Like in [Coibion et al. \(2012\)](#), it appears that the benefits of avoiding the zero lower bound are dominated by the welfare costs of price dispersion even for relatively low inflation targets.

There appears to be a negative empirical relationship between long-run inflation and the equilibrium real rate which supports my hypothesized channel. In recent years, inflation and the real interest rates have fallen across developed countries. This would contradict my channel if the fall in inflation was the only change that could have driven real interest rates lower.<sup>6</sup> However, many factors have been proposed that have lowered real interest rates for other reasons across developed countries.<sup>7</sup> I take this into account in my empirical analysis by looking at panel data

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<sup>4</sup>By average markup, I'm effectively referring to the average markup weighted by sales. This is why I get qualitatively different results to [Ascari and Sbordone \(2014\)](#) who consider the non-weighted average markup.

<sup>5</sup>Recent research by [Best et al. \(2018\)](#) suggests an intertemporal elasticity of substitution of 0.1.

<sup>6</sup>My channel would predict a rise in the equilibrium real rate when inflation falls.

<sup>7</sup>Stories include: demographic changes ([Carvalho et al. \(2016\)](#), [Gagnon et al. \(2016\)](#)), global savings glut ([Caballero and Farhi \(2017\)](#)), secular stagnation ([Eggertsson and Mehrotra \(2014\)](#)).

regressions of the real rate on long-run inflation controlling for country and time fixed effects in OECD countries. The time fixed effects allow me to control for any common change in real rates across countries. We find a 1p.p. rise in long-run inflation lowers the equilibrium real rate by 0.61p.p.

There also seems to be a negative empirical relationship between long-run inflation and the markup (just the first part of the channel) which fits my argument. Using the labor share as a proxy for the inverse of the markup, I conduct similar panel data regressions to the real rate on long-run inflation case. We find a 1p.p. rise in long-run inflation lowers the long-run markup by at least 0.46p.p.

There is a historical literature that looks at the impact of inflation on the equilibrium real rate through non-interest paying money balances but it may be less relevant today. [Mundell \(1963\)](#) and [Tobin \(1965\)](#) argued that a rise in inflation means it is costlier to hold money and therefore there is a portfolio shift in savings towards capital and thus a fall in the equilibrium real rate. A key assumption of this literature is that money does not pay interest (so agents shift away from it when inflation rises) however most developed central banks have now shifted to a framework where they control nominal interest rates by paying interest on reserves. Therefore, this literature appears less relevant to modern central banking. My proposed channel is very different because it does not rely upon money holdings in any way.<sup>8</sup>

Unlike common heterogeneous agents models, I consider the impact of an endogenous allocation of profits. Most heterogeneous agent models exogenously allocate profits i.e. agents are assigned to receive profits rather than buying shares in firms that pay profits. [Werning \(2015\)](#) considers the implications of how these exogenous profit allocations impact the marginal propensity to consume. I instead consider the case where agents only receive profits by owning shares in firms which get traded each period.

My channel implies that raising the inflation target can have long-run distributional effects. Changing the inflation target can have short-term inflation targets by redistributing from agents who hold nominal assets to those who have nominal borrowings. [Doepke et al. \(2015\)](#) consider these short-term redistributional effects in detail. My paper implies that there can actually be long-term redistributional effects as well. A rise in the inflation target reduces profits and thus the value of shares and total savings. This implies that old people, who rely upon savings, consume relatively less and young people consume relatively more.

I investigate optimal monetary policy within a New Keynesian model with OLG features. [Lepetit \(2017\)](#) shows that within a New Keynesian model with perpetual youth, it can be optimal to set a positive inflation target because heterogeneity can imply that private discounting is higher than social discounting. In this case, central banks raise inflation to lower average markups. My paper differs because Lepetit does not consider the impact of the effective lower bound on optimal

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<sup>8</sup>There are many other papers in this literature. [Stockman \(1981\)](#) proposed a reverse-Mundell-Tobin effect in which raising inflation raises the equilibrium real rate due to a cash in advance constraint on investment i.e. you need to take money out of the bank a period before investing. Papers in this literature tend to rely upon cash-in-advance constraints or money-in-utility.

inflation or the impact of feedback from a fall in the markup to a fall in the equilibrium real rate, which affects the frequency of hitting this bound.

There is evidence for the idea that raising the long-run inflation rate lowers the long-run real interest rate. [King and Watson \(1997\)](#) consider the impact of raising inflation upon the real interest rate and show that an increase in long-run inflation leads to a decrease in the long-run real interest rate regardless of the restrictions employed within a structural VAR model for US data. They find that a rise in of 1p.p. in long-run inflation lowers the equilibrium real rate by 0.66p.p. [Rapach \(2003\)](#) extends the analysis to 14 countries with a richer structural model. He demonstrates that a rise in long-run inflation leads a fall of between 0.94p.p. and 0.59p.p. in the equilibrium real rate.

There is other empirical evidence that raising the long-run inflation rate lowers the markup. [Bénabou \(1992\)](#) finds that raising inflation by 1p.p. lowers the markup by 0.36p.p. using a relatively reduced form approach with just US data. [Banerjee and Russell \(2001\)](#) apply a structural VAR approach to the G7 countries and Australia. They find that a 1p.p. rise in annual steady state inflation generates a fall of between 0.3p.p. and 2p.p. in the long-run markup.

In section 2, I outline a simple model that captures the key features found in the rest of the paper. I then outline the full model (section 3). I discuss the model solution and calibration in section 4. I use the full model to analyse how changing the inflation target will impact the steady state in section 5. I then consider the optimal inflation target in section 6. I discuss my supporting empirical results in section 7. Section 8 concludes.

## 2 Intuition through a Simplified Model

We break the intuition for the channel into two parts. First, we demonstrate how a rise in inflation will lower the average markup through firms' pricing decisions. Next, we consider how a fall in the markup will lower the equilibrium real rate once we introduce forms of household heterogeneity.

### 2.1 Relationship between the Inflation Level and the Markup

A firm's markup, denoted  $m_t$ , is its current price divided by its nominal marginal cost,  $MC_t$ :

$$m_t = \frac{P_t^*}{MC_t}$$

Firms have some optimal markup. Firms' profits depend upon their markup. If they set it too high, they will not make enough sales. If they set it too low, they will make a lot of sales but with too little profit on each sale. When firms have fully flexible prices, they can just set their price so that their markup is optimal each period.<sup>9</sup>

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<sup>9</sup>In the common case where firms face constant elasticity of demand, the optimal markup is just  $\frac{\sigma}{\sigma-1}$  where  $\sigma$  is the CES parameters.

Setting markups is more complex in the case with infrequent price adjustment. When firms can only change their price infrequently, they will no longer get to set their optimal flexible price markup each period. Let's consider what happens with positive inflation. There will be two important effects. Firstly, if firms do not get to change their price in a period then their markup will fall. This is because their nominal marginal costs ( $MC_t$ ) rise (due to the rise in the price level) while their price ( $P_t^*$ ) remains constant. Secondly, firms know this is a problem they face so they will set their markups to be higher than the optimal flexible price markup when they do get to change their price in anticipation that they may not get to change their price in the future and thus their markup will fall.

The impact of raising inflation on the markup depends upon the degree of discounting. In the case with no discounting, firms will weight their profits equally in current and future periods.<sup>10</sup> This leads to the special case where the markup is unaffected by changing the level of inflation since the two effects on the markup cancel out. However, when firms discount the future, they will weight their current period markup more in their decision-making. This implies that they set a lower markup when they get to change their price and thus that the average markup is lower with positive inflation. As the level of inflation rises, the strength of this effect will increase.

The frequency of price changes will not affect the relationship at low levels of inflation. If the frequency with which firms adjust prices increases, this would reduce the feedback from inflation to the markup.<sup>11</sup> However, [Gagnon \(2009\)](#) demonstrates that the frequency with which firms change their price does not appear to vary below annual rates of inflation of 10%. This makes sense because firms are likely to change their price for other reasons (like idiosyncratic demand or costs) than just inflation so the frequency of price changes doesn't need to change with low inflation.

The negative inflation-markup relationship also holds with price rigidities based upon adjustment costs. The relationship would hold in the case of menu costs (fixed costs of updating prices) or Rotemberg costs (convex adjustment costs of updating prices). The intuition is that firms prefer to pay the cost of updating their price in the future (with positive discounting) so they set a lower markup when inflation rises.

This is a general result. To get a negative relationship between inflation and the markup, we require that firms set their prices infrequently and discount the future. [Nakamura and Steinsson \(2008\)](#) demonstrates that firms have low frequencies of price changes. [Jagannathan et al. \(2016\)](#) demonstrates that firms discount the future significantly. It is also worth stressing that this relationship is present in the representative agent New Keynesian model - nothing here depends upon household heterogeneity.

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<sup>10</sup>Taking into account the probability that they will not have updated their price by future periods.

<sup>11</sup>Because firms would set their price for less long on average so firms would lower their average markup by relatively less.

## 2.2 Relationship between the Markup and the Equilibrium Real Rate

**Simple Model** We now take the markup as given and consider the real side of a simple model.

We consider an economy in which firms have linear production function given by equation 1. Their real marginal cost is just be the real cost of labor (equation 2) and their markup is just the inverse of the marginal cost (equation 3).<sup>12</sup> Total real profits are given by equation 4.

$$Y_t = L_t \quad (1)$$

$$\frac{MC_t}{P_t} = \frac{W_t}{P_t} \quad (2)$$

$$m_t = \frac{P_t}{MC_t} \quad (3)$$

$$\Omega_t = Y_t - \frac{W_t}{P_t} L_t \quad (4)$$

We can simplify equation 4 by inputting equation 1 and multiplying and dividing the first term on the right-hand side by  $m_t$ . This yields equation 5. We can then simplify equation 5 by inputting equation 3 into the denominator and then substituting using equation 2. This yields equation 6

$$\Omega_t = \frac{m_t}{m_t} L_t - \frac{W_t}{P_t} L_t \quad (5)$$

$$\Omega_t = (m_t - 1) \frac{W_t}{P_t} L_t \quad (6)$$

The only asset available for households to purchase and hold as savings is shares in firms. The total real value of firms is denoted by  $Z_t$ . Therefore, a household can purchase a share  $b$  of firms for price  $bZ_t$ . A share  $b$  of firms pays to the holder their proportional share of profits,  $b\Omega_t$ , each period. We leave the discussion of household's utility and budget constraints for the moment.

**Asset Supply** We're going to break down the solution into the supply and demand for assets. Asset supply is the amount of assets that are available for households to hold. Asset demand is the amount of assets that agents want to hold. We start by considering asset supply.

The total assets that are available for households to hold is just the total value of shares in firms (since shares are the only assets that households can save in). Therefore, asset supply, denoted  $A^s$ , is given by equation 7.

$$A^s = Z \quad (7)$$

Next, we rewrite the value of shares in the firm,  $Z$ , as profits. To do this, we note that by asset pricing the value of the firm must equal the sum of discounted future profits which we can rewrite

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<sup>12</sup>This step ignores price dispersion. I show the computations with price dispersion in APPENDIX.

more simply in steady state. This is given by equation 8.

$$\bar{Z} = \sum_{i=0}^{\infty} \frac{\bar{\Omega}}{(1 + \bar{r})^i} = \frac{\bar{\Omega}}{\bar{r}} \quad (8)$$

We can then input equation 8 and then equation 6 into equation 7 to get equation 9

$$\bar{A}^s = \frac{(\bar{m} - 1) \frac{\bar{W}}{\bar{P}} \bar{L}}{\bar{r}} \quad (9)$$

We define relative assets  $a$  to be assets in terms of labor income which is shown in equation 10.

$$a = \frac{A}{\frac{W}{P} L} \quad (10)$$

We rewrite equation 9 in terms of relative assets to get equation 11

$$\bar{a}^s = \frac{\bar{m} - 1}{\bar{r}} \quad (11)$$

We observe two features. Firstly, we see in equation 11 that a rise in  $\bar{r}$  lowers  $\bar{a}^s$ . This makes sense because higher discounting implies the discounted sum of profits is lower so the value of firms falls. We plot equation 11 in figure 1. The blue curve shows the relative asset supply for a markup of 1.3. Since raising  $\bar{r}$  lowers  $\bar{a}^s$ , the curve has a downward slope. It may appear strange that the supply curve is downward sloping but this is because we have the return on assets on the vertical axis. The return on assets is like the inverse of the price of assets (since as the price of assets rises, the return agents make on those assets falls). If we drew the curve with the price of assets on the vertical axis, we'd get the usual upward sloping supply curve.

Secondly, we see in equation 11 that a fall in  $\bar{m}$  lowers  $\bar{a}^s$  for any  $\bar{r}$ . This makes sense because when the markup falls, the value of firms falls and thus the value of owning shares in firms falls. We plot a fall in the markup in figure 1. The blue curve represents  $\bar{a}^s$  with  $\bar{m} = 1.3$  and the orange curve represents  $\bar{a}^s$  with  $\bar{m} = 1.2$ . We see that the fall in the markup shifts the relative asset supply curve left.

**Asset Demand: 1. Representative Agent** Next, we consider the shape of the asset demand under three different household structures: 1. Representative agent. 2. Heterogeneity through OLG features. 3. Heterogeneity through idiosyncratic labor.

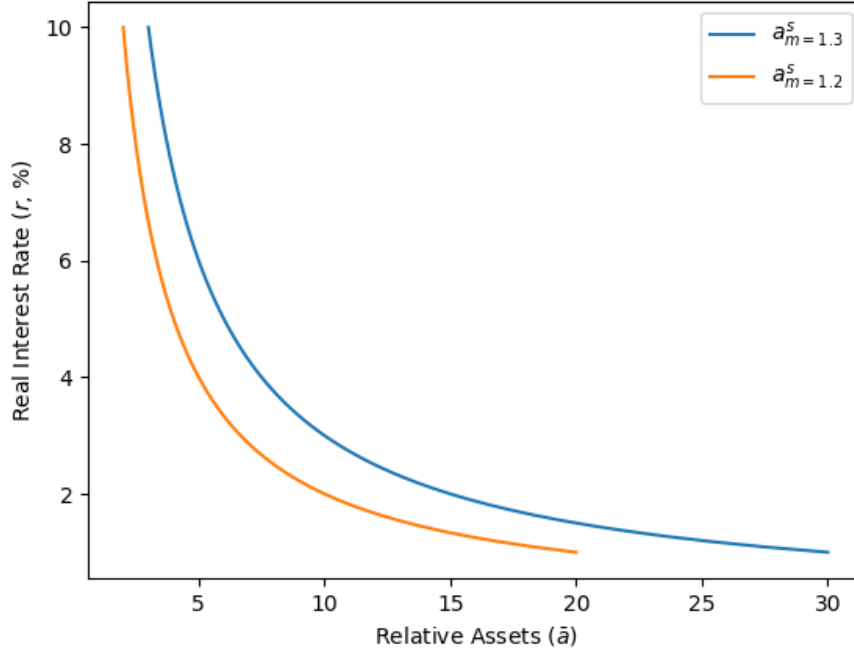
We consider a standard representative agent setup. Agents maximise some utility function (equation 12) subject to some budget constraint which we do not need to specify.

$$\max \sum_{t=0}^{\infty} \mathbb{E}_0[\beta^t u(C_t)] \quad (12)$$

A steady state equilibrium requires that a representative agent consumes the same amount



Figure 1: Relative Asset Supply under a Fall in the Markup



over time. The only way this is possible is if  $\bar{r} = \frac{1}{\beta} - 1$  as in equation 13 otherwise the agent has an incentive to raise/lower consumption over time.

$$u'(\bar{C}) = \beta(1 + \bar{r})u'(\bar{C}) \Rightarrow \bar{r} = \frac{1}{\beta} - 1 \quad (13)$$

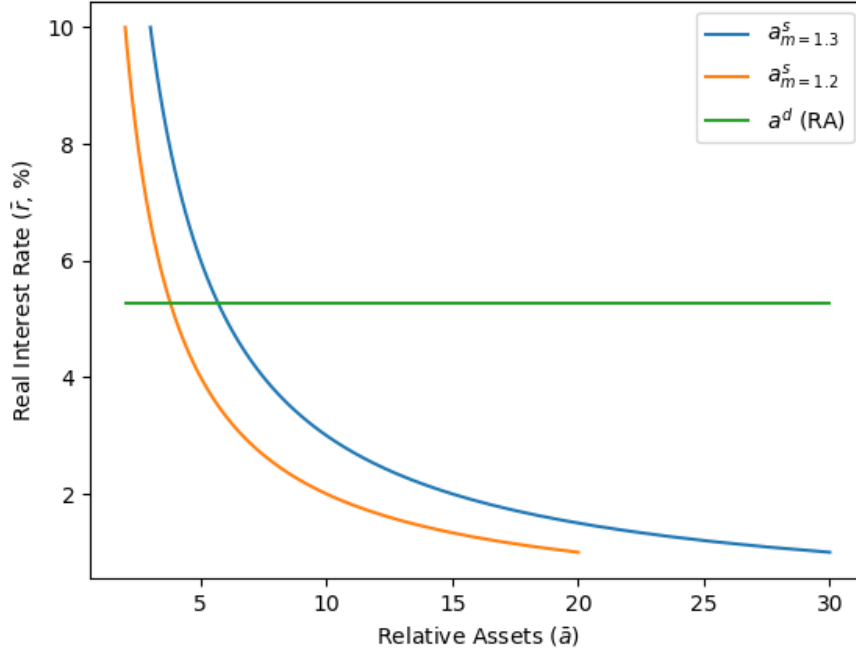
We plot equation 13 in figure 2 where we consider the impact of a fall in the markup (like in figure 1). We observe that the asset demand is just a horizontal line since  $\bar{r}$  is always pinned down. Thus, a shift left in the supply of assets lowers the amount of assets held by the household but has no impact upon  $\bar{r}$ .

We see that  $\bar{r}$  is unaffected by the level of assets. This is because in steady state the agent still will consume the same from one period to the next. Therefore, the assets held by the household have no impact upon the household's marginal utility over time. Therefore, a change in the level of assets has no impact upon the desire of agents to hold assets so the return of assets in equilibrium (the equilibrium real rate) stays the same.

**Asset Demand: 2. Overlapping Generations** Now, we consider the impact of introducing household heterogeneity. The implication in both cases we consider will be that the level of assets does impact the path of the household's marginal utility over time, meaning that the equilibrium real rate will be impacted by changing the markup.

Let's first consider a simple overlapping generations model based upon (Diamond, 1965). Ev-

Figure 2: Equilibrium under a Fall in the Markup: 2. Idiosyncratic Labor



every period a new generation is born. Each generation lives for two periods and then dies. Their utility is given by equation 14. We use log utility for simplicity. The budget constraint of the young is given by equation 15. Agents work one unit when they are young which they spend on either consumption  $C_{1,t}$  or assets  $A_{t+1}$ . The budget constraint of the old is given by equation 16. They merely consume  $C_{2,t+1}$  the value of their assets from when they were young on which they have earned a return of  $r_{t+1}$ .

$$\max_{C_{1,t}, C_{2,t+1}} \log(C_{1,t}) + \beta \log(C_{2,t+1}) \quad (14)$$

s.t.

$$C_{1,t} + A_{t+1} \leq \frac{W_t}{P_t} \quad (15)$$

$$C_{2,t+1} \leq (1 + r_{t+1})A_{t+1} \quad (16)$$

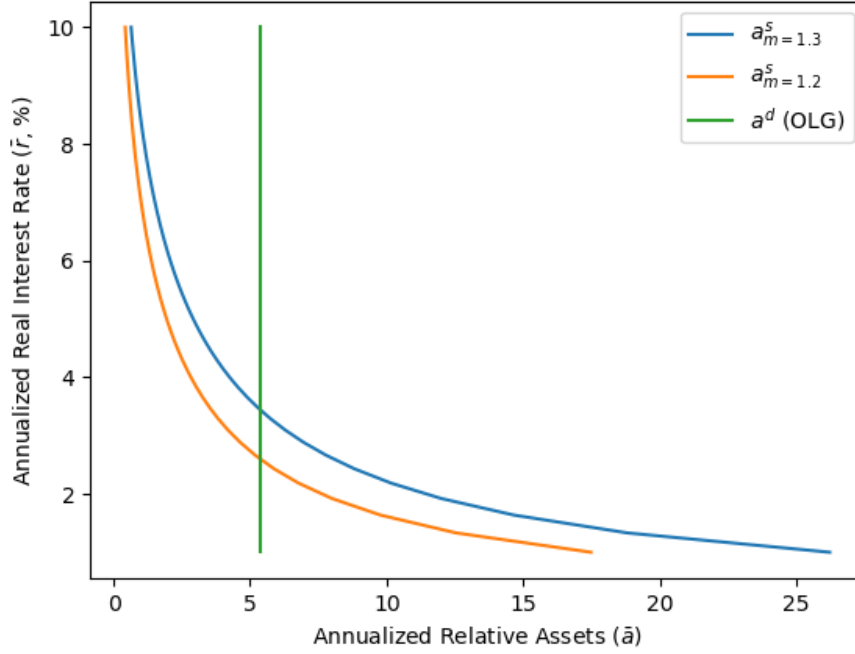
We can solve this by inputting equations 15 and 16 into equation 14. The steady state level of savings is given by equation 17. We divide by labor income to rewrite this as the demand for relative savings equation 18.

$$\bar{A} = \frac{\beta}{1 + \beta} \frac{\bar{W}}{\bar{P}} \quad (17)$$

$$\bar{a}^d = \frac{\beta}{1 + \beta} \quad (18)$$

In this case, the demand for savings is perfectly inelastic to changes in  $\bar{r}$ . This is something of

Figure 3: Equilibrium under a Fall in the Markup: 2. OLG



a special case (due to log utility and only having two periods). In our full model, we will not have perfectly inelastic demand. However, in general, once we move to an OLG framework, we always find that the elasticity of demand is not perfectly elastic and thus the real interest rate changes in response to a shift left in the demand for assets.

We plot equation 18 in figure 3 where we consider the impact of a fall in the markup (like in figure 1). We observe that the asset demand is just a horizontal line since  $\bar{a}^d$  is fixed. Thus, a shift left in the supply of assets lowers  $\bar{r}$  but has no impact upon the amount of assets demanded by the agent. This looks a bit different to previous asset demand/supply graphs since each period represents a generation and lasts for 25 – 30 years so it is necessary to rescale the curves to get back to an annual basis.<sup>13</sup>

This is effectively the opposite to the representative agent case. The reason the impact is so different is that a fall in the amount of savings held by the consumer affects the marginal utility of consumption of the young compared to the old. When assets fall, the old consume less relative to the young *ceteris paribus*. Thus, old people have a relatively higher marginal utility. Therefore, the price of assets rises since agents are keener to hold assets for when they are old. Consequently, the equilibrium real rate falls.

<sup>13</sup>We discuss the non-annualized case in appendix A.1.

**Asset Demand: 3. Idiosyncratic Labor** Within this paper, we will primarily examine the impact of heterogeneity through overlapping generations. However, we do explore an extension with idiosyncratic labor and it is worthwhile to demonstrate that a similar intuition explains why the channel holds in the case with idiosyncratic labor.

We use the standard idiosyncratic labor setup i.e. the same as Aiyagari households. Agents live forever and maximise their lifetime utility equation 19. They face some budget constraint equation 20. They get income from working some exogenous amount  $L_i$  with wage  $W$  and from their assets  $A_{i,t}$  which pay a return of  $r$ . They spend their income on consumption  $C_{i,t}$  and assets for the next period  $A_{i,t+1}$ . The key extra component is that they face some borrowing constraint equation 21 which limits the amount they may borrow each period - we set the limit to be 0. Note that there are no aggregate shocks hence why  $W, r$  have no time subscripts.<sup>14</sup>

$$\max \sum_{t=0}^{\infty} \mathbb{E}_0[\beta^t u(C_{i,t})] \quad (19)$$

s.t.

$$C_{i,t} + A_{i,t+1} = (1 + r)A_{i,t} + WL_{i,t} \quad (20)$$

$$A_{i,t+1} \geq 0 \quad (21)$$

where  $L_{i,t}$  follows some Markov process. We can solve this problem by value function iteration. We can then compute the asset demand  $\bar{a}^d$  for any equilibrium real rate  $\bar{r}$ .

We plot  $\bar{a}^d$  in figure 4 where we consider the impact of a fall in the markup (like in figure 1). A shift left in the asset supply due to a fall in the markup leads to a fall in the relative assets and a fall in the equilibrium real rate. As the level of assets increases, the shift left will lower relative assets more and lower the equilibrium real rate by less.

We again do not get the same as the representative agent case here because a fall in assets will lower the marginal utility in the next period by more than the current period since it means that more agents will face a binding borrowing constraint in the next period. This means that agents want to save more. In turn, this raises the price of assets and lowers their real return in equilibrium (the equilibrium real rate).

The degree to which a shift left in assets lowers the equilibrium real rate depends upon whether many agents are close the borrowing constraints. When the level of assets is high (low), a fall in

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<sup>14</sup> We do need to make one change from Aiyagari which is to rewrite the problem using relative assets (this has no substantive impact upon the results however):

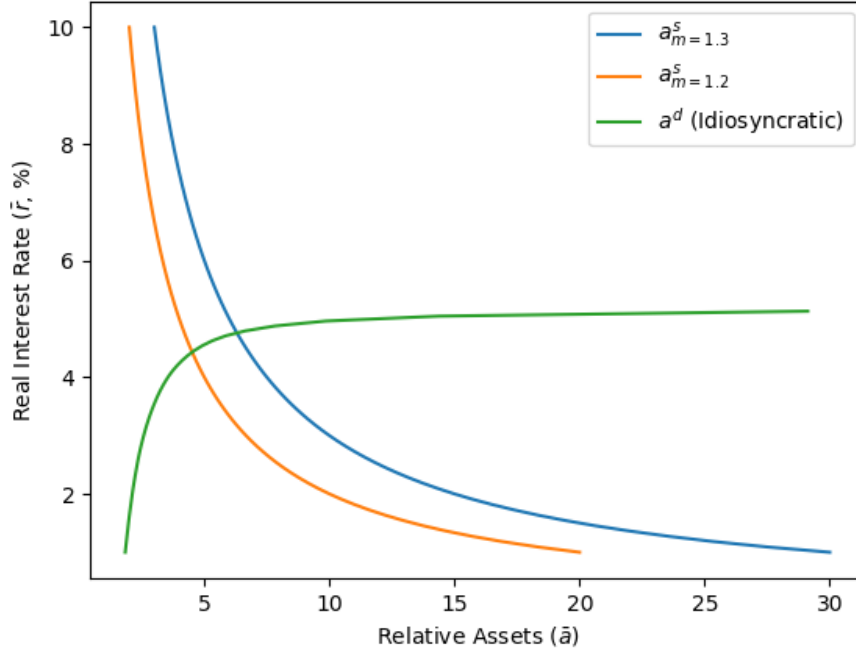
$$\max \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \right]$$

s.t.

$$a_{i,t+1} = (1 + r)a_t + \frac{L_{i,t}}{L_t} - c_{i,t}$$

$$a_{i,t+1} \geq 0$$

Figure 4: Equilibrium under a Fall in the Markup: 3. Idiosyncratic Labor



assets will increase a little (lot) the number of agents affected by the borrowing constraint so it will raise the demand for savings a little (lot) and thus lower the equilibrium real rate a little (lot).

### 3 Model

We now introduce the full model which we will use to assess the importance of the channel and to conduct welfare analysis.

#### 3.1 Households

We start by describing our general overlapping generations framework. Each period new agents are born. Each agent lives for  $M$  periods. Agents born in different periods overlap. We denote an agent by its age in periods so an agent born  $i$  periods ago is denoted  $i$ . Therefore, the  $M$  cohorts in any given period are denoted  $0, \dots, M-1$ . Each period: new agents are born (cohort 0), the oldest agents from the previous period (cohort  $M-1$  at time  $t-1$ ) have died and all other generations mature from cohort  $i$  to  $i+1$ .

We define that the population of the cohort born at time  $t$  is  $N_t$ . We define the total population to be  $\mathbb{N}_t$  and thus  $\mathbb{N}_t = \sum_{i=0}^{M-1} N_{t-i}$ . We assume that population grows at a constant rate of  $n$  so that  $N_{t+1} = (1+n)N_t$ . Thus, the total population also grows by  $1+n$  each year.

An agent of cohort  $i$  at time  $t$  has a budget constraint given by equation 96. An agent of cohort

$i$  consumes  $C_{i,t}$  at time  $t$ . An agent of cohort  $i$  works for  $L_{i,t}$ .  $W_t$  is the real wage paid at time  $t$  for each unit of work. An agent can invest in bonds, capital or shares in firms.  $B_{i,t}$ ,  $K_{i,t}$  are respectively the bonds and capital held by agents of cohort  $i$  at the start of period  $t$  (so they were chosen at  $t - 1$  when that agent was cohort  $i - 1$ ). The bond is in nominal terms and pays interest rate  $I_{t-1}$  at time  $t$  (denoted with a  $t - 1$  since the nominal interest rate is chosen at  $t - 1$ ). Capital is in real terms and agents get a real return of  $r_t$  from selling their capital to the firm at time  $t$ .  $\tilde{\omega}_{i,t}$  is the number of shares of the composite firm that agent  $i$  owns at the start of time  $t$ . We set that the total number of shares issued is 1 i.e.  $\sum_{i=0}^{M-1} \omega_{i,t} = 1$  so  $\tilde{\omega}_{i,t}$  represents the proportion of the firm owned by  $i$  at time  $t$ . The price of a share is  $\tilde{Z}_t$  and it pays out a proportional amount of the firm's total profits  $\tilde{\Omega}_t$  each period. We assume the agent starts with zero assets so  $K_{i,0} = B_{i,0} = \tilde{\omega}_{i,0} = 0$ .

$$C_{i,t} + \frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + \tilde{Z}_t \tilde{\omega}_{i+1,t+1} \leq W_t L_{i,t} + I_{t-1} \frac{B_{i,t}}{P_t} + (1 + r_t) K_{i,t} + (\tilde{\Omega}_t + \tilde{Z}_t) \tilde{\omega}_{i,t} \quad (22)$$

The agent's lifetime utility function when they are in cohort  $k$  is given by equation 23. We use CRRA utility. We allow for both endogenous and exogenous labor. In the exogenous labor case, we fix labor supply by each cohort so that  $L_{i,t} = L_i \forall t$  and the  $v$  disutility of labor term does not appear in the utility function. In the endogenous labor case, we allow for there to be disutility of labor with  $v_i(L_{i,t})$ .<sup>15</sup>

$$\mathbb{E}_t \left[ \sum_{i=0}^{M-k-1} \beta^i [u(C_{i,t}) - v(L_{i,t})] \right] \quad (23)$$

where:

$$\begin{aligned} u(C) &= \frac{C^{1-\gamma}}{1-\gamma} \\ v(L_{i,t}) &= \frac{1}{1+\eta} x_i L_{i,t}^{1+\eta} \end{aligned} \quad (24)$$

Therefore, an agent of cohort  $k$  faces the following problem:

$$\max_{\{C_{i,t+i}, B_{i+1,t+i+1}, K_{i+1,t+i+1}, \tilde{\omega}_{i+1,t+i+1}\}_{i=0}^{M-k-1}} \mathbb{E}_t \left[ \sum_{i=0}^{M-k-1} \beta^i [u(C_{i,t}) + u_b \left( \frac{B_{i,t}}{P_t} \right) - v(L_{i,t})] \right]$$

s.t.  $\forall i \in 0, \dots, M - 1$ :

$$C_{i,t} + \frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + \tilde{Z}_t \tilde{\omega}_{i+1,t+i+1} \leq W_t L_{i,t} + I_{t-1} \frac{B_{i,t}}{P_t} + (1 + r_t) K_{i,t} + (\tilde{\Omega}_t + \tilde{Z}_t) \tilde{\omega}_{i,t+i}$$

$$B_{M-k,t+M-k}, K_{M-k,t+M-k} \geq 0$$

We get the following first-order conditions (where we apply the fact that all agents of a given

<sup>15</sup>The disutility of labor has a subscript  $i$  and is thus dependent upon cohort since we want to allow agents of different cohorts to have different disutilities of labor (so old people want to work less).

cohort do the same so  $C_{i+1,t+1} = \bar{C}_{i+1,t+1}$  in equilibrium to get equation 26):  $\forall i \in 0, \dots, M-2$ :

$$u'(C_{i,t}) = \beta \mathbb{E}_t[u'(C_{i+1,t+1})(1 + r_{t+1})] \quad (25)$$

$$u'(C_{i,t}) = \beta \mathbb{E}_t[u'(C_{i+1,t+1}) \frac{I_t}{\Pi_{t+1}}] \quad (26)$$

$$\tilde{Z}_t u'(C_{i,t}) = \beta \mathbb{E}_t[u'(C_{i+1,t+1})(\tilde{\Omega}_{t+1} + \tilde{Z}_{t+1})] \quad (27)$$

With endogenous labor, we also get  $\forall i \in 0, \dots, M-1$ :

$$W_t u'(C_{i,t}) = v'(L_{i,t}) \quad (28)$$

Note that I have defined  $\Pi_t = \frac{P_t}{P_{t-1}}$ .

To make the model tractable, we rewrite the share holdings by generation in per capita terms. We define  $\omega_{i,t} = \mathbb{N}_t \tilde{\omega}_{i,t}$  so that  $\omega_{i,t}$  represents the proportional per capita holdings of an agent of cohort  $i$  at  $t$  of firm shares rather than the proportional total holdings of cohort  $i$  at  $t$ . We then define  $Z_t$  to be the price of a per capita share in firms i.e.  $Z_t = \frac{\tilde{Z}_t}{\mathbb{N}_t}$  and  $\Omega_t$  to be the profits paid by a per capita share in firms i.e.  $\Omega_t = \frac{\tilde{\Omega}_t}{\mathbb{N}_t}$ . Equations 27 and 96 become respectively:

$$Z_t u'(C_{i,t}) = \beta \mathbb{E}_t[u'(C_{i+1,t+1})(1 + n)(\Omega_{t+1} + Z_{t+1})] \quad (29)$$

$$C_{i,t} + \frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + Z_t \omega_{i+1,t+1} \leq W_t L_{i,t} + I_{t-1} \frac{B_{i,t}}{P_t} + (1 + r_t) K_{i,t} + (\Omega_t + Z_t) \omega_{i,t} \quad (30)$$

## 3.2 Firms

**Final Goods Firm** There is a single competitive final goods firm which aggregates production by firms in different industries which have weights  $a_j$ . It has CES production. It minimises its costs for a given level of output. Therefore, it faces the following problem:

$$\min_{\{Q_{j,t}\}_0^1} \int_0^1 Y_{j,t} P_{j,t} dj$$

s.t.

$$\left( \int_0^1 a_j^{\frac{1}{\sigma_2}} Y_{j,t}^{\frac{\sigma_2-1}{\sigma_2}} dj \right)^{\frac{\sigma_2}{\sigma_2-1}} = Y_t$$

The final goods firm has the usual CES demand for each industry good given by equation 34. The price aggregator also takes the usual form given by equation 34. Note that we need to add weights  $a_j$  for each industry.

$$Y_{j,t} = a_j \left( \frac{P_{j,t}}{P_t} \right)^{-\sigma_2} Y_t \quad (31)$$

$$P_t = \left( \int_0^1 a_j P_{j,t}^{1-\sigma_2} dj \right)^{\frac{1}{1-\sigma_2}} \quad (32)$$

**Industry Aggregator** We allow for different industries with different weights and degrees of price rigidity. We do this for two reasons. The primary reason is that allowing for different degrees of price rigidities increases the degree of monetary non-neutrality which is otherwise unrealistically low. See [Carvalho \(2006\)](#) for an explanation. It is also more realistic to allow for different industries with different degrees of price rigidity.

There are  $J$  industries in total, denoted  $1, \dots, J$ . A perfectly competitive firm aggregates intermediate goods of each industry  $j$  to produce the good for sector  $j$ . The sector firm has the following production function:

$$Y_{j,t} = \left( \int_0^1 Y_{i,j,t}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

This implies that the industry aggregator has the usual CES demand for each intermediate good given by equation 33. The price aggregator also takes the usual form given by equation 34.

$$Y_{i,j,t} = Y_{j,t} \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\sigma} \quad (33)$$

$$P_{j,t} = \left( \int_0^1 P_{i,j,t}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \quad (34)$$

**Intermediate Goods Firms** Within each industry, we have intermediate goods firms that face the usual Calvo problem.

Intermediate goods firms have Cobb Douglas production over capital ( $K_t^*$ ) and labor ( $L_t^*$ ). Real profits of an intermediate firm  $\Omega_t^*$  in a single period are given by ?? . They rent capital from consumers at real rate  $r_t$ . They also have to refund consumers for the depreciation  $\delta$  in capital. They pay workers a real wage  $W_t$  for each unit of labor. We allow for a tax (surplus)  $\tau$  on renting capital and labor. In equilibrium, we set that the lump sum transfer the firm receives each period is just the tax paid by the firm for its use of inputs (so the only impact of the tax is to adjust the markup at which the firm produces).<sup>16</sup>

$$Y_{i,j,t} = A_t K_{i,j,t}^\alpha L_{i,j,t}^{1-\alpha} \quad (35)$$

$$\Omega_{i,j,t} = \frac{P_{i,j,t} Y_{i,j,t}}{P_t} - (1 + \tau)((r_t + \delta)K_{i,j,t} + W_t L_{i,j,t}) + lumpsumtransfer_{i,j,t} \quad (36)$$

Intermediate firms minimise costs, which requires that equations 37 and 38 hold.  $MC_t$  represents the marginal cost of the firm before tax.

$$MC_t = \frac{r_t + \delta}{\alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}} \quad (37)$$

$$MC_t = \frac{W_t}{(1 - \alpha) A_t K_t^\alpha L_t^{-\alpha}} \quad (38)$$

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<sup>16</sup>We introduce the tax so we can set the equilibrium real rate to take a particular value.



Firms in each industry  $j$  have a  $\lambda_j$  probability of updating their price each period. When they do get to change their price, firms maximise equation 39 subject to the demand for their good from the industry aggregator firm equation 33. We allow for additional firm discounting  $\beta_f$  beyond the stochastic discount factor.

$$\max_{P_{j,t}^*, Q_{i,j,t}} \sum_{k=0}^{\infty} \beta_f^k M_{t,t+k} (1 - \lambda_j)^k \left[ \frac{P_{j,t}^* Q_{i,j,t+k}}{P_{t+k}} - MC_{j,t+k} Q_{i,j,t+k} \right] \quad (39)$$

The solution to the intermediate firm's problem is the FOC given by equation 40

$$\sum_{k=0}^{\infty} \beta_f^k M_{t,t+k} (1 - \lambda_j)^k Q_{j,t+k} P_{t+k}^{\sigma} \left[ \frac{P_{j,t}^*}{P_{t+k}} - \frac{\sigma}{\sigma - 1} MC_{j,t+k} \right] \quad (40)$$

Additionally, we define the price dispersion parameter for each industry in the usual manner in equation 41. We also get the process for price dispersion in each industry (equation 42 and a relationship between the chosen price and inflation in each industry (equation 43).

$$\nu_{j,t} = \int_0^1 \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\sigma} di \quad (41)$$

$$\nu_{j,t} = \lambda_j \left( \frac{P_{j,t}^*}{P_{j,t}} \right)^{-\sigma} + (1 - \lambda_j) \nu_{j,t-1} \Pi_{j,t}^{\sigma} \quad (42)$$

$$1 = \lambda_j \left( \frac{P_{j,t}^*}{P_{j,t}} \right)^{1-\sigma} + (1 - \lambda_j) \Pi_{j,t}^{\sigma-1} \quad (43)$$

We can aggregate equations 35 and 36 to get equations 44 and 45. It is also possible to write profits in the more usual form given in equation 46. Note that  $Y_t, K_t, L_t$  are all per capita variables so that they remain constant in steady state even with  $n \neq 0$ .<sup>17</sup>  $K_t$  specifically is the per capita value of consumption held by households at the start of  $t$ .

$$Y_t \nu_t = A_t K_t^{\alpha} L_t^{1-\alpha} \quad (44)$$

$$\Omega_t = Y_t - Y_t MC_t \nu_t \quad (45)$$

$$\Omega_t = Y_t - (r_t + \delta) K_t - W_t L_t \quad (46)$$

where:

$$\nu_t = \int_0^1 a_j \left( \frac{P_{j,t}}{P_t} \right)^{-\tau} \nu_{j,t} dj \quad (47)$$

The average markup  $m_t$  is defined to be the inverse of the marginal cost of producing one final good as in equation 48.<sup>18</sup> We rewrite equation 45 in terms of the markup in equation 49. Using

<sup>17</sup>We could either allow for the number of firms to change or the size of firms to change when  $n \neq 0$ . However, since we're typically interested in the  $n = 0$  case, we ignore these complications.

<sup>18</sup>This includes the degree of price dispersion because as the price dispersion increases, demand for intermediate

equation 44, we can also rewrite equations 37 and 38 in terms of  $m$  as equations 50 and 51

$$m_t = \frac{1}{MC_t \nu_t} \quad (48)$$

$$\Omega_t = (1 - \frac{1}{m_t}) Y_t \quad (49)$$

$$\frac{\alpha}{m_t} = \frac{(r_t + \delta) K_t}{Y_t} \quad (50)$$

$$\frac{1 - \alpha}{m_t} = \frac{W_t L_t}{Y_t} \quad (51)$$

### 3.3 Monetary and Fiscal Policy

When investigating the long-run equilibrium, we don't need to specify a monetary rule (since we're just computing the steady state). Therefore, we just note that the central bank holds inflation at some target  $\pi^*$ .

We assume that the government has no debt/savings:

$$B_t = 0$$

### 3.4 Other Conditions

In our main model, we set  $A_t = 1$ .

Total labor is just the population-weighted sum of labor given by ??.

$$L_t = \frac{\sum_{i=0}^{M-1} (\frac{1}{1+n})^i L_{i,t}}{\sum_{i=0}^{M-1} (\frac{1}{1+n})^i} \quad (52)$$

We will solve the model in a similar manner to section 2. We will compute the relative asset demand and relative asset supply and look for equilibria where they are equal. Assets are the amount of savings that agent have at the end of a given period which they save for the next period. Per capita assets at  $t$  are denoted by  $A_t$ . We define relative assets  $a_t$  to be total assets divided by labor income (??). The reason we use relative assets is because its easier to solve because then asset demand doesn't depend upon the wage.

$$a_t = \frac{A_t}{W_t L_t} \quad (53)$$

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goods with cheaper prices rises even though these goods contribute less to making a final good than less used goods with more expensive prices. Thus, more intermediate goods must be used to produce a final good than in the case where there is no price dispersion.

## 4 Model Solution and Calibration

### 4.1 Steady State

We compute the steady state of the full model in section 3 to how we computed the steady state in section 2. We break the solution into three parts. Firstly, we solve for the markup  $\bar{m}$  given the inflation target; this is explained in appendix C.1. Secondly, we solve for the demand for supply of relative assets  $\bar{a}^s$  given the markup; this is explained in appendix C.2. Thirdly, we solve for the demand for relative assets  $\bar{a}^d$ ; this is explained in appendix C.3. We can then find the steady state by looking for points where the supply and demand for relative assets intersect.

### 4.2 Calibration

We set that each period represents a quarter. We set standard parameters as follows:  $\alpha = 0.3, \beta = 0.98^{\frac{1}{4}}, \delta = 0.1^{\frac{1}{4}}, \gamma = 1$ .

We set  $M = 220$  to capture each quarter of life of an adult between the age of 24 and 78. We start at age 24 to avoid having to worry about how to capture college. We end at age 78 because the life expectancy of someone in the US is currently just under 79 years.

With exogenous labor supply, we set  $\bar{L}_i$  (hours worked by each age) to match the average hours worked of a person of that age in the American Time Use Survey between 2003 and 2016. With endogenous labor supply, we set  $x_i$  in the disutility of labor function (equation 24) so that when  $\beta\bar{R} = 1$  we have that  $\bar{L}_i$  matches the hours worked in the exogenous case.

We set the industry weights and frequencies of price adjustment to match regular prices in Table 2 of Nakamura and Steinsson (2008). The elasticity of substitution between varieties within industries ( $\sigma$ ) is set to be  $\sigma = 8$ . This is in between the lower and upper bounds used in Carvalho et al. (2016). The elasticity of substitution between industries ( $\sigma_2$ ) is set to 1 like in Shamloo (2010)<sup>19</sup>.

It is important that we get firm discounting right since it makes a difference for the size of the first part of the channel. We base the degree of firm discounting upon the Weighted Average Cost of Capital (WACC) which is the average a company is expected to pay to finance its assets.<sup>20</sup> Jaganathan et al. estimated that it was 8% in 2003 when the expected ten year rate on real bonds ( $r^e$ ) was 2.8p.p. Graham and Harvey estimated it was 10.0% in 2011Q1 when  $r^e$  was 2.2% Graham and Harvey estimated it was 2012Q2 in 9.3% when  $r^e = 1.3\%$ . We find that the average wedge between WACC and the expected real rate is 7p.p. Therefore, we apply a firm discount of  $\beta_f = \left(\frac{1}{1.07}\right)^{\frac{1}{4}}$ .

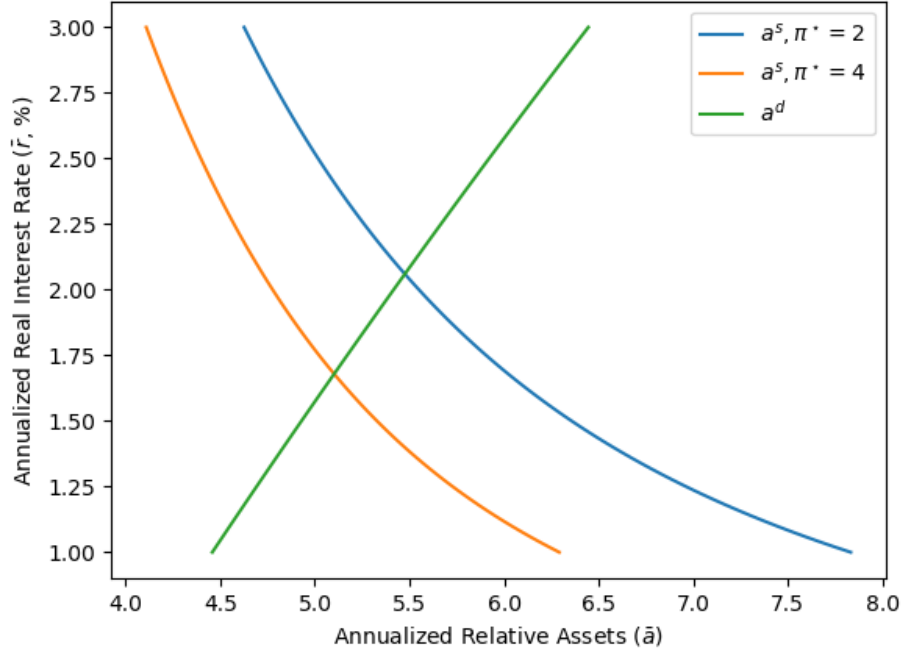
We set  $\bar{r} = 2.06$  when  $\pi^* = 2$ . This matches the average real interest rate on treasury bills between 1995 and 2007. We set  $\tau$  (the tax on the labor and capital inputs) to pin down  $\bar{r}$  at this level.

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<sup>19</sup>I actually set it to be 1.001 otherwise I'd have to rewrite the indices since 1 is a special limiting case.

<sup>20</sup>We use this since this is the cost to the firm of not obtaining funds earlier by setting a lower markup.

Figure 5: Baseline Calibration: Asset Supply and Demand



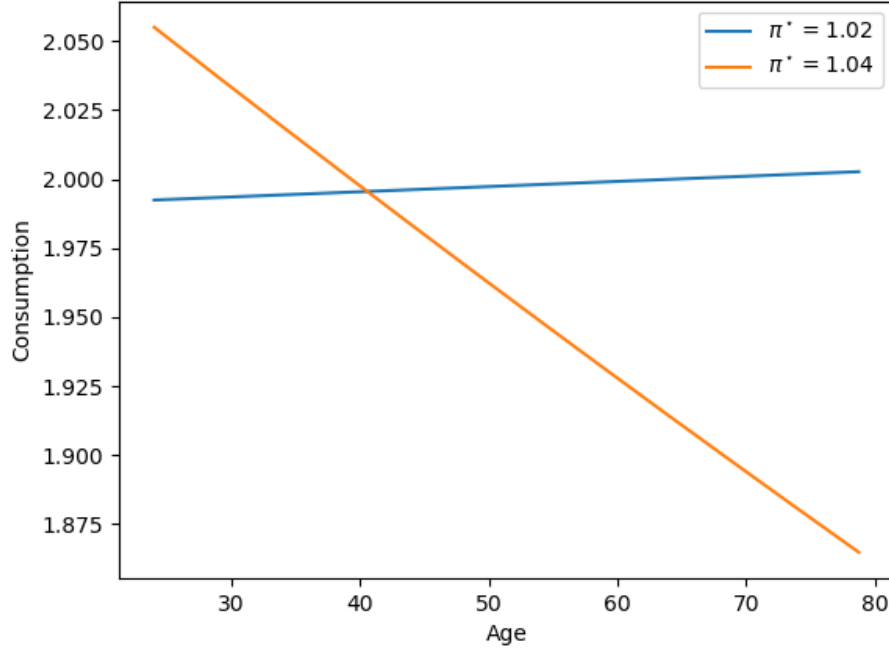
## 5 Impact of Raising the Inflation Target to 4 Percent

In this section, we consider the impact of raising the annual inflation target from 2% to 4%. We choose 2% to be the baseline level of the inflation target because that is the standard inflation target among developed countries. We choose to investigate the impact of raising the inflation target by 2p.p. to 4% since that is the most commonly proposed adjustment (Blanchard et al. (2010), Ball (2014), Krugman (2014)).

In figure 5, we observe the impact of the policy experiment on the supply and demand for relative assets. The impact of raising the inflation target has exactly the same qualitative impact as in section 2. The supply of assets shifts left since a lower markup lowers profits and thus the value of firms. Therefore, there are fewer assets available for households to hold. It has no impact upon the demand for assets by households. We observe that a shift left in the supply of savings lowers the equilibrium real rate and equilibrium relative assets. The intuition for the fall in the equilibrium real rate is also the same as in the simplified model. Households rely upon savings to consume when they are old. A fall in savings means that households consume relatively less when they are old so the price of saving rises which is equivalent to a fall in the equilibrium real rate.

Figure 6 shows the impact of the rise in inflation on the consumption path of agents across their lives. We see that a rise in the inflation target lowers the consumption of the old relative to the young. This is because agents save less for when they are old as a result of the lower supply

Figure 6: Baseline Calibration: Consumption Path



of assets.

In table 1, we show the numerical impact of the policy experiment with the default calibration.<sup>21</sup> We observe that a rise in the inflation target leads to a fall in the markup of 1.07p.p. This is just the first part of the channel where firms set a lower markup with higher inflation due to price rigidities. Ceteris paribus, a fall in savings reduces the ability of older agents to consume. Therefore, the consumption of the old falls relative to the young. The second row of table 1 shows that the consumption of the older half of consumers falls by 5.14 percent relative to the younger half of consumers. Next, we observe that agents hold 4.46 percent more capital (relative to the representative agent baseline case). Agents want to save more in capital try to reduce their loss of consumption when they are old. Since agents want to save more, the price of saving (the equilibrium real rate) falls. In this case, it falls by 0.38p.p.

Raising the inflation target would be less effective in reducing the probability of the effective lower bound. We observe that a rise in the inflation target of 2p.p. only raises the equilibrium nominal interest rate by 1.62p.p. in our default calibration, as opposed to the 2p.p. rise widely assumed and predicted by standard models. Since nominal interest rates would rise by less, this would give policymakers less room to cut in bad times before hitting the effective lower bound.

<sup>21</sup>By row, the mathematical expressions for what table 1 shows are:  $100\Delta_\pi m$ ,  $100\Delta_\pi(\log(\sum_{i=120}^{239} C_i) - \log(\sum_{i=120}^{240} C_i))$ ,  $100(\Delta_\pi(\log(K_{olg}) - \log(K_{ra})))$ ,  $100\Delta_\pi r$ . We consider capital relative to the baseline representative agent case because then we observe that capital always increases relatively to the representative case, even if it may fall in absolute terms - and we are interested in showing that agents want to hold more capital.

Table 1: Policy Experiment with Default Calibration

Defaults	
Change in markup (p.p.)	-1.07
Change in $\frac{C_{old}}{C_{young}}$ (%)	-5.14
Change in K (%)	4.46
Change in r (p.p.)	-0.38

In figure 7, we observe how lowering the intertemporal elasticity of substitution (IES) to 0.1 affects the supply and demand of relative assets. Recent estimates from Best et al. (2018) suggest that the IES is 0.1. Lowering the IES to 0.1 (from its baseline value of 0.5) causes the demand for assets to tilt backwards. The reason for this is because when agents have low IES, they have a stronger desire to consume the same each period. When the real interest rate rises, agents get a higher return on their savings allowing them to consume more when they are old. With a low IES, they will then reduce the amount they save to rebalance consumption back to when they are young. In this sense, the income effect of raising the real rate dominates when IES is high enough. A fall of the supply of savings implies that the relative consumption of the old falls. Since agents with low IES are keener to consume the same each period, the price of savings rises by more and thus the equilibrium real rate falls by more. We can also see this graphically by the fact that the gradient of the demand curve is much closer to the supply curve.

In table 2, we show the numerical impact of the policy experiment under different IES.<sup>22</sup> Note that the column with an IES of 0.5 is just our default calibration and matches the results in table 1. We observe that lower IES which pushes agents to consume the same in each period implies that the price of savings will rise by more and thus the return on savings falls by more. With an IES of 0.1, the equilibrium real rate falls therefore by 0.67p.p. compared to 0.38p.p. in the baseline case. With capital, agents can mitigate the fall in their savings through the lower markup (and thus lower profit) by raising their investment in capital. Since agents with low IES really want to consume the same over time, they invest more in capital hence why it rises by 8.02% relative to the representative agent case compared to 4.46% in the baseline calibration.<sup>23</sup>

In table 3, we explore the impact of allowing for endogenous labor supply.<sup>24</sup> Allowing for endogenous labor leads to a somewhat smaller fall in the real interest rate than in the baseline calibration with exogenous labor (figure 5). The reason for this is that when the markup falls and savings falls so that agents consume less when they are old relative to when they are young, agents can choose to work relatively more when they are old to substitute for the loss in consumption when they are old. The extent to which they do this depends upon their elasticity of labor supply.

<sup>22</sup>The rows have the same mathematical expressions as footnote 21.

<sup>23</sup>We find that the markup falls by less when IES is lower. This is because when IES is lower, the real interest rate falls by more which implies discounting falls and thus there's the impact of inflation on the markup is lessened slightly.

<sup>24</sup>Change in  $\frac{L_{old}}{L_{young}}$  has the following mathematical expression:  $100\Delta\pi(\log(\sum_{i=120}^{239} L_i) - \log(\sum_{i=120}^{240} L_i))$ . The other rows have the same mathematical expressions as footnote 21.

Figure 7: IES = 0.1: Asset Supply and Demand

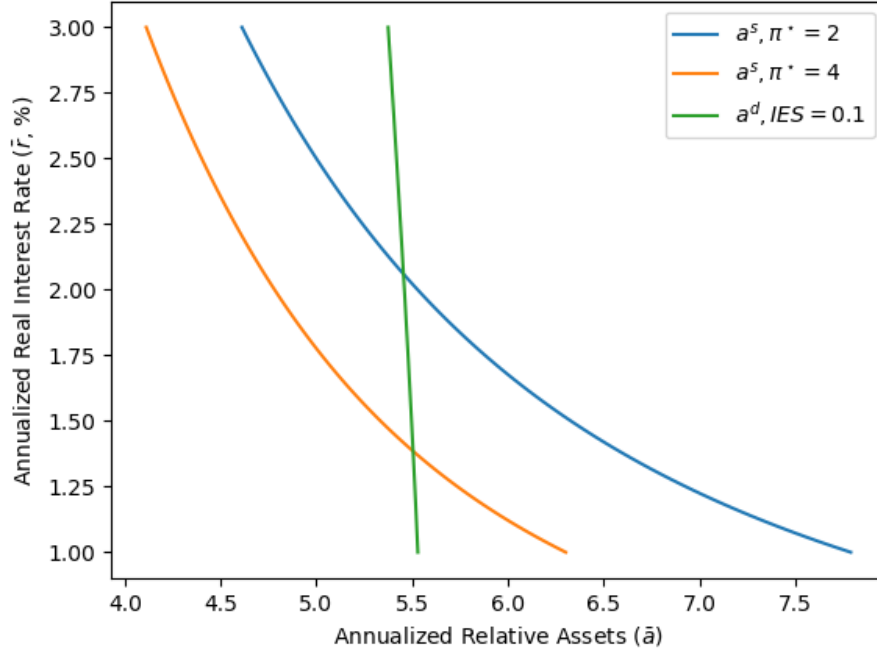


Table 2: Impact of Changing Intertemporal Elasticity of Substitution on Results of Policy Experiment

IES ( $\frac{1}{\gamma}$ )	0.1	0.2	0.3	0.5	1
Change in markup (p.p.)	-1.04	-1.05	-1.06	-1.07	-1.09
Change in $\frac{C_{old}}{C_{young}}$ (%)	-1.83	-3.11	-4.01	-5.14	-6.35
Change in K (%)	8.02	6.80	5.84	4.46	2.74
Change in r (p.p.)	-0.67	-0.58	-0.50	-0.38	-0.24

Table 3: Impact of Changing Elasticity of Labor Supply on Results of Policy Experiment

Elasticity of Labor Supply ( $\eta$ )	1	2	3	5	10
Change in $\frac{C_{old}}{C_{young}}$ (%)	-2.85	-3.68	-4.07	-4.45	-4.77
Change in $\frac{L_{old}}{L_{young}}$ (%)	4.65	3.01	2.22	1.45	0.78
Change in K (%)	2.17	2.91	3.28	3.67	4.02
Change in r (p.p.)	-0.21	-0.27	-0.30	-0.33	-0.35

With a low elasticity of labor supply, they work 4.65% relatively more when they are old than when they are young.<sup>25</sup> Consequently, with a low elasticity of labor supply, households also choose to invest relatively less in capital since they're replacing consumption when they are old through working when they are older instead. The fall in the equilibrium real rate lessens when households have a low elasticity of labor supply. As the elasticity of labor supply gets large, we converge back to the exogenous labor supply case in table 1.

We also explore the impact of idiosyncratic labor shocks. In appendix F, we examine how we can embed idiosyncratic labor shocks within the New Keynesian life cycle model of section 3. Figure 8 shows the impact of adding idiosyncratic labor shocks. The blue, orange and green curves are identical to figure 5. The blue and orange curves are the supply of relative assets respectively before and after the shift left in relative asset supply due to a rise in the inflation target. The green curve is the demand for relative assets with OLG households and no idiosyncratic labor shocks. The dashed red curve represents the OLG model with idiosyncratic labor included. We see that the only effective impact is that the demand for relative assets shifts out. The intuition for this is that households face more risk so they want to save more as a precaution against this risk. However, there does not appear to be a substantive impact on the degree to which the real interest rate falls following the shift left in the supply of assets.

## 6 Optimal Inflation Target

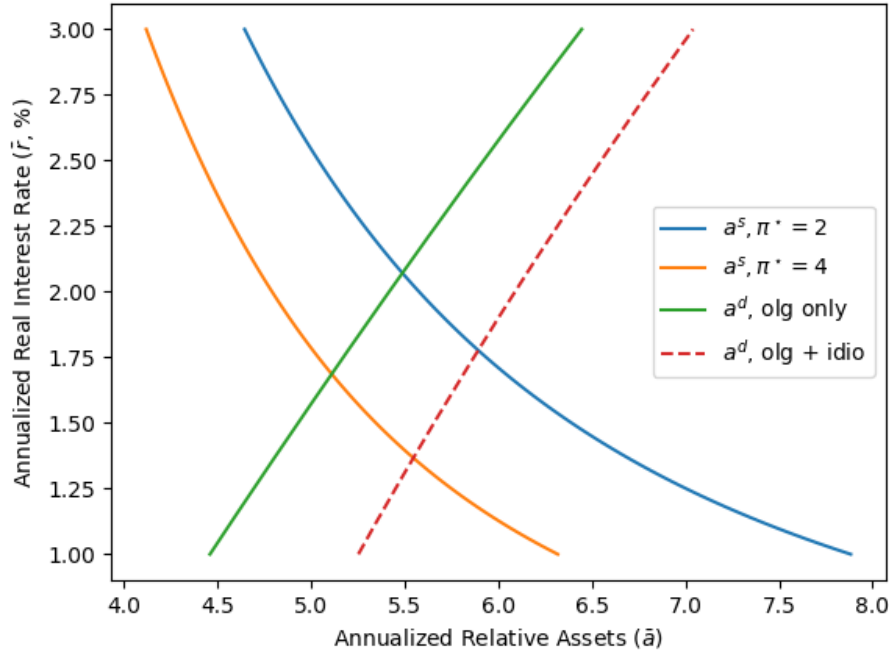
In this section, we consider optimal policy. There are two reasons why this is important. Firstly, it is useful for us to assess how allowing for raising the inflation target to lower the equilibrium real rate will affect the optimal inflation target. Secondly, it is interesting to explore more generally the implications of heterogeneity for optimal policy.

We allow for shocks within the model presented in section 3. We incorporate the same shocks as Coibion et al. (2012) into our main model. The model with log-linearised conditions is discussed in appendix G. To assess the optimal inflation target, we simulate the path of the economy over 2,000 periods under the same shocks for different inflation targets. We then find the optimal inflation target by computing which inflation target yields the highest welfare.

<sup>25</sup>We define old agents to be those above the average age and those who are young to be below the average age.



Figure 8: OLG with Idiosyncratic Labor Shocks: Asset Supply and Demand



We use a similar method to [Guerrieri and Iacoviello \(2015\)](#) to take account of the zero lower bound. For any period within a simulation, we guess when the zero lower bound would bind without further shocks. We solve backwards to get policy functions and verify whether or guess was correct. We iterate on the guess until it is correct.

The inflation target is low, like in the representative agent case. We find an optimal inflation target of 1.2%. The optimal inflation target is low because of the high costs of price dispersion. Even under a low inflation target, the costs of inflation through higher price dispersion seem to dominate the benefits of avoiding the zero lower bound. This also appears to be true in the representative case since [Coibion et al. \(2012\)](#) find the optimal inflation target in a representative agent model with the zero lower bound and find a similarly low optimal inflation target.

When we reduce the size of the welfare costs of inflation, optimal inflation is higher. If we conduct the analysis using Rotemberg pricing where only 20% of the costs of updating prices actually affect the broader economy (so 80% are just internalised costs like laziness which we ignore) then the optimal inflation target rises to 2.6%.<sup>26</sup>

<sup>26</sup>I will update this in a later draft.

## 7 Empirical Evidence

In this section, I provide reduced form empirical estimates of two key relationships in my paper to complement existing structural analysis.

We first consider the relationship between long-run inflation and the equilibrium real rate. The new channel I propose implies that a rise in long-run inflation lowers the equilibrium real rate. There is existing empirical evidence for this. Both [King and Watson \(1997\)](#) and [Rapach \(2003\)](#) find such a relationship. To complement this existing evidence, we conduct a reduced form analysis.

A problem with studying the reduced form relationship between long-run inflation and the equilibrium real rate is correlated trends. Inflation has trended down in recent years at the same time as the equilibrium real rate has fallen. If the fall in inflation was the only reason for the fall in the equilibrium real rate then this would imply my channel is incorrect. However, there are many other reasons why the equilibrium real rate has fallen.<sup>27</sup> Therefore, since real rates have fallen at the same time as inflation has fallen but for reasons other than the fall in inflation, a simple regression of the equilibrium real rate on inflation is likely to produce a positively biased coefficient.

To overcome common trends in inflation and real rates, we conduct panel data regressions with time fixed effects. Using time fixed effects allows us to control for the common global trend in real rates. Then we can assess whether higher relative inflation is associated with a positive or negative deviation from the global trend in real rates. If we assume that other factors that cause deviations from the global trend in real rates for a country are uncorrelated with that country's inflation level then this relationship is causal.

Regression: Equation 54 shows the regression relationship that we consider.  $\alpha_i$  represents country fixed effects i.e. whether the real interest rate is systematically higher in a country.  $\delta_t$  represents the time fixed effects.  $\beta$  is the coefficient of interest which represents the change in the real interest rate relative to the global trend associated with a 1p.p. rise in long-run inflation. We also allow for controls.

$$r_{i,t} = \alpha_i + \delta_t + \beta \overline{Inflation}_{i,t} + \Gamma Controls_t + u_{i,t} \quad (54)$$

Data: We limit our panel to just OECD members. We use annual data. We measure long-run inflation ( $\overline{Inflation}_{i,t}$ ) as the moving average of the current and previous four years of CPI inflation.<sup>28</sup> <sup>29</sup> We measure the real interest rate by a measure of the 10 year real rate. We use the 10 year real rate since there is more data availability and its likely to be a much less noisy measure of the equilibrium real rate. To measure the 10 year real rate we subtract our measure

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<sup>27</sup>See the introduction for a discussion of these factors.

<sup>28</sup>It may seem strange that we don't estimate the impact of changing the inflation target but nearly all inflation targets have not changed since they were introduced so the inflation target would be almost completely captured by the country fixed effects ( $\alpha_i$ ).

<sup>29</sup>Varying the measure to a different moving average does not appear to impact the results.

Table 4: Empirical Estimates of Relationship between Long-Run Inflation and Equilibrium Real Rate

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.167*** (0.040)	-0.196*** (0.044)	-0.607*** (0.069)	-0.904*** (0.149)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	1151	1151	1151	833

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

of long-run inflation from the nominal interest rate on 10 year government bonds.<sup>30</sup> We allow for business cycle controls.<sup>31</sup> We set the business cycle controls to be GDP growth and change in unemployment at  $t$  and  $t - 1$ .

The results are given in table 4. Without fixed or time effects a 1p.p. rise in long-run inflation is associated with a fall of  $-0.17$ p.p. in the real rate. This falls slightly once we add country fixed effects. We have already noted that inflation and the real rate both have a negative trend so it is unsurprising that once we add time fixed effects which remove this source of positive association, the coefficient drops a lot to  $-0.61$ p.p. Controls only make the result stronger. A causal interpretation of the regression without controls is thus that a rise of 1p.p. in long-run inflation lowers the equilibrium real rate by 0.61p.p.

The second relationship we are interested in is the relationship between long-run inflation and the long-run markup. The first part of the new channel I propose implies that a rise in long-run inflation lowers the long-run markup. There is existing empirical evidence for this. Both Bénabou (1992) and Banerjee and Russell (2001) find such a relationship. To complement this existing evidence we conduct a reduced form analysis.

We use the labor share as a proxy for the markup. We cannot directly measure the markup. However, we observe in equation 55 that a rise in the labor share of 1p.p. is equivalent to a  $\frac{1}{1-\alpha}$ p.p. rise in the inverse of the markup. And since  $m > 1$  a  $\frac{1}{1-\alpha}$ p.p. rise in the inverse of the markup equates to a greater than  $\frac{1}{1-\alpha}$ p.p. fall in the markup.<sup>32</sup> Therefore, if we compute that a 1p.p. rise in inflation raises the labor share by  $x$ p.p. then it should lower the markup by at least  $\frac{1}{1-\alpha}x$ p.p.

$$\frac{1}{\bar{m}} = \frac{1}{1-\alpha} \frac{\bar{W}\bar{L}}{\bar{Y}} \quad (55)$$

<sup>30</sup>Computing the measure of 10 year real interest rates by subtracting current inflation (rather than our measure of long-run inflation) from the nominal interest rate on 10 year government bonds yields similar results.

<sup>31</sup>We don't want to have controls that capture the long-run state of the economy since these could interfere in the long-run relationship between inflation and the real rate.

<sup>32</sup>For example, when  $m_0 = 2$ ,  $m_1 = 1.98$ , we see that  $\frac{1}{m_0} = 0.5$ ,  $\frac{1}{m_1} \approx 0.51$ . In this case, a 1p.p. rise in the inverse of the markup is equivalent to a 2p.p. fall in the markup.

Table 5: Empirical Estimates of Relationship between Long-Run Inflation and Long-Run Markup

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	0.209*	0.304***	0.281***	0.298
	(0.083)	(0.037)	(0.032)	(0.178)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	813	813	813	721

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

Regression: Equation 56 shows the regression relationship that we consider. We use the same basic panel data structure as for the inflation-real rate relationship.

$$LaborShare_{i,t} = \alpha_i + \delta_t + \beta \overline{Inflation}_{i,t} + \Gamma Controls_t + u_{i,t} \quad (56)$$

Data: To compute the labor share, we compute the percentage net value added in production that is received as compensation by employees for firms only. We get this data from National Accounts at the UN and the OECD.<sup>33</sup>

The results are given in table 5. They don't vary very much once we add in country fixed effects. It appears that a 1p.p. rise is associated with a 0.3p.p. rise in the markup. Assuming causality and  $\alpha = 0.35$ , we observe that a 1p.p. rise in the markup leads to a fall of more than 0.46p.p. in the markup.

We verify these two relationships are robust. In tables 7 and 8, we verify the relationships continue to hold with just OECD members that joined before 1975 (excluding a number of mostly Eastern European countries that joined from the 1990s onwards). In tables 9 and 10, we verify the relationships continue to hold under low inflation. In tables 11 and 12, we look at whether the relationships continue to hold before 2000. In tables 13 and 14, we look at whether the relationship continue to hold during/after 2000.

## 8 Conclusion

In this paper, I have proposed a new channel by which raising the inflation target lowers the equilibrium real rate. I have shown that a rise of 2p.p. in the inflation target lowers the equilibrium real rate by between 0.38p.p. in my default calibration. This suggests that raising the inflation target will be less effective than expected in reducing the probability of hitting the zero lower bound.

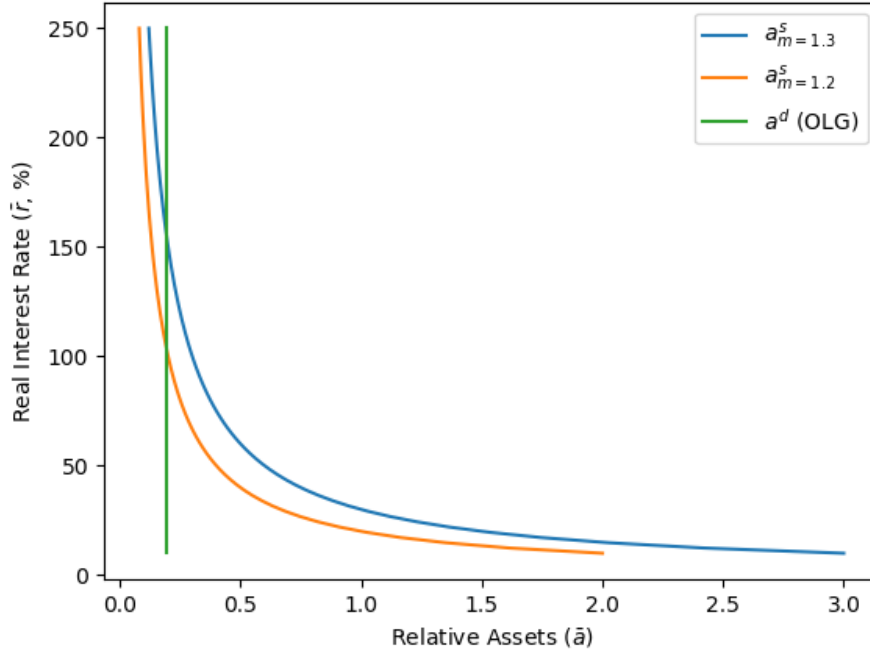
<sup>33</sup>Both the OECD and the UN National Accounts data for firms has some gaps. I take the UN data by default and fill in gaps with OECD data.

## A Intuition in a Simplified Model Details

### A.1 OLG in a Non-Annualized Model

We show the case where the OLG model is non-annualized in figure 9. We observe that the real interest rate is very high since it represents the return from one generation to the next.

Figure 9: Equilibrium under a Fall in the Markup: 2. OLG (Not Annualized)



## B Model Details

### B.1 Firms

**Cost Minimisation Derivation** Intermediate firms minimise their costs. They face the following problem:

$$\min_{K_{i,j,t}, L_{i,j,t}} (1 + \tau)(r_t K_{i,j,t} + W_t L_{i,j,t})$$

s.t.

$$Y_{i,j,t} = A_t K_{i,j,t}^\alpha L_{i,j,t}^{1-\alpha}$$

We note that  $r_t, W_t$  are real variables. Setting up a Lagrangean yields:

$$(1 + \tau)r_t = \lambda_{i,j,t} \alpha A_t K_{i,j,t}^{\alpha-1} L_{i,j,t}^{1-\alpha}$$

$$(1 + \tau)W_t = \lambda_{i,j,t}(1 - \alpha)A_t K_{i,j,t}^\alpha L_{i,j,t}^{-\alpha}$$

We observe that:

$$MC_t = \frac{r_t + \delta}{\alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}} \quad (57)$$

$$MC_t = \frac{W_t}{(1 - \alpha)A_t K_t^\alpha L_t^{-\alpha}} \quad (58)$$

We see that the ratio of each firm's capital to labor ( $\frac{K_{i,t}}{L_{i,t}}$ ) is determined by aggregate variables and is therefore the same across firms. Thus:  $\frac{K_{i,j,t}}{L_{i,j,t}} = \frac{K_t}{L_t}$ . We also observe that  $\lambda_{i,j,t}$  is the marginal cost after tax of production of the firm. This is constant for all firms and will be equal to the real marginal cost after tax of the firm  $MC_t^a$ . We define  $MC_t$  to be the marginal cost before tax so that  $MC_t^a = (1 + \tau)MC_t$ . Thus, we get equations 37 and 38

**Profit Condition** Applying the definition of marginal costs and inputting the lump sum transfer, we can rewrite equation 36 more simply as equation 59. We note that the lump sum transfer equals  $\tau(r_t K_{i,j,t} + W_t L_{i,j,t})$ .

$$\begin{aligned} \Omega_{i,j,t} &= \frac{P_{i,j,t} Y_{i,j,t}}{P_t} - (r_t + \delta)K_{i,j,t} + W_t L_{i,j,t} \\ \Omega_{i,j,t} &= \frac{P_{i,j,t} Y_{i,j,t}}{P_t} - MC_t Y_{i,j,t} \end{aligned} \quad (59)$$

**Aggregation Derivation** We rewrite output as follows:

$$\begin{aligned} Y_{i,j,t} &= A_t K_{i,j,t}^\alpha L_{i,j,t}^{1-\alpha} \\ Y_{j,t} \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\sigma} &= A_t \left( \frac{K_{i,j,t}}{L_{i,j,t}} \right)^\alpha L_{i,j,t} \end{aligned}$$

Taking integrals and noting that the ratio  $\frac{K_{i,j,t}}{L_{i,j,t}}$  is the same across  $i, j$ :

$$\begin{aligned} Y_{j,t} \nu_{j,t} &= A_t \left( \frac{K_t}{L_t} \right)^\alpha L_{j,t} \\ a_j^{\frac{1}{\tau}} Y_t \left( \frac{P_{j,t}}{P_t} \right)^{-\tau} \nu_{j,t} &= a_j A_t \left( \frac{K_t}{L_t} \right)^\alpha L_{j,t} \end{aligned}$$

Taking integrals again to get a condition with aggregate output:

$$Y_t \int_0^1 a_j \left( \frac{P_{j,t}}{P_t} \right)^{-\tau} \nu_{j,t} dj = A_t \left( \frac{K_t}{L_t} \right)^\alpha L_t$$

We can go through the same steps with real profits:

$$\Omega_{i,j,t} = \frac{P_{i,j,t}}{P_t} Y_{i,j,t} - MC_t Y_{i,j,t}$$

$$\begin{aligned}
\Omega_{i,j,t} &= \frac{P_{i,j,t}}{P_{j,t}} \frac{P_{j,t}}{P_t} Y_{i,j,t} - MC_t Y_{j,t} \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\sigma} \\
\Omega_{j,t} &= \frac{P_{j,t}}{P_t} Y_{j,t} - MC_t Y_{j,t} \nu_{j,t} \\
\Omega_{j,t} &= \frac{P_{j,t}}{P_t} Y_{j,t} - MC_t Y_{j,t} \nu_{j,t} \\
\Omega_t &= Y_t - MC_t a_j Y_t \left( \frac{P_{j,t}}{P_t} \right)^{-\tau} \nu_{j,t}
\end{aligned}$$

By definition of  $\nu_t$ :

$$\begin{aligned}
Y_t \nu_t &= A_t K_t^\alpha L_t^{1-\alpha} \\
\Omega_t &= Y_t - Y_t MC_t \nu_t
\end{aligned}$$

## C Steady State Details

### C.1 Markup and Inflation Target

Firstly, we note that in steady state:

$$\bar{\Pi} = \bar{\Pi}_j = \Pi^*$$

We can rewrite equation 43 to get a steady state equation for  $\frac{P^*}{P}$  i.e. equation 60. We can rewrite equation 42 to get a steady state equation for  $\bar{\nu}_j$  i.e. equation 61.

$$\overline{\left( \frac{P_j^*}{P_j} \right)} = \left( \frac{1 - \frac{1-\lambda_j}{\Pi^{*1-\sigma}}}{\lambda_j} \right)^{\frac{1}{1-\sigma}} \quad (60)$$

$$\bar{\nu}_j = \frac{1}{1 - (1 - \lambda_j) \Pi^{*\sigma}} \lambda_j \overline{\left( \frac{P_j^*}{P_j} \right)}^{-\sigma} \quad (61)$$

Next, we can rewrite equation 40 as equation 62. We can simplify equation 62 to get equation 63. We can then input  $\overline{\left( \frac{P_j^*}{P_j} \right)}$  from equation 60 to get equation 64

$$\sum_{k=0}^{\infty} (1 - \lambda_j)^k \left( \frac{\beta_f}{R} \right)^k \Pi^{*k\sigma} \left[ \overline{\left( \frac{P_j^*}{P} \right)} \frac{1}{\Pi^{*k}} - \frac{\sigma}{\sigma - 1} \bar{MC} \right] \quad (62)$$

$$\overline{\left( \frac{P_j^*}{P} \right)} = \frac{\sigma}{\sigma - 1} \frac{1 - (1 - \lambda_j) \beta \Pi^{*\sigma-1}}{1 - (1 - \lambda_j) \beta \Pi^{*\sigma}} \bar{MC} \quad (63)$$

$$\overline{\left( \frac{P_j}{P} \right)} = \frac{\sigma}{\sigma - 1} \frac{1 - (1 - \lambda_j) \beta \Pi^{*\sigma-1}}{1 - (1 - \lambda_j) \beta \Pi^{*\sigma}} \overline{\left( \frac{P_j^*}{P_j} \right)}^{-1} \bar{MC} \quad (64)$$

We can rewrite equation 34 as equation 65. We can then input  $\overline{\left( \frac{P_j}{P} \right)}$  from equation 64 into

equation 65 to get equation 66.

$$\int_0^1 a_j \left( \frac{P_{j,t}}{P_t} \right)^{1-\sigma_2} dj = 1 \quad (65)$$

$$\bar{M}C^{1-\sigma_2} \int_0^1 a_j \left[ \frac{\sigma}{\sigma-1} \frac{1-(1-\lambda_j)\beta\Pi^{\star\sigma-1}}{1-(1-\lambda_j)\beta\Pi^{\star\sigma}} \left( \frac{P_j^{\star}}{P_j} \right)^{-1} \right]^{1-\sigma_2} dj = 1 \quad (66)$$

We can back out  $\bar{M}C$  from equation 66. We can then find  $\left( \frac{P_j}{P} \right)$  from equation 64. We can then back get  $\bar{\nu}$  by its definition (equation 47). This allows us to get  $\bar{m}$  by its definition (equation 48)

## C.2 Relative Asset Supply

The total assets supplied for the household to hold are capital and the value of firms given in equation 67.<sup>34</sup>

$$\bar{A}^s = (1+n)\bar{K} + \bar{Z} \quad (67)$$

Applying equation 25 to equation 29 in the steady state allows us to get a standard equation for the value of firms equation 68.

$$\bar{Z} = \frac{(1+n)(\bar{\Omega} + \bar{Z})}{1+\bar{r}} \quad (68)$$

$$\bar{Z} = \frac{\bar{\Omega}}{\frac{1+\bar{r}}{1+n} - 1} \quad (69)$$

Inputting equation 69 into equation 67 yields equation 70. Inputting equation 46 into equation 70 yields equation 71

$$\bar{A}^s = \frac{(\bar{r}-n)\bar{K} + \bar{\Omega}}{\frac{1+\bar{r}}{1+n} - 1} \quad (70)$$

$$\bar{A}^s = \frac{\bar{Y} - (\delta+n)\bar{K} - \bar{W}\bar{L}}{\frac{1+\bar{r}}{1+n} - 1} \quad (71)$$

We can combine equations 50 and 51 to find equation 72. Dividing equation 71 by labor income and inputting equation 72 yields equation 73.

$$\bar{W}\bar{L} = \frac{1-\alpha}{\alpha}(\bar{r}+\delta)\bar{K} \quad (72)$$

$$\bar{a}^s = \frac{\frac{m}{1-\alpha} - \frac{\alpha}{1-\alpha} \frac{\delta+n}{\bar{r}+\delta} - 1}{\frac{1+\bar{r}}{1+n} - 1} \quad (73)$$

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<sup>34</sup>We need to multiply  $\bar{K}$  by the population growth from one period to the next since assets are the assets that agents hold going forward to the next period.  $\bar{K}$  represents the per capita capital held at the start of a period. To have  $\bar{K}$  at the start of the next period, households must save  $(1+n)\bar{K}$  at the end of the previous period.



### C.3 Relative Asset Demand

**Relative Labor** We define steady state savings by cohort  $i + 1$  to be the total savings that cohort  $i$  makes at time  $t$  for  $t + 1$ :

$$A_{i+1} = \frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + Z_t \omega_{i+1,t+i+1} \quad (74)$$

Applying arbitrage conditions on bonds and shares (equations 26 and 29) allows us to get an expression for the assets held by a cohort at the start of each period:

$$(1 + r) \bar{A}_i = I_{t-1} \frac{B_{i,t}}{P_t} + (1 + r_t) K_{i,t} + (\Omega_t + Z_t) \omega_{i,t+i} \quad (75)$$

We can input equations 74 and 75 into equation 30 to get the simplified steady state budget constraint (equation 91).

$$\bar{C}_i + \bar{A}_{i+1} = \bar{W} \bar{L}_i + \bar{R} \bar{A}_i \quad (76)$$

Next, we can iterate over equation 91 for a household from their first period of life to their last to get their intertemporal steady state budget constraint equation 77.

$$\sum_{i=0}^{M-1} \frac{\bar{C}_i}{\bar{R}^i} = \bar{W} \sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{R}^i} \quad (77)$$

We can rewrite the Euler condition as equation 78.

$$\bar{C}_{i+1} = (\beta \bar{R})^{\frac{1}{\gamma}} \bar{C}_i \quad (78)$$

Iterating over equation 78 yields equation 79.

$$\bar{C}_i = (\beta \bar{R})^{\frac{i}{\gamma}} \bar{C}_0 \quad (79)$$

Inputting this back into equation 77 and simplifying yields equation 80.

$$\bar{C}_0 = \left( \sum_{i=0}^{M-1} \beta^{\frac{i}{\gamma}} \beta \bar{R}^{\frac{i(1-\gamma)}{\gamma}} \right)^{-1} \bar{W} \sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{R}^i} \quad (80)$$

Therefore, applying equation 78 to equation 80 yields an expression for consumption in any period in terms of  $\bar{R}$ :

$$\bar{C}_i = (\beta \bar{R})^{\frac{i}{\gamma}} \left( \sum_{i=0}^{M-1} \beta^{\frac{i}{\gamma}} \beta \bar{R}^{\frac{i(1-\gamma)}{\gamma}} \right)^{-1} \bar{W} \sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{R}^i} \quad (81)$$

We define relative consumption for each cohort  $i$  to be consumption by that cohort divided by labor income, like in equation 53:

$$\bar{c}_i = \frac{\bar{C}_i}{\bar{W} \bar{L}_i}$$

Thus, we can rewrite equation 81 as equation 82

$$\bar{c}_i = (\beta \bar{R})^{\frac{i}{\gamma}} \left( \sum_{i=0}^{M-1} \beta^{\frac{i}{\gamma}} \beta \bar{R}^{\frac{i(1-\gamma)}{\gamma}} \right)^{-1} \sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{R}^i} \quad (82)$$

**Endogenous Labor Relative Labor Supply** In the case with exogenous labor, we have rewritten relative consumption for each cohort purely in terms of  $\bar{R}$ . However, in the case with endogenous labor  $\frac{\bar{L}_i}{\bar{L}}$  are endogenous so we need to rewrite the labor part of  $f$  (i.e.  $\sum_{i=0}^{M-1} \frac{\bar{L}_i}{\bar{L}} \frac{1}{\bar{R}^i}$ ) in terms of  $\bar{R}$  only.

Substituting the labor-leisure condition (equation 28) into the Euler condition (equation 25) yields the intertemporal labor supply condition:

$$\frac{v'(L_{i,t})}{W_t} = \beta R_{t+1} \frac{v'(L_{i,t+1})}{W_{t+1}} \quad (83)$$

Applying steady state and the disutility of working function to equation 83 yields:

$$x_i \bar{L}_i^\eta = \beta \bar{R} x_{i+1} \bar{L}_{i+1}^\eta \quad (84)$$

Rewriting equation 85:

$$\bar{L}_{i+1} = \left( \frac{1}{\beta \bar{R}} \right)^{\frac{1}{\eta}} \left( \frac{x_i}{x_{i+1}} \right)^{\frac{1}{\eta}} \bar{L}_i \quad (85)$$

Iterating over equation 85:

$$\bar{L}_i = \left( \frac{1}{\beta \bar{R}} \right)^{\frac{i}{\eta}} \left( \frac{x_0}{x_i} \right)^{\frac{i}{\eta}} \bar{L}_0 \quad (86)$$

Next, we note that we can find the (population weighted) total labor supply:

$$\bar{L} = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \bar{L}_i \quad (87)$$

Inputting equation 86 into equation 87 yields:

$$\bar{L} = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \left( \frac{1}{\beta \bar{R}} \right)^{\frac{i}{\eta}} \left( \frac{x_0}{x_i} \right)^{\frac{i}{\eta}} \bar{L}_0 \quad (88)$$

Inputting  $\bar{L}_0$  from equation 88 into equation 86 yields the relative labor supplied by each cohort given by equation 89.

$$\frac{\bar{L}_i}{\bar{L}} = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \left( \frac{1}{\beta \bar{R}} \right)^{\frac{i}{\eta}} \left( \frac{x_0}{x_i} \right)^{\frac{i}{\eta}} \right)^{-1} \left( \frac{1}{\beta \bar{R}} \right)^{\frac{i}{\eta}} \left( \frac{x_0}{x_i} \right)^{\frac{i}{\eta}} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \quad (89)$$

This has some economic intuition. When  $\bar{R}$  is higher, agents supply relatively less labor when

they are old since they are already getting a high return on their savings so they don't need to work as much.

**Relative Asset Demand** We define relative assets by cohort in the same way as the definition of relative assets (equation 53):

$$\bar{a}_i = \frac{\bar{A}_i}{\bar{W}\bar{L}} \quad (90)$$

We can rewrite equation 91 in terms of relative assets and relative consumption

$$\bar{c}_i + \bar{a}_{i+1} = \frac{\bar{L}_i}{\bar{L}} + \bar{R}\bar{a}_i \quad (91)$$

We note that  $\bar{a}_0 = \bar{a}_M = 0$  (since agents start with zero assets and have no need for assets when they are dead). Therefore, we have  $M - 1$  equations from equation 91 and  $M - 1$  unknowns  $\bar{a}_1, \dots, \bar{a}_{M-1}$ . Thus, we can solve for  $\bar{a}_i$  by iterating over equation 91 starting from the beginning or end.

Total assets  $\bar{A}$  must equal the weighted sum of assets by cohort:

$$\bar{A} = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \bar{A}_i \quad (92)$$

Next, we observe by dividing equation 92 and applying the definition of relative assets and relative assets by cohort (equations 53 and 90) that the total relative asset demand is just given by the weighted sum of the relative assets held by each cohort:

$$\bar{a}^d = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \bar{a}_i$$

Thus, we can solve for  $\bar{a}^d$  using this process.

## D Calibration Details

### D.1 Endogenous Labor Supply

Substituting the labor-leisure condition into the Euler condition yields the intertemporal labor supply condition:

$$\frac{v'(L_{i,t})}{W_t} = \beta R_{t+1} \frac{v'(L_{i,t+1})}{W_{t+1}} \quad (93)$$

Applying steady state and the disutility of working function to equation 93 yields:

$$x_i \bar{L}_i^\eta = \beta \bar{R} x_{i+1} \bar{L}_{i+1}^\eta \quad (94)$$

Table 6: Impact of Changing Firm Discounting on the Policy Experiment Results

Firm Additional Discount ( $\beta_f$ )	0.89	0.935	0.972	1
Change in markup (p.p.)	-1.45	-1.07	-0.68	-0.32
Change in $\frac{C_{old}}{C_{young}}$ (%)	-7.34	-5.12	-3.10	-1.39
Change in K (%)	6.43	4.44	2.66	1.19
Change in r (p.p.)	-0.54	-0.38	-0.23	-0.10

We rewrite equation 94 as:

$$x_{i+1} = \frac{1}{\beta \bar{R}} x_i \left( \frac{\bar{L}_i}{\bar{L}_{i+1}} \right)^\eta \quad (95)$$

Iterating over this yields:

$$x_i = \frac{1}{(\beta \bar{R})^i} x_0 \left( \frac{\bar{L}_0}{\bar{L}_i} \right)^\eta$$

We set  $x_0 = 1$ . To keep things simple, I just set  $x_i \forall i > 0$  so that we would get the same labor supply as the exogenous case when  $\beta \bar{R} = 1$  so:

$$x_i = \left( \frac{\bar{L}_0}{\bar{L}_i} \right)^\eta$$

where the  $\bar{L}$  ratios are the same as in the exogenous labor case.

## E Results Details

### F Idiosyncratic Labor Model

#### F.1 Households

We develop a model where agents have life cycle structure with idiosyncratic risk. To avoid further complications we assume there is no aggregate uncertainty.

We start by describing our general overlapping generations framework. Each period new agents are born. Each agent lives for  $M$  periods. Agents born in different periods overlap. We denote an agent born  $i$  periods ago as being a part of cohort  $i$  in time  $t$ . Therefore, the  $M$  cohorts in any given period are denoted  $0, \dots, M - 1$ . Each period: new agents are born (cohort 0), cohort  $M - 1$  at time  $t - 1$  has died and all other generations mature from cohort  $m$  to  $m + 1$ . If  $M = 2$ , we can imagine that these cohorts represent generations like working age and old age and, in this case, a generation would last for around 30 years (like in the standard [Diamond \(1965\)](#) OLG framework). However, we can also set  $M = 60$  in which case each cohort can represent one year of an agent's life from age 21 to 80.

We define that the population of the cohort born at time  $t$  is  $N_t$ . We define the total population

to be  $\mathbb{N}_t$  and thus  $\mathbb{N}_t = \sum_{i=0}^{M-1} N_{t-i}$ . We assume that population grows at a constant rate of  $n$  so that  $N_{t+1} = (1+n)N_t$ . Thus, the total population also grows by  $1+n$  each year. Since we have idiosyncratic shocks within cohorts, we need to consider how individuals within a cohort will respond. We consider an continuum of individuals denoted  $h$  between 0 and 1 for each cohort  $i$ .

An individual  $h$  of cohort  $i$  at time  $t$  has a budget constraint given by equation 96. An agent either spends their money on consumption  $C_{h,i,t}$  or saves  $S_{h,i+1,t+1}$  for the next period. An agent receives direct income from working an exogenously set amount  $L_{h,i,t}$  at time  $t$  for real wage  $W$ . Savings from the previous period pay a gross return of  $R$ .

$$C_{h,i,t} + S_{h,i+1,t+1} \leq WL_{h,i,t} + RS_{h,i,t} \quad (96)$$

The amount that each individual works is dependent upon whether that household is employed or unemployed:

$$L_{h,i,t} = \begin{cases} L_{i,t} & \text{if employed} \\ U_{i,t} & \text{otherwise} \end{cases}$$

Whether or not the individual is employed is a Markov process.

Since there is no aggregate uncertainty, all assets must return the same. This is why we don't need to specify exactly what assets agents hold for their savings. Instead, we can just specify that they save  $S_{h,i,t}$  without exactly specifying how. Also, note that we assume that agents born today start with zero assets ( $S_{h,0,t} = 0$ ).

The agent has Epstein-Zin utility (which allows me to separate out the effects of risk aversion and income elasticity of substitution) so their utility is defined recursively:

$$V_{i,t} = \left( (1-\beta)C_{h,i,t}^{1-\rho} + \beta(\mathbb{E}_t[V_{i,t+1}^{1-\alpha}]^{\frac{1-\rho}{1-\alpha}}) \right)^{\frac{1}{1-\rho}}$$

Therefore, an agent of cohort  $k$  faces the following problem:

$$\max_{\{C_{h,i,t+i}, S_{h,i+1,t+i+1}\}_{i=0}^{M-k-1}} \mathbb{E}_t \left[ \sum_{i=0}^{M-k-1} \beta^i u(C_{h,i,t}) \right] \quad (97)$$

s.t.  $\forall i \in 0, \dots, M-1$ :

$$C_{h,i,t} + S_{h,i+1,t+1} \leq WL_{h,i,t} + RS_{h,i,t} \quad (98)$$

$$S_{M-k,t+M-k} \geq 0$$

We take a value function approach due to the presence of heterogeneous agents. We could do this using equations 97 and 98 but then savings will be a function of the wage which is determined on the demand side. We don't want to have to update our supply of assets when we change the demand side. So we consider labor income adjusted variables which we call relative consumption and relative savings:

$$c_{h,i,t} = \frac{C_{h,i,t}}{WL}$$

$$s_{h,i,t} = \frac{S_{h,i,t}}{WL}$$

We can then rewrite the problem as<sup>35</sup>:

$$\max_{\{C_{h,i,t+i}, S_{h,i+1,t+i+1}\}_{i=0}^{M-k-1}} \mathbb{E}_t \left[ \sum_{i=0}^{M-k-1} \beta^i u(c_{h,i,t}) \right] \quad (99)$$

s.t.  $\forall i \in 0, \dots, M-1$ :

$$c_{h,i,t} + s_{h,i+1,t+1} \leq \frac{L_{h,i,t}}{L} + R s_{h,i,t} \quad (100)$$

$$s_{M-k,t+M-k}, K_{M-k,t+M-k} \geq 0$$

We can then express this as a series of value function problems. The value of savings and labor income in the final period of the agent's life is given by the utility of consuming all the remaining assets of the agent:

$$V_{M-1}(s_{M-1}, L_{M-1}) = u\left(\frac{L_{M-1}}{L} + R s_{M-1}\right)$$

Then, working backwards, we can compute the value of an agent of cohort  $i$ 's savings and labor income given the value of an agent of cohort  $i+1$ 's savings and labor income. This will just equal the utility of their choice of consumption in the current period plus the value of the assets they leave to the next period:

$$V_i(s_i, L_i) = \max_{s_{i+1}} u\left(\frac{L_{h,i,t}}{L} + R s_i - s_{i+1}\right) + \beta \mathbb{E}[V_{i+1}(s_{i+1}, L_{i+1})]$$

We observe that we are able to compute  $s_i \forall i$  given the value of  $R$ . Therefore, individual relative savings is effectively a function of  $R$ . Consequently, we have that aggregate relative savings also a function of the real interest rate and the population weighted sum of individual cohort labor supplies:

$$s(R) = \left( \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \right)^{-1} \sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \int_0^1 s_{h,i}(R) dh$$

## G Welfare Simulation Model

I will add more detail to this section in a later draft.

### G.1 Household Additional Results

We define the amount that agents of cohort  $i$  save at  $t$  for  $t+1$  as  $S_{i+1,t+1}$  and the amount that agents of cohort  $i$  have available at  $t$  from savings they made in  $t-1$  as  $T_{i,t}$ . Therefore: <sup>36</sup>

<sup>35</sup>We have simplified by cancelling a constant in the utility function.

<sup>36</sup>Note that  $T_{0,t}, S_{M,t} = 0$  which makes sense since agents don't hold assets when they are born or when they die.

$$S_{i+1,t+1} = \frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + Z_t \omega_{i+1,t+i+1} \quad (101)$$

$$T_{i,t} = I_{t-1} \frac{B_{i,t}}{P_t} + R_t K_{i,t} + (\Omega_t + Z_t) \omega_{i,t+i} \quad (102)$$

We see that the budget constraint (equation 30) can be rewritten as:

$$C_{i,t} + S_{i+1,t+1} \leq W_t L_{i,t} + T_{i,t} \quad (103)$$

We can find per capita savings, which we define to be  $S_t$ , by calculating the cohort-weighted sum of individual savings:

$$S_{t+1} = \frac{\sum_{i=0}^{M-1} N_{t-i} S_{i,t+1}}{\sum_{i=0}^{M-1} N_{t-i}} = \frac{\sum_{i=0}^{M-1} N_t \frac{1}{(1+n)^i} S_{i,t+1}}{\sum_{i=0}^{M-1} N_t \frac{1}{(1+n)^i}} = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} S_{i,t+1}}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}} \quad (104)$$

We also define  $T_t$  to be the per capita amount of assets agents have available at  $t$  from saving at  $t-1$ ,  $K_t$  to be per capita holdings of capital and  $B_t$  to be the per capita holdings of bonds. We note that the per capita holdings of  $\omega_{i,t}$  (per capita proportional holdings of firms) must by definition equal 1 since the total number of per capita shares is  $\mathbb{N}_t$ . Using these definitions, we can aggregate equations 101 and 102 to get:

$$S_{t+1} = \frac{B_{t+1}}{P_t} + K_{t+1} + Z_t \quad (105)$$

$$T_t = I_{t-1} \frac{B_t}{P_t} + R_t K_t + \Omega_t + Z_t \quad (106)$$

We define  $s_{i,t} = \frac{S_{i,t}}{S_t}$  and thus equation 104 becomes:

$$1 = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} s_{i,t}}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}} \quad (107)$$

Next, we set  $T_{i,t} = s_{i,t} T_t$ . This implies that the amount of total assets that a cohort holds at time  $t$  is proportional to their saving at time  $t-1$ . Equation 103 then can be rewritten as: <sup>37</sup> <sup>38</sup>

$$C_{i,t} + s_{i+1,t+1} S_{t+1} = W_t L_{i,t} + s_{i,t} T_t \quad (108)$$

---

<sup>37</sup> The log-linearised arbitrage conditions (equations 121 and 122) imply that all cohorts are indifferent between holding equivalently valued capital, bonds or shares (this is also true in the purely deterministic case) since they all give a real expected return of  $\mathbb{E}_t[\hat{R}_{t+1}]$  and we have abstracted from risk by employing a first order approximation. Thus, although we know the savings of each cohort, we do not know how the savings is comprised i.e. we know  $\frac{B_{i+1,t+1}}{P_t} + K_{i+1,t+1} + Z_t \omega_{i+1,t+i+1}$  but not  $\frac{B_{i+1,t+1}}{P_t}$  or  $K_{i+1,t+1}$  or  $\omega_{i+1,t+i+1}$ . Therefore, it is unclear how many assets a cohort will hold at time  $t$  if an agent is indifferent between saving capital or bonds but if there was a negative shock to the return on capital an agent who saved capital would get less.

<sup>38</sup> Setting  $T_{i,t} = s_{i,t} T_t$  is equivalent to a framework in which all cohorts save the same proportion of capital, bonds and shares in any given period so all cohorts get the same return on their savings regardless of shocks. This is the simplest way to model how agents hold assets which avoids shocks leading to unanticipated redistribution.

We can substitute  $S_{t+1}, T_t$  out of equation 108 using equations 105 and 106:

$$C_{i,t} + s_{i+1,t+1} \left[ \frac{B_{t+1}}{P_t} + K_{t+1} + Z_t \right] = W_t L_{i,t} + s_{i,t} \left[ I_{t-1} \frac{B_t}{P_t} + R_t K_t + \Omega_t + Z_t \right] \quad (109)$$

We see that we have simplified the household's problem so that it is summarised by Euler condition(s) (equation 25), two arbitrage conditions (equations 26 and 29), the sum of savings shares (equation 107) and simplified budget constraints (equation 109).

## G.2 Firm Differences

$$\max_{P_t^*} \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \beta^k (1-\lambda)^k \frac{M_{t,t+k}}{P_{t+k}} [P_t^* Q_{i,t+k} - Q_{i,t+k} \tau M C_{t+k} P_{t+k}] \right]$$

s.t.

$$Q_{i,t+k} = Q_t \left( \frac{P_{i,t}}{P_t} \right)^{-\sigma}$$

$$\max_{P_t^*} \sum_{k=0}^{\infty} (1-\lambda)^k \frac{M_{t,t+k}}{P_{t+k}} [P_t^{*1-\sigma} P_{t+k}^{\sigma} Q_{t+k} - P_t^{*-\sigma} P_{t+k}^{\sigma} Q_{t+k} \tau M C_{t+k} P_{t+k}]$$

Taking FOCs:

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} (1-\lambda)^k \frac{M_{t,t+k}}{P_{t+k}} [P_t^{*-\sigma} P_{t+k}^{\sigma} Q_{t+k} - P_t^{*- \sigma - 1} P_{t+k}^{\sigma} Q_{t+k} \tau M C_{t+k} P_{t+k}] \right] = 0$$

Multiplying by  $P_t^{*\sigma+1}$  and rearranging:

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} (1-\lambda)^k M_{t,t+k} Q_{t+k} P_{t+k}^{\sigma} \left[ \frac{P_t^*}{P_t} \frac{P_t}{P_{t+k}} - \tau M C_{t+k} \right] \right] = 0$$

We can then break this into recursive conditions which can be used within a DSGE model. We define:

$$U_t = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (1-\lambda)^k M_{t,t+k} Q_{t+k} \left( \frac{P_{t+k}}{P_t} \right)^{\sigma-1} \right]$$

$$V_t = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (1-\lambda)^k M_{t,t+k} Q_{t+k} \left( \frac{P_{t+k}}{P_t} \right)^{\sigma} \frac{\sigma}{\sigma-1} \tau M C_{t+k} \right]$$

Then we have that:

$$U_t \frac{P_t^*}{P_t} = V_t \quad (110)$$

$$U_t = Q_t + \mathbb{E}_t [\Pi_{t+1}^{\sigma-1} (1-\lambda) M_{t,t+1} U_{t+1}] \quad (111)$$

$$V_t = Q_t \frac{\sigma}{\sigma-1} \tau M C_t + \mathbb{E}_t [\Pi_{t+1}^{\sigma} (1-\lambda) M_{t,t+1} V_{t+1}] \quad (112)$$

Under Calvo pricing, prices are made up of the optimal price today and the remaining prices



that are unchanged from the previous period. Thus, we get:

$$1 = \lambda \left( \frac{P_t^*}{P_t} \right)^{1-\sigma} + (1-\lambda) \Pi_t^{\sigma-1} \quad (113)$$

In the Calvo pricing case, we get the following process for the price dispersion parameter:

$$\nu_t = \lambda \left( \frac{P_t^*}{P_t} \right)^{-\sigma} + (1-\lambda) \nu_{t-1} \Pi_t^{\sigma} \quad (114)$$

### G.3 Monetary Policy

We assume the central bank pursues some form of Taylor Rule:

$$I_t = I_{t-1}^{\rho_{i1}} I_{t-2}^{\rho_{i2}} (\bar{\Pi})^{\phi_{\pi}} \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_{\pi}} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_y} )^{1-\rho_{i1}-\rho_{i2}} \quad (115)$$

We can log linearise this to find:

$$\hat{I}_t = \rho_{i1} \hat{I}_{t-1} + \rho_{i2} \hat{I}_{t-2} + (1 - \rho_{i1} - \rho_{i2})(\phi_{\pi} \hat{\Pi}_t + \phi_y \hat{Y}_t) \quad (116)$$

### G.4 Other Differences from the Basic Model

I have not updated the welfare simulation after making changes to my main model. The parts that remain different are:

- We also allow for bonds to have value to the consumer. This allows us to introduce a wedge between the average returns on bonds and the average returns on capital and stocks without having to concern ourselves with the value of risky investments (this is not key to the analysis but helps us match real world features).<sup>39</sup> Then the overall utility function is given by equation 117 with the safe bond utility given by equation 118. This allows us to get the Fisher equation with a distortion term given by equation 119.

$$\mathbb{E}_t \left[ \sum_{i=0}^{M-k-1} \beta^i [u(C_{i,t}) + u_b \left( \frac{B_{i,t}}{P_t} \right) - v(L_{i,t})] \right] \quad (117)$$

$$u_b \left( \frac{B_t}{P_t} \right) = v_b I_{t-1} \frac{B_t}{P_t} u'(\bar{C}_{i,t}) \quad (118)$$

$$u'(C_{i,t}) = \beta \mathbb{E}_t [u'(C_{i+1,t+1}) \frac{I_t}{\Pi_{t+1}} (1 + v_b)] \quad (119)$$

- Calvo with only one sector with  $\lambda = 0.2, \sigma = 11$ .

<sup>39</sup>We set  $u_b$  so that the wedge simplifies easily in the Fisher equation but it makes some logical sense. The higher is the return  $I_t$ , the higher the value of bonds is likely to be. We denote average consumption by agent  $i$  at time  $t$  as  $\bar{C}_{i,t}$  (which does not equal  $C_{i,t}$  when there are multiple agents in a given cohort) and it could make sense that as consumption of their cohort falls, agents receive a higher marginal utility from bonds.

## G.5 Full Conditions

Common conditions are the cost minimisation conditions (equations 37 and 38), the definition of output (??) the definition of profits (??). Under Calvo, we also have pricing conditions (equations 110 to 113) and the price dispersion condition (equation 114). Under Rotemberg, we have a pricing condition (??). Under the empirical case, we have an additional condition between steady state marginal cost and inflation based upon the data.

## G.6 Log-Linearised Conditions

We can log linearise and simplify equations 25, 26 and 29 to get:

$$\mathbb{E}_t[\hat{C}_{i+1,t+1}] = \frac{1}{\gamma} \mathbb{E}_t[\hat{R}_{t+1}] + \hat{C}_{i,t} \quad (120)$$

$$\mathbb{E}_t[\hat{R}_{t+1}] = \hat{I}_t - \mathbb{E}_t[\hat{\Pi}_{t+1}] \quad (121)$$

$$\hat{Z}_t + \mathbb{E}_t[\hat{R}_{t+1}] = \frac{\bar{\Omega}}{\bar{\Omega} + \bar{Z}} \mathbb{E}_t[\hat{\Omega}_{t+1}] + \frac{\bar{Z}}{\bar{\Omega} + \bar{Z}} \mathbb{E}_t[\hat{Z}_{t+1}] \quad (122)$$

We can log-linearise the sum of savings shares (equation 107), the simplified budget constraints (equation 109) and the total savings:

$$0 = \frac{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i} \bar{s}_i \hat{s}_{i,t}}{\sum_{i=0}^{M-1} \frac{1}{(1+n)^i}} \quad (123)$$

$$\bar{C}_i \hat{C}_{i,t} + \bar{s}_{i+1} [\bar{S} \hat{s}_{i+1,t+1} + \frac{\bar{B}}{\bar{P}} \left( \frac{\widehat{B_{t+1}}}{P_t} \right) + \bar{K} \hat{K}_{t+1} + \bar{Z} \hat{Z}_t] = \bar{W} \bar{L}_i \hat{W}_t + \bar{s}_i [\bar{R} \bar{S} \hat{s}_{i,t} + \bar{R} \frac{\bar{B}}{\bar{P}} \left( \frac{\widehat{B_t}}{P_{t-1}} \right) + \bar{R} \bar{K} (\hat{R}_t + \hat{K}_t) + \bar{\Omega} \hat{\Omega}_t + \bar{Z} \hat{Z}_t] \quad (124)$$

We log linearise to get:

$$\hat{U}_t + \frac{\widehat{P_t^*}}{\bar{P}_t} = \hat{V}_t \quad (125)$$

$$\bar{U} \hat{U}_t = \bar{Y} \hat{Y}_t + \bar{P} i^{\sigma-1} (1 - \lambda) \beta_f \bar{U} ((\sigma - 1) \mathbb{E}_t[\hat{\Pi}_{t+1}] + \mathbb{E}_t[\hat{M}_{t,t+1}] + \mathbb{E}_t[\hat{U}_{t+1}]) \quad (126)$$

$$\bar{V} \hat{V}_t = \frac{\sigma}{\sigma - 1} \bar{Y} \bar{M} \bar{C} (\hat{Y}_t + \hat{M} \hat{C}_t) + \bar{\Pi}^\sigma (1 - \lambda) \beta_f \bar{V} (\sigma \mathbb{E}_t[\hat{\Pi}_{t+1}] + \mathbb{E}_t[\hat{M}_{t,t+1}] + \mathbb{E}_t[\hat{V}_{t+1}]) \quad (127)$$

$$0 = \lambda \left( \frac{\bar{P}^*}{\bar{P}} \right)^{1-\sigma} (1 - \sigma) \left( \frac{\widehat{P_t^*}}{\bar{P}_t} \right) + (1 - \lambda) \bar{\Pi}^{\sigma-1} (\sigma - 1) \hat{\Pi}_t \quad (128)$$

$$\bar{\nu} \hat{\nu}_t = -\sigma \lambda \left( \frac{\bar{P}^*}{\bar{P}} \right)^{-\sigma} \left( \frac{\widehat{P_t^*}}{\bar{P}_t} \right) + (1 - \lambda) \bar{\nu} \bar{\Pi}^\sigma \hat{\nu}_{t-1} \quad (129)$$

We can log-linearise equations 37 and 38 and ???? to find:

$$\hat{M} \hat{C}_t = \hat{r}_t - \hat{A}_t + (1 - \alpha) \hat{K}_t \quad (130)$$

$$\hat{M}C_t = \hat{W}_t - \hat{A}_t - \alpha \hat{K}_t \quad (131)$$

$$\hat{Y}_t + \hat{\nu}_t = \hat{A}_t + \alpha \hat{K}_t \quad (132)$$

$$\hat{\Omega}_t = \hat{Y}_t - \frac{\bar{M}C\bar{\nu}}{1 - \bar{M}C\bar{\nu}}(\hat{M}C_t + \hat{\nu}_t) \quad (133)$$

## H Empirics Robustness

**OECD Members Pre-1975** We only consider countries that were members of the OECD before 1975.

Table 7: Inflation-Real OECD Original Members

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.142** (0.047)	-0.169*** (0.048)	-0.546*** (0.072)	-0.909** (0.280)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	966	966	966	651

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

Table 8: Inflation-LaborShare OECD Original Members

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	0.269 (0.406)	0.517* (0.235)	0.164 (0.441)	0.351 (0.974)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	584	584	584	500

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

**Low Inflation (< 10%)** We exclude data points where long-run inflation exceeded 10%.

Table 9: Inflation-Real Low Inflation

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.033 (0.063)	-0.059 (0.067)	-0.643*** (0.085)	-0.948*** (0.171)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	1088	1088	1088	808

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

Table 10: Inflation-LaborShare Low Inflation

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.061 (0.512)	0.719** (0.242)	0.753* (0.312)	0.514 (0.416)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	745	745	745	678

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

**Pre-2000** We only consider years which were before 2000.

Table 11: Inflation-Real Pre-2000

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.323*** (0.042)	-0.354*** (0.034)	-0.426*** (0.049)	-0.451 (NaN)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	558	558	558	253

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

Table 12: Inflation-LaborShare Pre-2000

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	0.180*	0.211***	0.200***	0.074
	(0.080)	(0.043)	(0.048)	(NaN)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	286	286	286	198

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

**Post-2000** We only consider years after 2000 (including 2000).

Table 13: Inflation-Real Post-2000

$RealRate_{i,t}^{10yr}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.228**	-0.414***	-0.868***	-1.038***
	(0.078)	(0.093)	(0.108)	(0.153)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	593	593	593	580

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

Table 14: Inflation-LaborShare Post-2000

$LaborShare_{i,t}$	(1)	(2)	(3)	(4)
$Inflation_{i,t-4,t}$	-0.886	0.867***	1.049***	0.856*
	(0.804)	(0.254)	(0.282)	(0.339)
country dummies		*	*	*
year dummies			*	*
econ controls				*
N	527	527	527	523

Notes: \*, \*\* and \*\*\* respectively represent  $p < 0.05$ ,  $p < 0.01$  and  $p < 0.001$ . The numbers in parentheses represent the clustered standard errors.

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