Housing and Consumption Volatility*

Wonmun Shin†

Columbia University

This version: November 28, 2019

Job Market Paper

Link to the latest version

Abstract

Business cycles in emerging economies exhibit both greater volatility of housing prices and relative consumption compared to business cycles in rich countries. This paper provides evidence of a positive relationship between housing price and relative consumption volatility across countries, and explores that linkage by building a real business cycle model of a small open economy with both housing and rental markets. While housing consumption, as measured through rental prices, is a non-negligible portion of total consumption, the role of the rental market has largely been overlooked in studies of consumption volatility. By explicitly modelling separate housing and rental sectors, this paper is able to explain some new stylized facts that emerge when housing and non-housing consumption are disaggregated: first, housing consumption is more volatile than non-housing consumption in emerging countries; and, second, even after controlling for housing consumption volatility, non-housing consumption in emerging economies is still more volatile than that in rich countries. Simulation results suggest that cross-country variation in the volatility of shocks to credit prices and availability is a driving force in generating the observed relationship between house price and relative consumption volatility. The model also suggests that a financial friction stemming from constraints in housing-collateralized credit can explain excess non-housing consumption volatility in emerging countries, while rental market frictions may account for the greater housing consumption volatility observed.

Keywords: Consumption Volatility, Housing Consumption, House Prices, Housing Rental Market, Emerging Countries

JEL Classification: C82, E10, E32, F41, F44

*I wish to thank numerous professors and colleagues for useful comments. In particular, I am grateful to Stephanie Schmitt-Grohé, Martín Uribe, Jesse Schreger, Stijn Van Nieuwerburgh, Andres Drenik, Émilien Gouin-Bonenfant and all participants in Economic Fluctuations colloquium and Monetary Economics colloquium in Columbia University, Midwest Macro Spring Meeting 2019, Georgetown Center for Economic Research Conference 2019, Asian Meeting of the Econometric Society 2019, Young Economists Symposium 2019, and Midwest Macro Fall Meeting 2019.

†Email: wonmun.shin@columbia.edu, Personal webpage: https://www.wonmunshin.com/

1 Introduction

A well-established stylized fact in international macroeconomics is that emerging countries exhibit high output volatility and even higher consumption volatility compared to rich countries. Previous literature has emphasized that consumption is significantly more volatile than output in emerging countries, which is termed *excess consumption volatility* for emerging economies. This observation has attracted attention because it might seem at odds with consumption smoothing, which is an essential implication of optimizing business cycle models. Aguiar and Gopinath (2007), who documented the excess volatility of consumption, suggested that non-stationary shocks to productivity in the frictionless RBC model can well explain this puzzle, but Garcia-Cicco, Pancrazi and Uribe (2010) argued that financial frictions in a model with several shocks are assigned a primary role in understanding business cycles in emerging countries. The present paper also attempts to explain higher relative consumption volatility in emerging countries by introducing a household borrowing constraint and a debt-elastic interest rate to mainly focus on household consumption fluctuations. Financial frictions in the form of a borrowing constraint are specified by housing collateral, reflecting the strong correlation between house prices and aggregate consumption documented in several empirical studies (including Case, Quigley and Shiller (2005) and Campbell and Cocco (2007)). Furthermore, this paper suggests novel stylized facts for emerging countries by disaggregating total consumption expenditures into *housing consumption* and *non-housing consumption*. The new findings are then examined through a theoretical model framework that this paper develops and through the analysis of cross-country data that supports the model's predictions.

Considering the higher house price volatility observed in emerging countries, it is sensible to consider house prices and the housing market when investigating consumption volatility in emerging countries. Cesa-Bianchi, Cespedes and Rebucci (2015) first compared house price volatility in emerging and rich economies and found that it is much higher among the former. I also confirm the empirical finding of Cesa-Bianchi, Cespedes and Rebucci (2015) by constructing a cross-country data set of 10 emerging countries and 14 rich countries covering the period from 1970 Q1 to 2017 Q4 subject to data availability. Based on the stylized facts cited above—excess consumption volatility and higher housing price volatility—we can conjecture that a country where house prices are more volatile will tend to have higher consumption volatility relative to output volatility. However, the literature has not supported or rejected thus hypothesis. This paper fills this gap by providing robust cross-country evidence of a positive relationship between house price volatility and relative consumption volatility. Therefore, to account for excess consumption volatility in emerging countries, this paper primarily seeks to explain the positive cross-country correlation between the two volatilities of interest by constructing a theoretical framework with financial frictions. By determining what drives the positive relationship between housing prices and relative consumption volatility across countries, this paper asserts that understanding this driving force is crucial in comprehending consumption volatility in emerging countries.

To develop an appropriate model for answering this paper's research question, I examine disaggregated household consumption expenditures: housing service expenditures, namely, *housing consumption*, and other expenditures, including both durable and non-durable goods consumption, namely, *non-housing consumption*. This paper considers data at both a quarterly frequency and an annual frequency due to significantly limited data availability. The unbalanced disaggregated consumption data set covers 3 emerging countries and 6 rich countries between 1970 and 2017. I obtain several new findings from the data set. First, both relative housing consumption volatility and relative non-housing consumption volatility show a positive correlation with house price volatility, as does relative total consumption volatility. Second, housing consumption is more volatile than non-housing consumption in emerging countries, whereas this is not the case in rich countries. Finally, even after controlling for housing consumption volatility, non-housing consumption in emerging economies is still more volatile than that in rich countries. This then leads to the question of whether a model in which financial frictions take the form of a constraint on housing collateral is capable of reproducing the above findings. While the housing collateral constraint means that house price changes affect the amounts of both housing services and non-housing goods consumed, it might be silent on housing consumption, which provides information on the amount of housing services consumed as well as their prices. Unfortunately, we only observe housing consumption, as measured through rental prices, in the data, not the amount of housing services. Additionally, since housing consumption is a non-negligible portion of total consumption (more than 20 percent in most sample countries), the model should explicitly account for it. Therefore, this paper incorporates the housing rental market and its prices to investigate fluctuations in housing consumption, which have largely been overlooked in studies of consumption volatility.

The model borrows building blocks from Iacoviello (2005), Iacoviello and Neri (2010), and Monacelli (2009) and adjusts them for the small open economy RBC model presented in Schmitt-Grohé and Uribe (2003). A newly introduced feature is that the model explicitly captures the housing rental market, which is formed by the interaction between two types of households in the simplified theoretical world – *homeowners* and *renters*. The two groups are basically identical except that homeowners are able to purchase houses. Homeowners access a secured loan against housing collateral from international financial markets, subject to a borrowing constraint. The price of debt on international financial markets is given by an external debt-elastic interest rate, following Schmitt-Grohé and Uribe (2003). In the case of renters, they do not access the financial market because they do not have collateralizable houses. On the production side, there are three types of firms: non-housing consumption goods producers, house construction firms, and housing rental agencies. In particular, the existence of rental agencies brings rental price rigidity into the model.

The rigid adjustment of rental prices is a key friction introduced by the model. The rigidity in rental price changes, designed following Rotemberg (1982), allows the model to generate stable rental prices, as observed in the actual data. Based on this paper's cross-country data on rental prices and house prices, we observe that rental price volatility is lower than house price volatility in all sample countries, although the magnitude varies across countries. Moreover, several studies provide evidence of substantial price rigidity in housing rents (including Genesove (2003), Shimizu, Nishimura and Watanabe (2010), Aysoy, Aysoy and Tumen (2014), and Verbrugge and Gallin (2017)). Rental market frictions, which take the form of rental price rigidity in the model, affect housing consumption volatility because housing consumption is measured by rental prices and also has an effect on non-housing consumption volatility by controlling the relative price of non-housing consumption goods. More important, it provides a source of cross-country variation in explaining the higher housing consumption volatility observed in emerging countries, which will be discussed in the following paragraphs along with this paper's results.

This paper considers six structural shocks throughout the model framework to determine which shock is crucial in generating the second moments of interest: a non-housing productivity shock, a housing productivity shock, an intertemporal preference shock, a housing preference shock, a credit availability shock, and an interest rate shock. The simulation results of the calibrated model for a representative emerging economy (South Korea) qualitatively capture well the empirical findings this paper suggests: excess consumption volatility, higher housing consumption volatility, excess non-housing consumption volatility, and lower rental price volatility. In particular, the excess consumption volatility generated by the model relies on the role of credit shocks and the housing collateral constraint, and house prices play a primary role as a channel for the transmission and an amplifier of shocks. I also found that the non-housing productivity shock and credit shocks account for most of house price volatility. However, the variation in non-housing productivity shock volatility generates a negative relationship between house price volatility and relative consumption volatility, which is not consistent with the cross-country evidence. In contrast, the variation in credit shock volatility—both for the credit availability shock and the interest rate shock—successfully generates a positive relationship, which implies that a country with a more volatile credit shock tends to exhibit higher house price volatility and higher relative consumption volatility. Therefore, this paper provides sources of cross-country variation in credit shock volatility, especially focusing on the difference between emerging and rich countries: higher interest rate volatility and higher sensitivity to an international credit supply shock in emerging countries. The latter is elaborated using a higher share of variable-rate housing loans in emerging countries.

The simulated model results also account for higher relative non-housing consumption volatility and even the far higher housing consumption volatility observed in emerging countries. The former is explained by the structural parameter related to financial frictions, which is a proportion of homeowners in the economy. The credit market effect arising from the housing collateral constraint is stronger when the proportion of homeowners is larger, which primarily contributes to generating excess consumption volatility but also generates excess non-housing consumption volatility in the model. Therefore, the observation of a higher homeownership rate in emerging countries provides an explanation for the higher relative non-housing consumption volatility in those countries. Second, higher housing consumption volatility relative to non-housing consumption volatility can be explained by a cross-country difference in rental price rigidity; a lower degree of rental price rigidity in emerging countries can explain why housing consumption is more volatile than non-housing consumption. To qualitatively compare the degree of rental market frictions between emerging and rich countries, this paper constructs a *rent rigidity index* for each country based on the survey data on housing rental market characteristics across countries and finds that the index is smaller in emerging countries on average. Finally, this paper summarizes all of the implications of the model predictions by showing that higher credit shock volatility, a higher homeownership rate, and a lower degree of rental price rigidity jointly account for excess consumption volatility and higher house price volatility in emerging countries.

The paper mainly relates to two strands of literature. In addition to Aguiar and Gopinath (2007) and Garcia-Cicco, Pancrazi and Uribe (2010), Neumeyer and Perri (2005) and Alvarez-Parra, Brandao-Marques and Toledo (2013) highlight financial frictions to explain the puzzle. The former stresses a working capital borrowing requirement, and the latter suggests a borrowing premium counter-cyclically responding to the output gap as a necessary financial friction to explain the puzzle. As noted above, while my paper's view accords with the financial friction literature in a broad sense, the specific focus is completely different; my paper neglects investment and concentrates on household consumption behavior subject to financial frictions in the form of housing collateral constraint. Next, Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011) explains the volatility of consumption in emerging economies with shocks to the volatility of the borrowing premium. In summary, in Neumeyer and Perri (2005), Garcia-Cicco et al. (2010), Alvarez-Parra et al. (2013), and Fernández-Villaverde et al. (2011), interest rate disturbances play a significant role. The interest rate shock also matters in my paper, but I suggest that a credit availability shock, or a leverage shock, is as important as or more important than a credit price shock. A few other papers provide alternative explanations for excess consumption volatility in emerging economies. Boz, Daude and Durdu (2008) extends Aguiar and Gopinath (2007) to include imperfect information, and Restrepo-Echavarria (2014) explores the role of the informal economy. The present paper also suggests a new perspective on excess consumption volatility by shedding light on housing, but housing is a more important and relevant issue in understanding consumption fluctuations once one considers the share of housing assets on household balance sheets and the proportion of housing service expenditures in household expenditures.

A second strand of literature has explored the effect of house price changes on aggregate consumption through the DSGE model framework. Iacoviello (2005) and subsequent Iacoviello and Neri (2010), which are renowned studies in the macroeconomics literature that model the housing sector, assign an essential role to the housing collateral effect in explaining the effect of house prices on household consumption. The model in the present paper borrows the specification of collateral constraint from Iacoviello (2005) and features the same transmission channel, but it discusses the effect in the context of international comparison. Specifically, the housing collateral effect is stronger in emerging countries because credit shock volatility and homeownership rates are higher in this country group. In contrast, Iacoviello (2005) and Iacoviello and Neri (2010) assume that the representative household is a homeowner, so their model does not feature variation in the degree of financial frictions, which is captured by the proportion of homeowners in the economy. Although we investigate variations in the standard deviations of shocks in the model of Iacoviello (2005) and Iacoviello and Neri (2010), the effect of the variation on consumption volatility relative to output volatility is washed out by a central bank's response in a closed economy setting. Thus, their model generates an almost flat relationship between house price volatility and relative consumption volatility. The present paper overcomes the failure of their models by building a small open economy model that is characterized by a debt-elastic domestic interest rate. While several papers, including Funke and Paetz (2013), Ferrero (2015), and Gete (2018), investigate the effect of house prices on consumption in an open economy framework, they lack a rental market, which limits the effect of house prices. However, the model in the present paper explicitly accommodates a housing rental market and discusses its role in connecting house prices and consumption.

The remainder of the paper is organized as follows. In the next section, this paper constructs the cross-country dataset on the macroeconomic variables of interest and documents several stylized facts regarding cyclical fluctuations in house prices and consumption in emerging countries.

In Section 3, I present the model. Section 4 describes the calibration and presents the results of the baseline model and counterfactual analyses. The subsequent Section 5 discusses the sources of cross-country variations to explain the empirical findings of this paper, based on the model results. Finally, Section 6 concludes the paper.

2 Empirical Facts: Cross-Country Evidence of Consumption Volatility

This paper constructs a cross-country data set containing key macroeconomic variables of interest: house prices, output, and consumption. First, for the house price data, we try to consider quarterly data over a period that is as long as possible in order to avoid the small sample bias in observing business cycle facts.[1] This paper initially collects house price indices from the BIS property price database to control the multiple source issue and then cross-checks them with the indices from each sample country's central bank or national statistics.[2] Controlling for the heterogeneity in cross-country house price data is a difficult task. Basically, this paper undertakes considerable effort to ensure comparable house price indices across countries by confining the definition of the house price index; the index for residential buildings, covering all transactions — new buildings, as well as existing buildings, and the nationwide index or urban area index.[3] For each country's output and consumption, the source of both indicators is the OECD national account database. Since the quarterly national account data in OECD statistics begin from 1950 Q1, the sample period for this paper's data set depends on the availability of house price data in the BIS database. The resulting data set is an unbalanced panel of 54 quarterly time series — house prices (HP_t), GDP (Y_t), and consumption (C_t) for 24 countries — with varying coverage from 1970 Q1 to 2017 Q4. The mode coverage is 1970 Q1 to 2017 Q4 (192 observations) for 11 countries, and the minimum coverage is from 2001 Q1 to 2017 Q4 (68 observations) for Brazil.

[1] One contribution of Cesa-Bianchi et al. (2015) is the construction of a quarterly house price data set for a substantial number of countries — 33 emerging countries and 24 advanced countries. However, its sample coverage is from 1990 Q1 to 2012 Q4, thus providing a house price time series with at most 92 observations. Even in the case of emerging economies, approximately half of the countries cover less than 68 observations (starting from 1996 Q1), and the minimum coverage is from 2006 Q1 to 2012 Q4 with only 28 observations. This is one weakness of the Cesa-Bianchi et al. (2015) data set in that it can cause an overestimation problem of the standard deviation.

[2] Cesa-Bianchi et al. (2015) mainly refers to the Federal Reserve of Dallas international house price database, OECD house price database, and BIS property price data set for the sources of their house price data set. In fact, the Dallas Fed international house price database is not a raw data set, as its house price indices are produced and updated based on its own methodology (Mack and Martinez-Garcia (2011)). OECD house price data is a raw data set, but I conclude the BIS data set is superior to the OECD data set in terms of the number of countries covered and the length of the time series.

The sample countries included in the data set considering data availability are categorized as either an emerging country or a rich country according to the classification of Schmitt-Grohé and Uribe (2017)[4]: ten emerging countries (Brazil, Colombia, Greece, Israel, Malaysia, New Zealand, Portugal, South Africa, South Korea, and Spain) and 14 rich countries (Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom, and the United States).

The raw data of HP_t, Y_t, and C_t are collected in nominal terms. After deflating each series by the GDP deflator, I implement seasonal adjustment to only house price series by X-12-ARIMA because HP_t is non-seasonally adjusted data, whereas the raw data of Y_t and C_t are already seasonally adjusted. Then, I conduct detrending by first log-differencing, which implies the raw series for the house prices; GDP and consumption are assumed to be nonstationary. In other words, I eliminate the trend component of each series by inducing stationarity by first-differencing the series.[5] Before detrending, in the case of Y_t and C_t, they are divided by population to obtain a per capita measure. The quarterly population is for all ages and both sexes, which is linearly interpolated from annual population data of the UN population division.[6] Next, to characterize the average business cycle facts for the respective country group, I compute second moments of interest for each individual country and then take a population-weighted average (as of 2010) across countries.

2.1 Business Cycle Facts and Cross-Country Evidence

In Table 1, this paper reports the individual country's statistics and the weighted averages for emerging countries and rich countries. The stylized facts that we can observe in Table 1 are twofold. First, housing price volatility is much larger in emerging countries than in rich

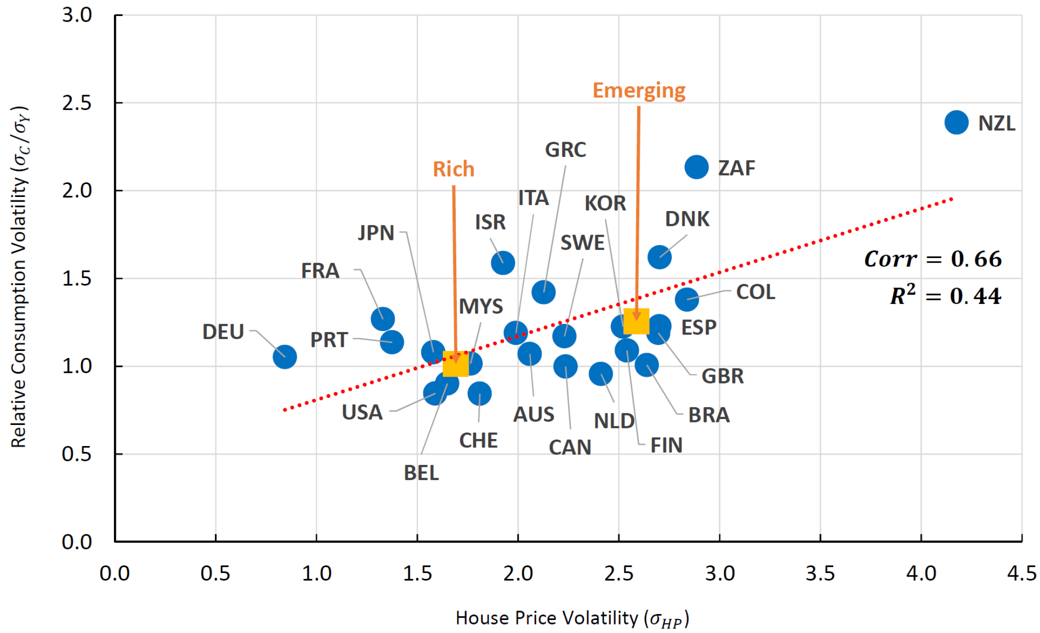
[3] There is a relatively high correlation between the national house price index and individual city index — for the United States, Canada, and the United Kingdom; therefore, this paper takes the urban area house price index if there is no available national index — for Australia and Japan. Cesa-Bianchi et al. (2015) also states that to the extent to which domestic housing markets are driven by common country factors, using a particular house price indicator should be less problematic.

[4] Schmitt-Grohé and Uribe (2017) defines the country group with the average PPP converted GDP per capita: poor country (less than \$3,000), emerging countries (between \$3,000 and \$25,000), and rich countries (above \$25,000). Please refer to Chapter 1 in Schmitt-Grohé and Uribe (2017), pp.10-11.

[5] I also checked alternative detrending methods, such as an HP filter and a quadratic filter. Detrending by the HP filter with a constant of 1,600 and the quadratic filter show similar results to what I obtain from a first-difference, so I confirm robustness. Among the three options for detrending that I took into account, I choose the first-differencing filter considering comparability with Cesa-Bianchi et al. (2015). I would also like to note that the HP filter might be problematic as Hamilton (2016) points out: as most of the macroeconomic variables of interest follow martingales or near martingales, the cyclical component characterized by the HP filter might generate a spurious pattern regardless of the underlying data generating process.

[6] I first tried to make use of OECD population data (for all ages), but it was suggested to instead use the UN database due to the discontinuity that occurred when it stopped collecting population data. Thus, the annual population data is from "World Population Prospects: The 2015 Revision" published by UN population division.

Figure 1: House Price and Relative Consumption Volatility



countries. This finding was also presented by Cesa-Bianchi et al. (2015). Second, emerging countries present higher relative volatility of consumption, or less consumption smoothing, than rich countries. While σ_C/σ_Y for rich countries is 1.01, it is 1.26 in the case of emerging countries. The higher relative consumption volatility — in particular, σ_C/σ_Y greater than one — is known as the excess consumption volatility puzzle, which is declared in several papers (Aguiar and Gopinath (2007), Alvarez-Parra et al. (2013), Schmitt-Grohé and Uribe (2017), among others).[7] The above two are established stylized facts in terms of the business cycle across countries, but there has been no paper connecting two volatilities across countries. Therefore, this paper, for the first time, provides cross-country evidence between house price volatility and relative consumption volatility.

If we plot the sample countries in the data set on a plane that takes σ_{HP} and σ_C/σ_Y as a horizontal and a vertical axis, respectively, then it is easily observed that there is a positive correlation between two volatilities across countries, as Figure 1 presents. That is, a country

[7] In *Appendix A*, I compare several empirical facts from this paper's cross-country data set with well-established results in the literature. First, we compare business cycle facts in terms of consumption and output with those presented in Schmitt-Grohé and Uribe (2017) and find relatively high correlation between them; second, we compare house price volatilities in this paper with those in Cesa-Bianchi et al. (2015); lastly, we compare the correlation between house prices and consumption — though it is not main interest of this paper — with Calza et al. (2007). Please refer to *Appendix A* for the details. Moreover, in *Appendix B*, I show that the two stylized facts of interest — higher σ_{HP} and higher σ_C/σ_Y in emerging countries — are robust to different sample periods and different detrending filters in this paper's data set.

Table 1: Business Cycles in Emerging and Rich Countries

	σ_{HP}	σ_C/σ_Y	Period[1]		σ_{HP}	σ_C/σ_Y	Period[1]
Emerging Countries				***Rich Countries***			
Brazil	2.64	1.01	2001 Q1-	Australia	2.06	1.07	1970 Q1-
Colombia	2.84	1.38	2000 Q1-	Belgium	1.65	0.90	1970 Q1-
Greece	2.13	1.42	1994 Q1-	Canada	2.24	1.00	1970 Q1-
Israel	1.93	1.59	1995 Q1-	Denmark	2.70	1.62	1970 Q1-
Malaysia	1.76	1.02	1991 Q1-	Finland	2.54	1.09	1970 Q1-
New Zealand	4.18	2.39	1970 Q1-	France	1.33	1.27	1970 Q1-
Portugal	1.38	1.14	1988 Q1-	Germany	0.84	1.05	1970 Q1-
South Africa	2.89	2.13	1970 Q1-	Italy	1.99	1.19	1970 Q1-
South Korea	2.52	1.23	1975 Q1-	Japan	1.58	1.08	1970 Q1-
Spain	2.70	1.23	1971 Q1-	Netherlands	2.41	0.96	1970 Q1-
				Sweden	2.23	1.17	1970 Q1-
				Switzerland	1.81	0.84	1970 Q1-
				United Kingdom	2.70	1.19	1970 Q1-
				United States	1.59	0.84	1970 Q1-
Average[2]	**2.59**	**1.26**		**Average**[2]	**1.69**	**1.01**	

[1] Sample period ends in 2017 Q4 in all countries.
Sources: BIS property price database, OECD national accounts, Central banks, National statistics offices
[2] Population-weighted average Source: UN population division

whose house price volatility is high tends to have higher relative consumption volatility according to the scatter plot. Additionally, the weighted average for each country group — the square dots in Figure 1 — summarizes that emerging countries have higher house price volatility as well as higher relative consumption volatility than rich countries on average, as the literature suggests. We check the robustness of the cross-country evidence in a diverse dimension. The evidence is robust to different detrending filters, such as an HP filter and a quadratic filter, and robust to different sample periods — a shorter coverage from 1997. New Zealand is an outlier country in Figure 1, so we confirm the robustness of positive correlation in the samples excluding New Zealand. Lastly, we change the horizontal axis variable, σ_{HP}, to the relative house price volatility, σ_{HP}/σ_Y, considering the possibility of the comovement between house price volatility and output volatility. Although we cannot find any economic interpretation for the relative house price volatility, we can eliminate concern about the comovement effect of σ_Y on σ_{HP} by observing the positive relationship between σ_C/σ_Y and σ_{HP}/σ_Y across countries. Please refer to *Appendix B* for the cross-country scatter plots for the above-stated robustness check.

2.2 Further Evidence: Disaggregated Consumption Volatilities

I extend the quarterly cross-country data set to include disaggregated consumption data series: housing consumption (HC_t) and non-housing consumption (NHC_t). In the literature, a basic model setup with housing considers two types of consumption goods; one is housing services, and the other is consumption goods. However, there is no existing paper looking into cross-country observations from housing service expenditure data, which is the motivation in this paper to collect the disaggregated consumption data across countries. This paper defines housing consumption as expenditure on housing service and housing-related utilities. In the view of the national account, housing consumption is the item of final consumption expenditure on housing, water, electricity, gas, and other fuels, which is included in the item of expenditure on services. Non-housing consumption is defined as the rest of consumption, which is the sum of durable consumption and non-durable and service consumption, other than housing consumption.[8] Unfortunately, the sample size shrinks to cover only four countries due to the extremely limited data availability at the quarterly frequency (Canada, New Zealand, South Korea, and the United States). Therefore, this paper also takes annual frequency data into account in order to observe a cross-country relationship and business cycles with more sample countries. In annual data, there are ten countries available — six rich countries and three emerging countries: Australia, Canada, Denmark, Finland, France, and the United States (rich countries); and New Zealand, South Korea, and South Africa (emerging countries).[9] All series are transformed to per capita variables, deflated by the GDP deflator and detrended using the first-differencing filter. Additionally, we generate an annual house price index by taking an average of the quarterly house price index for each country, whose cyclical component is also extracted through a first-differencing filter.

Table 2 presents the key second moments of individual sample countries with group-average statistics. First, in quarterly data, we observe house prices are more volatile in New Zealand and South Korea than in the United States and Canada, and there is excess consumption

[8]Real term non-housing consumption (or housing consumption) is computed by multiplying the real consumption and a share of non-housing consumption (or a share of housing consumption). The share of non-housing consumption (or housing consumption) is calculated by dividing the current price of non-housing consumption (or housing consumption) by the current price consumption. The data sources for disaggregated consumption (housing consumption and non-housing consumption) are Statistics Canada for Canada, Statistics New Zealand for New Zealand, Bank of Korea for South Korea, and Bureau of Economic Analysis for the Unites States.

[9]Data sources: South African Reserve Bank for South Africa, and OECD national accounts for the other countries

Table 2: Business Cycles with Disaggregated Consumption

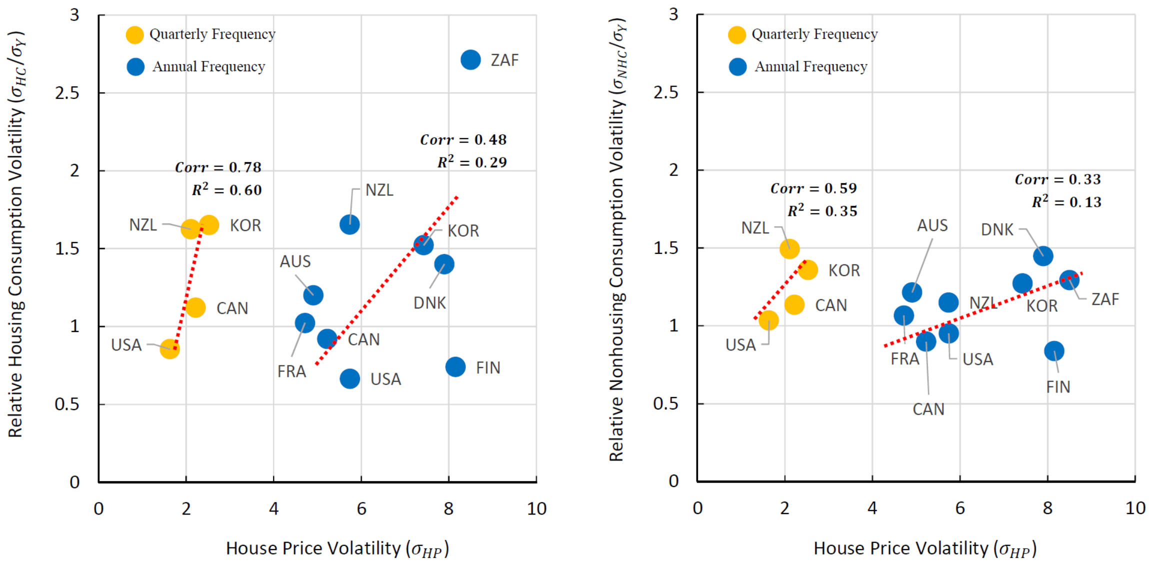
	Country	σ_{HP}	$\frac{\sigma_C}{\sigma_Y}$	$\frac{\sigma_{HC}}{\sigma_Y}$	$\frac{\sigma_{NHC}}{\sigma_Y}$	Period
		Quarterly Frequency				
Emerging	New Zealand	2.11	1.26	1.62	1.49	1987 Q3-2017 Q4
	South Korea	2.52	1.23	1.68	1.36	1975 Q1-2017 Q4
	Average	**2.49**	**1.23**	**1.65**	**1.37**	
Rich	Canada	2.24	1.00	1.12	1.14	1970 Q1-2017 Q4
	United States	1.59	0.84	0.85	1.04	1970 Q1-2017 Q4
	Average	**1.65**	**0.86**	**0.88**	**1.05**	
		Annual Frequency				
Emerging	New Zealand	5.74	1.07	1.66	1.15	1987-2017
	South Korea	7.42	1.13	1.52	1.27	1970-2017
	South Africa	8.50	1.26	2.71	1.29	1970-2017
	Average	**7.88**	**1.19**	**2.11**	**1.28**	
Rich	Australia	4.90	1.05	1.20	1.22	1970-2017
	Canada	5.22	0.72	0.92	0.90	1970-2017
	Denmark	7.90	1.12	1.40	1.45	1970-2017
	Finland	8.15	0.61	0.74	0.84	1975-2017
	France	4.72	0.90	1.02	1.07	1970-2017
	United States	5.74	0.85	0.66	0.95	1970-2017
	Average	**5.57**	**0.86**	**0.77**	**0.98**	

[1] Sources: BIS property price database, OECD national accounts, Central banks, National statistics offices

[2] The average of emerging or rich countries is the population-weighted average.

volatility in New Zealand and South Korea, which is the stylized business cycle fact for emerging economies. It is notable that relative housing consumption volatility is significantly larger than non-housing consumption volatility in emerging countries ($\sigma_{HC}/\sigma_Y = 1.62 > 1.49 = \sigma_{NHC}/\sigma_Y$ for New Zealand; $\sigma_{HC}/\sigma_Y = 1.68 > 1.36 = \sigma_{NHC}/\sigma_Y$ for South Korea). In contrast, in both the United States and Canada, the housing consumption volatility is similar to or smaller than non-housing consumption volatility ($\sigma_{HC}/\sigma_Y = 1.12 < 1.14 = \sigma_{NHC}/\sigma_Y$ for Canada; $\sigma_{HC}/\sigma_Y = 0.85 < 1.04 = \sigma_{NHC}/\sigma_Y$ for the United States). Another striking feature from Table 2 is that housing consumption volatilities for New Zealand and South Korea (1.62 and 1.68, respectively) are much greater than those for Canada and the United States (1.12 and 0.85,

Figure 2: House Price and Disaggregated Consumption Volatility



respectively). We also observe the same pattern in annual frequency data. House price volatility and relative consumption volatility are higher in emerging countries than in rich countries; housing consumption is far more volatile than non-housing consumption in the emerging country group $((\sigma_{HC}/\sigma_Y)_{EC} = 2.11 > 1.28 = (\sigma_{NHC}/\sigma_Y)_{EC})$, but it is not the case in rich countries $((\sigma_{HC}/\sigma_Y)_{RC} = 0.77 < 0.98 = (\sigma_{NHC}/\sigma_Y)_{RC})$. Moreover, housing consumption volatility in emerging countries is larger than in rich countries. The observation from housing consumption expenditure is one of the novel findings of this paper. Starting from this finding, this paper extends the argument that house prices in emerging countries matter in terms of their higher relative consumption volatility. Specifically, higher housing consumption volatility in emerging countries is one of the key sources explaining the channel from house price to consumption volatility. However, we also have to pay attention to non-housing consumption. Even if we consider higher housing consumption volatility, there is still excess volatility in the remaining part of consumption for emerging countries. That is, we can observe σ_{NHC}/σ_Y for emerging countries is greater than unity, which requires reasoning beyond higher housing consumption in explaining excess consumption volatility.

Let us consider the disaggregated consumption volatility across countries. Figure 2 shows the cross-country relationship between house price volatility and disaggregated consumption volatilities. The left panel is for housing consumption volatility with house price volatility, and the right panel is for non-housing consumption volatility. Both housing consumption and non-housing consumption volatility show a positive correlation with house price volatility across countries. In other words, a country whose house price volatility is higher tends to have higher housing consumption volatility and non-housing consumption volatility alike. As a result, I declare the above and the cross-country evidence in Figure 1 as *Finding 1.*

Finding 1 *There is a positive correlation between house price volatility and relative consumption volatility across countries. In addition, the relationship holds for disaggregated consumption volatility — housing consumption and non-housing consumption volatility.*

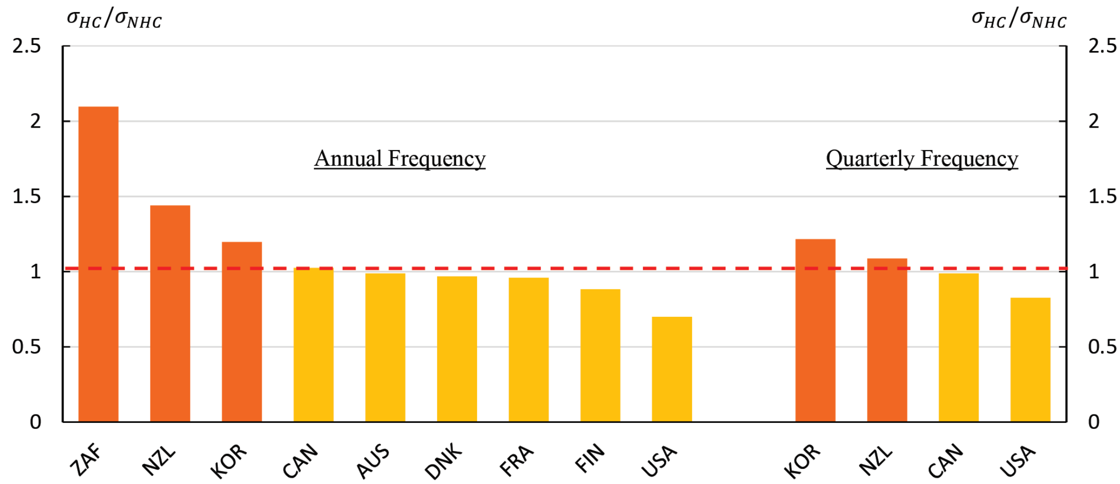
What if we investigate the ratios of housing consumption volatility to non-housing consumption volatility for sample countries? As Figure 3 shows, we can confirm the business cycle fact discussed in Table 2: housing consumption is more volatile in emerging countries than in rich countries. In annual frequency data, South Africa, New Zealand, and South Korea present σ_{HC}/σ_{NHC} greater than one, and in quarterly frequency, South Korea and New Zealand present $\sigma_{HC}/\sigma_{NHC} > 1$. In contrast, housing consumption is as volatile as, or less volatile than, non-housing consumption in rich countries. Therefore, we suggest that the ratio of housing consumption volatility to non-housing consumption volatility is greater in the emerging country group than in the rich country group ($(\sigma_{HC}/\sigma_{NHC})_{EC} > (\sigma_{HC}/\sigma_{NHC})_{RC}$). However, the higher relative consumption volatility or excess consumption volatility observed in emerging countries cannot be explained solely through the higher housing consumption volatility, as we discussed above. If it is fully accounted for by housing consumption volatility; there should be no excess volatility in non-housing consumption. However, the weighted average of relative non-housing consumption volatilities of emerging countries is 1.37 for quarterly data, and 1.28 for annual data, which are larger than those of rich countries (1.05 for quarterly data, and 0.98 for annual data). As a result, we summarize *Finding 2* and *Finding 3* as follows.

Finding 2 *Housing consumption is more volatile than non-housing consumption in emerging countries. In contrast, this is not the case in rich countries. In terms of the ratio between housing consumption volatility to non-housing consumption volatility, the ratio in emerging countries is larger than that in rich countries.*

Finding 3 *Relative non-housing consumption volatility is still larger in emerging countries than in rich countries. Moreover, in emerging countries, excess volatility still holds for non-housing consumption.*

Moreover, *Finding 1* and *Finding 2* jointly imply house price volatility likely matters for higher relative consumption volatility in emerging countries through their higher housing consumption volatility. Since housing consumption expenditure in the national account is measured

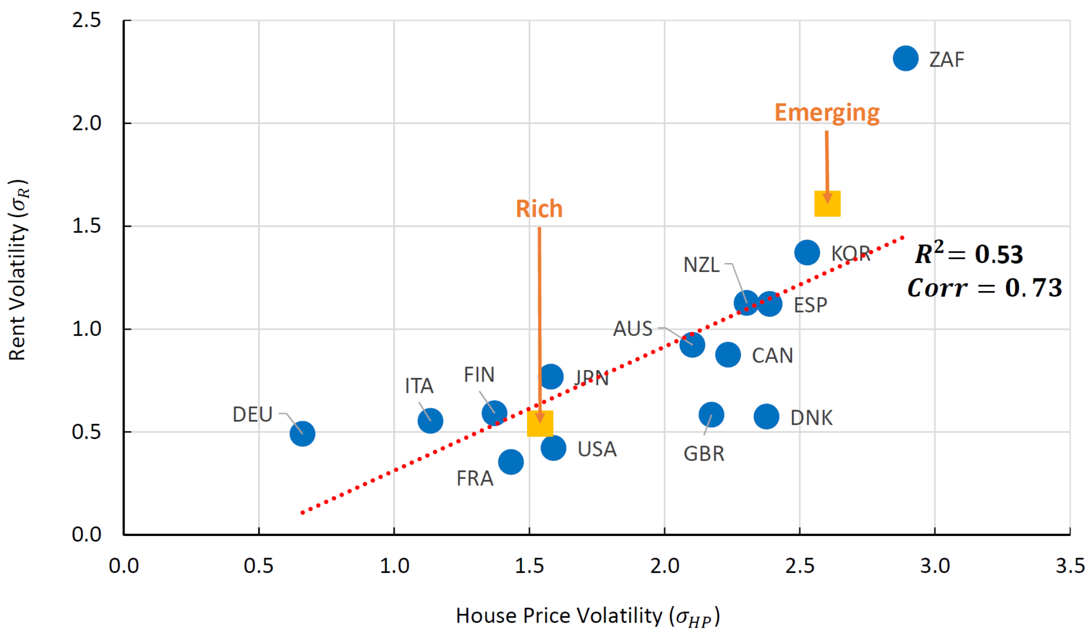
Figure 3: Ratio of Housing Consumption Volatility



by the sum of the rental value of tenant-occupied housing and imputed rental value of owner-occupied housing, the housing rental price emerges as a key factor that links house price volatility and housing consumption volatility. At the same time, the rental price determines both a relative change in price for non-housing consumption for renters and a source of rental incomes for homeowners, so we can conjecture that it also substantially affects non-housing consumption. Taking its importance into account, this paper collected the housing rent index for data-available sample countries[10] and found that there is a positive correlation between house price volatility and rental price volatility across countries, as Figure 4 shows. More importantly, Figure 5 suggests housing rents, generally, are less volatile than house prices in most countries. This fact is clear if I compute the second moment for each country; the ratio of rent volatility to house price volatility varies across countries, but all of them are located between zero and one[11]. The weighted average of ratios is 0.42. Therefore, this paper declares the facts related to rental prices across countries as *Finding 4*:

Finding 4 *Rental price volatility shows a positive relationship with house price volatility across countries. Generally, this volatility is less than house price volatility, though the magnitude is different across countries.*

Figure 4: House Price and Rent Volatility

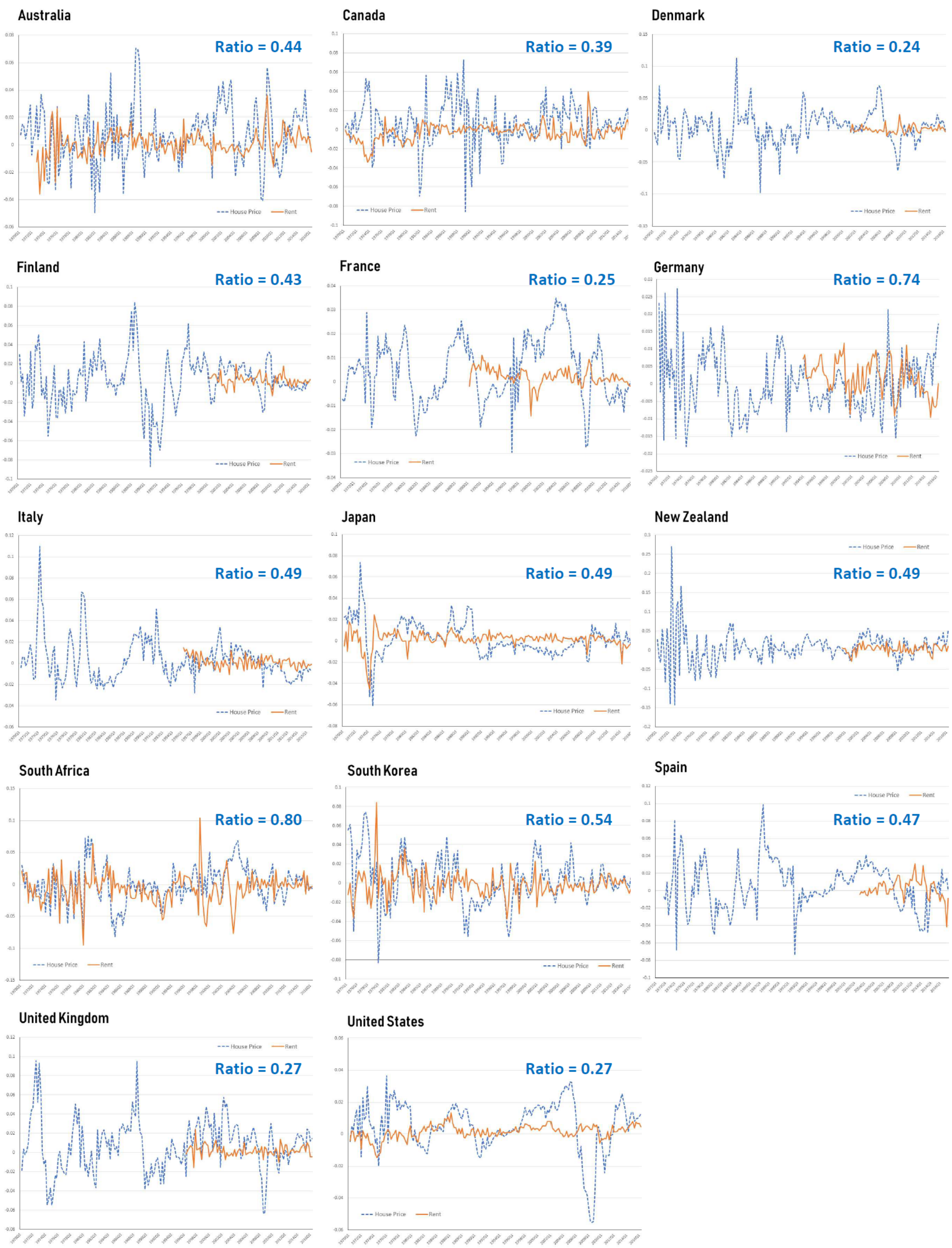


Next, according to *Finding 1* and *Finding 3*, we should consider the link between house prices and non-housing consumption in aggregate. Note that there is no traditional wealth effect on consumption from house prices because houses are different from other assets for two reasons. First, people usually live in their house and directly value the services provided by their house. Therefore, the benefit of an increase in house prices is directly offset by an increase in the opportunity cost of housing services. Second, houses are not widely traded internationally. Thus, one country's homeowners in aggregate cannot realize their capital gains on houses to increase their consumption. That is, all homeowners cannot simultaneously move out of homeownership. Therefore, the gain to a last-time seller is also a loss to a first-time buyer, who will usually be the country's customer. This contrasts with capital gains on financial assets. However, there is another explanation in the literature. House prices may have a direct impact on non-housing consumption via the credit market effect, the so-called housing collateral effect. Houses

[10] I extracted the housing rent index from CPI for each country. The rental price data set is an unbalanced panel, considering data availability. The sample countries with coverage and source are as follows: Australia (1973 Q1-2017 Q4, Australian Bureau of Statistics), Canada (1970 Q1-2017 Q4, Statistics Canada), Denmark (2001 Q1-2017 Q2, Statistics Denmark), Finland (2000 Q1-2017 Q7, Statistics Finland), France (1990 Q1-2017 Q4, INSEE), Germany (1994 Q1-2017 Q4, DESTATIS), Italy (1996 Q1-2017 Q4, IStat), Japan (1970 Q1-2017 Q4, Statistics Bureau), New Zealand (1999 Q1-2017 Q4, Statistics New Zealand), South Africa (1970 Q1-2017 Q4, Statistics South Africa), South Korea (1975 Q1-2017 Q4, Bank of Korea), Spain (2002 Q1- 2017 Q4, INE), the United Kingdom (1996 Q1-2017 Q4, INS), and the United States (1970 Q1-2017 Q4, BLS).

[11] The ratio of $\sigma_R ent/\sigma_H P$ for each country is as follows: 0.44 for Australia, 0.39 for Canada, 0.24 for Denmark, 0.43 for Finland, 0.25 for France, 0.74 for Germany, 0.49 for Italy, 0.49 for Japan, 0.49 for New Zealand, 0.80 for South Africa, 0.54 for South Korea, 0.47 for Spain, 0.27 for the United Kingdom, 0.27 for the United States.

Figure 5: House Price and Rent Fluctuation, by Country



†The solid line and the dashed line is for rents and house prices for each country, respectively.

represent collateral for homeowners, and borrowing on a secured basis against housing collateral is generally less expensive than borrowing on an unsecured basis. Thus, an increase in house prices makes more collateral available to homeowners, which in turn may encourage them to borrow more (for instance, in the form of mortgage equity withdrawal in the United States) in order to finance consumption. Then, the natural follow-up question is whether the collateral channel also works in emerging countries or not. The average of the mortgage-to-GDP ratio which is widely used as a mortgage depth indicator for emerging countries in this paper's sample is 33.0%.[12] Although this average is lower than the average for rich countries (58.6%), we can suggest that mortgage also matters in emerging countries, and the collateral effect exists.

3 The Business Cycle Model with Housing and Rental Markets

In this section, this paper builds up a small open economy business cycle model with housing, which is able to capture the empirical findings I suggested in the previous section. To be specific, we investigate the driving force to generate cross-country evidence of the positive relationship between house price volatility and relative consumption volatility through the theoretical model framework. To do so, the model features endogenous house price because a partial equilibrium model with exogenously given house price cannot explain what makes house prices more volatile in emerging countries, which is one of the key motivational facts of the paper.[13]

This paper borrows main building blocks from Iacoviello (2005) and subsequent Iacoviello and Neri (2010), which are undoubtedly renowned papers relating the DSGE model with housing. Basically, their model is a business cycle model with a housing collateral constraint. In detail, Iacoviello (2005) develops a monetary business cycle model to explain a relationship between house prices and economic fluctuation in a large closed economy such as the United States. The primary variations of this paper are twofold: I discuss the real economic fluctuations in this

[12] Badev et al. (2014) gauges the depth of each country's mortgage market by focusing on the total volume. It collects and shows each country's outstanding mortgage debt relative to GDP on average for the period 2006 to 2010. It also presents housing loan penetration indices, which is defined by the percentage of the adult population with an outstanding loan to purchase a home. In terms of both indicators, the mortgage market matters in emerging countries in this paper's sample.

[13] Shin (2018), my previous paper, develops a partial equilibrium model where agents act given an exogenously given house price process. Given the exogenous house price, an agent can purchase as many houses as she wants. That is, the economy has a demand-determined housing market, and housing supply is assumed to be perfectly elastic. The paper determines that the key mechanisms in understanding excess consumption volatility and higher housing consumption volatility are a collateral constraint effect and rental price pass-through. However, the paper mutes the issue of why house price volatility is high in emerging countries and thus has a limit to extending its argument to the fundamental source of the excess consumption volatility puzzle.

paper, so the model is a real business cycle model, and it does not include a monetary policy. Moreover, it is an open economy model, especially in that it considers a small open economy where most of the emerging countries are categorized. The small open economy environment is featured by an external debt elastic interest rate following Schmitt-Grohé and Uribe (2003). The agent can access the international financial market so that she can borrow money at the given interest rate. In the case of the domestic bond market, there is zero-sum domestic debt holding in the aggregate. As a result, all debt is international borrowing. I also borrow an idea of rental price determination suggested by Sommer et al. (2013). One contribution of the model is an explicit consideration of a housing rental market, and it is implemented by a two-agent setup of homeowners and renters. Last, the time unit of the model is a quarter.

3.1 Firms

There are three types of firms on the production side of the economy: non-housing consumption goods producers, house construction firms, and housing rental agencies. Consumption goods producers and house construction firms use labor, which is the only input factor supplied by households to produce non-housing goods and new houses, respectively. By assumption, they are owned by households. Note that non-tradable goods, or houses, are only purchased by homeowners, but tradable goods, or non-housing consumption goods, are consumed by both homeowners and renters. Rental agencies buy housing services from homeowners and sell them to renters after setting rental prices under the assumption of monopolistic competition between agencies. The existence of rental agencies brings housing rental price rigidity into the model. They are assumed to be owned by homeowners.

3.1.1 Non-housing Consumption Goods Producers

Consumption goods producers use only labor to produce non-housing consumption goods, y_t. They maximize the present value of profits given by

$$\max_{n_{c,t}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \varepsilon_t^b Profit_{c,t} \tag{1}$$

$$\text{s.t.} \quad Profit_{c,t} = y_t - Wage_{c,t} n_{c,t} - \Phi_c \left(n_{c,t} - n_{c,t-1} \right) \tag{2}$$

where y_t is domestic production of non-housing consumption goods. Households purchase non-housing consumption goods at price of one (numeraire). Note that profits are discounted using the factor $\beta^t \lambda_t / \lambda_0$, which is the value assigned by households to contingent payment of goods in period t in terms of units of goods in period 0. This way of discounting makes sense because households own the firms (both consumption goods producers and construction firms). In this theoretical world, there is no *representative* household; instead, there are two types of households: *homeowners* and *renters*. As the proportion of each type of household is assumed to be constant over time, and both agents own the firms, without loss of generality, λ_t denotes the average of marginal utility of consumption between two agent groups, say, $\lambda_t \equiv \omega \lambda_t^O + (1 - \omega) \lambda_t^R$. Next, ε_t^b represents intertemporal preference shock, which occurs because the discounting factor is derived from the households' marginal utilities. The production technology is

$$y_t = A_{c,t} (n_{c,t})^{\alpha_c} \tag{3}$$

The technology basically follows the Cobb-Douglas formation. The term $A_{c,t}$ measures productivity in the non-housing sector, and $n_{c,t}$ is for the amount of labor necessary to produce one unit of y_t. The corresponding marginal cost for one unit of labor for the non-housing sector is $Wage_{c,t}$. The parameter α_c is between zero and one. The non-housing productivity $A_{c,t}$ is a source of uncertainty that follows

$$\ln A_{c,t+1} = \rho_c \ln A_{c,t} + \sigma_c \xi_{c,t+1} \tag{4}$$

where ρ_c is a persistence of shock, σ_c is a standard deviation of innovation to shock, and $\xi_{c,t}$ follows $iid\,(0, 1)$. There is a quadratic adjustment cost $\Phi_c (\cdot)$ to moving labor across sectors, which is assumed to satisfy $\Phi_c (0) = \Phi_c' (0) = 0$ and $\Phi_c"(0) > 0$. The cost is paid in units of non-housing consumption goods. The functional form for adjustment cost is

$$\Phi_c (n_{c,t} - n_{c,t-1}) = \frac{\phi_c}{2} (n_{c,t} - n_{c,t-1})^2 \tag{5}$$

where the parameter ϕ_c is a positive number. The first-order condition for the non-housing goods producer's profit maximization problem is:

$$\alpha_c A_{c,t} (n_{c,t})^{\alpha_c - 1} - \phi_c (n_{c,t} - n_{c,t-1}) + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\varepsilon_{t+1}^b}{\varepsilon_t^b} \phi_c (n_{c,t+1} - n_{c,t}) \right] = Wage_{c,t} \tag{6}$$

3.1.2 House Construction Firms

Construction firms use labor to build new houses, and they maximize their profits given by

$$\max_{n_{h,t}} \ E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \varepsilon_t^b Profit_{h,t} \tag{7}$$

$$\text{s.t.} \quad Profit_{h,t} = q_t nh_{t-k} - Wage_{h,t} n_{h,t} - \Phi_h \left(n_{h,t} - n_{h,t-1} \right) \tag{8}$$

where nh_t denotes new houses built by firms and q_t denotes house price. Note that the construction firm's revenue at period t is related to nh_{t-k}, reflecting the delays in construction. In detail, resources for housing production must be committed at time $t-k$, but the newly constructed house will not be completed until time t. This time-to-build feature is introduced to capture the time required to construct houses in the real world. For simplicity, this paper set $k = 1$, which implies it takes a quarter to build new houses. Notice that the construction firm's problem also features the same discounting factor as that of the non-housing goods producer because both of them are assumed to be owned by households.

$$nh_t = A_{h,t} \left(n_{h,t} \right)^{\alpha_h} \tag{9}$$

The technology follows the Cobb-Douglas formation. The term $A_{h,t}$ measures productivity in the housing sector, which follows an AR(1) process with the persistence of ρ_h and the standard deviation of σ_h:

$$\ln A_{h,t+1} = \rho_h \ln A_{h,t} + \sigma_h \xi_{h,t+1} \tag{10}$$

where $\xi_{h,t}$ follows $iid\,(0,1)$. Next, $n_{h,t}$ is the amount of labor required to build one unit of nh_t, and α_h is a parameter that is less than one. Additionally, there is an adjustment cost of moving labor across sectors whose functional form is the same as the labor adjustment cost in the non-housing sector:

$$\Phi_h \left(n_{h,t} - n_{h,t-1} \right) = \frac{\phi_h}{2} \left(n_{h,t} - n_{h,t-1} \right)^2 \tag{11}$$

where $\phi_h > 0$. Note that the only choice variable in the construction firm's problem is the amount of labor demanded given its price, $Wage_{h,t}$. The first-order condition with $k = 1$ is:

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\varepsilon_{t+1}^b}{\varepsilon_t^b} \Big(\alpha_h q_{t+1} A_{h,t} \left(n_{h,t} \right)^{\alpha_h - 1} + \phi_h \left(n_{h,t+1} - n_{h,t} \right) \Big) \right] - \phi_h \left(n_{h,t} - n_{h,t-1} \right) = Wage_{h,t} \tag{12}$$

Labor Market Clearing The theoretical world is a full-employment economy since no disutility from labor supply by households is assumed. There is a continuum of measure one of households, so the labor market clearing condition is given by

$$n_{c,t} + n_{h,t} = 1 \tag{13}$$

Moreover, the full-employment assumption and labor allocation across two sectors make the value of one unit of labor equal across sectors in equilibrium, say, $Wage_{c,t} = Wage_{h,t}$). Let the equilibrium wage $Wage_t$, which gives the optimal condition for labor allocation, be

$$\begin{aligned}
&\alpha_c A_{c,t} \left(n_{c,t}\right)^{\alpha_c - 1} - \phi_c \left(n_{c,t} - n_{c,t-1}\right) + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\varepsilon^b_{t+1}}{\varepsilon^b_t} \phi_c \left(n_{c,t+1} - n_{c,t}\right)\right] \\
&\quad = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\varepsilon^b_{t+1}}{\varepsilon^b_t} \left(\alpha_h q_{t+1} A_{h,t} \left(n_{h,t}\right)^{\alpha_h - 1} + \phi_h \left(n_{h,t+1} - n_{h,t}\right)\right)\right] - \phi_h \left(n_{h,t} - n_{h,t-1}\right)
\end{aligned} \tag{14}$$

3.1.3 Housing Rental Agencies

I allow for price rigidity in the housing rental market to capture *Finding 4*. This feature is in line with the evidence of substantial price rigidity among housing rents provided by Genesove (2003), Verbrugge and Gallin (2017), Aysoy et al. (2014), Shimizu et al. (2010), among others. Price rigidity in the rental market is introduced by assuming monopolistic competition between rental agencies and adjustment costs of rental price following Rotemberg (1982) pricing. Specifically, homeowners supply their homogeneous housing services to rental agencies. The agencies buy housing services $s^R_t \left(= h_t - s^O_t\right)$ from homeowners at the price ρ^O_t in a competitive market, differentiate the rental services at no cost, and set rental prices. The CES aggregates of these services are converted back into homogeneous housing services, and they are sold at a markup Z_t.[14] As a result, renters buy housing services at the final rental price ρ_t, which we can observe, so $Z_t = \rho_t/\rho^O_t$. Note that the total amount of housing services the rental agencies deal with is ωs^R_t.

Formally, there is a continuum of rental agencies of mass 1, indexed by i. Each rental agency chooses a rental price ρ_{it}, taking $\rho^O_t \left(= \rho_t/Z_t\right)$ and the individual demand curve that each monopolistic competitive agency faces as given.

$$\max_{\rho_{it}} \; E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda^O_t}{\lambda^O_0} \varepsilon^b_t \left[\rho_{it} \omega s^R_{it} - mc_t \omega s^R_{it} - \frac{\theta}{2} \left(\frac{\rho_{it}}{\rho_{i,t-1}} - 1\right)^2\right] \tag{15}$$

[14] It is easy to understand if there is an imaginary headquarters of rental agencies. Retail level agencies offer the services to the headquarters that assemble differentiated rental services into the homogeneous services and provide them to renters.

$$\text{s.t.} \quad s_{it}^R = \left(\frac{\rho_{it}}{\rho_t}\right)^{-\nu} s_t^R \tag{16}$$

The discount factor is based on the assumption that the rental agencies are owned by homeowners. The individual demand curve s_{it}^R is derived under the monopolistic competition and CES aggregates for s_t^R which is $s_t^R = \left[\int_0^1 \left(s_{it}^R\right)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}}$, and for ρ_t which is $\rho_t = \left[\int_0^1 \rho_{it}^{1-\nu} di\right]^{\frac{1}{1-\nu}}$. The parameter ν represents an elasticity of substitution between differentiated rental services, and $\nu > 1$. The marginal cost the rental agency faces is the price it pays for purchasing one unit of housing service from homeowners, so $mc_t = \rho_t^O = \frac{\rho_t}{Z_t}$. The last term is a quadratic price adjustment cost function. If agency i decides to set $\rho_{it} = \rho_{i,t-1}$, clearly it pays no cost. Instead, if it chooses to set a ρ_{it} different from $\rho_{i,t-1}$, it does incur adjustment costs. Moreover, the cost is larger the further from the reference level, $\rho_{i,t-1}$. Due to the quadratic nature of the cost, adjustment costs are symmetric with respect to both price increases and price decreases. Note that the adjustment cost is a real cost – that is, it is denominated in terms of real non-housing consumption goods. As a result, we can solve the following maximization problem:

$$\max_{\rho_{it}} \; E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t^O}{\lambda_0^O} \varepsilon_t^b \left[\rho_{it} \left(\frac{\rho_{it}}{\rho_t}\right)^{-\nu} \omega s_t^R - \frac{\rho_t}{Z_t} \left(\frac{\rho_{it}}{\rho_t}\right)^{-\nu} \omega s_t^R - \frac{\theta}{2} \left(\frac{\rho_{it}}{\rho_{i,t-1}} - 1\right)^2\right] \tag{17}$$

The first-order condition is

$$\begin{aligned} &(\nu - 1) \left(\frac{\rho_{it}}{\rho_t}\right)^{-\nu} \omega s_t^R \\ &= \nu \frac{1}{Z_t} \left(\frac{\rho_{it}}{\rho_t}\right)^{-\nu-1} \omega s_t^R - \theta \left(\frac{\rho_{it}}{\rho_{i,t-1}} - 1\right) \frac{1}{\rho_{i,t-1}} + \beta\theta E_t \left[\frac{\lambda_{t+1}^O}{\lambda_t^O} \frac{\varepsilon_{t+1}^b}{\varepsilon_t^b} \left(\frac{\rho_{i,t+1}}{\rho_{it}} - 1\right) \frac{\rho_{i,t+1}}{\rho_{it}^2}\right] \end{aligned} \tag{18}$$

In equilibrium, all agencies behave identically. This means that they all charge the same price. Therefore, the equilibrium condition derived from the rental agency problem is that:

$$(\nu - 1)\, \omega s_t^R = \nu \frac{1}{Z_t} \omega s_t^R - \theta \left(\frac{\rho_t}{\rho_{t-1}} - 1\right) \frac{1}{\rho_{t-1}} + \beta\theta E_t \left[\frac{\lambda_{t+1}^O}{\lambda_t^O} \frac{\varepsilon_{t+1}^b}{\varepsilon_t^b} \left(\frac{\rho_{t+1}}{\rho_t} - 1\right) \frac{\rho_{t+1}}{\rho_t^2}\right] \tag{19}$$

3.2 Households

This theoretical world is populated by two types of agents, *homeowners* and *renters*. Every period in the economy, there are ω homeowners and $(1-\omega)$ renters. The parameter ω controls both the share of homeowners and their share in the consumption of goods. This parameter cannot change over time.[15] Two agent groups are basically identical except for the ability to purchase houses. Homeowners decide housing investment at each period and access a secured loan against housing collateral from the international financial market, subject to a collateral constraint. In the case of renters, they do not access the financial market because they do not have collateralizable houses. Specifically, this paper assumes all unsecured lending behavior between lenders and borrowers is captured as domestic financial market transactions, and domestic debt holding is zero-sum in the aggregate, as I mentioned above. Thus, the model does not consider an unsecured loan market. The only source of borrowing is foreign lenders, and it should be secured lending against housing collateral.[16] Both agents share the same preference representation, which is non-separable between two goods, which are non-housing consumption goods and housing consumption goods.

3.2.1 Homeowners

There are ω infinitely-lived identical homeowners with preferences described by the utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^b \ln X_t^O \tag{20}$$

where E_t denotes the expectation operator conditional on information available in period t, and X_t^O denotes a consumption aggregate. Note that the superscript "O" represents a homeowner. β is a subjective discount factor, and ε_t^b captures disturbance on impatience, or intertemporal preference shock. The periodic utility function takes the form of a logarithm. There is no disutility from labor supply, so the only source of happiness is the consumption bundle, X_t^O.

[15] Based on the fact that the homeownership rate is quite stable over time in the sample countries, the model assumes the constant ω because it focuses on aggregated economic fluctuation, especially the aggregate consumption volatility rather than the impact on individual household's status change. Therefore, the model mutes the household's decision-making problem between becoming a homeowner and staying as a renter or vice versa.

[16] Imagine commercial banks that finance funding from foreign creditors and domestic savers. Domestic savers are not necessarily homeowners; a domestic saver is either a homeowner or a renter. Banks extend unsecured lending to domestic borrowers, and their source is assumed to be domestic deposit without loss of generality. At the same time, banks extend secured lending against housing collateral, and its source is assumed to be foreign creditors. As a result, the model of this paper is the simplified model by removing financial intermediaries under the zero-sum domestic lending assumption.

Specifically, homeowners consume two goods to increase their utility: non-housing consumption goods, c_t^O, and housing services, s_t^O. The two consumption goods compose a consumption bundle X_t^O, which is a constant elasticity of substitution aggregator defined by

$$X_t^O \equiv \left[\gamma\left(c_t^O\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\,\varepsilon_t^s\left(s_t^O\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{21}$$

where $\gamma > 0$ is the share of non-housing consumption goods in the consumption aggregator, and $\eta \geq 0$ is the intratemporal elasticity of substitution between non-housing consumption goods and housing services. ε_t^s represents housing preference shock, or intratemporal preference shock, following Iacoviello and Neri (2010), which captures variation in social and institutional changes that shift preferences toward housing or in the availability of resources needed to purchase housing relative to other goods. Two preference shocks, ε_t^b and ε_t^s are assumed to follow an independent AR(1) process:

$$\ln \varepsilon_{t+1}^b = \rho_b \ln \varepsilon_t^b + \sigma_b \xi_{b,t+1} \tag{22}$$

$$\ln \varepsilon_{t+1}^s = \rho_s \ln \varepsilon_t^s + \sigma_s \xi_{s,t+1} \tag{23}$$

where ρ_b and ρ_s are the autoregressive parameters, σ_b and σ_s are the parameters for the standard deviation of innovation to corresponding shock, and $\xi_{b,t}$ and $\xi_{s,t}$ are independently and identically distributed innovation with mean zero and variance one.

The homeowners maximize their lifetime utility subject to the sequence of budget constraints

$$\begin{aligned} c_t^O + \rho_t^O s_t^O + q_t\left[h_t - (1-\delta)\,h_{t-1} + \Phi_h\left(h_t - h_{t-1}\right)\right] + \left(1 + r_{t-1}\right) d_t \\ = Wage_t\left(n_{c,t}^O + n_{h,t}^O\right) + d_{t+1} + \rho_t^O h_t + Div_t^O \end{aligned} \tag{24}$$

The left-hand side of the budget constraint, or the uses of funds for homeowners comprise four components: non-housing consumption goods given a numeraire price, housing services given a housing service sales price ρ_t^O, housing investment given a house price q_t, and repayment of debt. Note that $\rho_t^O = \rho_t/Z_t$ where Z_t represents a markup at which rental agencies provide renters with housing services. s_t^O denotes owner-occupied housing services, and h_t denotes the housing stock that the owner is holding at a period t, and δ is a quarterly depreciation rate. The homeowner spends funds on purchasing the housing stock at a period t given a house price q_t, and at the same time, she gets back funds as much as the current value of housing stock she holds from the previous period after depreciation. As a result, the terms $h_t - (1-\delta)h_{t-1}$

represent a housing investment that the owner determines at period t. Let us define ih_t as a housing investment, for convenience.

$$ih_t = h_t - (1-\delta)\, h_{t-1} \tag{25}$$

In regard to the housing investment, the owners are faced with investment adjustment costs. The function $\Phi_i(\cdot)$ is assumed to satisfy $\Phi_i(0) = \Phi_i'(0) = 0$ and $\Phi_i"(0) > 0$. The functional form of adjustment cost is quadratic, so

$$\Phi_i(h_t - h_{t-1}) = \frac{\phi_i}{2}(h_t - h_{t-1})^2 \tag{26}$$

with $\phi_i > 0$. d_t denotes the real debt whose maturity is time t, and r_t denotes the real interest rate on loans between t and $t{+}1$, so the term $(1 + r_{t-1})\, d_t$ is the amount of repayment at period t. Next, as we can see on the right-hand side of the resource constraint, the funds for homeowners come from four sources: wage income, international debt, housing service sales income, and dividend income from firm ownership. First, wage income is $Wage_t$ since $n^O_{c,t} + n^O_{h,t} = 1$. Regarding the term $\rho^O_t h_t$, the linear technology lets the homeowners produce the exactly the same amount of housing services from the amount of housing stock at period t at no cost; ρ^O_t times h_t is her "gross" housing service sales income. Another interpretation of that term is as follows. Since a homeowner is the provider of housing rental services as well as the consumer of owner-occupied housing services (purchasing the amount of s^O_t at time t), we can understand $\rho^O_t(h_t - s^O_t)$ is her "gross" housing service sales income. The last term of dividend income of homeowners comprises the lump-sum dividends received from consumption goods producers ($Profit_{c,t}$), those from construction firms ($Profit_{h,t}$), and those from rental agencies ($\frac{Profit_{r,t}}{\omega}$), so $Div^O_t \equiv Profit_{c,t} + Profit_{h,t} + \frac{Profit_{r,t}}{\omega}$. Here, ω is a share parameter that determines the proportion of homeowners in the economy. In other words, there is a continuum of households of mass one where ω homeowners and $(1-\omega)$ renters live, and ω is constant over time. As mentioned above, non-housing consumption good firms and construction firms are owned by both agents, so the amount of dividends "one" homeowner (or renter) obtains from each sector is equal to each sector's profit in this setup. In contrast, the dividend income one homeowner receives from rental agencies is the profit scaled by ω because the rental agencies are owned by homeowners, not renters.

Notice that the wage income ($Wage_t$), the dividend income from consumption good producers ($Profit_{c,t}$), and the dividend income from construction firms ($Profit_{h,t}$) are common income sources for homeowners and renters alike. For convenience, therefore, let us define the common income by:

$$\Pi_t \equiv Wage_t + Profit_{c,t} + Profit_{h,t} \tag{27}$$

which gives us a simpler version of the budget constraint:

$$c_t^O + \frac{\rho_t}{Z_t} s_t^O + q_t \left[i h_t + \frac{\phi_i}{2} \left(h_t - h_{t-1} \right)^2 \right] + \left(1 + r_{t-1}\right) d_t = \Pi_t + d_{t+1} + \frac{\rho_t}{Z_t} h_t + \frac{Profit_{r,t}}{\omega} \tag{28}$$

where $Profit_{r,t} = \rho_t \omega s_t^R \left(1 - \frac{1}{Z_t}\right) - \frac{\theta}{2}\left(\frac{\rho_t}{\rho_{t-1}} - 1\right)^2$ in equilibrium. In addition, we do not take care of the levels of wage because they are canceled out in equilibrium:

$$\begin{aligned} \Pi_t &\equiv Wage_t + Profit_{c,t} + Profit_{h,t} \\ &= y_t + q_t n h_{t-1} - \Phi_c \left(n_{c,t} - n_{c,t-1}\right) - \Phi_h \left(n_{h,t} - n_{h,t-1}\right) \end{aligned}$$

As in Iacoviello (2005) and Iacoviello and Neri (2010), I assume the homeowners are faced with the borrowing limit. Formally,

$$d_{t+1} \leq \frac{m q_t h_t}{1 + r_t} \varepsilon_t^m \tag{29}$$

where $0 \leq m \leq 1$ represents a loan-to-value (LTV) ratio. Equation (29) implies the borrowing amount is limited by the LTV ratio times the collateral value of housing stock the owner is holding at a period t. As a result, we can interpret the parameter m as the fraction of house value used as collateral and the term $q_t h_t$ as the market value of collateral based on the current house price (Kaplan et al. (2017), Sommer et al. (2013)).[17] I assume that the borrowing constraint always holds with equality near the steady state. This is because I also assume the agents in the economy are sufficiently impatient. Formally, I assume $\beta(1 + r^*) < 0$ where r^* is the steady-state level of interest rate. ε_t^m represents LTV shock, or credit availability shock, which captures financial policy change, variation in the lending standard of banks, and variation in the financial institutional environment affecting the total amount of credit in the economy.

Given initial values $\{h_{-1}, d_0, r_{-1}\}$, the homeowner chooses $\{c_t^O, s_t^O, h_t, d_{t+1}\}$ to maximize equation (20) subject to equations (24) and (29). The Lagrange multiplier for each constraint is defined as λ_t^O and $\lambda_t^O \mu_t$. Corresponding first-order conditions are as follows:

$$\gamma \left(X_t^O\right)^{\frac{1-\eta}{\eta}} \left(c_t^O\right)^{-\frac{1}{\eta}} = \lambda_t^O \tag{30}$$

[17] Several papers, including Kaplan et al. (2017), Sommer et al. (2013) suggest the collateral value in a borrowing constraint is based on the current price of houses, and I follow their view considering the collateral appraisal procedure using the current house price. At the same time, there are other papers whose view is different. For example, Iacoviello (2005), Iacoviello and Neri (2010), and Andrés et al. (2017) suggest that agents' maximum borrowing is given by the expected present value of their house times LTV ratio.

$$\frac{1-\gamma}{\gamma}\varepsilon_t^s\left(\frac{c_t^O}{s_t^O}\right)^{-\frac{1}{\eta}}=\frac{\rho_t}{Z_t} \tag{31}$$

$$q_t\left[1+\phi_i\left(h_t-h_{t-1}\right)\right]=\frac{\rho_t}{Z_t}+\beta E_t\left[\frac{\lambda_{t+1}^O}{\lambda_t^O}\frac{\varepsilon_{t+1}^b}{\varepsilon_t^b}q_{t+1}\left(\left(1-\delta\right)+\phi_i\left(h_{t+1}-h_t\right)\right)\right]+\frac{m\mu_t q_t}{1+r_t}\varepsilon_t^d \tag{32}$$

$$\lambda_t^O\varepsilon_t^b\left(1-\mu_t\right)=\beta\left(1+r_t\right)E_t\left[\lambda_{t+1}^O\varepsilon_{t+1}^b\right] \tag{33}$$

Equations (30) and (31) are the first-order condition with respect to the homeowner's non-housing consumption and housing services, respectively. Specifically, equation (31) represents the housing rental service supply condition because homeowners are housing rental service providers. Equation (32) is the first-order condition with respect to the homeowner's housing stock, and it equates the marginal utility of non-housing consumption to the shadow value of housing stock. The latter relies on three components: a direct gain from additional rental income, an expected gain from change in the value of the house realized at the next period, and a marginal utility of relaxing the collateral constraint. Last, equation (33) is an Euler equation, but it is not a standard one due to the existence of μ_t. Since I assume the collateral constraint is always binding, the multiplier $\lambda_t^O\mu_t$ is always positive. If the collateral constraint is slack, then μ_t is zero, and we can obtain the standard Euler equation of marginal utilities between time t and $t+1$.

House Market Clearing In the housing market at period t, the construction firms supply nh_{t-1}, which is started at $t-1$ and finished at t, and the ω proportion of households, or homeowners, purchase new houses as housing investment. Note that houses are non-tradable goods, and they are only purchased by homeowners, not renters. Therefore, the house market clearing conditions equate the above supply by firms and demand by homeowners.

$$nh_{t-1}=\omega\left[h_t-\left(1-\delta\right)h_{t-1}+\frac{\phi_i}{2}\left(h_t-h_{t-1}\right)^2\right] \tag{34}$$

3.2.2 Renters

Renters whose proportion in the economy is $(1-\omega)$ maximize their lifetime utility subject to a sequential budget constraint:

$$\max_{c_t^R,s_t^R}\; E_0\sum_{t=0}^{\infty}\beta^t\varepsilon_t^b\ln X_t^R \tag{35}$$

$$\text{s.t.} \quad c_t^R + \rho_t s_t^R = \Pi_t \tag{36}$$

where the superscript "R" is for renters, so X_t^R denotes the renter's consumption aggregate, c_t^R denotes her non-housing consumption, ρ_t is a rental price, and s_t^R denotes her housing services. Notice that the price for housing service is different from that the homeowner faces for owner-occupied housing services, so $\rho_t \neq \rho_t^O$, and the observable rental price is the final rental price in the housing rental market, ρ_t. Π_t is $Wage_t + Profit_{c,t} + Profit_{h,t}$ which is the common income. The consumption aggregate X_t^R is defined by:

$$X_t^R \equiv \left[\gamma \left(c_t^R\right)^{\frac{\eta-1}{\eta}} + (1-\gamma)\,\varepsilon_t^s \left(s_t^R\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{37}$$

Note that there is no intertemporal choice in the renter's problem. The sources of funds the renter has are the wage income and the dividend income she receives at every period. As a result, the solution to the renter's optimization problem is equal to solving her static utility maximization problem period by period. Additionally, it is worth noting that the parameters such as the discount factor, β, the share parameter, γ, and the elasticity parameter, eta, do not have superscripts, which implies that homeowners and renters are basically identical households except for homeownership.

The renter chooses optimal c_t^R and s_t^R at every period to maximize her periodic utility function $\ln X_t^R$ subject to her periodic resource constraint (36). By defining λ_t^R as the Lagrange multiplier for the resource constraint, the corresponding first-order conditions are followed:

$$\gamma \left(X_t^R\right)^{\frac{1-\eta}{\eta}} \left(c_t^R\right)^{-\frac{1}{\eta}} = \lambda_t^R \tag{38}$$

$$\frac{1-\gamma}{\gamma}\varepsilon_t^s \left(\frac{c_t^R}{s_t^R}\right)^{-\frac{1}{\eta}} = \rho_t \tag{39}$$

In equation (38), the renter's marginal utility of non-housing consumption is equated to the shadow value of relaxing the budget constraint (36). Equation (39) represents the housing rental service demand condition in the economy. This is because the renters are housing rental service demanders in the housing rental market.

Housing Rental Market Clearing The clearing condition of the rental market is to equate the aggregate amount of rental supply to the aggregate amount of rental demand. Since the economy consists of ω homeowners and $(1-\omega)$ renters, the market clearing condition is

$$\omega s_t^O + (1-\omega) s_t^R = \omega h_t \tag{40}$$

We derive the housing rental service supply condition (31) and demand condition (39) from the respective agent's optimization problem. As a result, we can solve to the rental market equilibrium using the supply and demand condition with the above market clearing condition (40). The equilibrium amount of housing rental service and the equilibrium housing rental price are

$$s_t^R = \frac{\omega c_t^R}{\omega Z_t^\eta c_t^O + (1-\omega)\, c_t^R} h_t \tag{41}$$

$$\rho_t = \frac{1-\gamma}{\gamma} \varepsilon_t^s \left(\frac{\omega Z_t^\eta c_t^O + (1-\omega)\, c_t^R}{\omega h_t} \right)^{\frac{1}{\eta}} \tag{42}$$

Consumption Goods Market Clearing The non-housing consumption goods are tradable goods, so the following market clearing condition shows that the households import the consumption goods from outside of the economy by borrowing internationally.

$$c_t + \omega\,(1 + r_{t-1})\, d_t = y_t + \omega d_{t+1} \tag{43}$$

where $c = \omega c_t^O + (1-\omega) c_t^R$ is the aggregate non-housing consumption.

3.3 External Debt Elastic Interest Rate

Assume that the interest rate faced by homeowner, r_t, is increasing in the economy's average level of debt, which we denote by $\tilde{d}_{t+1}$ as in Schmitt-Grohé and Uribe (2003). Formally, r_t is given by

$$r_t = r^* + \psi \left(e^{\tilde{d}_{t+1} - \bar{d}} - 1 \right) + e^{\varepsilon_t^r - 1} - 1 \tag{44}$$

where the first term r^* is the steady-state level of interest rate and the second term represents the interest rate premium. Note that $\psi > 0$ controls the interest rate adjustment, and $\bar{d}$ denotes the steady-state level of aggregate debt. That is, the interest rate relies on the degree of distance of the current debt from the steady state. This is an external debt elastic interest rate (EDEIR) because households take the cross-sectional average level of current debt as exogenously given. Since $\omega d_{t+1} = \tilde{d}_{t+1}$ holds in equilibrium, the above specification (44) is rewritten by:

$$r_t = r^* + \psi \left(e^{\omega d_{t+1} - \bar{d}} - 1 \right) + e^{\varepsilon_t^r - 1} - 1 \tag{45}$$

There is another source of uncertainty, ε_t^r, which captures a real interest rate shock, or credit cost shock. Therefore, the model takes two credit-related shocks — the credit availability shock (ε_t^m), and the credit cost shock (ε_t^r) — and they are assumed to follow independent stationary AR(1) processes:

$$\ln \varepsilon_{t+1}^m = \rho_m \ln \varepsilon_t^m + \sigma_m \xi_{m,t+1} \tag{46}$$

$$\ln \varepsilon_{t+1}^r = \rho_r \ln \varepsilon_t^r + \sigma_r \xi_{r,t+1} \tag{47}$$

where ρ_m and ρ_r are the autoregressive parameters, σ_m and σ_r are the parameters for standard deviation of innovation to corresponding shock, and $\xi_{m,t}$ and $\xi_{r,t}$ are independently and identically distributed innovation with mean zero and variance one.

3.4 Aggregation and Equilibrium

We construct the equilibrium process of aggregate variables such as total consumption, TC_t, housing consumption, HC_t, non-housing consumption, NHC_t, housing stock, H_t, international debt holdings, D_{t+1}, and output, Y_t. Note that we assign numeraire price to price for non-housing consumption goods (c_t or y_t), which implies all real variables are denoted by a unit of non-housing goods or a relative price to non-housing goods. Since the data in Section 2 are deflated using the GDP deflator, we also deflate aggregate variables using the GDP deflator for coherent comparison. To construct the GDP deflator, we first measure GDP in the theoretical world by the income approach. Since the income is generated from wages, rental incomes, and dividends, GDP is the aggregation of homeowners' and renters' right-hand sides of the respective resource constraints:

$$\begin{aligned} GDP_t &= Wage_t + \frac{\rho_t}{Z_t} \omega h_t + Profit_{c,t} + Profit_{h,t} + Profit_{r,t} \\ &= \Pi_t + \frac{\rho_t}{Z_t} \omega h_t + \rho_t \omega s_t^R \left(1 - \frac{1}{Z_t} \right) - \frac{\theta}{2} \left(\frac{\rho_t}{\rho_{t-1}} - 1 \right)^2 \end{aligned} \tag{48}$$

There are two relative price in Equation (48): house prices q_t in Π_t, and rental prices ρ_t. Therefore we construct a GDP deflator by dividing GDP_t in current relative prices (corresponding to nominal GDP) by GDP_t in the state-state level of relative prices (corresponding to real

GDP):

$$P_t = \frac{GDP_t|_{q_t,\rho_t}}{GDP_t|_{q_0,\rho_0}} = \frac{GDP_t|_{q_t,\rho_t}}{GDP_t|_{q_{ss},\rho_{ss}}} \tag{49}$$

Then we can construct the deflated aggregate variables of interest:

$$HC_t = \left(\frac{1}{P_t}\right) \times [\,\omega\rho_t s_t^O + (1-\omega)\rho_t s_t^R\,] = \frac{\omega\rho_t h_t}{P_t} \tag{50}$$

$$NHC_t = \left(\frac{1}{P_t}\right) \times [\,\omega c_t^O + (1-\omega) c_t^R\,] = \frac{c_t}{P_t} \tag{51}$$

$$TC_t = HC_t + NHC_t \tag{52}$$

$$H_t = \frac{\omega h_t}{P_t} \tag{53}$$

$$D_{t+1} = \frac{\omega d_{t+1}}{P_t} \tag{54}$$

$$Y_t = \frac{GDP_t}{P_t} = GDP_t|_{q_{ss},\rho_{ss}} \tag{55}$$

In addition, we need real house prices, HP_t, and real housing rental prices, R_t, deflated by the GDP deflator:

$$HP_t = \frac{q_t}{P_t} \tag{56}$$

$$R_t = \frac{\rho_t}{P_t} \tag{57}$$

As a result, an equilibrium in the basic model is then a set of processes of 31 endogenous variables $\{c_t^O,\, c_t^R,\, c_t,\, y_t,\, nh_t,\, ih_t,\, h_t,\, s_t^O, s_t^R,\, d_{t+1},\, X_t^O,\, X_t^R,\, n_{c,t},\, n_{h,t},\, \Pi_t,\, \lambda_t,\, \lambda_t^O,\, \lambda_t^R,\, \mu_t,\, q_t,$ $\rho_t,\, r_t,\, Z_t,\, TC_t,\, HC_t,\, NHC_t,\, H_t,\, D_{t+1}, Y_t,\, HP_t,\, R_t\}_{t=0}^{\infty}$ satisfying (2), (3), (5), (6), (8), (9), (11) to (14), (19), (21), (24) to (34), (36) to (43), (45) to (57), given the processes $\{A_{c,t},\, A_{h,t},\, \varepsilon_t^b,\, \varepsilon_t^s,$ $\varepsilon_t^m,\, \varepsilon_t^r\}_{t=0}^{\infty}$ and the initial condition $\{n_{c,0},\, n_{h,0},\, nh_{-1},\, h_{-1},\, d_0,\, \rho_{-1},\, r_{-1}\}$. Please see *Appendix C* for a full set of equilibrium conditions and the steady state of the economy.

4 Results of the Model

In this section, this paper presents the numerical results of the model described in Section 3. In the previous section, we constructed several variables in order to observe the aggregate economy's business cycle feature rather than that of the individual representative agents. Mainly, we are interested in the consumption volatilities relative to GDP volatility — the relative total consumption volatility, the relative housing consumption volatility, and the relative non-housing consumption volatility — and house price volatility. This section first describes the baseline calibration and shows the properties of the model focusing on business cycle facts. Then, we discuss the positive relationship between the house price volatility and the relative consumption volatility.

4.1 Calibration

This paper solves the model numerically through the first-order approximation. To implement the perturbation method, I calibrate the parameters of the basic model introduced in the previous section. The target country is South Korea, and the time unit is one quarter, as in the model.

I set the share of non-housing consumption parameter, $\gamma = 0.0193$, to match average housing consumption share in South Korea over the sample period, from 1975 Q1 to 2017 Q4, which is 15.2%. Together with the other calibrated parameters, it pins down the average housing consumption share. For the intratemporal elasticity of substitution between non-housing consumption goods and housing services, Flavin and Nakagawa (2008), Li et al. (2016), and Stokey (2009) estimated the elasticity using household-level data, and they found that it is less than one. That is, non-housing consumption goods and housing services are complements according to the elasticity. Song (2011, 2012) and Davidoff and Yoshida (2013) also estimated the elasticity to be less than one from macro-level aggregate consumption data. In this paper, I pick $\eta = 0.35$, which is the estimate Song (2012) provides.[18] The steady-state level of interest rate, r^*, is set to 0.01 to match the average long-run interest rate per annum in Korea, 3.91% over the sample period, which is from 1981 to 2017 considering data availability. The subjective discount factor is calibrated to $\beta = 0.95$ following Iacoviello (2005). Notice that $\beta(1 + r^*) < 1$ because households are assumed to be sufficiently impatient, which lets them bind a borrowing constraint. Quarterly housing depreciation rate, δ, is set to 0.0025 following Iacoviello and Neri (2010). In the case of the key parameter of the collateral constraint, m, the average loan-to-value (LTV) ratio for mortgages in Korea is considered. Due to the limited data availability for the actual LTV

Table 3: Calibration

Parameter		**Value**
Share of non-housing consumption	γ	0.0193
Intratemporal elasticity of substitution	η	0.35
Steady state of interest rate	r^*	0.01
Subjective discount factor	β	0.95
Housing depreciation rate	δ	0.0025
Loan-to-value ratio	m	0.5
Homeowner proportion	ω	0.55
Labor share	α_c, α_h	0.59
Interest rate premium adjustment	ψ	0.001
Labor adjustment: non-housing sector	ϕ_c	1.6
Labor adjustment: construction sector	ϕ_h	3.0
Housing investment adjustment	ϕ_i	1.9
Rent rigidity	θ	190
Steady state of rental agency markup	Z^*	1.15

ratio in South Korea, I make use of the average over the recent five years as a proxy. According to the Bank of Korea, the average LTV ratio from 2012 to 2017 is 40∼60%. Furthermore, the Korean Financial Supervisory Services reported that the average LTV ratio was 53% as of the end of 2016. Hence, I set $m = 0.5$ for the baseline calibration. Although a regulatory LTV ratio can be regarded as one candidate for m, there is a substantial gap between the regulatory ratio (70% since July 2014) and the actual ratio in Korea. The homeowner proportion parameter, ω, is set to 0.55 to match the average homeownership rate in Korea, 55% over the sample period between 1995 and 2016.,[19] Note that there is little variation in the homeownership rate over the sample period from 1995 to 2016. I set the labor share parameter for each sector's production function at 0.59. I assume the same labor share across sectors, and the calculated average labor income share for South Korea based on OECD database between 1975 and 2017 is 59.4%.

The adjustment parameters are set to match the second moments of the data of South Korea. The data sources are the Bank of Korea and Statistics Korea. I calibrate the interest rate premium adjustment parameter, $\psi = 0.001$, targeting a standard deviation of net external debt growth (1994-2017); the labor adjustment cost parameter, $\phi_c = 1.6$ for the non-housing consumption goods sector, with $\phi_h = 3.0$ for the construction sector, targeting the ratio of a standard deviation of the non-housing sector's domestic product to a standard deviation of the construction sector's domestic product (1975-2017) and the ratio of a standard deviation of the two sectors' employment change (1993-2017); and the housing investment adjustment cost parameter, $\phi_i = 1.9$, targeting a standard deviation of residential investment (1975-2017). The last two parameters are related to the rigid adjustment of the rental price. The parameter governing rental price adjustment cost is set to $\theta = 190$ to match the ratio of rent price volatility to house price volatility for South Korea, which is 0.54.[20] The steady-state level of the markup for rental agencies, Z^*, is 1.15 following Sun and Tsang (2017), which means the elasticity of substitution between differentiated rental services, ν, is set to 7.667.

Moving on to the parameters defining the stochastic processes of the exogenous driving forces, there are 6 autoregressive parameters and 6 standard deviation parameters for 6 structural shock processes (Equations (4), (10), (22), (23), (46), (47)) which are assumed to be independent AR(1) processes. Without substantial deviation from standard values, I assume that the persistence parameter for each shock (ρ_c, ρ_h, ρ_b, ρ_s, ρ_m, and ρ_r) is 0.95, and the standard deviation for innovation of each shock (σ_c, σ_h, σ_b, σ_s, σ_m, and σ_r) is 0.01. This paper assigns the same serial correlation and standard deviation for each shock, in order to determine which shock is crucial in generating the second moments of interest in the theoretical model.

[18]There is little consensus in the literature whether the elasticity is less than one or not. In contrast to the papers I mentioned in the main text, Piazzesi et al. (2007) and Davis and Martin (2009) argue $\eta > 1$ from the micro-level data estimation. Another branch of the literature ignores nonseparable complementarity between non-housing consumption and housing services by using separable preference specification (Iacoviello (2005), Iacoviello and Neri (2010), Calza et al. (2013), Sun and Tsang (2017), among others), or simply assumes the Cobb-Douglas preference which implies $\eta = 1$ (Sommer et al. (2013), Li and Yao (2007), Yang (2009), Aoki et al. (2004), Funke and Paetz (2013), among others).

[19]Homeownership rate in South Korea is a national statistic from two nationwide surveys: a biennial survey conducted by the Ministry of Land, Infrastructure, and Transport (2006, 2008, 2010, 2012, 2014, 2016), and a census conducted by Statistics Korea every five years (1995, 2000, 2005, 2010, 2015).

[20]We can recover the probability of fixed real rental price (say, θ_c) in Calvo-type specification using the relationship between θ and θ_c. The relationship is achieved by solving the isomorphic problem with the Calvo-type price rigidity:

$$\theta = \frac{(\nu - 1)\theta_c}{(1 - \theta_c)(1 - \beta\theta_c)}$$

Since the baseline calibration is $\theta = 190$, the corresponding θ_c is 0.848. The Calvo parameter is useful in interpretation of the rigidity. That is, $\theta_c = 0.845$ implies the duration of the rental contract with the unchanged real rental price is approximately 6.56 quarters.

Table 4: Selected Second Moments

	Data[1]	Baseline Model[2]	Model (Occ.Bin.)[3]
$\frac{\sigma_C}{\sigma_Y}$	1.23	1.30	1.23
$\frac{\sigma_{HC}}{\sigma_Y}$	1.68	1.51	1.42
$\frac{\sigma_{NHC}}{\sigma_Y}$	1.36	1.30	1.24
$\frac{\sigma_R}{\sigma_{HP}}$	0.54	0.54	0.55

[1] QoQ growth rate, South Korea, 1975 Q1-2017 Q4
[2] Under baseline calibration (Table 3)
[3] Model with occasionally binding borrowing constraint

4.2 Properties of Approximated Model

4.2.1 Business Cycle Properties

Under the baseline calibration, I approximate the model, simulate it 100,000 times and compute the average of second moments for the aggregate variables to compare them with the business cycle facts observed in the actual data. The variables of interest at first are the relative consumption volatility, including the relative disaggregated consumption volatilities, and the ratio of rental price volatility to house price volatility. Table 4 shows the numbers for the data and model. Note that the data is from South Korea because the target country of the calibration is this country, and it covers 1975 Q1 to 2017 Q4. Table 4 also presents the result from the model with an occasionally binding borrowing constraint to show the robustness of the result, which is discussed in detail in the paragraph that follows.

The total consumption volatility relative to GDP volatility for South Korea is 1.23, which represents the excessive consumption volatility observed in emerging countries. The model generates $\sigma_c/\sigma_Y = 1.27$ under the baseline calibration, so it successfully captures the excessiveness of total consumption. The higher volatility of total consumption compared to output by the model relies on a credit market effect, which is related to the role of credit shocks and the credit constraint. Specifically, the household's consumption is highly responsive to the credit price condition, which is represented by the interest rate shock. When the interest rate moves, an agent is faced with variation in the amount of repayment as well as the capability of borrowing for the present and the future, which leads to fluctuations of consumption. However, the interest rate shock is relatively less important in explaining GDP fluctuation because domestic production is affected by not only a change in the demand condition by a change in the interest rate shock but also by a change in the supply condition. Even a change in the supply condition is directly determined by a change in productivity shocks. In other words, domestic production is mainly affected by a productivity shock, but not by an interest rate shock. Unlike the production sector, household consumption is realized in both the domestic market and international market, so the international borrowing condition is important. Likewise, the credit availability condition, which is represented by the leverage shock, matters for higher consumption volatility relative to GDP volatility. As a result, the existence of credit shocks contributes to yield excess consumption volatility by the model. This view is in line with the literature focusing on the interest rate shock in explaining the excess consumption volatility puzzle (for example, Neumeyer and Perri (2005), Uribe and Yue (2006), among others). Next, there is an amplification effect by the housing collateral constraint. Since the housing stock is hardly volatile, the crucial component in housing collateral is the house price. Let us consider the house price increases responding to any shock occurrence. The increase in house price relaxes the homeowner's borrowing constraints through an increase in the value of a collateralizable asset, which makes homeowners have more funding and achieve a higher level of non-housing consumption. In addition, the increase in non-housing consumption stimulates the increase in housing rental price by Equation (42), which leads to a further increase in non-housing consumption in two ways. The relative price of non-housing consumption becomes less expensive; the house price is positively linked with the rental price, so the house price increases further, and the above process starts again. Note that the credit shocks and the collateral constraint are featured in the homeowner's problem. Therefore, the main driving group of this economy is the homeowners.

Additionally, the disaggregated consumption volatilities generated in the model capture the actual data qualitatively. The key finding from the data is that housing consumption volatility is higher than non-housing consumption volatility in the case of emerging countries (1.68 and 1.36 for South Korea, respectively), and the model also yields large housing consumption volatility compared to non-housing consumption volatility (1.44 and 1.28, respectively). Notice that the housing service sales price (ρ_t^O) is the corresponding price measure for the owner-occupied housing services (s_t^O) and the housing rental price (ρ_t) for the tenant-occupied housing services (s_t^R). However, in measuring aggregate housing consumption, we use ρ_t instead of ρ_t^O for the owner-occupied housing services because what we can observe is only ρ_t, which can be used as the imputed rent.[21] The item of the last row of Table 4 is the ratio of rent price volatility to house price volatility. The data and the result of the model are exactly the same because I set the price adjustment parameter, θ, to match the ratio.

The last column of Table 4 comes from an exercise with an occasionally binding borrowing constraint. The key mechanism between house prices and consumption relies on the role of the housing collateral constraint, but this paper assumes the constraint is always binding, and I relax the assumption in order to show the robustness of the result. The exercise follows the piecewise linear approximation methodology developed in Guerrieri and Iacoviello (2015). The key idea of their methodology is that we can treat a model with an occasionally binding constraint as the one with two regimes. Under one regime, the constraint binds; under the other regime, the constraint is slack. In this paper's model, the occasionally binding constraint specifies that the collateral constraint, Equation (29), binds in normal times, as we assume that homeowners are sufficiently impatient ($\beta(1+r^*) < 1$), which implies the multiplier $\mu_t > 0$ in normal times. The constraint is violated if, in the candidate solution, $\mu_t < 0$ is realized. Then, the algorithm shifts toward the other regime where the constraint is slack, which implies the multiplier $\mu_t = 0$. We return to the scenario of a binding constraint if the borrowing amount exceeds the value of housing collateral. With 100,000 simulations, the economy hits the housing collateral constraint in 71.1% of the simulations, so we have a difference of approximately 30% of the simulations from the baseline model approximation. However, the results for the selected moments of interest are not significantly different from the baseline result, as Table 4 presents, although the numbers are slightly small. The main reason is twofold. First, the economy does not stay long in the deleveraging period and reverts to a normal time within 1-3 quarters. Second, the response of aggregate variables during the deleveraging period does not significantly deviate from the baseline case.

4.2.2 Variance Decomposition

Table 5 presents the predicted contribution of each structural shock to explaining the volatility of aggregated consumption, housing consumption, non-housing consumption, housing stock, debt, GDP, house prices, and housing rental prices. Two main results arise from the variance decomposition. First, as I stated in the previous subsubsection, the excess consumption volatility depends on the credit shock volatility. The credit availability shock, ε^m, accounts for 8.8% of the total consumption volatility, which is five times as large as its contribution to the GDP volatility (1.8%). Similarly, the credit price shock, ε^r, accounts for 30.8% of total consumption

[21] Shin (2018) shows that housing consumption is twice as volatile as non-housing consumption and explains it as being due to the exceptionally highly volatile rental price generated by the model. Therefore, the present paper suppresses the rental price volatility by introducing rental price rigidity. However, the existence of rental price rigidity does not suppress the housing consumption volatility in the same magnitude due to the reason stated in the main text.

Table 5: Variance Decomposition

	TC	***HC***	***NHC***	***H***	***D***	***Y***	***HP***	***Rent***
A_c	0.5260	0.5628	0.5103	0.0280	0.2355	0.9024	0.3465	0.5406
A_h	0.0706	0.0475	0.0763	0.2729	0.1704	0.0145	0.1577	0.0767
ε^b	0.0036	0.0016	0.0039	0.0003	0.0004	0.0008	0.0007	0.0001
ε^s	0.0036	0.0643	0.0039	0.0113	0.0044	0.0075	0.0122	0.0682
ε^m	0.0880	0.0464	0.0939	0.0033	0.2240	0.0180	0.0544	0.0168
ε^r	0.3083	0.2774	0.3118	0.6843	0.3652	0.0568	0.4285	0.2977

volatility, which is also more than five times as large as its contribution to the GDP volatility (5.7%). In contrast, in the case of the non-housing productivity shock, A_c, it cannot be a driving source of excess consumption volatility because its contribution to GDP volatility is much more significant than to total consumption volatility, although it explains approximately half of total consumption volatility.

Second, we can investigate what the main cause of higher house price volatility is in the model through variance decomposition. The second to the last column in Table 5 shows that credit shocks explain half of the house price fluctuation, and productivity shocks explain another half. The importance of credit shocks in explaining house price fluctuation in the small open economy has been discussed in several papers, including Bian and Gete (2015) and Ferrero (2015), but the productivity shock, particularly the non-housing sector productivity shock, has not been highlighted in the literature. Note that I am not arguing that this paper suggests that non-housing productivity shock matters in explaining observed house price fluctuation because the results of this paper are based on the assumed shock processes. We need estimation to discuss whether the productivity shock matters or not. However, this paper at least raises attention to the productivity shock in explaining high house price volatility in emerging countries because we fully know the productivity shock is greater in emerging economies. To study how the non-housing productivity shock affects house price volatility in this paper, I rewrite the house price equilibrium condition on the demand side, Equation (32), without the adjustment cost terms:

$$U_{c,t}q_t = U_{c,t}\rho_t + \beta E_t\left[U_{c,t+1}\frac{\varepsilon^b_{t+1}}{\varepsilon^b_t}q_{t+1}\right] + \frac{m\mu_t\varepsilon^m_t}{1+r_t}U_{c,t}q_t \tag{58}$$

Equilibrium house prices equate the marginal utility of one unit of housing purchase to the shadow value of housing services according to the above equation. In detail, the right-hand side of Equation (58) comprises three parts: the direct gain from additional rental income, the gain from the change in the value of the house next period, and the marginal utility of relaxing the collateral constraint. In the case of the first two terms, they are related to the present value of future rental income flows, so the housing rental price is the key to understanding house prices in equilibrium. Recalling the equilibrium rental price, Equation (42), the critical component is the relative size of non-housing consumption compared to the housing stock. However, the housing stock does not fluctuate substantially, so the fluctuation in non-housing consumption is essential. Then, let us think about the credit shocks and the non-housing productivity shock in explaining house price volatility. First, the credit shocks are directly related to the last term of Equation (58), and at the same time, they affect the non-housing consumption through the change in the amount of borrowing. As a result, they obtain a significant portion in explaining house price fluctuation in the above two ways. Next, in the case of the non-housing productivity shock, it affects the non-housing consumption in two dimensions; first, more dividend income for consumption; second, changes in the volume of domestic production of non-housing consumption goods. Therefore, the non-housing productivity shock matters through the link between house price and rental price, which is a crucial mechanism in the present model incorporating the explicit housing rental market.

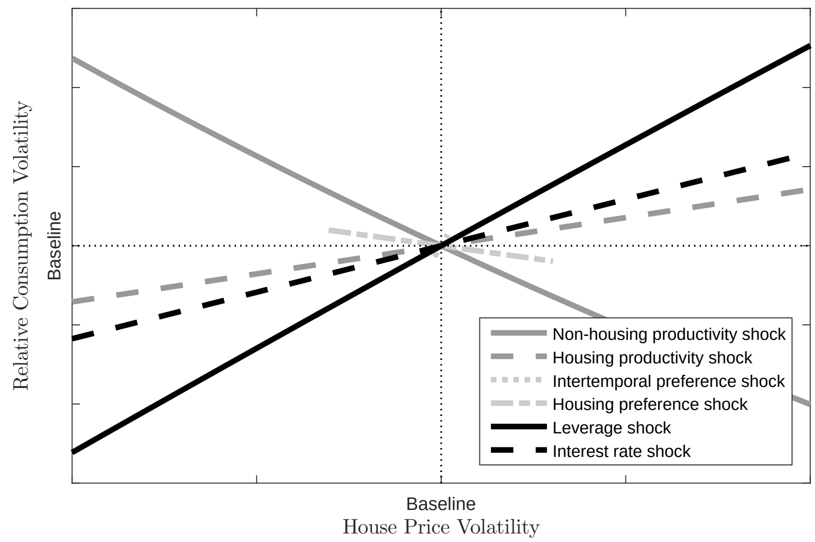
4.3 House Price Volatility and Relative Consumption Volatility

One of the main research interests of this paper is to determine the driving force to explain the observed relationship between relative consumption volatility and house price volatility across countries, presented in Figure 1. To answer the question, this paper simulates the model with respect to a change in variance of each shock, leaving other shocks' variance at the baseline level, which is $(0.01)^2$. Formally, the domain of simulation is:

$$\sigma^2_{i\in\{c,h,b,s,m,r\}} \in \{0 \leq \sigma_i^2 \leq 2\sigma_i^2\}, \quad \text{given } \; \sigma^2_{-i}$$

Then, I compute the simulated second moments of interest (with 100,000 simulations) — here, σ_C, σ_Y, and σ_{HP} correspond to the different levels of variance of each shock, and I plot the pairs of relative consumption volatility and house price volatility on the same plane, and Figure 6 presents the result. Note that the horizontal axis is truncated in order to compare the lines for each shock clearly around the baseline house price volatility. This is because the substantially different lengths of each line in terms of the horizontal axis depend on each shock's contribution to house price volatility, making it difficult to compare each other visually. As a result, the plot is presented in a window of [*Baseline* σ_{HP} $\pm 2\%$] for the horizontal axis and [*Baseline* σ_C/σ_Y $\pm 3\%$] for the vertical axis.

Figure 6: House Price Volatility and Relative Consumption Volatility



To begin, the dotted line for the intertemporal preference shock and the dash-dot line for the housing preference shock are ignorable due to their lower contribution to house price volatility. They are exceedingly short in terms of the horizontal axis even though we zoom-in the plots within $\pm 2\%$ of the baseline window. In the case of the gray dashed line for the housing productivity shock and the black dashed line for the interest rate shock, their variance change generates a positive relationship between house price volatility and relative consumption volatility, though the lines show weakly positive slopes. Next, the variation in non-housing productivity shock volatility fails to generate a positive relationship, as the gray solid line shows. In contrast, the leverage shock volatility change successfully generates a sufficiently positive relationship, as the solid black line shows in Figure 6. Since house price volatility is positively related to all shock volatility, the negative slope of the solid gray line means that the relative consumption volatility is negatively correlated with the non-housing productivity shock volatility. The reason is quite simple: non-housing productivity shock accounts for most of GDP fluctuation in the model, so the variation in its volatility contributes to GDP volatility more than to consumption volatility. Therefore, the non-housing productivity shock fails to generate a positive relationship in the model even if it matters for house price volatility, as we discussed in the previous subsubsection. In contrast, leverage shock affects consumption fluctuation more than GDP fluctuation because it is directly related to the credit market effect from the housing collateral constraint, which is a key in the amplification effect of house prices on consumption. Thus, the bottom line is that the positive relationship between house price volatility and the relative consumption volatility is attributed to leverage shock volatility, interest rate shock volatility, and housing productivity shock volatility, in the order of their influence. Note that the top two shocks are categorized as credit shocks based on this paper's interpretation — a credit availability shock and a credit price shock. In the context of the cross-country relationship, the above result is interpreted as the country whose credit shock is more volatile will tend to show higher house price volatility and higher relative consumption volatility alike. Therefore, this paper provides sources of cross-country variation in credit shock volatility in the paragraphs that follow and discusses that the sources are attributed to the higher relative consumption volatility and higher house price volatility observed in emerging countries.

Sources of Cross-Country Variation in Credit Shock Volatility

The credit shock volatility in the model is represented by two parameters: σ_m and σ_r. First, we can intuitively approach a cross-country variation in σ_r by observing an interest rate volatility for each country because we know the interest rate volatility is significantly higher in emerging countries than in rich countries. Fernández and Gulan (2015) rigorously showed that interest rates are more than twice as volatile in the former group of countries than in the latter. I sort out the countries overlapping this paper's sample from theirs and present each country's interest rate volatility observed in their data set in Table 6. Note that there are eight countries (six rich, two emerging countries) that do not have the volatility in Table 6 because Fernández and Gulan (2015) does not consider those countries. The weighted average of interest rate volatility for emerging countries is 0.95, and that for rich countries is 0.63. This result implies that an emerging country faces higher σ_r, which is a source of higher house price volatility and relative consumption volatility, according to this paper's argument. In contrast to σ_r, we do not have direct information regarding σ_m. Therefore, I investigate indirect approach in order to figure out what is related to the size of σ_m as well as σ_r of a country in terms of fundamental. Specifically, this paper highlights the country's sensitivity when an international credit supply shock occurs. In a situation of a global credit crunch, or credit expansion, each country will face a different size of shock depending on its sensitivity to the same size of the international shock. In other words, even if there is the same size of innovation in equation (46) or (47) across countries (or, $\xi_{m,t+1}$ or $\xi_{r,t+1}$ are identical across countries), the size of shock on ε^m_{t+1} or ε^r_{t+1} for each country is not identical depending on the country's own σ_m or σ_r. In the model, σ_m and σ_r affect the economy through the credit market effect, and note that the amount of credit or the amount of borrowing is determined by the interaction between credit suppliers and credit demanders in equilibrium. The credit supplier is a foreign lender in the model because this paper assumes that debts are transacted in only the international financial market. Therefore, the idea of the international credit supply shock inherently covers the credit supply side. The credit demander is obviously homeowners in the model, which provides a demand-side margin in examining a

Table 6: Sources of Cross-country Variation in Credit Shock Volatility

	Interest rate volatility[1]	Share of Variable-rate Housing Loans[2]		
		Share (%)	Coverage	Sources[3]
Australia	0.72	88.3	1991-2018	RBA
Belgium	0.63	17.9	2003-2018	ECB
Brazil	1.19	88.9	2013-2015	BCB
Canada	0.69	27.6	2009-2018	CMHC
Colombia	0.78		n.a.	
Denmark	0.70	42.9	2003-2018	ECB
Finland	0.68	95.8	2003-2018	ECB
France	-	12.6	2003-2018	ECB
Germany	-	15.1	2003-2018	ECB
Greece	-	74.5	2003-2018	ECB
Israel	-	70.7	2011-2018	BOI
Italy	-	63.0	2003-2018	ECB
Japan	-	38.1	2011-2018	JHFA
Malaysia	0.66	69.1	2009-2018	BNM
Netherlands	0.54	21.5	2003-2018	ECB
New Zealand	0.49	91.0	2016-2018	RBNZ
Portugal	0.61	62.5	2010-2018	EMF
South Africa	0.66	60	2015	City Press
South Korea	0.91	74.1	2001-2018	BOK
Spain	0.79	74.5	2003-2018	ECB
Sweden	0.54	68.3	2009-2018	EMF
Switzerland	0.38	26.0	2012-2014	BG (2015)
United Kingdom	-	23.0	2009-2018	EMF
United States	-	27.0	1985-2009	FHFA

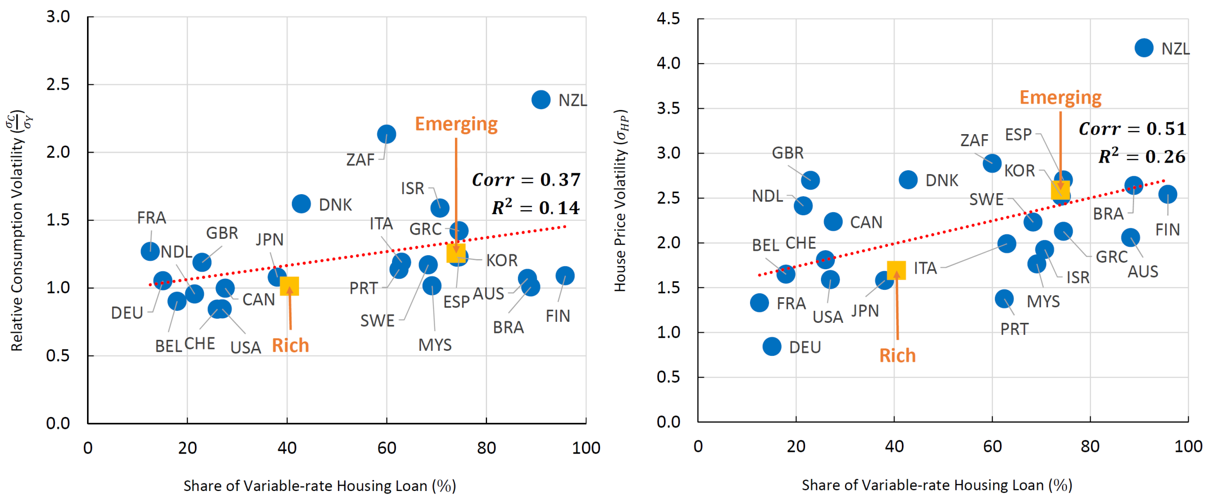
[1] The source is Fernandez and Gulan (2015) "Interest Rates, Leverage, and Business Cycle in Emerging Economies: The Role of Financial Friction" published in *American Economic Journal: Macroeconomics*. It constructs international interest rate data based on EMBI spreads for emerging countries, and 90-day corporate commercial papers, call money rates, or interbank lending rates for developed countries. Please refer to the data appendix of Fernandez and Gulan (2015) for the details.

[2] Loans with variable interest rates are loans extended at floating rates or an initial period of rate fixation of up to one year. If the fixation period is longer than one year, the housing loan is considerd a fixed rate loan.

[3] Sources for the share of variable-rate mortgage: Banco Central do Brasil (BCB), Reserve Bank of Australia (RBA), European Central Bank (ECB) MFI interest rate statistics, Canada Mortgage and Housing Corporation (CMHC), Bank of Israel (BOI), Japan Housing Finance Agency (JHFA), Bank Negara Malaysia (BNM), Reserve Bank of New Zealand (RBNZ), European Mortgage Federation (EMF), Bank of Korea (BOK), survey data for Brown and Guin (2015), Federal Housing Finance Agency (FHFA. For South Africa, the only available statistics is from a news article of City Press.

Figure 7: Share of Variable-rate Housing Loan

(a) Relative Consumption Volatility (b) House Price Volatility



country's sensitivity.

This paper focuses on a share of the variable-rate housing loan as a response to the idea — what fundamental is related to a country's sensitivity when an international credit supply shock occurs. I define the variable-rate loans as loans extended at floating rates or an initial period of rate fixation up to one year, and Table 6 presents the average share of variable-rate housing loans for each country over the corresponding sample period. Since the variable-rate mortgage imposes a risk that homeowners should endure when the interest rate becomes volatile, the share of variable-rate mortgages affects the sensitivity of the interest rate borrowers are facing at first. In addition, it affects the sensitivity of the leverage borrowers want to gain, considering an increased risk they are facing, which is the narrative for the demand side of credit. Therefore, this paper argues that a higher share of variable-rate housing loans is related to higher σ_m and σ_r, so it is attributed to higher relative consumption volatility and higher house price volatility alike.

Figure 7 shows the cross-country relationship between volatilities of interest and the source of credit shock volatility variation this paper suggests. The horizontal axes of the two panels of Figure 7 are identical, which is the share of variable-rate housing loans. We observe a positive cross-country relationship between the share and the relative consumption volatility in Figure 7a with a correlation of 0.37. Additionally, we see the same pattern in Figure 7b for the house price volatility (correlation = 0.51). Furthermore, there is an explicit difference between the emerging country group and rich country group; emerging countries have a higher share of variable-rate housing loans than do rich countries. Therefore, this paper asserts emerging countries have higher credit shock volatility, which explains why emerging countries have higher relative consumption volatility and higher house price volatility.

4.4 Counterfactual Analysis

In this subsection, I present the counterfactual analysis concerning the critical effects of the model: a credit market effect and a rental market effect. The credit market effect arises from the housing collateral constraint, which is financial friction of the economy. It is related to the homeowners' borrowing capacity affected by house price changes and generates an amplification effect contributing to higher relative volatility of consumption. The latter effect is related to an explicitly accommodated housing rental market, which allows a transmission channel of shocks through a link between the rental price and house price. Additionally, there is a friction in the rental market, which is a rigid adjustment of rental price, and it is introduced to avoid extreme volatility of the rental price. Therefore, this subsection investigates how the key frictions matter in the model in order to clarify key mechanisms.

4.4.1 Credit Market Effect: Does Housing Collateral Constraint Matter?

First, I rewrite the model without the collateral constraint and remove the relevant credit shocks, which are the leverage shock and the interest rate shock. The selected second moments for the rewritten model are presented under the heading of Model [I] in Table 7. Note that the header of Model [B] is for the baseline model result presented in Table 4. The Model [I] does not generate excess consumption volatility and there is no excess non-housing consumption volatility, either (0.79 and 0.69, respectively). The ratio of rent volatility to house price volatility is almost one since house price volatility becomes lower due to the absence of the amplification effect generated by the housing collateral constraint. More importantly, the model without the collateral constraint fails to yield the positive relationship between relative consumption volatility and house price volatility, as Figure 8a shows. Even the housing productivity shock, which generates a weakly positive relationship between two volatilities in the baseline model, produces almost a flat relationship, as the dashed gray line shows. Therefore, we can confirm that the financial friction is a crucial feature in explaining excess consumption volatility and the positive relationship between the two volatilities of interest.

Next, in order to determine the effect of the collateral constraint clearly, I break down the condition of Model [I] into two separate conditions: removal of only the collateral constraint and removal of only the interest rate shock. For the former condition, I add the interest rate shock to Model [I], namely, Model [II]. For the latter condition, I remove the interest rate shock

Table 7: Counterfactual Analysis 1: Collateral Constraint

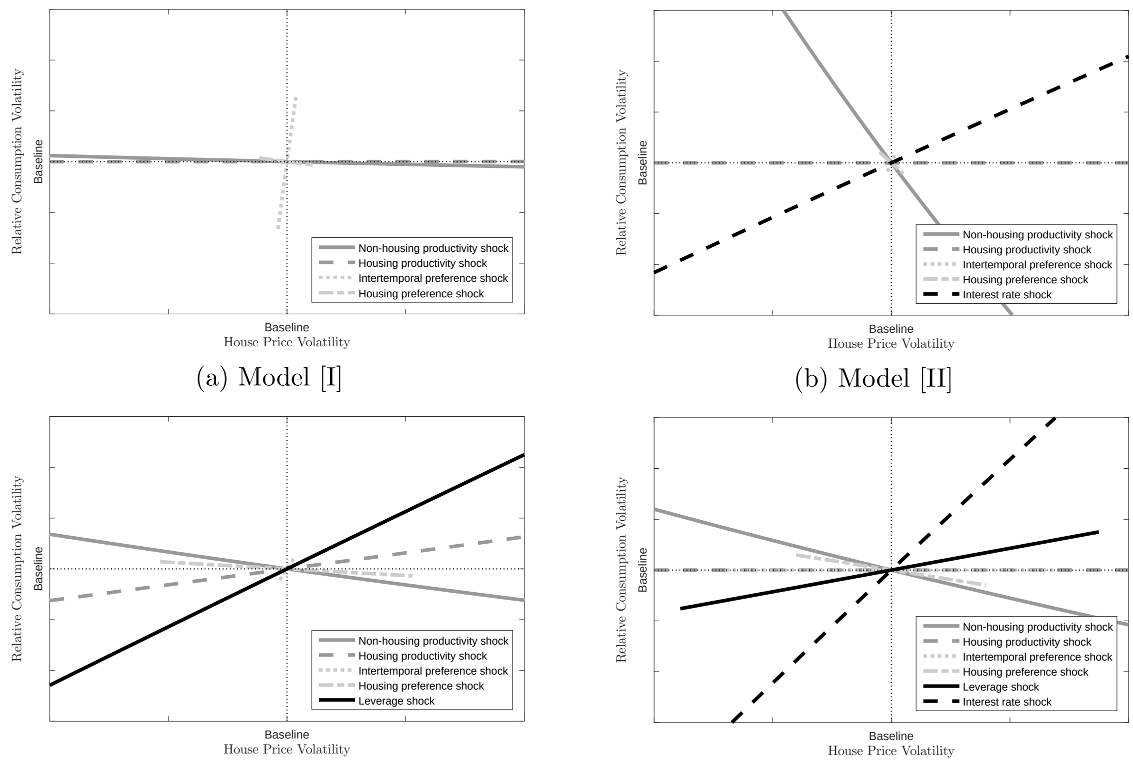
	Data	**Model [B]**[1]	**Model [I]**[2]	**Model [II]**[3]	**Model [III]**[4]	**Model [IV]**[5]
$\frac{\sigma_C}{\sigma_Y}$	1.23	1.30	0.79	1.64	1.11	1.04
$\frac{\sigma_{HC}}{\sigma_Y}$	1.68	1.51	1.53	2.51	1.32	1.62
$\frac{\sigma_{NHC}}{\sigma_Y}$	1.36	1.30	0.69	1.54	1.11	0.97
$\frac{\sigma_R}{\sigma_{HP}}$	0.54	0.54	0.98	1.08	0.60	0.81

[1] Baseline model
[2] Model without collateral constraint: neither LTV shock and interest rate shock is included.
[3] Model without collateral constraint: there exists interest rate shock.
[4] Model with collateral constraint: there is no interest rate shock.
[5] Model with different specification of borrowing constraint: housing collateral is excluded.

from the baseline model, namely, Model [III]. According to Table 7, Model [II] generates a highly volatile economy in terms of both relative consumption volatility and disaggregated consumption volatility, which implies the collateral constraint prevents the model from the excessive response of the economy to the interest rate shock. Notice that the housing consumption volatility surges in Model [II], which is due to the increase in volatility of the rental price ($\sigma_R/\sigma_{HP} = 1.08$). In contrast, the column of Model [III] is close to the baseline case though its magnitude is slightly smaller due to the absence of the interest rate shock. The σ_R/σ_{HP} is 0.60, which supports the argument that the collateral constraint plays a key role in generating sufficiently high house price volatility. In regard to the positive relationship between house price volatility and relative consumption volatility, Figures 8b and 8c suggest that the credit shock — the credit price shock for Model [II] or the credit availability shock for Model [III] — yields the positive relationship. One difference between the two panels is the result concerning the housing productivity shock. While the dashed gray line for the housing productivity shock is almost flat in Figure 8b, that in Figure 8c has a positive slope. This is because the housing collateral is a channel of transmission of the housing productivity shock to consumption.

The last column of Table 7 is for the model with an alternative specification of the borrowing constraint that takes a form of:

Figure 8: Counterfactual Anlysis 1: Collateral Constraint



(a) Model [I] (b) Model [II] (c) Model [III] (d) Model [IV]

$$d_{t+1} \leq \frac{\bar{d}}{\omega} + e^{\varepsilon_t^m - 1} - 1$$

where $\bar{d}$ is the steady-state level of aggregate debt, which is set to the baseline steady-state realization, 2.4012. The credit availability shock, ε_t^m, still follows Equation (46) as in the baseline model. The objective of this exercise is to investigate the role of housing collateral, $q_t h_t$. In other words, the agents of Model [IV] are faced with the borrowing constraint, but it is not related to the value of housing they hold at all. Since there is no amplification effect in Model [IV], the non-housing consumption volatility and total consumption volatility become lower than that of the baseline case. In the case of the ratio of rent price volatility to house price volatility, the ratio increases because house price volatility falls due to the missing credit condition margin in determining the equilibrium house price. Last, Figure 8d shows that the credit availability shock in Model [IV] (the solid black line) is not crucially important for house price volatility and its relationship with relative consumption volatility, compared to its critical role in the baseline model.

Table 8: Counterfactual Analysis 2: Housing Rental Market

	Data	**Model [B]**	**Model [V]**[1]	**Model [VI]**[2]
$\frac{\sigma_C}{\sigma_Y}$	1.23	1.30	1.75	1.74
$\frac{\sigma_{HC}}{\sigma_Y}$	1.68	1.51	4.39	4.31
$\frac{\sigma_{NHC}}{\sigma_Y}$	1.36	1.30	1.29	1.29
$\frac{\sigma_R}{\sigma_{HP}}$	0.54	0.54	2.24	2.54

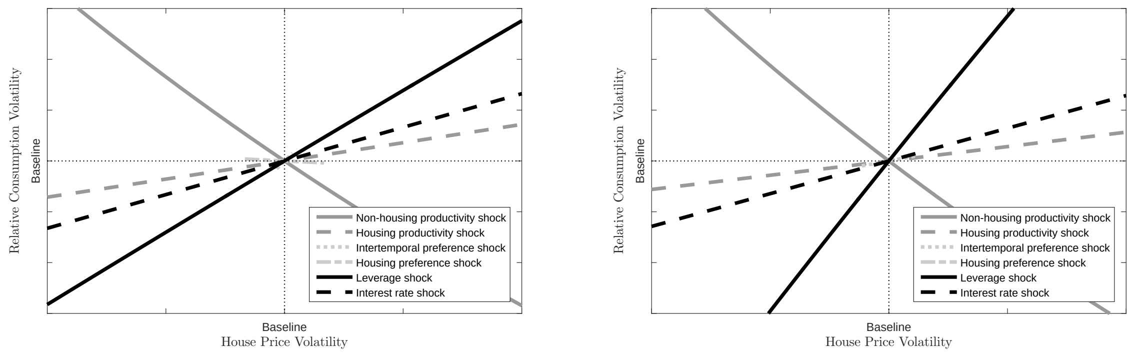
[1] Model without rigid adjustment of rental price
[2] Model without housing rental market

4.4.2 Rental Market Effect: Does Rigid Adjustment of Rental Price Matter?

Another key friction this paper introduces is the rigid adjustment of housing rental prices. To study its role in the performance of the model, I modify the baseline model, excluding the rental agency problem, namely, Model [V]. As a result, the corresponding price measure for housing consumption is only ρ_t for both homeowners and renters, there is no dividend income from rental agencies for homeowners, and GDP comprises the common income, Π_t, and the homeowners' rental income, $\rho_t \omega h_t$.

The column of the header of Model [V] in Table 8 presents a substantial increase in relative consumption volatility compared to the baseline case. Since the relative non-housing consumption volatility for Model [V] is almost the same as that for the baseline model, the larger σ_C/σ_Y is attributed to the excessive housing consumption volatility — σ_{HC}/σ_Y for Model [V] is 4.39, which is three times as large as σ_{HC}/σ_Y for Model [B] which is 1.51. The excessive housing consumption volatility is a result of the excessive housing rental price volatility — σ_R/σ_{HP} for Model [V] is 2.24. Therefore, without the rigidity feature, the model generates a significantly highly volatile rental price, which is not consistent with actual data, suggested in *Finding 4* and Figure 5. Next, the last column of Table 8 is for the model without the housing rental market. Specifically, in this version of the model, there are no renters; all household agents are homeowners. Since the explicitly considered housing rental market makes room for introducing the rental price rigidity, the modified Model [VI] also does not feature the rigid adjustment of

Figure 9: Counterfactual Anlysis 2: Housing Rental Market



(a) Model [V] (b) Model [VI]

the rental price. Note that the price measure for housing consumption is implicitly defined by the marginal rate of substitution between housing services and non-housing consumption goods. Obviously, the selected second moments presented in Table 8 are similar between the columns for Model [V] and [VI]. In addition, according to Figures 9a and 9b, both Model [V] and [VI] do not affect the model's performance regarding the positive relationship between house price volatility and relative consumption volatility. A notable difference between two modified models is observed outside of the table and figure. Recall, the baseline model result suggests the non-housing productivity shock matters for house price volatility, which is named the *revisited* productivity shock, and the key mechanism behind it is the link between house price and rental price. Since Model [VI] does not accommodate the rental market explicitly, the non-housing productivity shock in Model [VI] does not account for house price volatility at all — it contributes 0.1% in explaining house price volatility according to variance decomposition. In contrast, the non-housing productivity shock recovers its influence over house price fluctuation in Model [V] (32.3%) because Model [V] takes the rental market explicitly into account even though it does not have the rental price rigidity feature.

5 Sources of Cross-Country Variation in Explaining Consumption Volatility: Emerging and Rich Countries

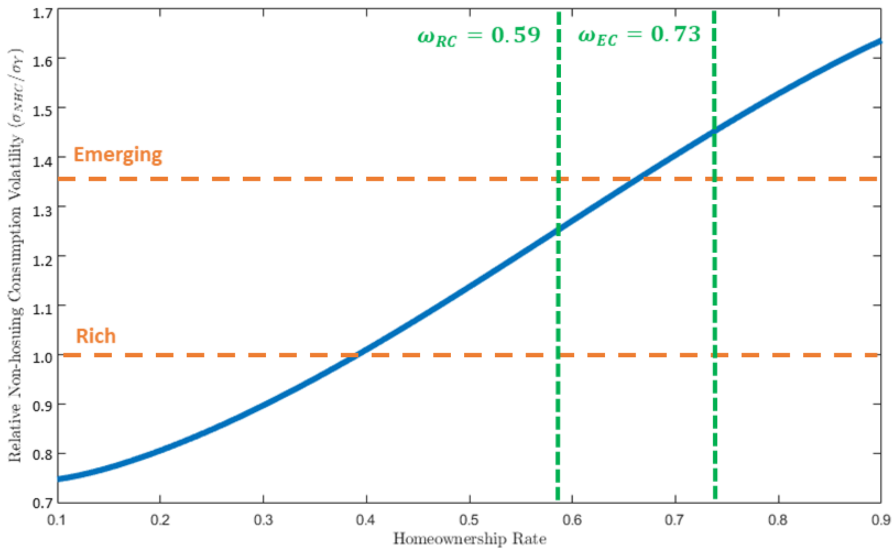
This paper suggests new stylized facts for emerging countries from cross-country data on disaggregated consumption, which are housing consumption and non-housing consumption. First, housing consumption is more volatile than non-housing consumption in emerging countries, but it is as volatile as or less volatile than non-housing consumption in rich countries, which is *Finding 2* described in Section 2. In other words, the ratio of housing consumption volatility to non-housing consumption volatility is higher in emerging countries than in rich countries ($(\sigma_{HC}/\sigma_{NHC})_{EC} > (\sigma_{HC}/\sigma_{NHC})_{RC}$). Second, even though we remove the volatile part of housing consumption from total consumption expenditure, we still observe higher relative volatility in non-housing consumption in emerging countries than in rich countries ($(\sigma_{NHC}/\sigma_Y)_{EC} > (\sigma_{NHC}/\sigma_Y)_{RC}$). Moreover, excess consumption volatility for emerging countries also holds in non-housing consumption ($(\sigma_{NHC}/\sigma_Y)_{EC} > 1$), which is *Finding 3*. This section provides explanation for the above-mentioned facts using the theoretical model developed in Section 3, focusing on key parameters as sources of cross-country variation.

5.1 Higher Non-housing Consumption Volatility in Emerging Countries: Degree of Financial Friction

In addition to excess consumption volatility for emerging countries, this paper found evidence of excess non-housing consumption volatility for emerging countries (*Finding 3*). Notably, in terms of the comparison between country groups, we observe higher relative non-housing consumption volatility in emerging countries than in rich countries. As we discussed in subsection 4.4, the model explains that the credit market effect is the crucial mechanism in explaining higher relative non-housing consumption volatility, as well as higher relative total consumption volatility. Note that the credit market effect relies on financial friction, whose form is a housing collateral constraint; hence, the corresponding key parameter in the model is homeownership rate, ω, because only homeowners are facing the collateral constraint in the theoretical world. In other words, a degree of financial friction, which is a key in non-housing consumption volatility, depends on the abovementioned parameter value. Therefore, we highlight the cross-country variation in ω to explain the gap of relative non-housing consumption volatility, or relative consumption volatility, between emerging countries and rich countries. Hereafter, I will not distinguish relative non-housing consumption volatility and relative total consumption volatility because they show almost the same patterns.

The model yields higher relative non-housing consumption volatility when the homeownership rate is higher, as 10 presents. It implies that a country that has a higher degree of financial friction has both higher σ_{NHC}/σ_Y according to the upward sloping curves. Recalling *Finding 3*, the weighted average of relative non-housing consumption volatilities of emerging countries is 1.37, and that of rich countries is 1.05 for quarterly data, which are shown as horizontal dashed

Figure 10: Relative Non-housing Consumption Volatility and Degree of Financial Friction



lines in Figure 10 (1.28 and 0.98 in annual data, respectively). The model predicts that this higher relative non-housing consumption volatility in emerging countries is explained if emerging countries have a higher degree of financial friction, or higher homeownership rate. Thus, I collect homeownership rate data across countries, which are presented in Table 9. I extend the baseline sample countries (10 emerging countries, 14 rich countries) to discuss differences between country groups in general, so Table 9 covers 16 emerging countries and 16 rich countries, including the baseline sample countries. The homeownership rate is varying from 48% (Colombia) to 90% (Hungary) in emerging countries and from 40% (the Netherlands) to 69% (Luxembourg) in rich countries. As we can conjecture from the range of homeownership rates for each country group, the rate is higher in emerging countries than in rich countries on average (73% for emerging countries, 59% for rich countries). It implies ω_{EC} is greater than ω_{RC}. Therefore, larger non-housing consumption volatility in emerging countries can be explained by their higher homeownership rate if other factors are controlled as the model predicts. Indeed, when we plot the baseline sample countries on the plane whose horizontal axis is homeownership rate and vertical axis is relative consumption volatility, we can observe positive correlation across countries, as Figure 11 describes. Note that emerging countries have higher homeownership rate, as well as higher relative consumption volatility in the figure.

Table 9: Homeownership Rate: Emerging and Rich Countries

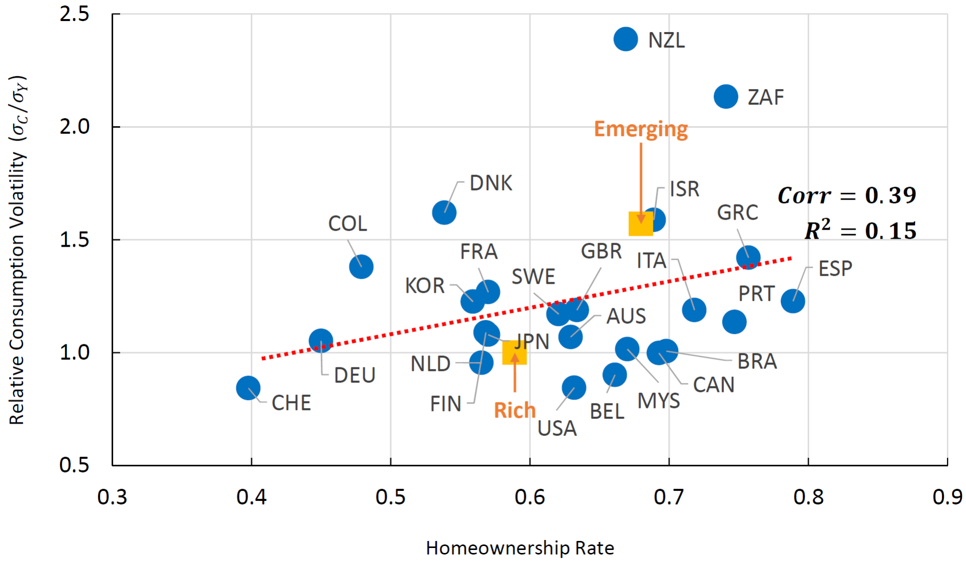
Emerging Countries		***Rich Countries***	
Hungary	0.90	Norway	0.76
India	0.87	Italy	0.72
Poland	0.84	Canada	0.69
Philippines	0.80	Luxembourg	0.69
Spain	0.79	Belgium	0.66
Greece	0.76	United Kingdom	0.63
Portugal	0.75	United State	0.63
South Africa	0.74	Australia	0.63
Mexico	0.72	Sweden	0.62
Thailand	0.70	France	0.57
Brazil	0.70	Japan	0.57
Israel	0.69	Finland	0.57
Malaysia	0.67	Netherlands	0.56
New Zealand	0.67	Denmark	0.54
South Korea	0.56	Germany	0.45
Colombia	0.48	Switzerland	0.40
Average	0.73	**Average**	0.59

[1] As of 2017 for most sample countries. The homeownership rate is little varying over time; Cesa-Bianchi, Ferrero, and Rebucci (2018) presents the average homeownership rates over 2005-2014 period, and the correlation between theirs and this paper's is 0.81. Sources: OECD affordable housing database, Housing Finance Information Network (HOFINET), EuroStat, Brazil National Household Sample Survey, DANE (Colombia National Statistics Service), Census of India, Israel Central Bureau of Statistics, Statistics New Zealand, South Africa General Households Survey, Statistics Korea, Thailand Real Estate Information Centre, Statistics Bureau of Japan

5.2 Higher Housing Consumption Volatility in Emerging Countries: Degree of Rental Market Friction

This paper suggests that housing consumption is more volatile than non-housing consumption in emerging countries, and the former is as volatile as or less volatile than the latter in rich countries, which is *Finding 2*. Basically, housing consumption is defined by the sum of the rental value of tenant-occupied housing and the imputed rental value of owner-occupied housing. The first part is clear because it captures the actual expenditure amount of housing services. For the second part, national accounts treat the owner-occupant as if the one rents to himself; then, how much he pays for his own housing service is imputed. In other words, national accounts impute the owner-occupied housing service value based on the rents charged for similar tenant-occupied housing. Therefore, we can say that the observed housing consumption in national accounts consists of two components: housing rental prices and the number of housing units in the economy. Note that the number of housing units is not affected by rental price but is affected by house prices.

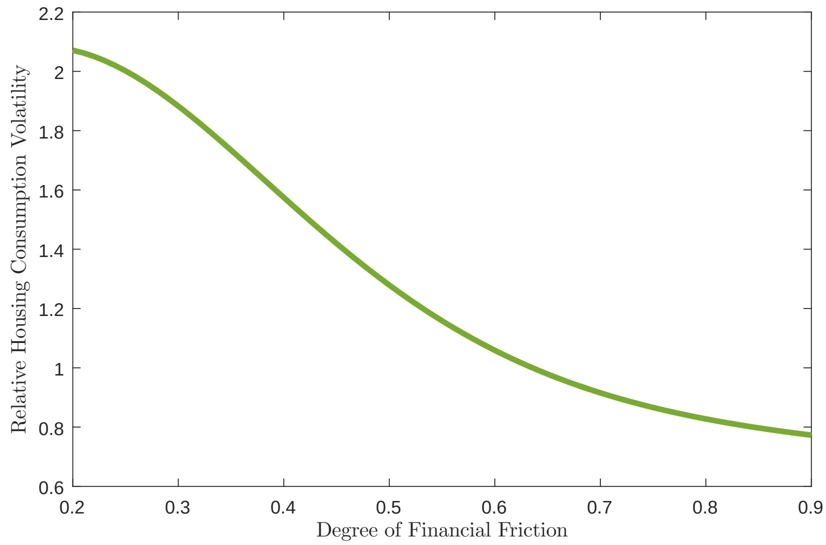
Figure 11: Relative Consumption Volatility and Homeownership Rate



Based on the understanding of the observed housing consumption, its higher volatility compared to that of non-housing consumption in emerging countries could be approached in two ways: the first hypothesis is that higher house price volatility in emerging countries leads to higher housing stock volatility, which might matter for higher housing consumption volatility; and second, rental price volatility is relatively higher in emerging countries, as shown in Figure 4 and *Finding 4*, which affects the measured housing consumption volatility.

Regarding the first story, what we can only observe related to the amount of housing stock in the economy is the residential investment in national accounts. The volatility of residential investment of sample countries is strongly correlated with house price volatility, which implies the high housing stock volatility in emerging countries can explain their high housing consumption volatility. However, what we want to determine is not the level of housing consumption volatility but the relative size of housing consumption volatility compared to non-housing consumption volatility. Note that the amount of housing stock and its fluctuation affects non-housing consumption through a financial friction channel. In other words, the effect of higher housing stock volatility on the relative size of housing consumption volatility is ambiguous because non-housing consumption also becomes volatile. Moreover, we can observe a higher degree of financial friction in emerging countries than that in rich countries, as discussed in the previous subsection, which implies it is difficult to explain higher housing consumption volatility relative to non-housing consumption volatility in emerging countries using the first housing stock hypothesis. Figure 12 shows the model result of the change in the ratio of housing consumption volatility to non-housing consumption volatility with respect to the change in the degree of financial

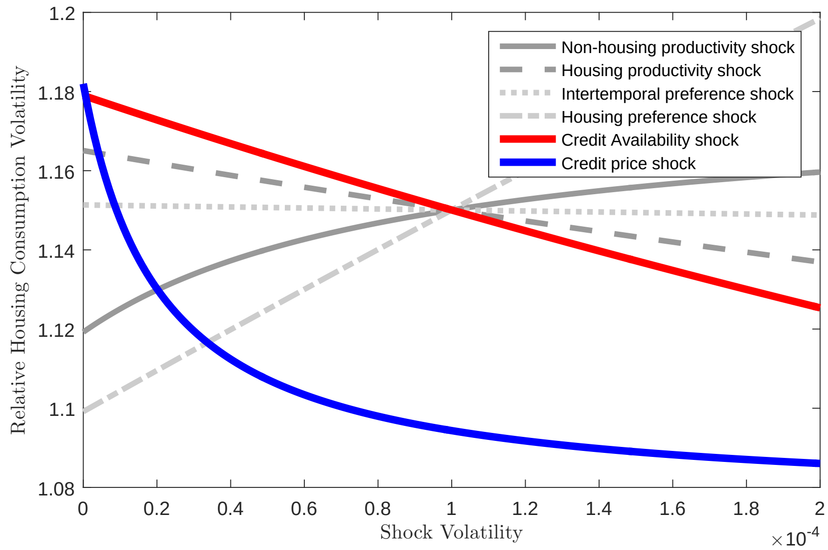
Figure 12: Relative Housing Consumption Volatility and Degree of Financial Friction



friction. The stronger the financial friction's effect is, the stronger the effect of housing stock fluctuation on non-housing consumption fluctuation, so the ratio falls as the degree increases. Even if we consider another source of cross-country variation we discussed, which is the credit shock volatility, the ratio shows a downward sloping curve, as shown in Figure 13. As a result, we conclude that we cannot explain *Finding 2* through the higher housing stock volatility.

How about the alternative story of higher rental price volatility? Recalling *Finding 4*, we know there is a positive relationship between rental price volatility and house price volatility across countries. However, this means the first story of housing stock volatility kicks in again, so simply higher rent volatility does not explain the higher ratio of housing consumption volatility to non-housing consumption volatility. Another observation from rental price data in *Finding 4* is that housing rental price volatility is generally less than house price volatility, but the magnitude is varying across countries. Specifically, the weighted average ratio of rental price volatility to house price volatility is 0.42. Moreover, we can observe different ratios between emerging countries and rich countries, which is room for finding a source for explaining higher σ_{HC}/σ_{NHC} in emerging countries. We observe a higher ratio for emerging countries than for rich countries in the sample: 0.62 for emerging countries and 0.35 for rich countries. Note that the model accommodates the low volatility of rental price through the rigid adjustment of the rental price setting, and the degree of rigid adjustment is governed by the adjustment cost parameter, θ, in Equation (15). Therefore, owing to the simplified assumption of the Rotemberg setup, we can discuss the varying magnitude of low rental price volatility compared to house price volatility across countries using the variation in the one rental market friction parameter, which is θ. In

Figure 13: Relative Housing Consumption Volatility and Shock Volatility



other words, a low level of θ means a high σ_R/σ_{HP}, and a high level of θ means a low σ_R/σ_{HP}. The model predicts, as shown in Figure 14[22], low θ or low degree of rental market friction results in higher σ_{HC}/σ_{NHC}. As a result, the different degrees of rental market friction between emerging countries and rich countries can explain the gap between the two horizontal dashed lines in Figure 14, which are from actual quarterly data (σ_{HC}/σ_{NHC} for emerging countries is 1.20 and for rich countries is 0.84). Unfortunately, we cannot directly observe θ for each country, so instead, this paper surveys the housing rental market characteristics that are related to the rental price friction across countries, particularly focusing on the difference between emerging countries and rich countries, and then discusses the degree of rental market friction qualitatively.

The housing rental market is different from the housing market in terms of what is traded in the market. While housing structure or housing stock is traded in the housing market, the housing rental market is for the service flow generated by housing structure, so its price is subject to contracts that are negotiable between a homeowner and a renter. Since the negotiability implies the housing rents can be tailored to a particular renter based on the homeowner's subjective evaluation, the housing rental market is exposed to the intervention of policymakers who are interested in enhancing social welfare. Therefore, we should investigate the regulations in the private housing rental market in order to discuss rental market friction. Moreover, there are several terms of rental contracts, which are related to the degree of rigid adjustment of

[22] The plot is a result of model simulation with respect to the different values of θ between 100 and 1,000. The left-end value or the smallest value of $\theta = 100$ implies a duration of fixed price is 4.7 quarters if we recover the Calvo parameter for interpretation ($\theta_{Calvo} = 0.79$). The right-end value or the largest value of $\theta = 1,000$ implies a duration of fixed price is 16 quarters ($\theta_{Calvo} = 0.94$).

Table 10: Housing Rental Market Characteristics: Emerging and Rich Countries

	Rent control[1]		Rental contract features[2]		Real estate agent commission[3]	**Rent rigidity index (1 to 4)**[4]
	Initial rent	Rent increase	Common lease period	Deposit (*in equivalent of monthly rent*)		
Emerging Countries						
Brazil	free		1 year	3 months	5%	**3**
Chile	free		1 year	1 month	1.5-3%	**1**
Colombia	free	O	6 months or 1 year	prohibited	3%	**2**
Czech Republic	free	O	1 year	maximum 6 months	2-5%	**3**
Estonia	free		1 year	up to 3 months, usually 1-2 months	2-4%	**1**
Greece	free		3 years (minimum by law)	1-2 months	2-4%	**2**
Hungary	free		1 year or more	up to 3 months, usually 1-2 months	3-5%	**2**
Israel	free		1 year	3 months	2%	**2**
Latvia	free		2 years	usually 2-3 months	3-5%	**3**
Lithuania	free		6-12 months	1 month	1.5-3%	**1**
Mexico	regulated[5]		1 year or more	1-2 months	4-8%	**3**
Malaysia	free		1 year	2 months	2-3%	**1**
New Zealand	free	O	20 months	up to 4 weeks	2-4%	**2**
Poland	free	O	6-12 months	up to 6 months, usually 1 month	1.5-3%	**2**
Portugal	free	O	1 year or more	maximum 3 months	3-6%	**4**
Slovak Republic	free		1 year or more	1, 3 or 6 months	2-5%	**2**
South Korea	free	O	1 or 2 years	different arrangements apply[6]	0.5-1%	**2**
South Africa	free		1-2 years	4-6 weeks	5-7.5%	**2**
Spain	free		3 years or more	minimum 1 month, usually 2-3 months	1.5-5%	**3**
Rich Countries						
Australia	free		6-12 months	4-6 weeks	2-3.5%	**1**
Austria	regulated[5]	O	3 years	maximum 6 months	3%	**4**
Belgium	free		3, 6 or 9 years	1-3 months	3%	**3**
Canada	regulated[5]	O[5]	1 year or more	2 weeks, 1 month or 6 months	3-7%	**4**
Denmark	regulated[5]	O	mostly open-ended	usually up to 3 months	1.5-3%	**5**
Finland	free	O	mostly open-ended	usually 1 month	4-7%	**4**
France	regulated[5]	O	3-6 years	maximum 1 month	5-10%	**5**
Germany	regulated[5]	O	open-ended	maximum 3 months	3.5-7%	**5**
Ireland	free	O	4 years	usually 1 month	1.5-2.5%	**3**
Italy	free	O	3+2 or 4+4 years	maximum 3 months	4-8%	**4**
Japan	free		1-2 years or 3-5 years	3-6 months[7]	3-5%	**4**
Luxembourg	free	O	1 year	maximum 3 months	3%	**2**
Netherlands	regulated	O	open-ended	usually 1 month	0.5-2%	**4**
Sweden	regulated	O	open-ended	not commonly used	3-5%	**5**
Switzerland	free	O	6 years	maximum 3 months	3-5%	**4**
United Kingdom	free		usually initial 6 months	usually 1 month	1-3%	**2**
United States	regulated[5]	O	1-2 years	usually 1-3 months	5-6%	**5**

[1] Sources: OECD affordable housing database, O'Sullivan and Pascal De Decker (2007) "Regulating the private rental housing market in Europe", each country's landlord tenant law (as of 2018)

[2] Sources: OECD affordable housing database, Just Landed housing portal (international housing information service provider), Global Property Guide (as of 2018)

[3] Sources: Surefield, Tranio (international property broker), Mortgage calculator (international real estate agency information provider), Delcoure and Miller (2002) "International residential real estate brokerage fees and implications for the US brokerage industry", each country's local real estate agencies (as of 2018)

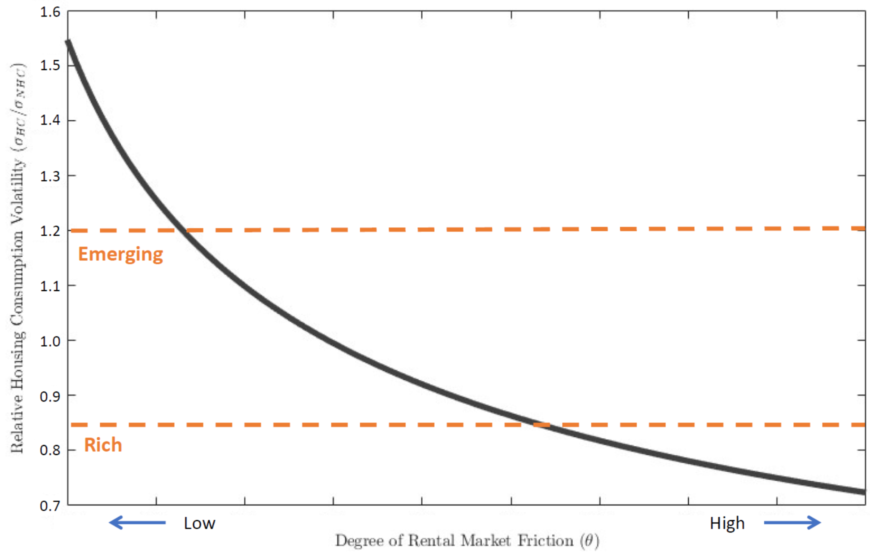
[4] The index indicates the number of characteristics toward the rental price rigidity in each country, so it is between one and six. The characteristic related to rigidity is: (a) existence of control on initial level of rent price, (b) existence of control on rental price increase, (c) relatively long common lease period ($\geq$ 2 year), (d) relatively large amount of deposit ($\geq$ 2 month-rents), and (e) relative high real estate agent commission rates ($\geq$ 3.75%).

[5] Rent control in the private rental market usually does not apply uniformly across the entire rental sector. In the case of Austria, Canada, Denmark, and the United States rent control applies to an older part of the housing stock. In Germany, rent control applies to all dwellings except newly constructed ones. Also, in some countries, the regulation is varying across regions; France introduces rent control in areas where demand exceeds supply, and in Mexico, the control of initial rent levels is varying across states, and the rent increases are differently controlled across states in United States and Canada.

[6] In South Korea, different arrangements apply in the rental sector: 1) *Jeonse* is a unique housing rental system of South Korea. Under the *Jeonse* system the renter pays the homeowner a substantial deposit without paying any monthly rent, and the renter can stay in the dwelling for the duration of the contract; 2) there is also the standard housing rental system where renters pay monthly rent and a deposit (usually 1 month rent); and 3) there is a "mixed" system with a large deposit and a relatively small monthly rent.

[7] In Japan, the renters pay two types of key money when the housing rental contract is concluded: 1) *Shikikin* is the typical deposit which is refundable and usually the same as one to two months of rent; 2) *Reikin* is a mandatory payment to the homeowner that is often the same amount as the *Shikikin*, but this money is considered as a gift to the homeowner, so non-refundable.

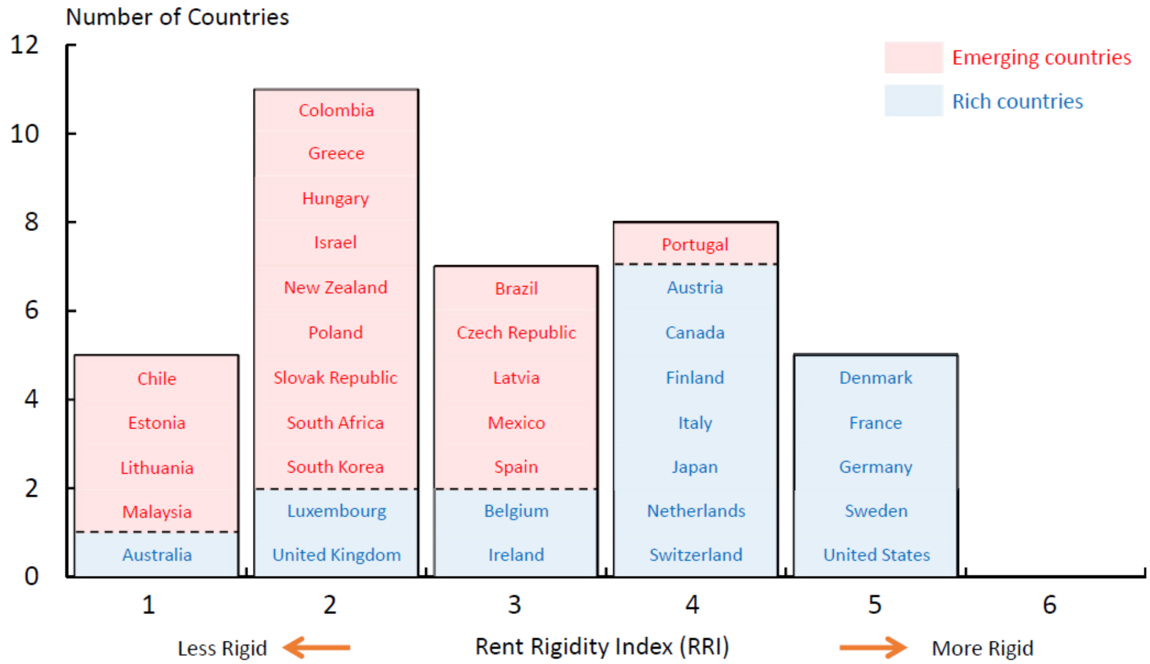
Figure 14: Relative Housing Consumption Volatility and Degree of Rental Market Friction



aggregate rental prices; for instance, the typical duration of the contract and the amount of deposit a tenant has to pay when the contract is initially agreed upon. Therefore, I collect the above information across 19 emerging countries and 17 rich countries, considering data availability in order to determine a structural difference in rental market characteristics between two country groups. I also gather the real estate agent commission for each country as a proxy for information acquisition cost or negotiation cost in determining housing rental prices. In the presence of these costs, the rental price adjustment becomes costly, so it intensifies the degree of housing rental market friction. Considering the commission information on a housing rental agent is available in a few countries, the general real estate agent commission is collected for comparability across countries. The collected information on housing rental market characteristics is presented in Table 10.

The first two columns of Table 10 present the existence of rent control in each sample country. We observe more rich countries implement the control of initial rent levels as well as of rent increases, as of 2018. In detail, only one country, Mexico, conducts the initial rent level control among 19 emerging countries (5.3%), while eight rich countries use the control among 17 countries (47.1%). Although more emerging countries implement the control of rent increases compared to the control of initial rent (6 countries, 31.6%), the proportion is still significantly smaller than the proportion of rich countries conducting the rent increase control (13 countries, 76.6%). Therefore, in terms of rental market intervention, there is a higher degree of rental market friction in the case of rich countries. Second, we can discuss the rental price rigidity

Figure 15: Rent Rigidity Index



through the typical features of rental market contracts for each country. The common lease period and the amount of deposit are varying across countries, as on the third and the fourth column of Table 10. However, we can generalize them in view of the country-group comparison between emerging and rich countries. First, the typical lease period is longer in rich countries; 52.6% of emerging countries have a standard lease period less than 1 year, 42.1% have the period between 1 and 2 years, and 5.3% have the period longer than 3 years. In rich countries, 17.6% have a standard lease period less than 1 year, 17,6% the period between 1 and 2 years, and 64.8% have the period longer than 3 years. In addition, there are five rich countries whose common lease period is mostly open-ended (Denmark, Finland, Germany, the Netherlands, and Sweden). We also observe a similar pattern in the amount of deposit. The amount of 1-month rent or less is usually required in 26.3% of emerging countries, 1-2-month rent is required in 42.1%, and 3-month rent or more is required in 26.3% of emerging countries. In contrast, in rich countries, almost half of countries show 3-month rent or more is required for rental contraction (47.1%), which means that there is a larger deposit amount in rich countries than in emerging countries. Thus, we also observe a higher degree of rental price rigidity or rental market friction in rich countries in terms of the typical rental contract. Next, Table 10 provides each country's real estate agent commission as of 2018. The commission is higher in rich countries than in emerging countries on average. Since we have a range of typically imposed commission rates for each country, I compare them in terms of median, minimum, and maximum of the range. The average of medians of usual commission ranges is 3.4% for emerging countries and 4.1% for rich countries. It is similar in the case of minimum or maximum; the average of minimums is 2.4% for emerging countries and 3.1% for rich countries; the average of maximums is 4.3% for emerging countries, and 5.1% for rich countries. As I illustrated in the previous paragraph, a higher commission rate indicates a higher degree of rental market friction, which implies it is the case for rich countries. Based on the observations from rent controls and rental contract features, this paper summarizes them by suggesting an index related to rent rigidity, as shown in the column under the heading of "Rent rigidity index" in Table 10. The index indicates the number of characteristics toward the rental price rigidity in each country, so it is between one and six. The classification rule is that the characteristic is related to rental price rigidity (a) if there is a control on initial level of rental price, (b) if there is a control on rental price increases, (c) if a commonly observed lease period is relatively long ($\geq$ 2 years, which is a cross-country average), (d) if the typical amount of deposit is relatively large ($\geq$ 2-month rent, which is a cross-country average), and (e) if the real estate agent commission rate is relatively high ($\geq$ 3.75%, which is a cross-country average). For example, Denmark has four characteristics related to the rigid adjustment of rent, so the rent rigidity index for Denmark is 5. Figure 15 visually illustrates the distribution of each country's rent rigidity index. We can see that emerging countries tend to be on the left-hand side, and rich countries tend to be on the right-hand side. On average, an emerging country shows a lower index, which is 2.16, than that for a rich country, which is 3.76. Therefore, the index clearly suggests that rich countries have more rental market characteristics toward rental price rigidity than emerging countries have. The index is a simple suggestion but provides a good reference point, which is able to be extended in a more comprehensive manner. As a result, this paper argues that the key parameter, θ, is different between emerging and rich country groups, and it is higher for rich countries ($\theta_{EC} < \theta_{RC}$).

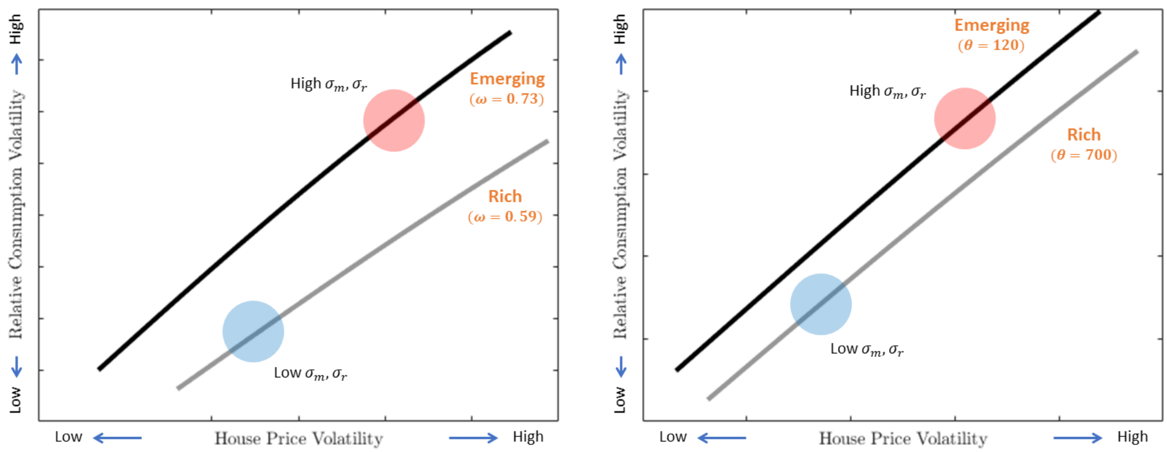
5.3 Revisited Cross-Country Evidence: Higher House Price Volatility and Relative Consumption Volatility in Emerging Countries

To summarize the implications of this paper so far, we revisit the cross-country evidence of the positive relationship between house price volatility and relative consumption volatility. In Section 4, this paper showed that credit shock volatility variation could explain the cross-country evidence. In the context of a comparison between the emerging country group and the rich country group, it implies that emerging countries that have higher volatility in credit availability shock (σ_m) and credit price shock (σ_r) have higher house price volatility and higher relative consumption volatility alike. We revisit the implication to jointly consider it with the discussion

Figure 16: Key Parameters and Relationship between Volatilities

(a) Homeownership Rate

(b) Rigid Adjustment of Rental Price



in this section — particularly, higher homeownership rate and lower degree of rental price rigidity observed in emerging countries.

Figure 16 presents the model simulation results with respect to different levels of financial friction (the left panel, Figure 16a) and to different levels of rental market friction (the right-pane, Figure 16b). To focus on the variation in credit shock volatility, the model is simulated with respect to the variation of both σ_m and σ_r, leaving other shocks' volatility at the baseline level of 0.0001. First, in Figure 16a, the solid black line is generated with $\omega = 0.73$, which is the average homeownership rate of emerging countries. The line for emerging countries is above the line for rich countries (the solid gray line generated with $\omega = 0.59$, which is the average homeownership rate of rich countries). The reason is that relative consumption volatility increases and house price volatility falls when ω becomes higher. The credit market effect becomes stronger, which drives higher relative consumption volatility. The supply of housing increases when the proportion of homeowners in the economy grows, and since the housing stock volatility is quite stable, the increased housing supply affects house price volatility negatively. As a result, we can comprehend higher house price volatility and relative consumption volatility in emerging countries through a joint-effect of their higher homeownership rate and higher credit shock volatility — a red circle in Figure 16a is located northeastward compared to a blue circle. Next, in Figure 16b, we can also observe that the line for emerging countries is above the line for rich countries. The solid black line is the simulation result with $\theta = 120$, and the solid gray line is that with $\theta = 700$. Each θ is set to match each country-group's average ratio of rent volatility to house price volatility for comparison (0.62 for the emerging, 0.35 for the rich country group). The reason the solid black line is above is that lower θ yields higher relative consumption volatility and lower house price volatility. Note that the degree of rigid adjustment of rental price controls housing consumption volatility, so low θ contributes to higher total consumption volatility. The degree of rental price rigidity also affects house price volatility through a direct link between house prices and rental prices illustrated in the subsubsection 4.2.2. The weaker the rental price rigidity is, the weaker the contribution of financial constraint margin (the last term of Equation (58)) to house price fluctuation is. Therefore, house price volatility decreases when θ is lower. As a result, we observe higher relative consumption volatility and higher house price volatility in emerging countries, which have both lower θ and lower (σ_m, σ_r) — a red circle in Figure 16b is located northeastward compared to a blue circle.

6 Conclusion

This paper linked the two stylized facts of the business cycle observed in emerging countries — higher house price volatility and relative consumption volatility — by providing evidence of a positive relationship between the two volatilities across countries. I built up a real business cycle model with housing for the small open economy by incorporating new features into the existing theoretical frameworks to explain it. Specifically, the model explicitly accommodated a housing rental market and its prices, whose role had been overlooked in explaining consumption volatility, though housing consumption, whose corresponding price measure is the rent, accounts for a nonnegligible portion of total consumption expenditure. Moreover, the disaggregation of housing and non-housing consumption introduced new stylized facts for emerging countries. First, housing consumption is more volatile than non-housing consumption in emerging countries. Second, non-housing consumption still shows excessiveness in emerging countries even if we remove the volatile part of housing consumption from total consumption. The result of the model suggested that the variation of credit shock volatility is a driving force in generating the positive relationship between house price volatility and relative consumption volatility, and this paper provided qualitative evidence for cross-country discussion. The mechanism relied on financial friction of the housing collateral constraint, and the degree of the friction gave an account of excess consumption volatility, as well as excess non-housing consumption volatility in emerging countries. I also discussed another key friction — rental market friction, which explained higher housing consumption volatility for emerging economies. In particular, one contribution of this paper is the construction of the rent rigidity index based on the surveyed data about housing rental market characteristics across countries, which presents that the rent rigidity is less severe in emerging countries. To conclude, while this paper sheds light on *housing* — house prices, rental prices, housing consumption, housing collateral constraints, and rental market friction — to explain the differences in consumption volatility across countries, there might be many other country-specific factors related to housing that this paper does not fully take into account. I hope that this paper provides a benchmark to researchers trying to study housing and consumption across countries.

References

Aguiar, Mark and Gita Gopinath, "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, 2007, *115* (1), 69–102.

Alvarez-Parra, Fernando, Luis Brandao-Marques, and Manuel Toledo, "Durable Goods, Financial Frictions, and Business Cycles in Emerging Economies," *Journal of Monetary Economics*, 2013, *60*, 720–736.

Andrés, Javier, Óscar Arce, and Carlos Thomas, "Structural Reforms in a Debt Overhang," *Journal of Monetary Economics*, June 2017, *88*, 15–34.

Aoki, Kosuke, James Proudman, and Gertjan Vlieghe, "House Prices, Consumption, and Monetary Policy: A Financial Accelerator Approach," *Journal of Financial Intermediation*, October 2004, *13* (4), 414–435.

Aysoy, Cem, Cevriye Aysoy, and Semih Tumen, "Quantifying and Explaining Stickiness in Housing Rents: A Turkish Case Study with Micro-level Data," *Journal of Housing Economics*, September 2014, *25*, 62–74.

Badev, Anton, Thorsten Beck, Ligia Vado, and Simon Walley, "Housing Finance Across Countries: New Data and Analysis," *World Bank Policy Research Working Paper Series 6756*, 2014.

Bian, Timothy Yang and Pedro Gete, "What Drives Housing Dynamics in China? A Sign Restrictions VAR Approach," *Journal of Macroeconomics*, December 2015, *46*, 96–112.

Boz, Emine, Christian Daude, and Ceyhun Bora Durdu, "Emerging Market Business Cycles Revisited: Learning about the Trend," *FRB International Finance Discussion Papers*, April 2008, (927).

Calza, Alessandro, Tommaso Monacelli, and Livio Stracca, "Mortgage Markets, Collateral Constraints, and Monetary Policy: Do Institutional Factors Matter?," *Centre for Economic Policy Research (CEPR) Discussion Paper No.6231*, 2007.

_ , _ , and _ , "Housing Finance and Monetary Policy," *Journal of the European Economic Association*, 2013, *11* (S1), 101–122.

Campbell, John Y. and Joao F. Cocco, "How Do House Prices Affect Consumption? Evidence from Micro Data," *Journal of Monetary Economics*, 2007, *54*, 591–621.

Case, Karl E., John M. Quigley, and Robert J. Shiller, "Comparing Wealth Effects: The Stock Market versus the Housing Market," *Advances in Macroeconomics*, 2005, *5* (1), 1–32.

Cesa-Bianchi, Ambrogio, Andrea Ferrero, and Alessandro Rebucci, "International Credit Supply Shocks," *Journal of International Economics*, May 2018, *112*, 219–237.

_ , Luis Felipe Cespedes, and Alessandro Rebucci, "Global Liquidity, House Prices, and the Macroeconomy: Evidence from Advanced and Emerging Economies," *Journal of Money, Credit and Banking*, 2015, *47* (1), 301–335.

Ciarlone, Alessio, "House Price Cycles in Emerging Economies," *Studies in Economics and Finance*, March 2015, *32* (1), 17–52.

Davidoff, Thomas and Jiro Yoshida, "Estimating Consumption Substitution between Housing and Non-Housing Goods using Macro Data," 2013.

Davis, Morris A. and Robert F. Martin, "Housing, Home Production, and the Equity- and Value-Premium Puzzles," *Journal of Housing Economics*, June 2009, *18* (2), 81–91.

Fernández, Andrés and Adam Gulan, "Interest Rates, Leverage, and Business Cycles in Emerging Economies: The Role of Financial Frictions," *American Economic Journal: Macroeconomics*, July 2015, *7* (3), 153–188.

Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, Juan F Rubio-Ramírez, and Martin Uribe, "Risk Matters: The Real Effects of Volatility Shocks," *American Economic Review*, October 2011, *101* (6), 2530–2561.

Ferrero, Andrea, "House Price Booms, Current Account Deficits, and Low Interest Rates," *Journal of Money, Credit and Banking*, March 2015, *47* (S1), 261–293.

Flavin, Marjorie and Shinobu Nakagawa, "A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habit Persistence," *American Economic Review*, February 2008, *98* (1), 474–495.

Funke, Michael and Michael Paetz, "Housing Prices and the Business Cycle: An Empirical Application to Hong Kong," *Journal of Housing Economics*, March 2013, *22* (1), 62–76.

Garcia-Cicco, Javier, Roberto Pancrazi, and Martin Uribe, "Real Business Cycles in Emerging Countries?," *American Economic Review*, 2010, *100*, 2510–2531.

Genesove, David, "The Nominal Rigidity of Apartment Rents," *Review of Economics and Statistics*, November 2003, *85* (4), 844–853.

Gete, Pedro, "Expectations and the Housing Boom and Bust: An Open Economy View," 2018.

Guerrieri, Luca and Matteo Iacoviello, "OccBin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily," *Journal of Monetary Economics*, March 2015, *70*, 22–38.

Hamilton, James D., "Why You Should Never Use the Hodrick-Prescott Filter," 2016.

Iacoviello, Matteo, "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, 2005, *95* (3), 739–764.

_ **and Stefano Neri**, "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics*, April 2010, *2* (2), 125–164.

Kaplan, Greg, Kurt Mitman, and Giovanni Violante, "The Housing Boom and Bust: Model Meets Evidence," Technical Report August 2017.

Li, Wenli and Rui Yao, "The Life-cycle Effects of House Price Changes," *Journal of Money, Credit and banking*, 2007, *39* (6), 1375–1409.

_ **, Haiyong Liu, Fang Yang, and Rui Yao**, "Housing over Time and over the Life Cycle: A Structural Estimation," *International Economic Review*, 2016, *57* (4), 1237–1260.

Low, Simon, Matthew Sebag-Montefiore, and Achim Dubel, *Study on the Financial Integration of European Mortgage Markets*, European Mortgage Federation, 2003.

Mack, Adrienne and Enrique Martinez-Garcia, "A Cross-Country Quarterly Database of Real House Prices: A Methodological Note," *Federal Reserve Bank of Dallas Globalization and Monetary Policy Institute Working Paper No. 99*, 2011.

Monacelli, Tommaso, "New Keynesian Models, Durable goods, and Collateral Constraints," *Journal of Monetary Economics*, March 2009, *56* (2), 242–254.

Neumeyer, Pablo A. and Fabrizio Perri, "Business Cycles in Emerging Economies: The Role of Interest Rates," *Journal of Monetary Economics*, 2005, *52*, 345–380.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel, "Housing, Consumption and Asset Pricing," *Journal of Financial Economics*, March 2007, *83* (3), 531–569.

Restrepo-Echavarria, Paulina, "Macroeconomic Volatility: The Role of the Informal Economy," *European Economic Review*, August 2014, *70*, 454–469.

Rotemberg, Julio J., "Sticky Prices in the United States," *The Journal of Political Economy*, December 1982, *90* (6), 1187 – 1211.

Schmitt-Grohé, Stephanie and Martín Uribe, "Closing Small Open Economy Models," *Journal of International Economics*, October 2003, *61* (1), 163–185.

_ **and** _ , *Open Economy Macroeconomics*, Princeton University Press, 2017.

Shimizu, Chihiro, Kiyohiko G. Nishimura, and Tsutomu Watanabe, "Residential Rents and Price Rigidity: Micro Structure and Macro Consequences," *Journal of The Japanese and International Economies*, June 2010, *24* (2), 282 – 299.

Shin, Wonmun, "A New Angle on Excess Consumption Volatility in Emerging Countries: Does House Price Matter?," 2018.

Sommer, Kamila, Paul Sullivan, and Randal Verbrugge, "The Equilibrium Effect of Fundamentals on House Prices and Rents," *Journal of Monetary Economics*, October 2013, *60* (7), 854–870.

Song, In Ho, "Essays on House Prices and Consumption." PhD dissertation, The Ohio State University 2011.

_ , "House Prices and Monetary Policy: Focus on Elasticity of Intra-temporal Substitution between Housing and Consumption," *KDI Policy Study*, 2012, *10.*

Stokey, Nancy L., "Moving Costs, Nondurable Consumption and Portfolio Choice," *Journal of Economic Theory*, November 2009, *144* (6), 2419–2439.

Sun, Xiaojin and Kwok Ping Tsang, "What Drives the Owner-Occupied and Rental Housing Markets? Evidence from an Estimated DSGE Model," *Journal of Money, Credit and Banking*, March 2017, *49* (2-3), 443–468.

Uribe, Martin and Vivian Z. Yue, "Country Spreads and Emerging Countries: Who Drives Whom?," *Journal of International Economics*, 2006, *69*, 6–36.

Verbrugge, Randal J. and Joshua Gallin, "A Theory of Sticky Rents: Search and Bargaining with Incomplete Information," May 2017.

Yang, Fang, "Consumption over the Life Cycle: How Different is Housing?," *Review of Economic Dynamics*, 2009, *12* (3), 423–443.

A Empirical Facts: Comparison with Literature

A.1 Business Cycles: Comparison with Schmitt-Grohé and Uribe (2017)

Table A1: Business Cycles with Schmitt-Grohé and Uribe (2017)

	Shin (2019)			**SGU (2017)**		
	σ_Y	σ_C/σ_Y	$\sigma_{Y,C}$	σ_Y	σ_C/σ_Y	$\sigma_{Y,C}$
Australia	0.92	1.07	0.42	0.77	1.11	0.25
Belgium	0.73	0.90	0.62	0.67	0.92	0.52
Brazil	1.30	1.01	0.67		n.a.	
Canada	0.81	1.00	0.41	0.78	0.91	0.35
Colombia	0.88	1.38	0.16		n.a.	
Denmark	1.08	1.62	0.45	1.26	1.16	0.61
Finland	1.46	1.09	0.60	1.43	0.85	0.32
France	0.57	1.27	0.65	0.50	1.26	0.59
Germany	0.98	1.05	0.57	0.95	1.05	0.49
Greece	1.46	1.42	0.60		n.a.	
Israel	0.91	1.59	0.30	1.64	1.42	0.26
Italy	0.86	1.19	0.53	0.75	1.23	0.57
Japan	1.09	1.08	0.70	0.96	0.94	0.73
Malaysia	2.60	1.02	0.43		n.a.	
Netherlands	1.13	0.96	0.35	0.87	1.26	0.36
New Zealand	1.58	2.39	0.22	1.77	1.02	0.47
Portugal	0.94	1.14	0.63	2.07	1.10	0.89
South Africa	0.93	2.13	0.36	0.87	1.89	0.51
South Korea	1.65	1.23	0.69	1.60	1.27	0.71
Spain	0.79	1.23	0.58	0.92	1.34	0.43
Sweden	1.16	1.17	0.40	1.25	0.78	0.48
Switzerland	0.79	0.84	0.64	0.78	0.68	0.45
United Kingdom	0.94	1.19	0.62	0.79	1.31	0.61
United States	0.81	0.84	0.68	0.75	0.92	0.69

In Table A1, I report second moments of interest for each country: the standard deviations of house prices and output, standard deviations for consumption relative to those for corresponding output, and correlation coefficients between output and consumption. Those under the heading "Shin (2019)" are the second moments calculated using this paper's data set which is described in Section 2, and those under the heading "USG (2017)" are corresponding numbers in Schmitt-Grohé and Uribe (2017). Since data source for both GDP and private consumption as well as methodology such as deflating and detrending are same between this paper's baseline work and Schmitt-Grohé and Uribe (2017), the only difference is sample period. My data set covers from 1970 Q1 to 2017 Q4 on average, while the data in Schmitt-Grohé and Uribe (2017) covers from 1980 Q1 to 2012 Q4. As a result, this paper replicates the work in Schmitt-Grohé and Uribe (2017) successfully, and a variation is observed in several countries due to different sample period. In fact, the correlation between two works is 0.67 (0.94 for output, 0.52 for consumption, and 0.54 for correlation of consumption with output). The result is much clearer when we present the weighted averages of the corresponding individual country statistics, by country group, in Table A2. We confirm, with this paper's data set, the stylized facts in the business cycle of emerging countries documented in Schmitt-Grohé and Uribe (2017); business cycle in rich countries is less volatile than in emerging countries, and there is excess consumption volatility in emerging countries.

Table A2: Business Cycles with Schmitt-Grohé and Uribe (2017)

	Shin (2019)			**SGU (2017)**		
	σ_Y	σ_C/σ_Y	$\sigma_{Y,C}$	σ_Y	σ_C/σ_Y	$\sigma_{Y,C}$
Emerging Countries	1.27	1.26	0.55	1.15	1.50	0.55
Rich Countries	0.88	1.01	0.62	0.81	1.03	0.61

Table A3: Business Cycles with Cesa-Bianchi et al. (2015)

	Shin (2019)			**Cesa-Bianchi et al. (2015)**		
	σ_{HP}	σ_C	σ_Y	σ_{HP}	σ_C	σ_Y
Emerging Countries	2.58	1.54	1.27	4.77	2.43	2.10
Rich Countries	1.69	0.89	0.88	1.94	1.10	1.09

A.2 House Price Volatilities: Comparison with Cesa-Bianchi et al. (2015)

Those under the heading "Cesa-Bianchi et al. (2015)" are the standard deviations for each variable of interest Cesa-Bianchi et al. (2015) presents[1], and again those under the heading "Shin (2019)" are corresponding results in my data work. The main observation by authors of Cesa-Bianchi et al. (2015) is that house prices in emerging countries are more than twice as volatile as in advanced economies ($\sigma_{HP}^{EC} = 4.8 > 1.9 = \sigma_{HP}^{RC}$). This paper's data set captures that finding with a slightly smaller relative size — house price volatility in emerging economies is 1.5 times bigger than that in rich countries ($\sigma_{HP}^{EC} = 2.6 > 1.7 = \sigma_{HP}^{RC}$). Hence, this paper's data set supports the empirical fact about house prices in emerging countries suggested by Cesa-Bianchi et al. (2015) as the stylized fact. However, as I discussed the contribution of Cesa-Bianchi et al. (2015) in Section 2, there is in their work a trade-off between an increase in the number of countries — especially, emerging countries — and shorter samples the data covers with lower reliability. Since a shorter sample has a possibility of exaggerating the standard deviation for each indicator, we can understand why the gap in σ_{HP} between emerging and rich countries is greater in Cesa-Bianchi et al. (2015). Also, even if Cesa-Bianchi et al. (2015) does not highlight, excess consumption volatility is also observed in their work but it shows slight excessiveness ($(\sigma_C/\sigma_Y)_{EC} = 1.1 > 1.0 = (\sigma_C/\sigma_Y)_{RC}$). The main reason for the slightness is due to higher output volatility in emerging countries in Cesa-Bianchi et al. (2015). It is mainly because there are some countries whose output volatility is extremely high in the sample of Cesa-Bianchi et al. (2015), for example, Latvia (5.0%) and Lithuania (4.1%), and the authors fail to control the size of each sample country because their methodology is simple average.

A.3 Correlation between House Price and Consumption: Comparison with Calza et al. (2007)

When recalling the research question of the project — the relationship between house price and consumption volatility, the natural way of thinking is to investigate the correlation between house prices and private consumption. Calza et al. (2007) is a good reference in that it reports the correlation for several countries (mainly advanced countries). The authors observe that the correlation is generally positive, but it varies significantly across countries. To replicate their findings, I modified my data set, and the details are as follows. I cut the sample period to 1980 Q1

[1] Cesa-Bianchi et al. (2015) reports summary statistics for the log-difference of real house prices, real private consumption, and real GDP for the two groups of countries. Three points need to be mentioned; first, they use log-difference to detrend, second, variables are deflated using CPI, and third, they present mean, median, standard deviation, auto-correlation, and pairwise correlation as the summary statistics. Besides, they report summary statistics for equity price in order to compare prices for two substitutable assets: housing as a non-financial asset and equity as a financial asset.

Table A4: Correlations between House Price and Consumption

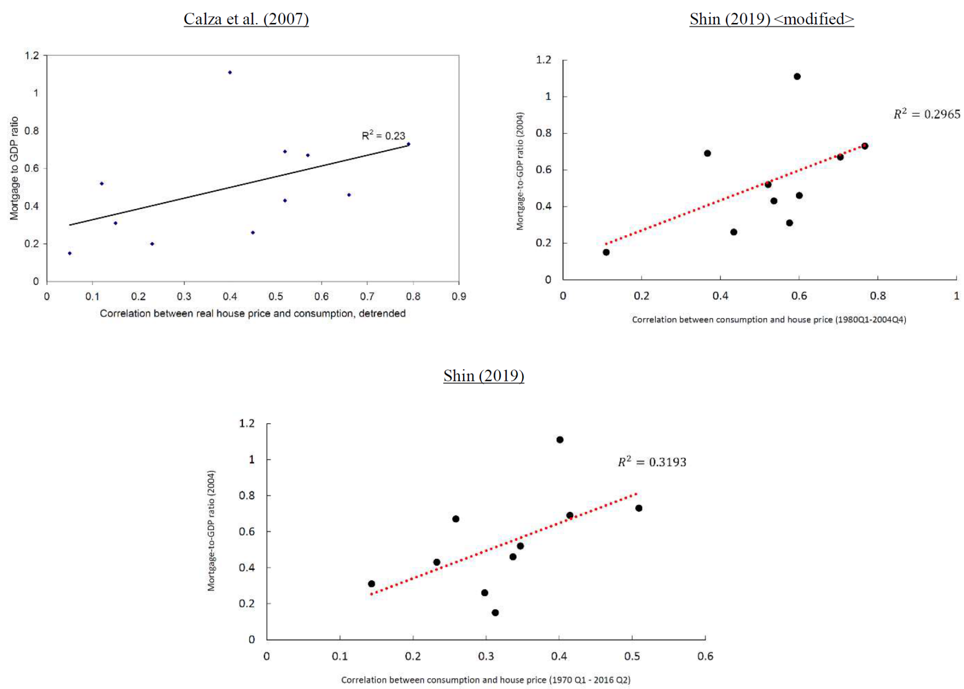
	Calza et al. (2007)	**Shin (2019)** $\langle m \rangle$	**Shin (2019)**
U.K.	0.79	0.77	0.51
Spain	0.66	0.60	0.34
Denmark	0.57	0.70	0.26
Canada	0.52	0.54	0.23
U.S.	0.52	0.37	0.41
France	0.45	0.43	0.30
Netherlands	0.40	0.59	0.40
Belgium	0.15	0.58	0.14
Germany	0.12	0.52	0.35
Italy	0.05	0.11	0.31
Correlation with Calza et al. (2007)		**0.63**	**0.47**

* Shin (2019) $\langle m \rangle$ uses the modified data set. Please refer to the text in *Appendix A.3* for details.

- 2004 Q4 to match the sample for Calza et al. (2007). House prices and private consumption series for each country are deflated using CPI and detrend by HP filter with constant 1,600 following Calza et al. (2007). Table A4 shows the comparison. The first column under the heading "Calza et al. (2007)" is the correlation for ten countries Calza et al. (2007) selects, and the other columns are from my data work. The column whose heading has $\langle m \rangle$ is produced using the modified data set explained above, and the last column follows this paper's original data set.

When comparing the first column and the second column, I am not able to conclude this paper's data set successfully replicates the correlations of Calza et al. (2007) in terms of quantitativeness because there are some countries whose correlations are significantly different, for instance, Belgium and Germany are that cases. Since I controlled the gap of detrending and deflating methodology, the source of variation is strongly suspected to be the data source. However, the second column is qualitatively similar to the first column in two-fold; First, the correlation between two columns is 0.63; Second, Shin (2019) $\langle m \rangle$ captures the cross-country patterns suggested in Calza et al. (2007). Calza et al. (2007) describes that correlation shows a significant increasing pattern against the indicator of development in mortgage markets such as mortgage-to-GDP ratio and the degree of completeness in mortgage markets.[2] The second column of Table A4 also shows same pattern against the indicators of Calza et al. (2007).[3] Please refer to Figures A1 and A2. Then the natural follow-up question in interest is whether this paper's original data set with longer periods and different deflating and detrending shows a similar result or not. The answer is yes, so it is robust in the original data set. The correlation of the last column with the first column is 0.47, and it also has an increasing pattern against the mortgage market indicator, as in Figures A1 and A2.

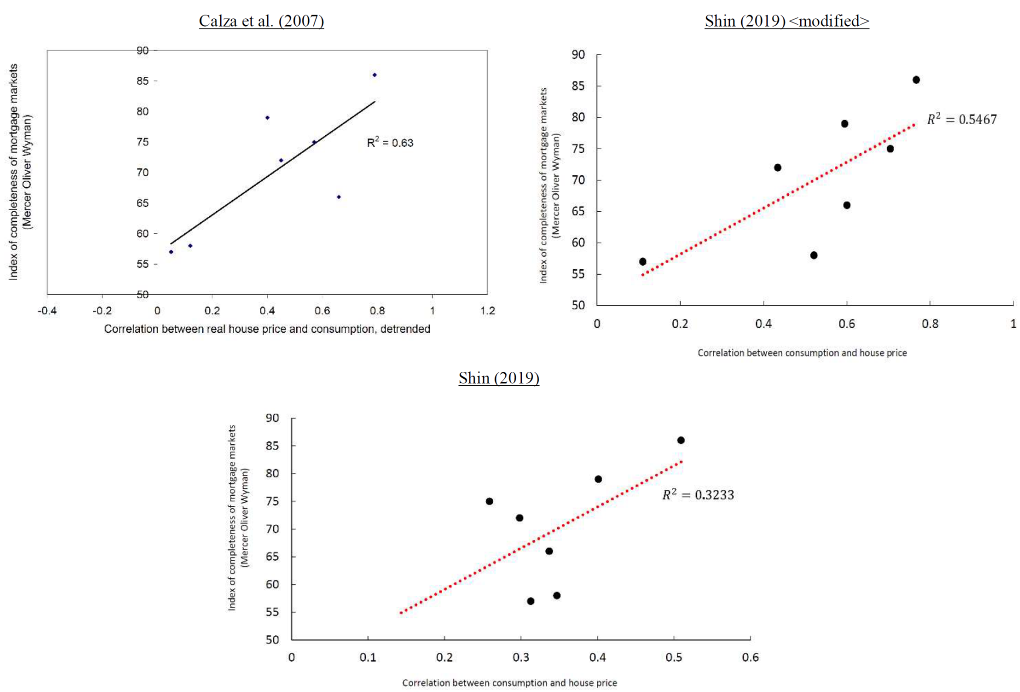
Figure A1: Correlation $\sigma_{HP,C}$ and Mortgage-to-GDP Ratio



[3]Mortage-debt-to-GDP ratio and the completeness index of mortgage market are the indicator the authors choose for measuring each country's mortgage market development. Because the sample period in Calza et al. (2007) ends in 2004 Q1, they report each country's mortgage-to-GDP ratio as of 2004, which is computed by dividing the outstanding amount of mortgage debt by GDP. The completeness index proposed by Mercer Oliver Wyman (Low et al. (2003)) mainly measures the number of mortgage products available in a given market, but the index is only available for several EU countries and is one-time index as of 2003.

[4]The published version of this paper is Calza et al. (2013), and the authors drop the finding related to the correlation between house price and private consumption in the published version. Also, Calza et al. (2013) presents the mortgage market index constructed by IMF WEO (2008) instead of the completeness index in Calza et al. (2007) as the indicator of measuring each country's mortgage market development.

Figure A2: Correlation $\sigma_{HP,C}$ and MOW Completeness Index



B Robustness of Empirical Facts

B.1 Robustness of Business Cycle Facts

Table B1: Robustness: Different Detrending Filters

	σ_{HP}	σ_Y	σ_C/σ_Y	$\sigma_{Y,C}$
		Log Difference		
Emerging	2.58	1.27	1.26	0.53
Rich	1.69	0.88	1.01	0.62
		HP filter		
Emerging	5.31	1.79	1.30	0.66
Rich	3.70	1.46	0.84	0.79
		Log Quadratic filter		
Emerging	17.39	5.27	1.12	0.76
Rich	10.99	3.06	0.92	0.84

Table B2: Robustness: Different Sample Periods

	Baseline (1970 Q1 to 2017 Q4)			**Before Global Financial Crisis** (1970 Q1 to 2006 Q4)		
	σ_{HP}	σ_Y	σ_C/σ_Y	σ_{HP}	σ_Y	σ_C/σ_Y
Emerging	2.58	1.27	1.26	2.77	1.33	1.27
Rich	1.69	0.88	1.01	1.59	0.87	1.02

	Before Asian Financial Crisis (1970 Q1 to 1997 Q2)			**Changing the starting date** (1980 Q1 to 2017 Q4)		
	σ_{HP}	σ_Y	σ_C/σ_Y	σ_{HP}	σ_Y	σ_C/σ_Y
Emerging	2.91	1.38	1.16	2.36	1.20	1.24
Rich	1.63	0.96	1.02	1.58	0.79	1.03

B.2 Robustness of Cross-Country Evidence

Figure B1: Robustness: Different Detredning Filters (HP Filter)

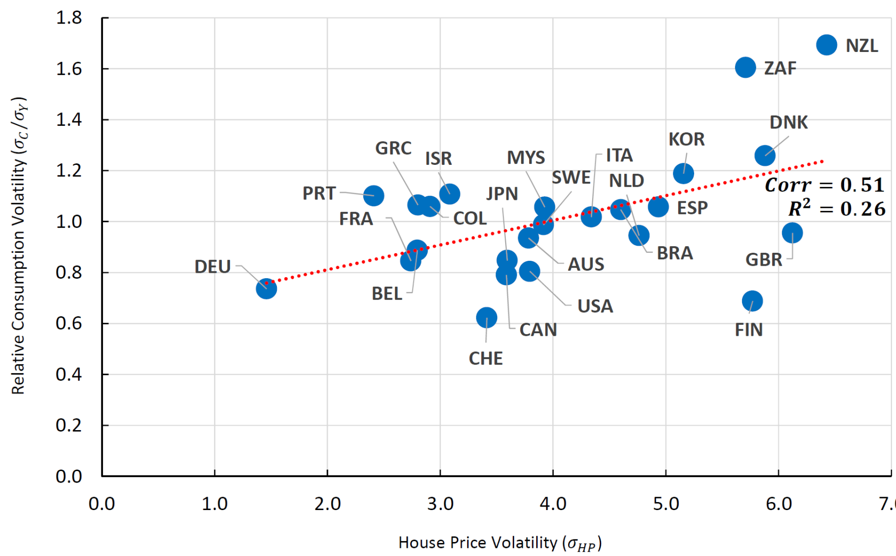


Figure B2: Robustness: Different Detredning Filters (Quadratic Filter)

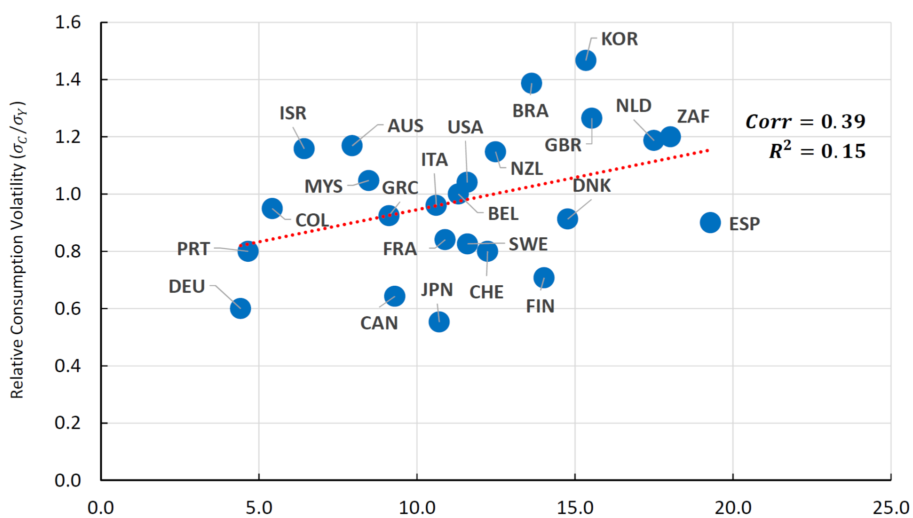


Figure B3: Robustness: Different Sample Periods (1997-2017)

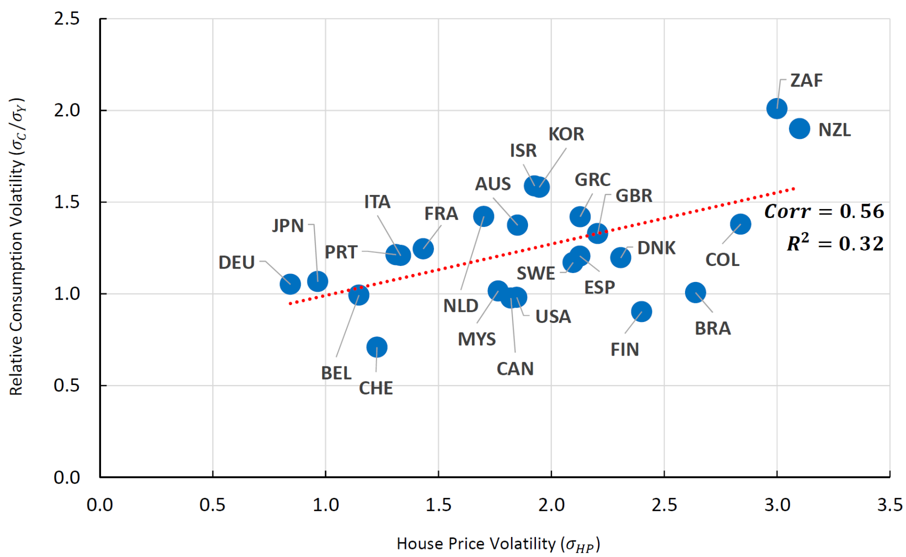


Figure B4: Robustness: Without Outlier (New Zealand)

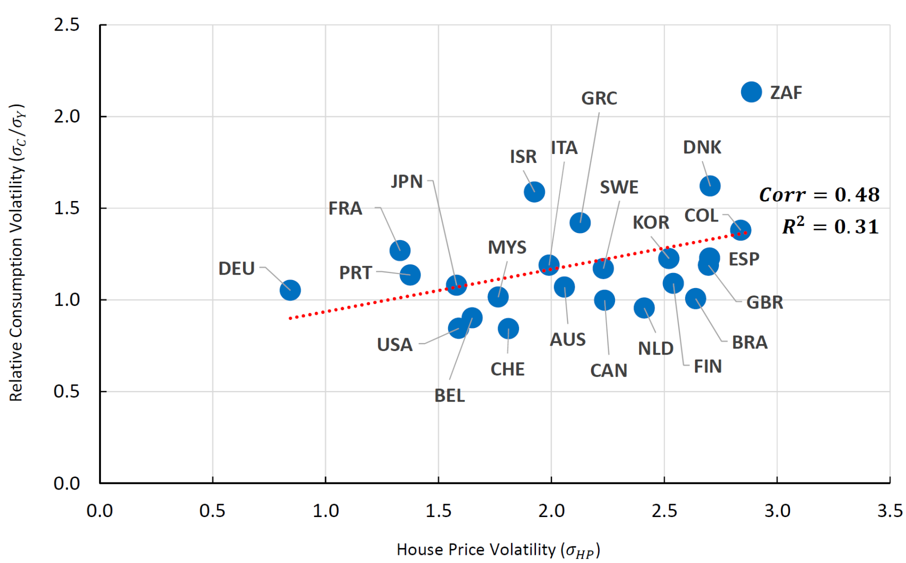
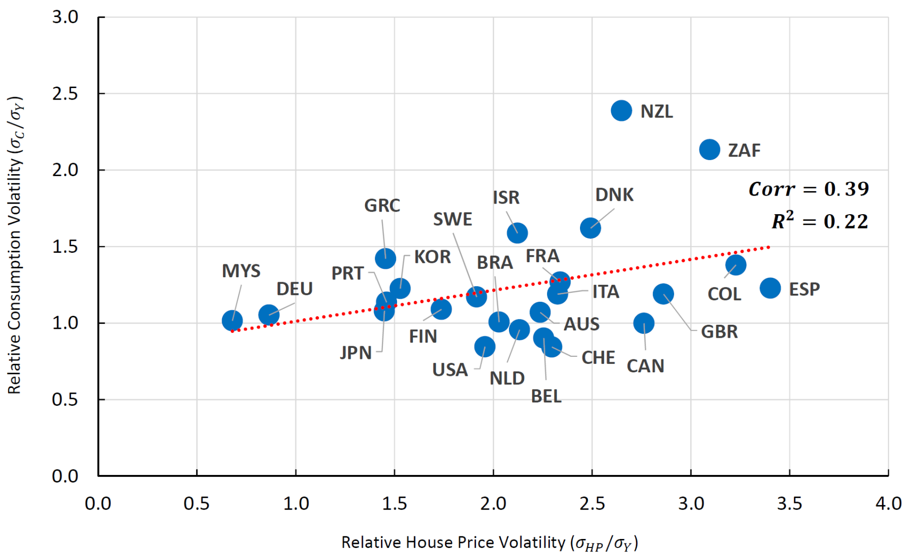


Figure B5: Robustness: Relative House Price Volatility (σ_{HP}/σ_Y



C Equilibrium Conditions and the Steady States

An equilibrium in the full model is then a set of processes of $\{c_t^O$, c_t^R, c_t, y_t, nh_t, ih_t, h_t, s_t^O, s_t^R, d_{t+1}, X_t^O, X_t^R, $n_{c,t}$, $n_{h,t}$, Π_t, λ_t, λ_t^O, λ_t^R, μ_t, q_t, ρ_t, r_t, Z_t, TC_t, HC_t, NHC_t, H_t, D_{t+1}, Y_t, HP_t, $R_t\}_{t=0}^{\infty}$ satisfying (2), (3), (5), (6), (8), (9), (11) to (14), (19), (21), (24) to (34), (36) to (43), (45) to (57), given the processes $\{A_{c,t}$, $A_{h,t}$, ε_t^b, ε_t^s, ε_t^m, $\varepsilon_t^r\}_{t=0}^{\infty}$ and the initial condition $\{n_{c,0}$, $n_{h,0}$, nh_{-1}, h_{-1}, d_0, ρ_{-1}, $r_{-1}\}$. Therefore, there are 31 endogenous variables, and we need 29 equations to determine equilibrium. Also, we need 6 equations which specify exogenous variable processes.

$$\gamma\left(X_t^O\right)^{\frac{1-\eta}{\eta}}\left(c_t^O\right)^{-\frac{1}{\eta}}=\lambda_t^O \tag{C.1}$$

$$\gamma\left(X_t^R\right)^{\frac{1-\eta}{\eta}}\left(c_t^R\right)^{-\frac{1}{\eta}}=\lambda_t^R \tag{C.2}$$

$$\lambda_t=\omega\lambda_t^O+(1-\omega)\,\lambda_t^R \tag{C.3}$$

$$c_t=\omega c_t^O+(1-\omega)\,c_t^R \tag{C.4}$$

$$y_t=A_{c,t}\left(n_{c,t}\right)^{\alpha_c} \tag{C.5}$$

$$nh_t=A_{h,t}\left(n_{h,t}\right)^{\alpha_h} \tag{C.6}$$

$$\Pi_t=y_t+q_t nh_{t-1}-\frac{\phi_c}{2}\left(n_{c,t}-n_{c,t-1}\right)^2-\frac{\phi_h}{2}\left(n_{h,t}-n_{h,t-1}\right)^2 \tag{C.7}$$

$$\rho_t=\frac{1-\gamma}{\gamma}\varepsilon_t^s\left(\frac{\omega Z_t^\eta c_t^O+(1-\omega)\,c_t^R}{\omega h_t}\right)^{\frac{1}{\eta}} \tag{C.8}$$

$$(\nu-1)\,\omega s_t^R=\nu\frac{1}{Z_t}\omega s_t^R-\theta\left(\frac{\rho_t}{\rho_{t-1}}-1\right)\frac{1}{\rho_{t-1}}+\beta\theta E_t\left[\frac{\lambda_{t+1}^O}{\lambda_t^O}\frac{\varepsilon_{t+1}^b}{\varepsilon_t^b}\left(\frac{\rho_{t+1}}{\rho_t}-1\right)\frac{\rho_{t+1}}{\rho_t^2}\right] \tag{C.9}$$

$$s_t^R=\frac{\omega c_t^R}{\omega Z_t^\eta c_t^O+(1-\omega)\,c_t^R}h_t \tag{C.10}$$

$$\omega s_t^O+(1-\omega)\,s_t^R=\omega h_t \tag{C.11}$$

$$ih_t=h_t-(1-\delta)\,h_{t-1} \tag{C.12}$$

$$nh_{t-1}=\omega\left[ih_t+\frac{\phi_i}{2}\left(h_t-h_{t-1}\right)^2\right] \tag{C.13}$$

$$\lambda_t^O\varepsilon_t^b\left(1-\mu_t\right)=\beta\left(1+r_t\right)E_t\left[\lambda_{t+1}^O\varepsilon_{t+1}^b\right] \tag{C.14}$$

$$q_t\left[1+\phi_i\left(h_t-h_{t-1}\right)\right]=\frac{\rho_t}{Z_t}+\beta E_t\left[\frac{\lambda_{t+1}^O}{\lambda_t^O}\frac{\varepsilon_{t+1}^b}{\varepsilon_t^b}q_{t+1}\left(\left(1-\delta\right)+\phi_i\left(h_{t+1}-h_t\right)\right)\right]+\frac{m\mu_t q_t}{1+r_t}\varepsilon_t^d \tag{C.15}$$

$$\begin{aligned}&\alpha_c A_{c,t}\left(n_{c,t}\right)^{\alpha_c-1}-\phi_c\left(n_{c,t}-n_{c,t-1}\right)+\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{\varepsilon_{t+1}^b}{\varepsilon_t^b}\phi_c\left(n_{c,t+1}-n_{c,t}\right)\right]\\&=\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{\varepsilon_{t+1}^b}{\varepsilon_t^b}\left(\alpha_h q_{t+1}A_{h,t}\left(n_{h,t}\right)^{\alpha_h-1}+\phi_h\left(n_{h,t+1}-n_{h,t}\right)\right)\right]-\phi_h\left(n_{h,t}-n_{h,t-1}\right)\end{aligned} \tag{C.16}$$

$$n_{c,t}+n_{h,t}=1 \tag{C.17}$$

$$X_t^O=\left[\gamma\left(c_t^O\right)^{\frac{\eta-1}{\eta}}+\left(1-\gamma\right)\varepsilon_t^s\left(s_t^O\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{C.18}$$

$$X_t^R=\left[\gamma\left(c_t^R\right)^{\frac{\eta-1}{\eta}}+\left(1-\gamma\right)\varepsilon_t^s\left(s_t^R\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{C.19}$$

$$\begin{aligned}&c_t^O+\frac{\rho_t}{Z_t}s_t^O+q_t\left[ih_t+\frac{\phi_i}{2}\left(h_t-h_{t-1}\right)^2\right]+\left(1+r_{t-1}\right)d_t\\&=\Pi_t+d_{t+1}+\frac{\rho_t}{Z_t}h_t+\frac{1}{\omega}\left[\rho_t\omega s_t^R\left(1-\frac{1}{Z_t}\right)-\frac{\theta}{2}\left(\frac{\rho_t}{\rho_{t-1}}-1\right)^2\right]\end{aligned} \tag{C.20}$$

$$d_{t+1}=\frac{mq_t h_t}{1+r_t}\varepsilon_t^d \tag{C.21}$$

$$c_t^R+\rho_t s_t^R=\Pi_t \tag{C.22}$$

$$r_t=r^*+\psi\left(e^{\omega d_{t+1}-\bar{d}}-1\right)+e^{\varepsilon_t^r-1}-1) \tag{C.23}$$

$$HC_t=\left(\frac{1}{P_t}\right)\times\left[\,\omega\rho_t s_t^O+(1-\omega)\rho_t s_t^R\,\right]=\frac{\omega\rho_t h_t}{P_t} \tag{C.24}$$

$$NHC_t=\left(\frac{1}{P_t}\right)\times\left[\,\omega c_t^O+(1-\omega)c_t^R\,\right]=\frac{c_t}{P_t} \tag{C.25}$$

$$TC_t=HC_t+NHC_t \tag{C.26}$$

$$H_t=\frac{\omega h_t}{P_t} \tag{C.27}$$

$$D_{t+1}=\frac{\omega d_{t+1}}{P_t} \tag{C.28}$$

$$Y_t=\frac{GDP_t}{P_t}=GDP_t|_{q_{ss},\rho_{ss}} \tag{C.29}$$

$$HP_t=\frac{q_t}{P_t} \tag{C.30}$$

$$R_t=\frac{\rho_t}{P_t} \tag{C.31}$$

$$\ln A_{c,t+1}=\rho_c\ln A_{c,t}+\sigma_c\xi_{c,t+1} \tag{C.32}$$

$$\ln A_{h,t+1}=\rho_h\ln A_{h,t}+\sigma_h\xi_{h,t+1} \tag{C.33}$$

$$\ln \varepsilon^b_{t+1} = \rho_b \ln \varepsilon^b_t + \sigma_b \xi_{b,t+1} \tag{C.34}$$

$$\ln \varepsilon^s_{t+1} = \rho_s \ln \varepsilon^s_t + \sigma_s \xi_{s,t+1} \tag{C.35}$$

$$\ln \varepsilon^m_{t+1} = \rho_m \ln \varepsilon^m_t + \sigma_m \xi_{m,t+1} \tag{C.36}$$

$$\ln \varepsilon^r_{t+1} = \rho_r \ln \varepsilon^r_t + \sigma_r \xi_{r,t+1} \tag{C.37}$$

Note that we need intermediate variables (GDP_t, P_t) for Equations (C.24) to (C.31):

$$GDP_t = \Pi_t + \frac{\rho_t}{Z_t}\omega h_t + \rho_t \omega s^R_t \left(1 - \frac{1}{Z_t}\right) - \frac{\theta}{2}\left(\frac{\rho_t}{\rho_{t-1}} - 1\right)^2$$
$$P_t = \frac{GDP_t|_{q_t,\rho_t}}{GDP_t|_{q_{ss},\rho_{ss}}}$$

The Steady States of the Economy

To begin with, r^* is the steady state level of interest rate, by definition. We can easily earn the steady state level of stochastic shock processes from (C.32) and (C.37).

$$A^*_c = A^*_h = \varepsilon^*_b = \varepsilon^*_s = \varepsilon^*_m = \varepsilon^*_r = 1$$

From the labor market clearing condition (C.17),

$$n^*_h = 1 - n^*_c \tag{C.i}$$

Then, substituting the above condition (C.i) into the steady state version of (C.15) (hereafter, equilibrium condition label denotes the steady state version of it.) gives us

$$q^* = \frac{\alpha_c}{\alpha_h}\frac{(1-n^*_c)^{1-\alpha_h}}{n^{*1-\alpha_c}_c} \tag{C.ii}$$

Also, plugging (C.i) into (C.6) yields

$$nh^* = (1-n^*_c)^{\alpha_h}\bar{l}^{\alpha_l} \tag{C.iii}$$

Similarly, (C.i) and (C.12) gives us

$$ih^* = \delta h^* \tag{C.iv}$$

The housing market clearing condition at the steady state from (C.13) is

$$nh^* = \omega\delta h^* \tag{C.v}$$

Let's move on to the households, from (C.14),

$$\mu^* = 1 - \beta(1 + r^*)$$

Since $\beta(1 + r^*) < 0$ is assumed, the shadow price parameter μ^* at the steady state is greater than zero. As we know μ^*, we can derive the steady state relationship between house price and rental price from (C.15)

$$\rho^* = Z^* q^* \left[1 - \beta(1 - \delta) - \frac{m\mu^*}{1 + r^*}\right] \tag{C.vi}$$

Here, Z^* is the steady state level of rental agency markup which is $\nu/(\nu - 1)$ from (C.9). Let define M as $\left[1 - \beta(1 - \delta) - \frac{m\mu^*}{1+r^*}\right]$, then (C.vi) can be rewritten by $\rho^* = Z^* M q^*$. Next, from (C.21),

$$d^* = \frac{mq^* h^*}{1 + r^*} \tag{C.vii}$$

Note that the parameter for the steady state level of aggregate debt, $\bar{d}$ must be equal to ωd^* by (C.23). Moving on to (C.8), we can earn the expression for the term ωh^*

$$\omega h^* = \left(\frac{\gamma}{1 - \gamma}\rho^*\right)^{-\eta} [\omega Z^{*\eta} c_O^* + (1 - \omega) c_R^*]$$

Using the above expression, s_R^* can be re-written from (C.10) by

$$s_R^* = \left(\frac{1 - \gamma}{\gamma}\right)^{\eta} \rho^{*-\eta} c_R^* \tag{C.viii}$$

By plugging (C.viii) into (C.22), we get the steady state level of renter's non-housing consumption, c_R^*

$$c_R^* = \frac{\Pi^*}{1 + \left(\frac{1-\gamma}{\gamma}\right)^{\eta} \rho^{*1-\eta}} \tag{C.ix}$$

Note that the quadratic adjustment costs vanish at the steady state, and $y^* = n_c^{*\alpha_c}$ from (C.5). As a result, (C.7) gives us the steady state level of Π_t

$$\Pi^* = n_c^{*\alpha_c} + \omega\delta q^* h^* \tag{C.x}$$

We earn the expression for c_O^* by rearranging (C.8)

$$c_O^* = \left(\frac{\gamma}{1-\gamma}\right)^{\eta}\left(\frac{\rho^*}{Z^*}\right)^{\eta} h^* - \frac{1-\omega}{\omega}\frac{1}{Z^{*\eta}} c_R^* \tag{C.xi}$$

Also, we earn the expression for s_O^* by rearranging (C.11) using (C.viii)

$$s_O^* = h^* - \frac{1-\omega}{\omega}\left(\frac{1-\gamma}{\gamma}\right)^{\eta} \rho^{*-\eta} c_R^* \tag{C.xii}$$

By putting (C.iv),(C.vi), (C.vii), (C.viii), (C.ix),(C.x), (C.xi), and (C.xii) into (C.20),

$$\begin{aligned}\left[\left(\frac{\gamma}{1-\gamma}\right)^{\eta}\frac{M^{1-\eta}}{q^{*1-\eta}} + \frac{mr^*}{1+r^*} + (1-\omega)\,\delta\right] q^* h^* = n_c^{*\alpha_c} + \frac{n_c^{*\alpha_c} + \omega\delta q^* h^*}{1+\left(\frac{1-\gamma}{\gamma}\right)^{\eta}(Z^* M q^*)^{1-\eta}}\cdot \\ \left[\frac{1-\omega}{\omega}\frac{1}{Z^{*\eta}}\left(1+\left(\frac{1-\gamma}{\gamma}\right)^{\eta}(Mq^*)^{1-\eta}\right) + \frac{Z^*-1}{Z^*}\left(\frac{1-\gamma}{\gamma}\right)^{\eta}(Mq^*)^{1-\eta}\right]\end{aligned} \tag{C.xiii}$$

whose undetermined steady state variables are three: q^*, h^*, and $n_c^{*\alpha_c}$. Recall (C.iii) and (C.v), which gives us

$$h^* = \frac{(1-n_c^*)^{\alpha_h}}{\omega\delta} \tag{C.xiv}$$

Since q^* and h^* are expressed as functions of n_c^* by (C.ii) and (C.xiv), respectively, the equation (C.xiii) can determine the steady state level of labor allocation to consumption goods production sector, n_c^*. Actually, we cannot derive the closed form of n_c^*, so we should count on the computation program to solve (C.xiii).

Therefore, we can get a set of constant sequences $c_t^O = c_O^* > 0$, $c_t^R = c_R^* > 0$, $c_t = c^* > 0$, $y_t = y^* > 0$, $nh_t = nh^* > 0$, $ih_t = ih^* > 0$, $h_{t-1} = h_t = h^* > 0$, $s_t^O = s_O^* > 0$, $s_t^R = s_R^* > 0$, $d_t = d_{t+1} = d^* > 0$, $X_t^O = X_O^* > 0$, $X_t^R = X_R^* > 0$, $n_{c,t} = n_c^* > 0$, $n_{h,t} = n_h^* > 0$, $\Pi_t = \Pi^* > 0$, $\lambda_t = \lambda^* > 0$, $\lambda_t^O = \lambda_O^* > 0$, $\lambda_t^R = \lambda_R^* > 0$, $\mu_t = \mu^* > 0$, $q_t = q^* > 0$, $\rho_t = \rho^* > 0$, $r_t = r^*$, $Z_t = Z^*$, $TC_t = TC^* > 0$, $HC_t = HC^* > 0$, $NHC_t = NHC^* > 0$, $H_t = H^* > 0$, $D_{t+1} = D^* > 0$, $Y_t = Y^* > 0$, $HP_t = HP^* > 0$, and $R_t = R^* > 0$. The multiplier for the collateral constraint should be positive since the constraint is binding at the steady state. Shocks are also constant at the steady state, $A_{c,t} = A_c^*$, $A_{h,t} = A_h^*$, $\varepsilon_t^b = \varepsilon_b^*$, $\varepsilon_t^s = \varepsilon_s^*$, $\varepsilon_t^m = \varepsilon_m^*$, and $\varepsilon_t^r = \varepsilon_r^*$.

$$A_c^* = A_h^* = \varepsilon_b^* = \varepsilon_s^* = \varepsilon_m^* = \varepsilon_r^* = 1$$

$$Z^* = \frac{\nu}{\nu - 1}$$

n_c^* such that

$$\left[\left(\frac{\gamma}{1-\gamma}\right)^{\eta} \frac{M^{1-\eta}}{q^{*1-\eta}} + \frac{mr^*}{1+r^*} + (1-\omega)\,\delta\right] q^* h^* = n_c^{*\alpha_c} + \frac{n_c^{*\alpha_c} + \omega\delta q^* h^*}{1 + \left(\frac{1-\gamma}{\gamma}\right)^{\eta} (Z^* M q^*)^{1-\eta}} \cdot \left[\frac{1-\omega}{\omega}\frac{1}{Z^{*\eta}}\left(1 + \left(\frac{1-\gamma}{\gamma}\right)^{\eta} (Mq^*)^{1-\eta}\right) + \frac{Z^*-1}{Z^*}\left(\frac{1-\gamma}{\gamma}\right)^{\eta} (Mq^*)^{1-\eta}\right]$$

$$n_h^* = 1 - n_c^*$$

$$q^* = \frac{\alpha_c}{\beta\alpha_h}\frac{(1-n_c^*)^{1-\alpha_h}}{n_c^{*1-\alpha_c}}$$

$$h^* = \frac{(1-n_c^*)^{\alpha_h}}{\omega\delta}$$

$$nh^* = \omega\delta h^* \quad \text{or} \quad nh^* = (1-n_c^*)^{\alpha_h}$$

$$ih^* = \delta h^*$$

$$\Pi^* = n_c^{*\alpha_c} + \omega\delta q^* h^*$$

$$d^* = mq^* h^*$$

$$\mu^* = 1 - \beta\,(1 + r^*) \quad > 0$$

$$\rho^* = Z^* M q^* = Z^* q^* [1 - \beta(1-\delta) - m\mu^*] \quad > 0$$

$$c_R^* = \frac{\Pi^*}{1 + \left(\frac{1-\gamma}{\gamma}\right)^{\eta} \rho^{*1-\eta}} \quad > 0$$

$$c_O^* = \left(\frac{\gamma}{1-\gamma}\right)^{\eta}\left(\frac{\rho^*}{Z^*}\right)^{\eta} h^* - \frac{1-\omega}{\omega}\frac{1}{Z^{*\eta}} c_R^* \quad > 0$$

$$c^* = \omega c_O^* + (1-\omega) c_R^*$$

$$y^* = (n_c^*)^{\alpha_c}$$

$$s_O^* = h^* - \frac{1-\omega}{\omega}\left(\frac{1-\gamma}{\gamma}\right)^{\eta}\frac{c_R^*}{\rho^{*\eta}} \quad \text{and} \quad s_R^* = \left(\frac{1-\gamma}{\gamma}\right)^{\eta}\frac{c_R^*}{\rho^{*\eta}}$$

$$X_O^* = \left[\gamma(c_O^*)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_O^*\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \quad \text{and} \quad X_R^* = \left[\gamma(c_R^*)^{\frac{\eta-1}{\eta}} + (1-\gamma)\left(s_R^*\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$\lambda_O^* = \gamma\left(X_O^*\right)^{\frac{1-\eta}{\eta}}\left(c_O^*\right)^{-\frac{1}{\eta}}, \quad \lambda_R^* = \gamma\left(X_R^*\right)^{\frac{1-\eta}{\eta}}\left(c_R^*\right)^{-\frac{1}{\eta}}, \quad \text{and} \quad \lambda^* = \omega\lambda_O^* + (1-\omega)\,\lambda_R^*$$

$$HC^* = \omega\rho^* h^*, \qquad NHC^* = c^*, \quad \text{and} \quad TC^* = HC^* + NHC^*$$

$$H^* = \omega h^*, \qquad D^* = \omega d^*, \quad \text{and} \quad Y^* = \Pi^* + \left(\frac{\rho^*}{Z^*}\right)\omega h^* + \rho^*\omega s_R^*\left(1 - \frac{1}{Z^*}\right)$$

$$HP^* = q^* \quad \text{and} \quad R^* = \rho^*$$