Macroprudential Policy and Asset Liquidity*

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Abstract

This paper develops a dynamic model to study optimal liquidity regulations for multiple assets with differing levels of liquidity. I show that optimal macroprudential policies are affected by both asset liquidity and the multi-asset structure. Lower asset liquidity amplifies drops in asset prices and tightens the collateral constraint during financial crises, thus raising macroprudential taxes to discourage holding. With multiple assets, the marginal benefit of investing in one asset is affected by the future crossprice elasticities of all assets. The effects of cross-price elasticities depend on future trading positions and the tightness of the collateral constraint. Quantitatively, optimal macroprudential policies favor a portfolio with more liquid assets and less borrowing. In the constrained-efficient equilibrium, agents decrease leverage by 9.4% and increase the liquid share of the balance sheet by 2.6% compared with the unregulated equilibrium. The optimal policy lowers the probability of encountering financial crises by 8% and increases consumption by 0.99%. Finally, I provide theoretical and quantitative analyses on the efficacy of the Basel III reform. The Basel III reform increases agents' liquid holdings and decreases the probability of crises. However, it deteriorates welfare, as agents overaccumulate liquid assets.

Keywords: Financial crises; Macroprudential policy; Liquidity; Fire sale. *JEL Codes*: D62, E32, E44, G01, G23, G28.

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1 INTRODUCTION

Following the Great Recession, policy discussions have focused on managing liquidity to prevent future financial crises.¹ The policies' objective, from a macroprudential perspective, is to manage the size and composition of banks' balance sheets in normal times to prevent financial crises.² The macroprudential policy is justified and determined by externalities that lead to inefficient equilibria. Although debt-related externalities are well studied in the literature, no dynamic framework exists that emphasizes the role of assets—and in particular, the role of heterogeneous liquidity that affects externalities and optimal policies. Moreover, the availability of multiple assets may also affect the optimal policies, as the externality of one asset is affected by the cross-price elasticities of other assets.

This paper aims to answer the following questions: How does asset composition affect externalities and optimal macroprudential policies? How can assets with differing liquidity be optimally managed? How effective are the optimal policies and current regulations proposed by Basel III?

This paper develops and quantifies a model with two assets, liquid and illiquid, and an occasionally binding collateral constraint that generates nonlinear amplification. There are two sectors: domestic agents and foreign investors. Domestic agents own banks and invest in assets by raising outside deposits and obtaining some exogenous equity capital. Domestic agents also own firms, whose production relies on domestic holdings of the illiquid asset. Foreign investors demand the two assets and acquire them at lower prices during financial crises. Liquid assets, which can easily be liquidated under financial distress, can be a better buffer against crises. Quantitatively, the government should impose macroprudential policies that raise holdings of the liquid asset by 2.6% of the total assets more than in the unregulated case. The policies reduce agents' leverage by 9.4%, which is much higher than values in the literature on the pecuniary externality.³ Regarding welfare improvement, optimal macroprudential policies lower the probability of undergoing a financial crisis by 8% and increase consumption-equivalent utility by 0.99%. The probability that optimal policies in the two-asset model are of opposite signs compared with the one-asset model is 23.8%.

¹For example, Basel III reform introduced the liquidity coverage ratio (LCR), which is designed to maintain banks' short-term resilience by accumulating high-quality liquid assets. Another policy innovation is regulation of the net stable funding ratio (NSFR), which guarantees banks' long-term resilience through sufficient stable funding.

²See, e.g., Bernanke (2009) and BIS (2008).

³Excess borrowing by decentralized agents is also known as "overborrowing." Bianchi (2011) estimates that overborrowing concerning the debt-to-GDP ratio is around 0.6%. In Benigno et al. (2010), overborrowing ranges from 0.1% to 0.3% in an endowment economy. Bianchi and Mendoza (2011) study a DSGE model with a working capital constraint and measure overborrowing by comparing the ergodic distribution of the leverage ratio, and conclude that overborrowing is less than 1%. Other papers conclude that overborrowing may not exist, and can be replaced instead by underborrowing under certain conditions. (See, e.g., Uribe (2006b); Benigno et al. (2010); Davila and Korinek (2018); and Schmitt-Grohe and Uribe (2018))

To highlight the importance of asset liquidity and its policy implications, I document the empirical dynamics of the liquid share, which is defined as the ratio of liquid holdings to total assets, following sudden stop events and financial crises in both emerging and advanced economies. The observation that the liquid share dropped in those events suggests that financial institutions tend to sell liquid assets when they are forced to liquidate assets. This trend in asset sales implies that liquid assets are a better buffer against financial crises and should be accumulated ex-ante from a macroprudential perspective.

The normative perspective I consider emphasizes the pecuniary externality,⁴ which operates via the collateral constraint and refers to the effects of the individual trades on future asset prices. The pecuniary externality can be divided into two parts. First, changes in future prices can benefit or harm agents, depending on whether agents are asset buyers or sellers. Second, agents benefit from higher asset prices, as they loosen the future collateral constraint. The social planner improves welfare by internalizing the externality via macroprudential policies that guide the competitive equilibrium (hereafter, CE) toward the solution of the social planner (hereafter, SP).

To study policies that affect the asset composition, I extend the model of Mendoza and Smith (2006) by adding two key features: (1) heterogeneous liquidity across assets and (2) a two-asset structure. Both features affect optimal policies. The two assets differ in their liquidity, which is characterized by market and funding liquidity. Market liquidity measures the ease with which assets can be traded and is captured by a quadratic transaction cost paid by investors during transactions. The transaction cost can be interpreted as either the real cost of finding a trading counterpart or a premium that compensates the asymmetric information. Funding liquidity captures the ease with which assets can be liquidated for funding and is proxied by the collateral value of assets. The liquid asset features both higher market and funding liquidity,⁵ but a lower gross dividend, as the illiquid asset can also be used for production.

In the competitive equilibrium, domestic agents invest in two assets based on the optimality conditions of assets where demand of the illiquid asset is higher, as it also serves as working capital that generates production. However, the illiquid asset provides a lower buffer to prevent crises, as it features a larger price drop during crises. Specifically, when

⁴Other externalities exist that lead to inefficiencies, such as fire-sale externalities (Stein (2012)); aggregate demand externalities (Farhi and Werning (2016)); and externalities that result from strategic complementarities (Ruckes (2004) and Dell'Ariccia and Marquez (2006)). See De Nicol et al. (2012) for a detailed survey of the literature.

⁵With the two kinds of liquidity being either low or high, assets can, in principle, be classified into four groups. However, these two kinds of liquidity mutually reinforce each other (Brunnermeier and Pedersen (2009)), and therefore assets with high funding liquidity also tend to have high market liquidity. This argument simplifies the analysis of liquidity, since I will now focus only on two assets: those with both higher funding and market liquidity, and those with both lower funding and market liquidity.

an asset has lower market liquidity, the transaction cost becomes higher. A negative shock to the initial wealth in the future period, such as a decrease in the current asset holding, provides an incentive for agents to sell more assets to compensate for the transaction cost. As asset demand declines, the asset price drops and the collateral constraint tightens. The social planner, who takes into account the effect of prices on the collateral constraint, tends to hold fewer illiquid assets compared with the decision of domestic agents.

Asset liquidity matters for the design of macroprudential policies precisely because it affects future price elasticities with respect to the current investment. Market illiquidity magnifies the price elasticity and affects optimal policies, depending on the future trading positions of assets and the future tightness of the collateral constraint. Moreover, I show that asset liquidity influences not only macroprudential policies in normal times but also the asset dynamics during financial crises. With funding liquidity in place, the asset dynamics are not trivial as the two kinds of liquidity generate opposite effects on agents' willingness to sell the liquid asset under adverse shocks. Market liquidity encourages agents to sell the liquid holding because of its low transaction cost, whereas funding liquidity discourages banks from selling since it will shrink agents' borrowing capacity. The relative strength of the two forces will determine the adjustment in the asset composition.

I also show analytically that the two-asset structure matters because the optimal macroprudential policy of one asset is affected by its pecuniary externality, which not only contains the effect of the investment on its own future price but also on the future price of the other asset.⁶ Depending on the future trading position of the other asset and the future tightness of the collateral constraint, the cross-price elasticity affects the marginal benefit of purchasing the asset and the optimal level of holdings. For example, suppose agents will be sellers in the next period in the one-asset model; then, they will benefit from a higher future price. The government can achieve this goal by subsidizing the current purchase of the asset that increases future wealth through dividends and eventually raises the asset demand and the price. If agents tend to sell both assets in the future, adding an additional asset magnifies the optimal subsidy, as the increase in future wealth can now benefit agents by increasing both assets' prices.

More importantly, I show that the additional asset offsets or switches the sign of the optimal policy when agents tend to be sellers of one asset and buyers of the other. For example, suppose agents tend to be sellers in the one-asset model; the model predicts that

⁶Some papers that focus on intermediation have proposed two-asset models (see, e.g., Brunnermeier and Sannikov (2014); He and Krishnamurthy (2011); Elenev et al. (2018); Diamond and Kashyap (2016); Kashyap et al. (2017)). However, those two-asset models do not allow analyses of the pricing effects between two assets for three reasons: (1) the intermediary either invests in only one asset and borrows from the other, implying that the model is, in fact, a one-asset model; (2) the assumption that at least one asset's price is exogenous; and (3) the assumption that at least one asset is in zero-supply.

the central planner should subsidize the asset purchase in the current period. With the additional asset in place, the rate of the subsidy shrinks if agents happen to be buyers of this asset in the future. The fact that agents benefit from buying the additional asset at a lower price offsets the benefit of achieving higher future wealth through a current subsidy. Moreover, the optimal policy for the initial asset may, in fact, switch from a subsidy to a tax when the benefit of lowering the price of the additional asset outweighs the benefit of raising the price of the initial asset.

The scenario in which agents buy the illiquid asset and sell the liquid asset occurs when their wealth is not too high or too low. If agents are wealthy, they tend to raise consumption and purchase both assets. As agents' wealth shrinks, they lower the purchase of both assets. There exists a turning point at which agents try to sell assets, but foreign investors can only pick up the asset demand of the liquid asset because the transaction cost of the illiquid asset is too high. It is only when agents are poor that they are willing to sell off the illiquid asset at a very low price to compensate for the transaction cost. Empirically, this concern about the policy switch is valid, as agents tend to sell the liquid asset and buy the illiquid asset during sudden stop events. Quantitatively, the probability that the two assets generate externalities of opposite signs and the effect of the illiquid asset outweighs the liquid asset is 23.8%.

Next, I calibrate the model to data from Argentina as an example of a small open economy. The most critical parameters are coefficients that control market liquidity and funding liquidity. Market liquidity, which affects the frequency of asset sales, is estimated by targeting the second moment of asset values. Funding liquidity, which captures the collateral values of assets, is estimated by margin data from the US tri-repo market due to data limitations. I then demonstrate the performance of the model by running event studies of the Argentine 2002 crisis from 2000 to 2004 and the US Great Recession from 2007 Q3 to 2009 Q4. By assuming exogenous sequences of agents' equity capital that replicate the data, the model matches the empirical dynamics of the deposit, the liquid asset, and the illiquid asset as well as the liquid share. Both the data and the model show decreases in deposits, asset holdings, and the liquid share in response to a decrease in agents' equity. The simulated series of the total asset captures 42.5% of the drop in the Argentine 2002 crisis. I also validate the model by comparing the empirical and simulated non-targeted moments of the liquid share and leverage.

I then evaluate the optimal liquidity regulation by comparing the solutions of the CE and the SP. The liquid share the SP owns is 2.6% higher than the level in the CE. To achieve efficient equilibrium, the SP can implement dividend taxes in which the average dividend tax on the liquid asset equals -21.4 basis points, and the average dividend tax on the illiquid asset is 1.73 basis points. Regarding the liability side, the SP borrows 2.24% less compared

with the solution of the CE in terms of the deposit-to-GDP ratio. Average leverage in the solution of the SP is 9.43% lower compared with the CE. The SP can guide the borrowing level of agents by tightening the margin requirement by 1.5% or imposing an average deposit tax that equals 7.24 basis point. By borrowing less and maintaining a more liquid portfolio, the SP lowers the probability of crises from 8% to zero. Benefiting from the lower probability of crises, the SP can on average increase the consumption-to-GDP ratio by 0.98%. With higher consumption, the SP then raises the consumption-equivalent utility by 0.99%. The government should focus not only on shrinking the size of agents' liabilities but also on incentivizing agents to own a more liquid portfolio.

Finally, I explore the effectiveness of the current regulation and compare it with the optimal regulation. Under Basel III, banks must ensure that their liquidity coverage ratio (LCR) and net stable funding ratio (NSFR) are both higher than 100%. The LCR, which is defined as the ratio of high-quality liquid assets to expected cash outflows over the next 30 days, is meant to preserve banks' short-term resilience. This policy provides an incentive to hold more liquid assets. The NSFR is defined as the ratio of the available amount of stable funding. This policy lowers agents' illiquid holding, as illiquid assets typically require stable funding.

I develop a model that incorporates the two policies and perform theoretical and numerical analyses. With the regulations in place, the model predicts that the LCR increases the liquid holding and the NSFR decreases the illiquid holding. I show quantitatively that in isolation, the LCR and NSFR indeed increase the liquid share and reduce the probability of crises. Regarding welfare improvement, the LCR and the NSFR can mitigate the welfare loss of the competitive equilibrium by 23% and 60%, respectively.

The complete Basel regulation that contains both policies, however, leads to welfare deterioration due to overaccumulation of the liquid asset. That is, agents become too cautious and do not take advantage of the higher dividend of the illiquid asset. The resulting consumption-equivalent welfare loss is 0.5% larger than the CE. To achieve the social optimum, the government should relax the lower bounds when both policies are implemented. Quantitatively, Basel policies can be on average equivalent to the optimal macroprudential policies if, under a standard LCR, the NSFR is looser and has a lower bound that equals roughly 20%.

Related Literature

This paper is related to several strands of literature. First, it complements studies that focus on the optimal macroprudential policy, which corrects overborrowing due to the pecuniary externality. Bianchi (2011) develops a model with an occasionally binding constraint to quantify and evaluate the optimal macroprudential policy. Bianchi and Mendoza (2011) and Bianchi and Mendoza (2018), on the other hand, build up models with a stock collateral constraint. In both papers, the SP reduces the magnitude and the probability of financial crises by taxing borrowing.⁷ However, they get numerically small overborrowing, which casts doubt on the importance of macroprudential policies. Regarding macroprudential policies on assets, Davila and Korinek (2018) use a finite-period model and derive optimal taxes imposed on purchases of financial instruments based on their state-contingent payoffs. Other examples that focus on the pecuniary externality include Uribe (2006a,b); Lorenzoni (2008); Benigno et al. (2010); Korinek (2012); Schmitt-Grohe and Uribe (2018); and Jeanne and Korinek (2019). I contribute to this literature by analyzing macroprudential policies that jointly regulate asset composition and the size of debts. Moreover, by considering the degree of asset liquidity and a multi-asset structure, I quantitatively obtain large overborrowing and justify the importance of macroprudential policies.

The second literature concerns the determination of liquidity and its relationship with bank runs in static or finite-period models. The classic paper is by Diamond and Dybvig (1983), who describe the real cost of bank runs, which are the result of the liquidity mismatch of the illiquid asset and short-term debt. Other papers that aim to pin down the optimal level of the liquid asset include the work of Diamond and Kashyap (2016), who use a twoasset model with incomplete information to derive the level of optimal liquidity to deter runs. Some studies analyze liquidity regulations (see, e.g., Ennis and Keister (2006); Vives (2014); Farhi et al. (2009); and Kashyap et al. (2017)) or the mechanism that drives liquidity hoarding (Heider et al. (2009); Acharya and Skeie (2011); and Gale and Yorulmazer (2013)) via models with finite period. Several papers have also proposed stylized frameworks that solve for the optimal allocation of liquidity (see, e.g., Perotti and Suarez (2011); Holmstrom and Tirole (1998); and Caballero and Krishnamurthy (2004)). My paper differs from the literature by providing a dynamic framework with infinite periods that can quantify the dynamics of asset holdings, asset prices, welfare, and the severity of crises.

The third literature is related to the role of bank intermediation and the nonlinearity in dynamic models with liquid and illiquid assets. Recent works include Brunnermeier and Sannikov (2014); He and Krishnamurthy (2011); and Elenev et al. (2018). In their models, the liquid asset is typically cash, which financial intermediaries tend to borrow to invest in the illiquid asset. This setup, therefore, is more like a one-asset-one-debt model. My twoasset model differs from the literature by featuring one debt with two assets , in which the values of holdings and prices are both endogenous. This framework allows me to study new implications on optimal macroprudential policies—the current holding of one asset affects not only its future price, but also the prices of other assets. The changes in prices then affect agents' utility via expected asset sales and purchases. More importantly, the multi-

⁷Similar definitions and the analysis of overborrowing are also adopted in the works of Benigno et al. (2010) and Korinek (2012).

asset structure with deposit is an appropriate setup to study existing policies, which jointly regulate both asset composition and the size of borrowing.

The fourth literature considers asset dynamics and the measurement of liquidity and risk. Some papers particularly examine the order and relative size of asset sales during financial distress. Scholes (2000) and Brunnermeier (2009) study the point at which financial institutions tend to sell liquid assets first to avoid transaction costs or significant collapses in prices due to asset illiquidity. Ben-David et al. (2012) show that hedge funds tend to sell more assets with higher liquidity, measured by the Amihud ratio (Amihud (2002)), in the 2007-2009 financial crisis. Anand et al. (2013) provide empirical evidence on the selling activities of institutional investors in which the share of illiquid securities (measured by the noise beta) of total asset sales dropped during 2008. Other papers have constructed liquidity indices that describe banks' degree of liquidity and mismatch between assets and liabilities (Berger and Bouwman (2009); Brunnermeier et al. (2012); Brunnermeier et al. (2014); and Bai et al. (2018)). Bai et al. (2018) find that banks' liquidity, measured by the Liquidity Mismatch Index, drops during the Great Recession, which supports the argument that the liquid asset is more likely to be sold under financial shocks. I contribute to this literature by documenting country-level and bank-level changes in the liquid share during sudden stop events.

Finally, the paper is related to theoretical and empirical papers on the effects and efficacy of liquidity regulations in Basel III. The most closely related paper is the work of Kashyap et al. (2017), who use a dynamic model with finite periods to compare the private equilibrium, the SP solution, and the regulated equilibrium under either the LCR or the NSFR. They show that both policies raise banks' liquidity, shrink the amount of loans, lower the crises' probability, and erode banks' welfare. These results are in line with my quantitative analyses. I contribute to the literature by further quantifying an infinite-period model with calibrated parameters. My results are also in agreement with empirical papers (see, e.g., DeYoung and Jang (2016); Duijm and Wierts (2016); Fuhrer et al. (2017); EBA (2017); and Banerjee and Mio (2018)) in which liquidity regulations raise banks' holdings of the high-quality liquid asset.⁸

The remainder of the paper is organized as follows. Section 2 provides empirical evidence of asset sales during financial crises and sudden stop events. Section 3 introduces the model, the analysis of liquidity, and its implication in a two-asset model. I then derive the optimal liquidity regulation by comparing the equilibrium solution of the CE and the SP. Section 4 calibrates the model and compares the simulated results in the regulated and the unregulated cases. Section 5 solves the equilibrium under the Basel III reform and presents welfare analyses of various policies, and Section 6 concludes.

⁸See Popoyan (2016) for a detailed summary of the literature.

2 EMPIRICAL EVIDENCE

Before introducing the theoretical framework, I first highlight the importance of the liquidity regulation by documenting the trading behavior of financial institutions. I use data from the IMF Internal Investment Position (IIP) and the IMF Financial Soundness Indicators (FSIs) to observe asset dynamics during sudden stop events. I then examine asset dynamics during the Great Recession by using the Flow of Funds data. Finally, I analyze bank-level adjustments in balance sheets using micro data from Bankscope.

Sudden stop events typically feature significant drops in the asset holdings of countries and are formally defined by sudden reversals in capital inflow, output contractions, and increases in the spreads of emerging market bonds. To examine how agents sell assets based on asset liquidity, I use two datasets that can distinguish between the holding of liquid and illiquid assets. I then merge those data with annual sudden stop events identified by Calvo et al. (2006) and Korinek and Mendoza (2014).

The first data is the IMF IIP, which is either annual or quarterly series from 1980 to 2017. Following similar categorization as in the Flow of Funds data, I define liquid assets as the sum of currency, deposits, and debt securities. Merging these data with the dates of sudden stop events, the unbalanced panel contains 54 events, in which 25 occurred in emerging markets and 29 in advanced economies. Table A.3 lists the matched events.

Using the IIP data, the left panel in Figure 1 shows normalized average trends for the liquid share in emerging and advanced economies during systemic sudden stop events. The liquid share drops by around 10% in emerging economies and by 2% in advanced economies. On average, the liquid share declines by more than 6% from the peak to the trough of sudden stop events. The trends are robust when considering a much narrower definition of the liquid asset: the sum of the currency and the deposit. This implies that other definitions of safe shares (see, e.g., Gorton et al. (2012) and Lenel et al. (2019)) which lie within the initial and narrower liquid share should also feature similar patterns.

Similar patterns can be observed using the IMF FSIs, which contains annual information from 2000 to 2017. The data defines an official liquid asset ratio, which equals the sum of currencies, deposits, short-term financial assets, and securities that are traded in the liquid market divided by total assets of the economies. There are 13 matched systemic sudden stop events (8 from emerging countries and 5 from advanced countries).⁹

To understand financial institutions' trading behavior, it is also necessary to examine data on asset transactions, because nominal asset values are subject to revaluation due to exchange rates fluctuations or renegotiation of debt contracts. Since changes in the liquid

⁹Figure A.4 plots the changes in the liquid shares in emerging markets during several financial crises and sudden stops. Figure A.5 plots the value of liquid shares using these two datasets and shows that emerging countries tend to hold a more liquid portfolio.



Figure 1: Normalized liquid share in sudden stop events

Notes: The horizontal axis denotes a 5-year window, where t indicates the period when a systemic sudden stop is identified. The vertical axis denotes the cross-sectional average of the normalized liquid share with t-2 being the base year. Assets in the left panel include currency and deposits, debt securities, loans, insurance and pension, accounts receivables, and gold. The sum of the first two assets is defined as the liquid asset. The right panel plots the liquid share calculated by using the IMF FSIs. See paragraph 4.80 in the IMF's complication guide for detailed definitions. Countries are divided into two groups: advanced and emerging economies. See Table A.3 for details of group types. Source: IMF Internal Investment Position (IIP) and Financial Soundness Indicators (FSIs).

share are more drastic in emerging markets, I will focus on the volumes of asset transactions in those markets. Figure 2 plots aggregate sales of liquid and illiquid assets. To distinguish sudden stop events from the influence of the US Great Recession, I define financial crises as sudden stop events that occurred during 2007 to 2009. During the crisis period, countries become sellers of liquid assets. Although the amount of the purchase of illiquid assets shrinks, countries still slightly raise their illiquid holdings.¹⁰ The relative magnitude of sales of liquid and illiquid assets is consistent with the observation that the liquid share drops. The bottom right panel of Figure 2 shows that the debt level rises before crises and declines when crises occur. This observation is in agreement with the literature, in which debt is procyclical during episodes of sudden stops. As a robust check, I also document the dynamics of the liquid share during the US recession in Appendix 7.3. The result also shows a declining pattern of the liquid share.

The relative decrease in liquid holdings during asset-selling events can also be studied

¹⁰To further explore changes in balance sheet items, Figure A.6 plots transaction patterns of main liquid and illiquid assets. Regarding the liquid asset, its negative growth is mainly explained by the decrease in cash and the deposit holding. Purchases of debt securities also shrink during crises. As for the illiquid asset, a shift from holding loans to holding account receivables occurs. Restructuring illiquid assets does not lead to aggregate sales of illiquid assets.



Figure 2: Adjustment in balance sheets during crises: Emerging countries

using bank-level data. Unlike country-level data, in which effects of sector-specific shocks can not be identified when assets are traded within domestic sectors, the effect of the financial shock to asset holdings can be better identified using bank-level data. Using micro data from Bankscope, which covers global banks during the period 1981 to 2016, I analyze banks' holdings on asset sales by regressing the change in the liquid share on the growth rate of asset holdings and other bank controls from the last period. The regression is given by

$$\Delta LS_{b,t} = \alpha_0 AssetGrowth_{b,t} + \beta Z_{b,t-1} + F_b + F_t + e_{b,t},\tag{1}$$

where $Z_{b,t-1}$ represents banks' characteristics. F_b and F_t indicate the bank and time fixed effect. α_0 measures the degree with which liquid share changes under 1% of asset growth. Results are shown in Table 1. Positive α_0 in column (1) and (2) support the previous observation that banks sell relatively more liquid assets when facing the pressure of selling assets. With the model in column (7), the liquid share will drop by about 0.03% when asset holding decreases by 1%. Relevant bank characteristics include the core tier 1 regulatory capital ratio, the ratio of impaired loans to gross loans, the log of loan loss provisions, and leverage. The core tier 1 regulatory capital ratio measures the rate of core capital to total risk-weighted assets and indicates the financial strength of the holder. Banks that have a high core tier 1 capital rate, and therefore have more stable funding sources, should be able to maintain a more illiquid portfolio, reflecting the capability to pursue high yields. On the other hand, banks become less healthy when holding more impaired loans, which then restrains them from possessing an illiquid portfolio. The amount of loan loss provision also provides information regarding banks' expected loss from lending. Higher loan loss provision implies a larger risk and a more liquid portfolio, which banks tend to hold in upcoming periods. Lastly, leverage, as a widely considered feature of banks regarding risk, is included but generates an insignificant effect.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$Asset growth_{b,t}$	0.0285^{**}	0.0254^{**}	0.037^{**}	0.0332^{***}	0.0294^{***}	0.0254^{**}	0.0299^{***}
	(0.029)	(0.033)	(0.016)	(0.001)	(0.001)	(0.033)	(0.001)
$Tier 1 capital_{b,t-1}$			-0.0697**				-0.154**
			(0.018)				(0.013)
$Impaired \ loan_{b,t-1}$				0.0486^{**}			-0.0194
				(0.010)			(0.558)
$ln(Provision_{b,t-1})$					0.900**		0.766^{**}
					(0.021)		(0.034)
$Leverage_{b,t-1}$						-0.006	-0.0285*
						(0.464)	(0.058)
R^2	0.106	0.155	0.159	0.159	0.239	0.155	0.242
N	$46,\!395$	$46,\!382$	$45,\!144$	$45,\!478$	$20,\!584$	$46,\!382$	19,362
Bank FE	Υ	Υ	Υ	Υ	Υ	Υ	Υ
Year FE	Ν	Υ	Υ	Υ	Υ	Υ	Υ

Table 1: Asset sales and liquid share

Notes: p-values in parentheses * p < 0.10, ** p < 0.05, *** p < 0.01. Standard errors clustered by type of financial institutions. Liquid assets equal the sum of trading securities and at FV through Income, loans and advances to banks, reverse repos and cash collateral, cash and due from banks, and mandatory reserves. Illiquid assets equal the sum of corporate and commercial loans, other consumer/retail loans, senior debt maturing after 1 year, other assets, other intangibles, and held to maturity securities. Source: Bankscope.

Across all specifications, the change in the liquid share is increasing in the asset growth, implying a decline in the liquid share when banks liquidate assets. In sum, empirical trends for asset sales during systemic sudden stops and the Great Recession support the argument that the magnitude of the asset sales vary across assets with different liquidity. Moreover, market liquidity plays a key role because liquid assets tend to be sold more often compared with illiquid assets. This argument implies that liquid assets should be a better buffer against financial shocks, and therefore should be accumulated.

3 A TWO-ASSET MODEL

The purpose of the model is to introduce a dynamic framework to quantify and evaluate optimal macroprudential policies, which guide asset composition and the size of agents' balance sheets. It extends the model of Mendoza and Smith (2006) with a stock collateral constraint by allowing multiple assets that differ in their liquidity. The normative theory that I provide emphasizes the pecuniary externality and the fire sale mechanism. A pecuniary

externality is produced when agents fire-sale assets to meet their obligations, which decreases aggregate prices and further tightens the collateral constraint.

Asset liquidity is captured by a pair of parameters that governs assets' market and funding liquidity. Market liquidity is modeled with a quadratic transaction cost paid by foreign investors during asset transactions. The transaction cost can be interpreted as either a real cost of finding a trading counterpart or a premium that compensates the asymmetric information. Funding liquidity is proxied by the collateral value of an asset—that is, the amount of debt one can raise by collateralizing a specific asset. The liquid asset, by definition, is assumed to have higher market and funding liquidity compared with the illiquid asset.

This section describes the model and the solutions of the CE and the SP. I also provide theoretical analysis on how liquidity affects the relative adjustment of assets and whether overborrowing implies underinvesting in the liquid asset. Finally, I illustrate the necessity of building a two-asset model by proving that the optimal policies differ compared with a one-asset model.

3.1 Setup

I develop a dynamic model of a small open economy with three sectors: a continuum of measure unity of identical domestic agents, foreign investors, and the government. Domestic agents raise outside deposits and invest in liquid and illiquid assets. They then produce output by using the illiquid asset. Foreign investors maximize their cash flows by investing in the two assets. The government taxes or subsidizes the purchase of assets and borrowing. Policies are financed by the tax on equity capital. The supply of the two assets is assumed to be fixed. Prices are endogenously determined by the demand of domestic agents and foreign investors, whereas the interest rate of the deposit is assumed to be exogenous in the small open economy.

3.1.1 Domestic Agents

There is a continuum of identical agents. The representative agent owns banks and produces using the illiquid asset.¹¹ Specifically, agents obtain the illiquid asset from the bank department and produce via the firm sector. The bank has access to both assets, and finances the investment by raising deposits from outside lenders. Agents' borrowing capacity is subject to a collateral constraint governed by the assets' haircuts, which measures funding liquidity. Each bank is assumed to be managed by an agent that maximizes and consumes profit by raising outside deposit d_{t+1} to invest in two assets: liquid asset a_{t+1}^L and illiquid asset a_{t+1}^I .

¹¹This setting is similar to the assumption of a firm-household made in Mendoza (2010) and Bianchi and Mendoza (2018). Bianchi and Mendoza (2018) show that the competitive equilibrium of the representative firm-household is equivalent to the case in which households and firms are separately modeled.

While I consider two assets, the deposit is the only inside money that agents issue.¹² The optimization problem of the agent can be characterized as

$$\max_{\{\pi_t, \{a_{t+1}^j\}, d_{t+1}\}} U_t = E_t [\sum_{s=1}^{\infty} \beta^s u(c_{t+s})]$$

subject to

$$\pi_t + \sum_{j=I,L} (1+\tau_t^j) q_t^j a_{t+1}^j - \frac{1+\tau_t^d}{1+r} d_{t+1} = (1-\tau_t)\omega_t + \sum_{j=I,L} [(q_t^j + z)a_t^j] - d_t + y_t, \quad (2)$$

$$c_{t+s} = \pi_{t+s},\tag{3}$$

$$\frac{d_{t+1}}{1+r} \le \sum_{j=I,L} \kappa^j a_{t+1}^j q_t^j, \tag{4}$$

$$y_t = A \times a_t^I K,\tag{5}$$

where the utility is CRRA and β is the subjective discount factor. Equation (2) is the budget constraint. The total deposit at time t equals $d_{t+1}/(1+r)$, which is in the form of a discounted deposit. $\{\tau_t^j, \tau_t^d, \tau_t\}$ are taxes implemented by the government to guide the equilibrium. τ_t is a lump-sum tax or subsidy on equity capital taxed by the government to finance potential macroprudential taxes. ω_t represents the newly injected equity capital, which is assumed to be equal across all agents. z measures the exogenous dividend that assets provide when investors hold them for 1 year.

Equation (3) indicates that the agent consumes the remaining wealth after investing in assets and raising deposits. Equation (4) is the occasional binding collateral constraint in which the nominal value of the deposit cannot exceed the sum of the nominal values of assets.¹³ κ^{j} is the asset haircut, which measures short-term funding liquidity. The interest rate r_{t} is assumed to be exogenous. Equation (5) is the production function, and it is linear in the capital level that was accumulated by the end of the last period. The time lag implies that it takes firms 1 period to produce the output. A is the exogenous technology level. The illiquid asset a_{t+1}^{I} can be viewed as the domestic share of a fixed amount of capital K, which can be owned by foreign investors and domestic agents. The role of the illiquid asset as a production input can be motivated by the fact that illiquid assets can serve as loans for working capital. Total capital K is normalized to be unity.

 $^{^{12}}$ This assumption is consistent with the fact that banks are mostly financed by deposits (Hanson et al. (2015)).

 $^{^{13}}$ Financial institutions here are similar to the shadow banking sector, in which the deposit is not protected by the deposit insurance. The reason to focus on uninsured financial institutions is that they were the most affected sector that sold assets during financial crises, as documented by He et al. (2010).

With the assumption that K = 1 and equation (5), the budget constraint (2) can also be written as

$$\pi_t + \sum_{j=I,L} q_t^j a_{t+1}^j - \frac{d_{t+1}}{1+r} = (1-\tau_t)\omega_t + \sum_{j=I,L} [(q_t^j + \tilde{z}^j)a_t^j] - d_t,$$
(6)
$$\tilde{z}^j = \begin{cases} z & j = L \\ z+A & j = I \end{cases},$$

where the gap between "gross" dividends \tilde{z}^L and \tilde{z}^I captures the technology level, which is the additional value of the illiquid asset as an input for production.

Table 2: Balance sheet

$$\begin{array}{c|c} \text{Asset} & \text{Liability} \\ \hline \sum_{j=I,L} q_t^j a_{t+1} & d_{t+1}/(1+r) \\ & e_t \end{array}$$

Table 2 presents the balance sheet, in which the aggregate value of assets equals the deposit plus the equity. Since the net worth is defined as the difference between the asset and the liability side, equity capital should then equal the net worth by construction.

$$n_{t} = \sum_{j=I,L} q_{t}^{j} a_{t+1}^{j} - \frac{d_{t+1}}{1+r},$$
$$e_{t} = n_{t}.$$

The evolution of the net worth is given by

$$n_t = n_{t-1} + \sum_{j=I,L} (q_t^j - q_{t-1}^j) a_t^j - (1 - \frac{1}{1+r}) d_t + \sum_{j=I,L} \tilde{z}^j a_t^j - \pi_t + \omega_t,$$
(7)

where the current net worth n_t equals the sum of previous net worth n_{t-1} plus the capital gain of asset holdings $\sum_j (q_t^j - q_{t-1}^j) a_t^j$, the gain from asset dividends $\sum_j \tilde{z}^j a_t^j$, and newly injected equity capital ω_t minus the interest payment of the deposit and the consumed profit π_{it} .

3.1.2 Foreign Investors

There is a continuum of representative foreign investors who have access to both assets. They pick up the asset that agents sell and provide a downward-sloping demand to pin down the asset price. The asset supply is assumed to be fixed, such that $a_t^j + a_{f,t}^j = \bar{A}^j$ under the symmetric equilibrium. The primary purpose of introducing foreign investors is to obtain non-constant asset holdings of agents under a fixed asset supply.¹⁴ As will be discussed later, a non-constant asset holding is important, as it changes the pecuniary externality via trading positions and further drives the difference between optimal policies in the one-asset model and the multiple-asset model.

Foreign investors are assumed to be risk-neutral and maximize their discounted cash flow. The maximization problem is given by

$$\max_{\{a_{f,t+1}^j\}} U_t^f = E_t \sum_{s=0}^{\infty} \beta_f^s(c_{t+s}^f)$$

subject to

$$c_t^f = \sum_{j=I,L} [a_{f,t}^j (\tilde{z}^j + q_t^j) - a_{f,t+1}^j q_t^j - \frac{1}{2} B^j q_t^j (a_{f,t+1}^j - a_{f,t}^j + \theta^j)^2],$$
(8)

where B^j , $\theta^j > 0$. β_f is the subjective discount factor of foreign investors. Equation (8) is the budget constraint whereby consumption equals the remaining wealth after investing in the two assets. Following a similar setup as in Mendoza and Smith (2006), the asset transaction is subject to an asset-specific quadratic transaction cost, which is governed by a positive scale parameter B^j that controls market liquidity.¹⁵ The additional cost on asset transactions can be viewed as some tangible costs from real transactions or asymmetric information, which must be compensated by a lower purchasing price. A higher B^j thus leads to lower liquidation value, and therefore agents who suffer from negative shocks liquidate more assets to smooth the consumption. As asset demand drops, the asset price falls by more and further tightens the collateral constraint (4), which may trigger a spiral deflation when it binds. As a result, the absolute value of the price elasticity is increasing in B^j . The SP, who concerns the drastic price collapse with high price elasticity, avoids holding assets with high B^j ex-ante.

 θ^{j} is the recurrent entry cost that captures the asymmetry in the marginal cost between buying and selling assets. A positive θ^{j} implies a higher transaction cost of selling one unit of the asset compared with buying. Since both foreign investors and domestic agents demand a fixed amount of assets, this implies that foreign investors pay a higher transaction when agents sell assets. This asymmetric cost can ensure that the solution of asset holdings is

¹⁴Note that non-constant equilibrium holdings can also be achieved by not fixing the asset supply.

¹⁵The transaction cost of equity or asset trades are often assumed to be quadratic in the transaction volume (see, e.g., Niehans (1992); Heaton and Lucas (1996); Aiyagari and Gertler (1999); and Herdegen et al. (2019)). Gertler and Kiyotaki (2015), although they do not directly incorporate a transaction cost, focus on a two-asset model and introduce a quadratic household management cost of capital to capture the household's lack of expertise relative to banks in managing an investment.

within the selected bounds. Otherwise, agents may further liquidate assets when they have less wealth and fewer asset holdings, resulting in solutions of current assets that are less than the lower bound. Similar to domestic agents, foreign investors are assumed to own the foreign production sector in which they hold the illiquid asset, as a foreign share of fixed capital K supports foreign production.

3.1.3 Government

The government implements macroprudential policies and balances the budget by taxing or subsidizing agents in a lump-sum fashion, according to equation (6). The government implements and finances taxes such that

$$\sum_{j=I,L} \tau_t^j q_t^j a_{t+1}^j - \frac{\tau_t^d d_{t+1}}{1+r} = \tau_t \omega_t.$$

3.2 Unregulated Competitive Equilibrium

In the unregulated equilibrium in which policies $\{\tau_t^j, \tau_t^d, \tau_t\}$ are null, domestic agents choose $\{c_t, \{a_{t+1}^j\}, d_{t+1}\}_{t\geq 0}$ to maximize the discounted profit subject to the budget constraint (6) and the collateral constraint (4) by taking prices $\{q_t^j\}$ as given. Foreign investors choose $\{a_{f,t+1}^j\}$ to maximize their discounted cash flows. The equilibrium can be characterized by optimality conditions (9), (10), (11), and (12), as well as the market-clearing condition (13).

$$q_t^j = \frac{\tilde{z}^j}{1 - \beta^f} \frac{1}{1 + B^j (a_{f,t+1}^j - a_{f,t}^j + \theta^j)} \qquad \forall j \in \{I, L\},$$
(9)

$$u'(c_t)q_t^j = E_{w'|w}[\beta u'(c_{t+1})(q_{t+1}^j + \tilde{z}^j)] + q_t^j \kappa^j \lambda_t \qquad \forall j \in \{I, L\},$$
(10)

$$u'(c_t) = \beta(1+r)E_{w'|w}[u'(c_{t+1})] + \lambda_t,$$
(11)

$$0 = \lambda \left[\sum_{j=I,L} \kappa^j a_{t+1}^j q_t^j - \frac{d_{t+1}}{1+r}\right],\tag{12}$$

$$\bar{A}^j = a^j_t + a^j_{f,t} \qquad \forall j \in \{I, L\}.$$
(13)

Equation (9) is the asset demand of foreign investors. Prices of assets are decreasing in the asset investment $(a_f^{j'} - a_f^j)$, and price elasticities are affected by the coefficient of market liquidity, B^{j} .¹⁶ If market illiquidity increases (i.e., B^{j} increases), drops in asset prices are magnified for a given unit of asset investment. Agents should then sell assets at lower prices

¹⁶Here, market and funding liquidity formally follow the definition provided by Brunnermeier and Pedersen (2009) in which market liquidity, denoted as Λ^{j} , is defined as the gap between the transaction price and the fundamental price, which is given by

to provide enough incentive for foreign investors to invest.

Equation (10) is the asset demand of domestic agents that equates the marginal benefit and the marginal cost of asset investment. By giving up marginal utility of consumption $u'(c_t)q_t^j$, agents earn capital gains and the benefit of loosening the collateral constraint. The value of loosening the collateral constraint is captured by funding liquidity κ^j , which measures the extent to which a unit of asset j enhances borrowing capacity. Therefore, an asset with higher funding liquidity will be more valuable and purchased by agents at a higher price.

Equation (11) equates the marginal benefit of current consumption and saving. Decreasing borrowing by one unit provides a marginal benefit of savings λ as it loosens the collateral constraint. Equation (12) is the complementary slackness condition.

A fire-sale spiral is triggered when the collateral constraint (4) is binding. In a one-asset model with only asset j, agents sell one unit of asset j and repay κ unit of the deposit, and therefore loosen the collateral constraint by $(1 - \kappa)\%$. In this two-asset model, the amount each asset must be sold for is, however, determined by liquidity. On the one hand, selling the liquid asset has the advantage that agents obtain funding without paying a substantial transaction cost due to lower market illiquidity. On the other hand, maintaining the holding of the liquid asset provides agents with a higher borrowing capacity because of its high funding liquidity. As will discussed later, the trade-off between the market and funding liquidity determines the relative size of the fire sale across two assets.

To formalize the CE, I characterize the recursive unregulated problem as follows.

3.2.1 Equilibrium Definition

Let $V(\{a^j\}, d, \Omega; \omega)$ be the value function of the representative agent. $\Omega = \{\{A^j\}, D\}$ is a set of aggregate variables where $\{A^j\}$ is the aggregate asset holding of domestic agents and D is the aggregate deposit. The representative agent's optimization problem is

$$V(\{a^{j}\}, d, \Omega; \omega) = \max_{\{a^{j'}\}, d', c} u(c) + \beta E_{\omega'|\omega}[V'(\{a^{j'}\}, d', \Omega')]$$

subject to

$$\Lambda^j = \frac{\tilde{z}^j}{1-\beta^f} \frac{1}{1+B^j(a_f^{j^{\prime\prime}}-a_f^j+\theta^j)} - \frac{\tilde{z}^j}{1-\beta} > 0.$$

Funding liquidity $\kappa^j \lambda_t$ is defined as the product of the asset margin and the shadow value of relaxing the collateral constraint. Two kinds of liquidity mutually reinforce each other. If funding liquidity declines, the asset is less valuable, and thus the equilibrium price drops and the gap Λ^j widens, implying a decrease in market liquidity. If market liquidity falls (i.e., $|\Lambda^j|$ rises), the equilibrium price is supported only when u'(c) increases and funding liquidity $\kappa^j \lambda_t$ declines, as shown in equation (10).

$$c + \sum_{j=I,L} [q^{j}(\Omega;\omega)a^{j'}] - \frac{d'}{1+r} = \omega + \sum_{j=I,L} (q^{j}(\Omega;\omega) + \tilde{z}^{j})a^{j} - d,$$

$$\frac{d'}{1+r} \leq \sum_{j=I,L} \kappa^{j}q^{j}(\Omega;\omega)a^{j'},$$

$$a^{j'} + a^{j'}_{f} = \bar{A}^{j},$$

$$d' = D',$$

$$\Omega' = \Gamma(\Omega,\omega),$$

$$a^{j'}_{f} = -[1 - \frac{\tilde{z}^{j}}{(1-\beta)q^{j}(\Omega;\omega)}]\frac{1}{B} - a^{j} - \theta^{j} \quad \forall j \in \{I,L\}, \quad (14)$$

where equation (14) is the asset demand of foreign investors. The equilibrium solution is characterized by decision rules $\hat{d}'(\{a^j\}, d, \Omega; \omega), \hat{c}(\{a^j\}, d, \Omega; \omega), \{\hat{a}^{j'}(\{a^j\}, d, \Omega; \omega)\}$, and $\hat{\lambda}'(\{a^j\}, d, \Omega; \omega)$, which imply an actual law of motion of aggregate assets $\hat{a}^{j'}(\{A^j\}, D, \Omega; \omega)$ and the aggregate deposit $\hat{d}'(\{A^j\}, D, \Omega; \omega)$. The above solution is associated with actual price functions $\{\hat{q}^j(\Omega; \omega)\}$, which can be solved by equation (10). In equilibrium, the perceived law of motion Γ of the states should converge to the actual law of motion of the states. The explicit price function of asset j is given by

$$q^{j}(\Omega,\omega) = E_{\omega'|\omega} \left[\frac{\beta u'(\hat{c}(\Gamma(\Omega,\omega),\omega'))}{u'(\hat{c}(\Omega;\omega)) - \lambda(\Omega;\omega)\kappa^{j}} (q^{j}(\Gamma(\Omega,\omega),\omega) + \tilde{z}^{j'}) \right] \qquad \forall j \in \{I,L\}.$$

Having described the agents' optimization problem, I then characterize the recursive competitive equilibrium as follows.

Definition. (Recursive unregulated competitive equilibrium) An unregulated recursive equilibrium is given by a set of asset price functions $\{q^j(\Omega, \omega)\}$, a perceived law of motion of assets and the deposit $\Gamma(\Omega, \omega)$, the solution $\hat{d}'(\{a^j\}, d, \Omega; \omega)$, $\hat{c}(\{a^j\}, d, \Omega; \omega)$, $\{\hat{a}^{j'}(\{a^j\}, d, \Omega; \omega)\}$, $\hat{d}'(\{a^j\}, d, \Omega; \omega)$, and the value function $V(\{a^j\}, d, \Omega; \omega)$ such that

The decision rules d̂'({a^j}, d, Ω; ω), ĉ({a^j}, d, Ω; ω), and {â^{j'}({a^j}, d, Ω; ω)}; the shadow value λ({a^j}, d, Ω; ω); the associated foreign holdings {â^{j'}_f({a^j}, d, Ω; ω)}; and V({a^j}, d, Ω; ω) solve the optimization problem of domestic agents, taking {q^j(Ω, ω)} and Γ(Ω, ω) as given.

2. The budget constraint holds:

$$\hat{c}(\{A^j\}, D, \Omega; \omega) + \sum_{j=I,L} [\hat{q}^j(\Omega; \omega) \hat{a}^{j'}(\{A^j\}, D, \Omega; \omega)] - \frac{\hat{d}'(\{A^j\}, D, \Omega; \omega)}{1+r}$$
$$= \omega + \sum_{j=I,L} (\hat{q}^j(\Omega; \omega) + \tilde{z}^j) A^j - D.$$

- 3. The perceived law of motion of assets and the deposit and the price function coincide with the actual law of motion and actual price functions such that $\Omega' = \Gamma(\Omega, \omega)$ and $q^j(\Omega; \omega) = \hat{q}^j(\Omega; \omega).$
- 4. Each asset market clears such that $\hat{a}^{j'}(\{A^j\}, D, \Omega; \omega) + \hat{a}^{j'}_f(\{A^j\}, D, \Omega; \omega) = \bar{A}^j$.

3.2.2 Asset Dynamics and Liquidity

Asset dynamics are affected by the trade-off of two kinds of asset liquidity: Market illiquidity requires agents to pay an additional cost or sell assets at a lower price to provide an incentive for buyers to purchase the asset. This discourages agents from liquidating the asset with low market liquidity. On the other hand, agents are encouraged to accumulate assets with high funding liquidity because they provide holders with high borrowing capacity.

Since the liquid asset features both higher market and funding liquidity, it is essential to compare the two opposite forces that control the asset sale. In particular, I analyze how liquidity coefficients κ^{j} and B^{j} affect the trade-off. I will focus first on the binding equilibrium as I am interested in the asset sale during financial crises. The asset sales can be represented as follows:

$$q_t^j \Delta a_{t+1}^j = (\frac{1}{B^j} + a_t^j + \theta^j) q_t^j - \frac{\tilde{z}^j}{1 - \beta^f} \frac{1}{B^j} - a_t^j q_t^j$$
(15)

$$=\frac{\tilde{z}^{j}}{B^{j}}(M_{t}^{j,CE}-\frac{1}{1-\beta^{f}})+\theta^{j}q_{t}^{j},$$
(16)

where

$$M_t^{j,CE} = E_t \left[\sum_{k=0}^{\infty} \prod_{i=0}^k m_{t+1+i}^j\right]$$

$$= E\left[\beta \frac{u'(c_{t+1})}{u'(c_t) - \lambda_t \kappa^j} + \beta^2 \frac{u'(c_{t+1})}{u'(c_t) - \lambda_t \kappa^j} \frac{u'(c_{t+2})}{u'(c_{t+1}) - \lambda_{t+1} \kappa^j} + \ldots\right]$$

$$< \frac{1}{1+r} + \left(\frac{1}{1+r}\right)^2 + \left(\frac{1}{1+r}\right)^3 + \ldots = \frac{1}{r}.$$
(17)
(17)

By using the asset demand (9), the sale of the asset j can be written as equation (15). After plugging in the forward-looking price, the asset sale can be further given by (16), where $M_t^{j,CE}$ is the stochastic discount factor of asset j and $q_t^{j,CE} = M_t^{j,CE} z^j$, as shown in equation (17). Equation (16) states that domestic agents will raise their investment if their discount factor is higher than the foreign investors' discount factor, which equals the geometric sum of the discount factor β^f .

Note that the stochastic discount factor is bounded by the sum of the discount factors of the risk-free bond, as shown in equation (18). The discount factor $M^{j,CE}$ is strictly increasing in the collateral margin κ^j when the collateral constraint binds. The reason is that the benefit of loosening the collateral constraint is higher when the margin κ^j increases. This implies a higher discount of future gains and a higher price that corresponds to higher asset accumulation.

To simplify the comparison between sizes of the two asset sales, I assume there exists no asymmetric effect between accumulating or selling assets; that is, $\theta^{j} = 0$. The relative asset sale between the liquid and the illiquid asset is given by

$$\frac{q_t^L(a_{t+1}^L - a_t^L)}{q_t^I(a_{t+1}^I - a_t^I)} = \frac{\frac{\tilde{z}^L}{B^L}(M_t^{L,CE} - \frac{1}{1 - \beta^f})}{\frac{\tilde{z}^I}{B^I}(M_t^{I,CE} - \frac{1}{1 - \beta^f})}.$$
(19)

Three observations emerge from this ratio. First, to gain positive funding when liquidating assets in binding states, the discount factor must be smaller than $1/(1 - \beta^f)$. This can be guaranteed by the condition $1/r < 1/(1 - \beta^f)$, which indicates that the foreign investor should be patient enough such that the desirable prices for him to purchase assets are not significantly low, such that the liquidation value of the asset sale turns negative. Second, the asset sale is decreasing in the adjustment cost B^j , as the cost erodes the gain in sales. Third, the trade-off provided by funding liquidity can be observed from the stochastic discount factor $M_t^{j,CE}$ where a higher collateral margin leads to lower asset sales because the asset becomes valuable as it enhances agents' borrowing capacity.

The relative asset sale, therefore, depends on the relative magnitude of the adjustment costs and margins. A sufficient condition, which ensures a drop in the liquid share that is consistent with the empirical finding, can be established by using the bounds of $M_t^{j,CE}$ where $M_t^{j,CE} \in (0, 1/r)$ and is given by

$$(\frac{\tilde{z}^{I}B^{L}}{\tilde{z}^{L}B^{I}} - 1)\frac{1}{1 - \beta^{f}} < (\frac{\tilde{z}^{I}B^{L}}{\tilde{z}^{L}B^{I}} \times 0 - \frac{1}{r}),$$

$$\Rightarrow B^{L} < (1 - \frac{1 - \beta^{f}}{r})\frac{\tilde{z}^{L}}{\tilde{z}^{I}}B^{I}.$$
 (20)

For arbitrary values of collateral margins (κ^{I} , κ^{L}), condition (20) guarantees a decrease in the liquid share when the collateral constraint binds. The intuition behind (20) is to ensure that the market liquidity of the liquid asset is significantly lower than the illiquid asset, such that it always dominates the effect of funding liquidity. This condition becomes easier to achieve under a higher relative dividend of the liquid asset $\tilde{z}^{L}/\tilde{z}^{I}$, which results in a higher relative price of the liquid asset and thus implies that the liquid asset is a better object to liquidate for funding. One special case is the parameterization in which κ and z are identical and the adjustment cost is heterogeneous; that is, $B^{I} > B^{L}$. In this example, the fire sale of the liquid asset will be larger than the illiquid asset, as the only concern is the difference in market liquidity.

The empirical evidence in Section 2 and, as will discussed later, the calibrated simulation in Section 4 show that the effect of market liquidity dominates funding liquidity.

3.3 Constrained-efficient Equilibrium

To derive the optimal macroprudential policy, this subsection characterizes the constrainedefficient equilibrium in which the SP faces the same borrowing capacity as the representative agent but internalizes the fact that current holdings of assets and the deposit affect asset prices in the future.¹⁷ The SP chooses $\{c_t, \{a_{t+1}^j\}, d_{t+1}\}_{t\geq 0}$ to maximize U_t subject to the optimality condition of foreign investors (9), the budget constraint (6), and the collateral constraint (4), taking price functions $\{q_t^j\}$ as given. The recursive optimization problem of the SP is characterized as follows.

3.3.1 Equilibrium Definition

The SP solves

$$V_{sp}(\Omega;\omega) = \max_{\{a^{j'}\},\{d'\},\{\pi\}} u(c) + \beta E_{\omega'|\omega} V_{sp}(\Omega';\omega')$$

subject to

$$c + \sum_{j=I,L} [q^j(\Omega;\omega)a^j] - \frac{d'}{1+r} = \omega + \sum_{j=I,L} (q^j(\Omega;\omega) + \tilde{z}^j)a^j - d,$$

¹⁷As emphasized in the literature, the assumption that the social planner has the same borrowing capacity (and thus current portfolio choices only affect future prices) as the decentralized equilibrium is imposed to avoid the time-inconsistency problem. If the planner's current portfolio choices affect current prices, which are forward-looking, as shown in equation (10), the planner will have an incentive to renege in the next period (see Bianchi and Mendoza (2018) for further elaboration). To relax the assumption that the current portfolio decision cannot affect current asset prices, Bianchi and Mendoza (2018) propose another time-consistent social planner problem that maximizes the objective function subject to the price function that is market-determined, but in which the debt level is chosen by the social planner.

$$\frac{d'}{1+r} \le \sum_{j=I,L} \kappa^j q^j(\Omega;\omega) a^{j'},$$
$$a^{j'} + a^{j'}_f = \bar{A}^j,$$

and foreign investors' optimality condition (14). The constrained-efficient equilibrium is characterized by decision rules $\{\hat{a}^{j'}(\Omega;\omega)\}, \{\hat{a}^{j'}_{f}(\Omega;\omega)\}, \hat{d'}(\Omega;\omega), \hat{c}(\Omega;\omega), \hat{\lambda}(\Omega;\omega), \text{ and price}$ functions $\{q^{j}(\Omega;\omega)\}$, which are derived from the solution of the competitive equilibrium. Having described the recursive optimization problem of the SP, the recursive constrainedefficient equilibrium can be defined as follows.

Definition. (Recursive constrained-efficient equilibrium) The recursive constrained-efficient equilibrium is defined by decision rules $\{\hat{a}^{j'}(\Omega;\omega)\}, \hat{d}'(\Omega;\omega), \hat{c}(\Omega;\omega), \text{ price functions } \{q^{j}(\Omega;\omega)\},$ and the value function $V_{sp}(\Omega;\omega)$ such that decision rules $\{\hat{a}^{j'}(\Omega;\omega)\}, \hat{d}'(\Omega;\omega), \text{ and } \hat{c}(\Omega;\omega);$ the associated foreign holdings $\{\hat{a}_{f}^{j'}(\Omega;\omega)\};$ and $V_{sp}(\Omega;\omega)$ solve the SP's optimization problem, taking as given price functions $\{q^{j}(\Omega;\omega)\}.$

The SP mainly decides her portfolio according to equations (21) and (22), where equation (21) equates the marginal cost and the marginal benefit of the deposit and equation (22) equates marginal cost and the marginal benefit of asset j. Key differences between the solutions of the CE and the SP are pecuniary externalities in (21) and (22):

$$u'(c_{i,t}) = \beta(1+r)E\{u'(c_{i,t+1}) + \sum_{s=I,L} \underbrace{[u'(c_{i,t+1})(\frac{\partial q_{t+1}^s}{\partial d_{i,t+1}}\Delta a_{i,t+2}^s) - \lambda_{i,t+1}\kappa^s a_{t+2}^s \frac{\partial q_{t+1}^s}{\partial d_{i,t+1}}]\} + \lambda_{i,t}, \quad (21)$$

$$\underbrace{u'(c_{i,t})q_t^j = \beta E\{u'(c_{i,t+1})(q_{t+1}^j + \tilde{z}^j) + \sum_{s=I,L} \underbrace{[-u'(c_{i,t+1}')(\frac{\partial q_{t+1}^s}{\partial a_{i,t+1}^j}\Delta a_{i,t+2}^s) + \lambda_{i,t+1}(a_{i,t+2}^s\kappa^s \frac{\partial q_{t+1}^s}{\partial a_{i,t+1}^j})]\} + q_t^j\kappa^j\lambda_{i,t}}_{Pecuniary \ externality \ on \ asset \ j}$$

$$(22)$$

The pecuniary externality on the deposit controls the size of excess borrowing. If the externality term is positive, the marginal benefit of saving in the solution of the SP is higher than in the CE, implying overborrowing in the CE. Similarly, the pecuniary externality on assets measures excess investment. If the externality term is positive, the marginal benefit of asset investment in the solution of the SP is higher than in the CE, implying underinvesting in the CE.

Each pecuniary externality can be decomposed into two parts: the trading and collateral effects. Depending on the future trading position, the trading effect influences future consumption through changes in prices. If agents tend to be asset sellers in the next period, they will benefit from higher future prices. Now, suppose future asset prices are increasing in current assets; the fact that agents benefit from higher future prices encourages the social planner to choose higher current assets.¹⁸ The second component of the pecuniary externality is the collateral effect, which affects the current balance sheet by providing a marginal benefit (cost) from loosening (tightening) the future collateral constraint. The sign of this effect depends on the sign of the price elasticity and the cross-elasticity of the asset demand. When pecuniary externalities on the deposit and assets are not zero, the government should impose macroprudential policies to achieve the solution of the SP.

The signs of pecuniary externalities heavily depend on the signs of price elasticities with respect to existing holdings of balance sheet items. The price elasticity of asset j with respect to the existing holding of asset j, $\partial q_t^j / \partial a_t^j$, is determined by the trade-off between a wealth effect and a higher transaction cost. The wealth effect, which leads to higher asset demand and prices, appears as wealth is increasing in existing assets. Note that an increase in a_t^j is equivalent to a decrease in a_f^j , which raises the convex transaction cost for a given $a_{f,t+1}^j$. With higher transaction cost, foreign investors are willing to purchase assets only at a lower price.¹⁹ When the set of price elasticities $\{\partial q_t^j / \partial a_t^i\}$ is positive, the cross-price elasticities of asset demand will also be positive due to the wealth effect.

Although the pecuniary externality on each balance sheet item depends mainly on price elasticities with respect to itself, pecuniary externalities are in fact not independent. Specifically, overborrowing implies relative underinvesting in the liquid asset (i.e., a lower liquid share in the CE) under certain conditions. Proposition 1 provides the conditions under which the above case occurs. This observation implies that the SP should simultaneously manage the asset and liability sides. In particular, the SP should optimally own a higher liquid share when her borrowing is less than agents'.

Proposition 1. (Overborrowing and underinvesting) In any given period t, overborrowing implies a lower liquid share in the competitive equilibrium when $\beta(1+r) < 1$ and the following condition holds:

$$B^{I}(a_{t}^{I} + \theta^{I}) \leq \frac{(1 - \frac{1 - \beta}{r})B^{L}(a_{t}^{L} + \theta^{L}) - 1}{1 + \frac{1 - \beta}{r} + B^{L}(a_{t}^{L} + \theta^{L})}.$$
(23)

Proof.

See Appendix 7.2.1.

¹⁸The trading effect is similar to the distributive externality in the work of Davila and Korinek (2018), where the distributive externality in their work sums up to zero as the effects from opposite trading positions cancel out across agents.

¹⁹As also emphasized by Davila and Korinek (2018), the sign of the price elasticity can be negative or positive, depending on the shape of the asset demand.

Condition (23) guarantees that $B^{I}(a_{t}^{I} + \theta^{I}) \leq B^{L}(a_{t}^{L} + \theta^{L})$, which implies that foreign investors face a higher transaction cost when investing in the liquid asset, and therefore agents hold more liquid assets in the solutions of both the CE and the SP. The solution of the SP, however, is further affected by pecuniary externalities. Note that in the case of overborrowing in which the trade effect tends to be positive, agents tend to be future asset sellers (i.e., $a_{t+2}^{j} - a_{t+1}^{j} < 0$) who benefit from higher future prices, which results from lower current borrowing. A relatively high holding of liquid asset a_{t+1}^{L} will magnify the future sale of the liquid asset, $a_{t+2}^{L} - a_{t+1}^{L}$, since a one unit increase in a_{t+1}^{L} does not lead to the same amount of increase in a_{t+2}^{L} , as wealth is also allocated to consumption and illiquid investment. As a result, condition (23) leads to a disproportionately high increase in the trading effect of the liquid asset, which implies underinvesting in the liquid asset.

In sum, the pecuniary externality is determined by a trading effect and a collateral effect, whose signs are affected by the price elasticity. The sign of the price elasticity depends on the relative magnitude of the wealth effect and the convex transaction cost. In equilibrium, the pecuniary externalities of balance sheet items are correlated, which implies that the government should jointly manage all of the balance sheet items.

3.4 Optimal Macroprudential Policy

The optimal policy is derived such that the regulated competitive equilibrium replicates the constrained-efficient equilibrium. By comparing optimality conditions (10) and (11)with optimality conditions (21) and (22), the macroprudential policies are asset-specific and given by

$$\tau_t^j = \frac{\beta E[u'(\pi_{t+1}) \sum_s (\frac{\partial q_{t+1}^s}{\partial a_{t+1}^j} \Delta a_{t+2}^s) - \lambda_{t+1} \sum_s (a_{t+2}^s \kappa^s \frac{\partial q_{t+1}^s}{\partial a_{t+1}^j})]}{u'(\pi_{i,t}) q_t^j},$$
(24)

$$\tau_t^d = \frac{\beta(1+r)}{u'(\pi_t)} E[u'(\pi_{t+1}) \sum_s (\frac{\partial q_{t+1}^s}{\partial d_{t+1}} \Delta a_{t+2}^s) - \lambda_{t+1} \sum_s \kappa^s a_{t+2}^s \frac{\partial q_{t+1}^s}{\partial d_{t+1}}].$$
 (25)

A positive asset tax is applied if the pecuniary externality on the asset is negative, which implies overinvesting in equilibrium, and vice versa. On the liability side, a positive deposit tax is implemented if the pecuniary externality on the deposit is positive, implying overborrowing in equilibrium, and vice versa. The above taxes are equivalent to other policies, such as dividend taxes on assets and a margin requirement. The dividend tax of asset j can be derived by comparing optimality condition (22) with the associated first-order condition, which is given by

$$q_t^j(u'(c_t) - \kappa^j \lambda_t) = \beta E[u'(c_{t+1})(q_{t+1}^j + \tilde{z}^j(1 - \zeta_t^j)]],$$

where

$$\zeta_t^j = \frac{E[u'(\pi_{t+1})\sum_s (\frac{\partial q_{t+1}^s}{\partial a_{t+1}^j} \Delta a_{t+2}^s) - \lambda_{t+1}\sum_s (a_{t+2}^s \kappa^s \frac{\partial q_{t+1}^s}{\partial a_{t+1}^j})]}{u'(\pi_t)} = \frac{\tau_t^j q_t^j}{\beta}$$
(26)

is the dividend tax. Equation (26) shows the functional form of the dividend tax and its relationship with the asset tax. Similar to the asset tax, the sign and magnitude of the dividend tax are determined by the sign of the pecuniary externality. The primary difference between the two kinds of taxes is that dividend taxes will not generate a first-order effect on the relative price of assets, which magnifies the effect of the policy.

The liability side can also be regulated by a more widely used policy, that is, a margin requirement. An optimal margin requirement with state-dependent margins is given by

$$\frac{d_{t+1}}{1+r} \le (1-\theta_t) (\sum_j \kappa^j q_t^j a_{t+1}^j),$$
(27)

where

$$\theta_t = \frac{\beta(1+r)[u'(\pi_{t+1})\sum_{s}(\frac{\partial q_{t+1}^s}{\partial d_{t+1}}\Delta a_{t+2}^s) - \lambda_{t+1}\sum_{s}\kappa^s a_{t+2}^s \frac{\partial q_{t+1}^s}{\partial d_{t+1}}]}{u'(\pi_t) - \beta(1+r)E[u'(\pi_{t+1})}$$

is the tightening of the margin. The margin is increasing in the pecuniary externality of the deposit and is positively correlated with the deposit tax. When the pecuniary externality on the deposit is positive, the marginal benefit of saving increases, and therefore the government supports the agent in borrowing less by tightening the collateral constraint. The sign of θ_t can, however, be negative when agents are expected to be asset buyers who benefit from lower future prices. In this case, the government encourages present borrowing by loosening the collateral constraint.

In sum, the magnitude and the sign of the optimal macroprudential policies are determined by the pecuniary externality of assets and the deposit. Numerical results of optimal macroprudential policies will later be provided in Section 4.3.

3.5 Asset Liquidity and Macroprudential Policy

Asset liquidity affects the pecuniary externality and macroprudential policies on both the liability and asset sides. Intuitively, the SP tends to borrow less when assets, become more illiquid because prices of assets with low liquidity collapse significantly during crises. As a result, the SP has an incentive to raise the tax on the deposit when asset liquidity is lower. Regarding policies that guide asset composition, the SP incentivizes agents to hold the liquid asset that provides a higher collateral value and can be liquidated for more funding in future periods. Therefore, the macroprudential tax on the liquid asset will be lower than on the illiquid asset, and it can potentially be a subsidy.

To identify the effect of asset liquidity, I will first focus on a one-asset model to study how market and funding liquidity affect the liability side and the asset side of the balance sheet. As will be discussed in Section 3.6, the number of asset also affects the sign and the size of macroprudential policies.

3.5.1 The Liability Side

The Effect of Market Liquidity

I begin by focusing on states in which the trading effect and the collateral effect are both non-zero in period t + 1; that is, states in which the probability of a future crisis is nonzero. For any future binding state in which $d'' = (1 + r)a''\kappa q'$, the agent's budget constraint is given by

$$c' + (1 - \kappa)q'a'' = \omega' + (q' + z)a' - d',$$
(28)

$$\Rightarrow \tilde{\omega}' = \omega' + za' - d', \tag{29}$$

$$\Rightarrow \tilde{\omega}' = c' + (1 - \kappa)q'(a'' - \frac{1}{1 - \kappa}a'), \tag{30}$$

where $\tilde{\omega}$ is the predetermined wealth, which equals the sum of newly injected equity capital plus the dividend, minus the existing deposit. Equation (30) shows that the predetermined wealth can be used to either invest or consume. To examine the role of market illiquidity, we first observe that the equilibrium can be characterized as follows:

$$q' = -\frac{1}{(1-\kappa)(\theta - \frac{\kappa}{1-\kappa}a' + \frac{1}{B})}c' + \frac{\tilde{\omega}' + (1-\kappa)\frac{1}{B}\frac{z}{1-\beta^{f}}}{(1-\kappa)(\theta - \frac{\kappa}{1-\kappa}a' + \frac{1}{B})},$$
 (31)

$$\frac{E[\beta u'(c'')(q''+z)]}{q'} = (1-\kappa)c'^{-\sigma} + \beta(1+r)\kappa E[u'(c'')],$$
(32)

where equation (31) combines the budget constraint (30) and the foreign investors' optimality condition (9). Equation (32) combines optimality conditions (10) and (11). The substitutability of consumption and asset investment is determined by the slope of equation (31), which depends on the sign of $(1 - \kappa)(\theta - \kappa a'/(1 - \kappa) + 1/B)$. The sign, in fact, reveals the sign of funding when liquidating the asset, which is given by

$$-\left(\frac{d''}{1+r} + a''q' - a'q'\right) = -(1-\kappa)\left[-\frac{1}{B}\frac{z}{1-\beta^f} + \left(\theta - \frac{\kappa}{1-\kappa}a' + \frac{1}{B}\right)q'\right],\tag{33}$$

$$\Rightarrow \frac{-\partial(\frac{d''}{1+r} - a''q' + a'q')}{\partial a''} = \frac{\partial((1-\kappa)\left[-\frac{1}{B}\frac{z}{1-\beta^f} + (\theta - \frac{\kappa}{1-\kappa}a' + \frac{1}{B})q'\right]\right)}{\partial q'} \times \frac{\partial q'}{\partial a''}$$
$$= \phi\left[\frac{z}{1-\beta^f}\frac{B}{(1-B(a''-a'-\theta))}\right],\tag{34}$$

where

$$\phi \triangleq (1-\kappa)\left(\theta - \frac{\kappa a'}{(1-\kappa)} + \frac{1}{B}\right) > 0.$$
(35)

The left-hand side of equation (33) is the provision of new funding. The asset investment and the deposit can be replaced by the asset demand $a'' = (1-z/[(1-\beta^f)q])/B+a'+\theta$ and the collateral constraint $d''/(1+r) = \kappa a''q'$. A unit of the asset liquidation, therefore, provides positive cash flow when the right-hand side of equation (34) is positive; that is, condition (35) holds. Condition (35) fails to hold when the state variable a' or the adjustment cost Bis high. In those situations, the asset price for a given level a'' appears to be low and thus hinders the gain of liquidation. Condition (35) ensures that in binding states, liquidating the asset provides positive funding even though the asset price drops. This assumption implies that asset investment and consumption are substitutes such that agents cannot pursue a future dividend without sacrificing current consumption.

Equation (31) is downward-sloping in a c' - q' space if (35) holds. The negative slope of equation (31) reveals the substitutability between consumption and investment in which higher consumption crowds out the resource for asset investment, and thus leads to a low asset price. Equation (32) is upward-sloping in a c' - q' space since the marginal benefit of the current consumption as well as the asset holding is equivalent in equilibrium. Specifically, an increase in consumption lowers the marginal benefit of consumption and correspondingly the marginal benefit of the asset holding, which can only be achieved via a high asset price.

An increase in market illiquidity generates two effects on the equilibrium solution via equation (31). First, it increases the absolute value of the slope of equation (31). The intuition is that the market value of the asset sale will be small when the adjustment cost is high following a unit decrease in the price. (Equivalently, if the adjustment cost is high, to provide an incentive for foreign investors to hold one additional unit of the asset, the price has to plunge significantly.) When agents sell the asset against a shock, the funding obtained from liquidation is decreasing in market illiquidity B. Thus, agents would rather decrease

consumption more, and therefore mitigate the drop in the asset price. The decline in the price elasticity with respect to wealth (or the existing deposit) then shrinks the pecuniary externality on the debt and subsequently shrinks overborrowing. I refer to the first effect as the "substitutability effect" to highlight the trade-off between consumption and investment.

Second, the magnitude of the fire sale is increasing in market illiquidity for a given level of consumption and a negative wealth shock. This can be observed in equation (31), where the fire-sale spiral continues until the point at which $\Delta \tilde{\omega}' = \phi \Delta q'$. When the adjustment cost is high, the market value of liquidating the asset (i.e., $-q'\Delta a''$) is low, and therefore it requires a larger fire sale, which leads to a further drop in price to fully compensate a given shock. I refer to the second effect as the "amplification effect" to emphasize the change in the magnitude of the fire sale.

To understand how market liquidity qualitatively affects overborrowing, it is crucial to determine the sign of the price elasticity with respect to the deposit, $\partial q'/\partial d'$, and its derivative with respect to market illiquidity, $\partial q'/(\partial d'\partial B)$. The price elasticity, $\partial q'/\partial d'$, is negative due to the wealth effect. The second derivative, $\partial q'/(\partial d'\partial B)$, is negative as long as the amplification effect dominates the substitutability effect. Under binding states in which agents are forced to sell the asset, the negative elasticity, $\partial q'/\partial d'$, implies overborrowing, as both the trading effect and the collateral effect are positive. Moreover, the size of overborrowing is increasing in market illiquidity when $\partial q'/(\partial d'\partial B)$ is negative.

Figure 3 provides a numerical example following a negative shock on the predetermined wealth $\tilde{\omega}'$ by increasing the existing deposit. To simplify the effect of the expectation terms and focus on asset liquidity, I consider a deterministic future state in which the asset and the deposit converge to a steady state right after the binding period (i.e., a''' = a'' and d''' = d''). The endowment under the binding state equals ω_L , and the endowment for periods onward is ω_F . The solid line is the baseline budget constraint (BC) with a low deposit, while the dashed line indicates the case under a positive deposit shock. The dash-dotted line represents equation (32), where the equilibrium solution is jointly determined with equation (31).

By comparing the two panels in Figure 3, the substitutability effect can be observed from the increase in the absolute slope of the budget constraint when B increases. The amplification effect is captured by the downward shift of the budget constraint, which is increasing in B. The magnitude of the drop in equilibrium price is larger in the right panel where B is higher, implying that the price elasticity and overborrowing are rising in market illiquidity. Table A.5 reports the values of parameters.

The Effect of Funding Liquidity

Funding liquidity, which is measured by κ , increases overborrowing by strengthening the marginal benefit of loosening the future collateral constraint. Using the same parameterization as in Table A.5, Figure 4 shows the relation between asset liquidity and overborrowing



Figure 3: Equilibrium under a positive deposit shock

Notes: This figure plots the equilibrium with low market illiquidity (left panel) and high market illiquidity (right panel). Curves with negative slopes represent agents' budget constraints (31) and the curve with a positive slope is the optimality condition (32). The dashed line indicates the budget constraint with a high initial deposit. The parallel shifts from the solid curves to the broken curves depict the positive deposit shock, which is equivalent to a 2.5% increase in the deposit.

in terms of leverage, which is a function of the liquidity coefficients κ and B. The solid curve indicates the equilibrium with low funding liquidity where $\kappa = 0.9$. The dashed line represents the case in which $\kappa = 0.905$. The magnitude of overborrowing is increasing in both the coefficient of market illiquidity B and the coefficient of funding liquidity κ .

3.5.2 The Asset Side

The Effect of Market Liquidity

In a one-asset model, the pecuniary externality can be written as the sum of the trading effect and the collateral effect, which is given by

Pecuniary externality =
$$E[-u'(c')\frac{\partial q'}{\partial a'}\Delta a'' + \lambda' a'' \kappa \frac{\partial q'}{\partial a'}].$$

To analyze the value of the pecuniary externality, I will focus on the case in which the collateral constraint binds with $\Delta a'' < 0$ and $\lambda' > 0$.

The sign of the price elasticity is mainly influenced by two effects: (1) the wealth effect of having a higher initial asset and (2) the change in asset prices to equate foreign demand. For example, an increase in the initial asset a' will provide a positive wealth effect, which tends to increase the holding of the current asset a''. However, for any given a'', it may lower the



Figure 4: Overborrowing and asset liquidity

asset price because the foreign purchase, $(a''_f - a'_f)$, increases. The trade-off can be observed by the price elasticity, which is given by

$$\frac{\partial q'}{\partial a'} = q' \frac{B}{\left[1 - B(a'' - a' - \theta)\right]} \left(\frac{\partial a''}{\partial a'} - 1\right),\tag{36}$$

where $\partial a'/\partial a$ captures the wealth effect, followed by the term, (-1), that captures the second effect. Note that the value of $\partial a''/\partial a'$ being greater than one implies that an increase in wealth leads to both asset purchases and consumption. With a positive price elasticity $\partial q'/\partial a'$, the value of the pecuniary externality is increasing in the price elasticity.

With the functional form in hand, higher market illiquidity B generates two effects: (1) insensitive asset dynamics in which $|\Delta a''|$ and $\partial a''/\partial a'$ are lower, as discussed in Section 3.2.2 and (2) larger price sensitivity due to transaction cost, which is captured by $B/[1 - B(a'' - a' - \theta)]$. The price elasticity is deceasing in B if the first effect dominates the second. Intuitively, assets that can be liquidated for more funding during downturns, captured by a more sensitive asset dynamic, tend to be more underinvesting. Note that in states in which the collateral constraint does not bind with $\Delta a'' > 0$ and $\lambda' = 0$, the transaction cost effect $B/[1 - B(a'' - a' - \theta)]$ tends to be more sensitive to B as $(a'' - a' - \theta)$ becomes less negative. In this case, the pecuniary externality is still decreasing in B.

The Effect of Funding Liquidity

Funding liquidity, on the other hand, decreases the optimal macroprudential tax. The reason is that funding liquidity raises the marginal utility of loosening the future collateral constraint, and therefore the purchase of the asset with higher funding liquidity should be subsidized. Intuitively, the SP encourages agents to hold assets that can boost their borrowing capacity.

However, the optimal macroprudential policies in a model with multiple assets are also affected by how the current holding of asset j affects the price of asset i, depending on the trading pattern of asset i and the extent to which the future holding of asset i loosens the future collateral constraint. The following section analyzes the influence of a multi-asset structure.

3.6 The Importance of Modeling Two Assets

This subsection shows that the coexistence of multiple assets generates additional general equilibrium effects, which will affect pecuniary externalities on both assets and the deposit, and thus influence the macroprudential policy. The additional general equilibrium effect cannot be captured by the one-asset model and its comparative statics of the liquidity parameters.

Using a stripped-down model that provides a closed-form solution, I show that the wealth effect of one asset can change the equilibrium price of the other asset. This wealth effect can switch the sign of the optimal policy, depending on the level of future wealth. In periods with low equity capital, agents begin to sell some assets to obtain funding to smooth consumption. The difference between the one-asset model and the two-asset model occurs when the representative agent is a seller of asset j and a buyer of the other asset i. The presence of the asset i can switch the policy from a subsidy to a tax if the negative trading effect of asset i dominates the positive pecuniary externality of asset j.

The stripped-down model is a four-period (i.e., from period 0 to period 3) framework with deterministic capital. The model is similar to the dynamic model with only a few changes: (1) newly injected equity capital is deterministic, (2) markets of assets and deposits end in period 2, and (3) agents simply consume their wealth with linear utility in period 3, in which consumption solves the forward-looking price in period 2.

Period 0 is the key timing that is regulated by optimal policies to correct the asset choices. Period 1 is the timing when agents trade under financial friction such that $d_2/(1+r) \leq \sum \kappa^j a_2^j q_1^j$. The collateral effect will affect the macroprudential policy in period 0 in states with low wealth. The purpose of introducing period 2 is to provide an incentive for saving and investment in period 1. Period 3 solves the final consumption and closes the model. The effect of the pecuniary externality is examined in period 0 by comparing solutions of the CE and the SP. The closed-form solution can be obtained by solving backward. See Appendix 7.1 for analytic details and model solutions.

The critical parts that determine the sign and magnitude of optimal policies are price elasticities with respect to balance sheet items, as shown in functional forms of optimal policies (24) and (25). Proposition 2 analyzes the sign of the price elasticity.

Proposition 2. (Price externalities) Suppose the collateral constraint in period 1 is not binding, there exists a threshold ω^* such that $\partial q_1^j / \partial a_1^j$ and $\partial q_1^j / \partial a_1^i$ are positive, and $\partial q_1^j / \partial d_1$ is negative when $\tilde{\omega} > \omega^*$, where

$$\tilde{\omega} = \omega_1 + \sum_{s=I,L} (1 + \frac{1}{1+r}) z^s a_1^s - d_1 + \frac{\omega_2}{(1+r)}$$
(37)

is increasing in the sum of initial wealth in period 1 and future capital ω_2 .

Proof.

See Appendix 7.2.2.

Condition (37) holds when agents are in wealthy states in which ω_1 , ω_2 and $\{a_1^s\}$ are large and d_1 is small. $\tilde{\omega}$ can be viewed as some lifetime wealth that includes future capital ω_2 . This condition implies that the marginal increase of the asset $\partial a_2/\partial a_1$ is large when agents are wealthy. The intuition is that agents with a concave utility will allocate more funding to asset purchases as u'(c) < 0. As the existing asset a_1 goes up, initial wealth increases. Agents who gain a smaller marginal utility from consumption would rather invest in assets such that $\partial a_2/\partial a_1$ rises.

Because the process of equity capital is deterministic, the agents' decision in period 0 can determine whether the collateral constraint binds in period 1 or not. Since the macroprudential policy does not play a role when future states bind with certainty, I will focus on the policy in period 0 when the collateral constraint in period 1 is nonbinding with certainty. In this situation, the collateral effect of the pecuniary externality is always zero. The remaining task is to determine how the trading effect is affected when we compare the one-asset model and the two-asset model. The sign of the trading effect (TE) that determines the sign of the macroprudential policy is characterized by Proposition 3.

Proposition 3. (Policy switch) There exist wealth thresholds $n^{**} < n^*$ such that a policy switch occurs when wealth $n \in [n^{**}, n^*]$ and condition (38), as well as (39), hold.

$$max\left[\frac{z^{j}}{1-\beta}\frac{1}{1-B^{j}(\underline{a}^{j}-\bar{a}^{j})+B^{j}\theta^{j}},\frac{z^{j}}{1+r}\right] < \frac{z^{j}}{1-\beta}\frac{1}{(1+B^{j}\theta^{j})}$$
(38)

$$\begin{cases} (1+\theta^{i}B^{i})(1-\beta) > 1+r\\ (1+\theta^{j}B^{j})(1-\beta) < 1+r \end{cases}$$
(39)

Proof. See Appendix 7.2.3. The interval of the initial wealth $[n^{**}, n^*]$ represents the bounds where trading effects in the one-asset model and the two-asset model are of opposite signs. Intuitively, a policy switch only occurs in states in which agents purchase one asset and sell the other. This happens when the initial wealth level is not too high or too low. If the initial wealth is too low, agents have an incentive to sell both assets to smooth consumption. On the other hand, if the wealth is too high, agents have an incentive to increase the holding of both assets and consumption. Condition (38) ensures that there exists a wealth level $n_1 \in (n^{**}, n^*]$ at which the levels of assets, $\{a_1^j\}$, are within the selected bounds of assets, $[\underline{a}^j, \overline{a}^j]$. The left-hand side of condition (38) is the minimum value of asset j's price, which is the maximum number between the lowest possible price within $[\underline{a}^j, \overline{a}^j]$ and the asset price $z^j/(1+r)$ when future consumption equals zero. The right-hand side is the price of asset j on the threshold that decides whether agents buy or sell asset j. Condition (39) implies that the transaction cost at which agents maintain the same level of the asset is higher for the illiquid asset, and that the gap between the two costs is large enough.

To highlight the difference between the one-asset model and the two-asset model, I provide a numerical example with the parameterization listed in Table A.6. The relative risk aversion σ is assumed to be one. Endowment capital is constant from period 0 to period 1 such that $y_0 = y_1 = \bar{y}$. The initial wealth is selected by moving \bar{y} across the interval [1.00, 2.35]. The discount parameter β follows a standard value 0.96 such that the quarterly discount rate equals 0.99. The collateral value of asset { κ^L, κ^I } is estimated from NY Fed Tri-Party/GCF Repo data, as described in Section 4.

Figure 5 shows the asset adjustment across different initial wealth. The solid line represents the adjustment of the liquid asset and the dashed line indicates the change in the illiquid asset. The unshaded region represents nonbinding states, while the shaded area indicates the binding states. By setting $\{a_1^s\}$ equals one, the vertical axis represents not only the actual level of the asset sale but also the percentage change in the asset level.

Changes in holdings of assets depend on the wealth level and asset liquidity. If agents are wealthy, they tend to raise consumption and purchase both assets. As agents' wealth shrinks, they lower the purchase of both assets. There exists a turning point at which agents try to sell assets, but foreign investors can only pick up the asset demand of the liquid asset because the transaction cost of the illiquid asset is too high. It is only when agents are poor that they are willing to sell off the illiquid asset at a very low price to compensate for the transaction cost, which is paid by the foreign investors. When changes in holdings of assets are of the same sign, optimal macroprudential policies are magnified when the second asset is incorporated. Importantly, when agents buy one asset and sell the other, policies may be offset or of opposite signs.

Within the binding states, agents are forced to sell both liquid and illiquid assets. The

fact that the liquid asset provides higher funding liquidity results in a portfolio adjustment by which the relative sale of the liquid asset becomes milder compared with the illiquid asset as wealth decreases.



Figure 5: Asset sales across wealth level

Figure 6: Optimal policies on the liquid asset

To elaborate on the difference in the sign and magnitude of the macroprudential policy between the two models, I show in Figure 6 the macroprudential policy on the liquid asset. The solid line represents the policy in the one-asset model, and the dashed line indicates the policy in the two-asset model. The blue shaded area (B) is the policy-switching region in which the trading effect of buying the illiquid asset dominates the trading effect of selling the liquid asset.

Within region (B), agents benefit from lower asset prices in period 2 in the two-asset model. Low asset prices can be achieved by low holdings of assets and a high level of the deposit in period 1, as shown by Proposition 2. As a result, the SP implements asset taxes to decrease investment and imposes a subsidy on the deposit (see Figure 7). In the model with only the liquid asset, agents—as net sellers of the asset—benefit from high asset price in period 2. Therefore, the optimal policy in the one-asset model is a subsidy, which is in contrast to the taxes in the two-asset model.

There exist some intervals in which policies in the two models are in the same direction. Region (A) is the reinforcing region in which the trading effects of the two assets have the same sign, and thus incorporating more assets reinforces the optimal policy. Region (C) is the offsetting region in which the trading effects of the two assets have opposite signs, and hence the magnitude of policies will be partially offset. Region (D) is again a reinforcing region in which agents become asset sellers of both assets. Region (E) indicates the binding region in which the solutions of the CE and the SP coincide. In this situation, the macroprudential policy is redundant. Figure 7 shows the macroprudential policy on the deposit. The above analysis regarding the sign of the policy and the pecuniary externality is similar. The only difference is that the policy features an opposite sign due to the fact that the price elasticities with respect to the assets and the deposit are of opposite signs.





Figure 8: Optimal policies on assets

Adopting the two-asset model, Figure 8 compares the values of the macroprudential policies imposed on different assets. It is worth noting that the sign of the policies on both assets is the same regardless of the level of the initial wealth, because the price elasticities with respect to assets are both positive. However, the policy rate on the illiquid asset appears to be higher. The reason is that the wealth effect provided by the investment of the illiquid asset is stronger than the liquid asset due to its high dividend.

4 NUMERICAL RESULTS

This section first calibrates the model to match Argentina data and solves policy functions of CE and the equilibrium of the SP. To highlight the difference between the financial crisis and regular downturns, I enter small and large equity shocks, whereby the former leads to a financial crisis and the second does not. I then simulate the model and compare simulated results under CE and the equilibrium of the SP. Finally, to validate the model, I run an event study of the Argentine 2002 crisis and compare empirical and simulated moments of the liquid share and leverage.

4.1 Calibration

The model is solved by applying the endogenous grid method with an occasionally binding constraint and three endogenous state variables $\{a_t^L, a_t^I, d_t\}$, which are discretized into 50 grids individually. The approach is similar to the method adopted by Hintermaier and
Koeniger (2010), who study the algorithm with one endogenous state variable.

Equity capital is an exogenous state variable, which is calculated as the HP-filtered sum of Argentine banks' capital using micro data from Bankscope from 1991 to 2015. The log of equity capital is assumed to be an AR(1) process $ln\omega_t = \alpha ln\omega_{t-1} + \epsilon_t$ and is discretized via the Tauchen (1986) method. $ln\omega_t$ is discretized into eight grids (which implies 1,000,000 states in total) from 3 standard deviations to -3 standard deviations.²⁰ The endogenous solution is interpolated by Delaunay interpolation.²¹ (See Appendix 7.5 for a detailed description of the algorithm.)

Values of parameters and their sources, as well as the calibrated targets, are listed in Table 3. The annual discount factor $\beta = 0.96$ is standard, such that the quarterly β equals 0.99. Because of the data limitations on the Argentine asset market, asset margins κ^j are estimated from NY Fed Tri-Party/GCF Repo data, which collects asset haircuts in the US tri-party repo market. The data are monthly from September 2010 to July 2011.²² I estimate the margin of the illiquid asset by the time-average haircut value of the wholesale loans. The margin of the liquid asset is estimated by averaging over time the volume-weighted mean of the haircut for all other assets.²³ Values of asymmetric factor θ^j are set such that there exists a unique steady state in which the steady-state prices of assets are equivalent between the solutions derived from the first-order conditions of agents and foreign investors.

Regarding calibrated parameters, dividends z^j are calibrated to the ratios of the average nominal value of asset j to average equity capital. The intuition is that dividends reflect on asset prices and asset volumes. The coefficients of market liquidity B^j , which affect the frequency of asset transactions, are calibrated to match the ratio of the standard deviation of asset j to the standard deviation of equity capital. The interest rate r, which guides the inter-temporal decision of consumption and savings, is calibrated to the probability of crises calculated by Reinhart and Rogoff (2009). Table (3) shows the targeted moments derived from the data and the model.

4.2 Balance sheet decisions

To understand how the representative agent adjusts his portfolio subject to the collateral constraint, this subsection analyzes agents' decisions on balance sheet items and nonlinearity

²⁰The AR(1) coefficient α is 0.17, and the variance of the residual ϵ_t is 4.56 × 10⁻⁴. The grids of the $ln\omega_t$ constructed by the bandwidth that equals 3 standard deviations are [0.9370, 0.9546, 0.9725, 0.9907, 1.0093, 1.0283, 1.0476], which correspond to \pm [0.425, 1.285, 2.145, 3] standard deviations.

²¹A similar approach has been adopted by Ludwig and Schon (2018), and Brumm and Grill (2014).

 $^{^{22}}$ Margin data on the liquid asset are available from May 2010 to July 2019 (present), whereas the margin of loans is only available for the period September 2010 to July 2011. I therefore select the latter range of dates to get balanced margin data.

²³Liquid assets include treasuries, agency securities, ABS, CMO, corporate bonds, equities, international securities, money market, and municipality debt.

 Table 3: Parameters

Fixed parameters								
β	0.9600	Standard value	κ^{I}	0.9000	Tri-repo data			
σ	2.0000	Standard value	θ^L	0.4398	$(1/\beta - 1)/B^L$			
κ^L	0.9720	Tri-repo data	θ^{I}	0.0822	$(1/\beta - 1)/B^I$			

Calibrated parameters

		Targeted Moments	Data	CE
\tilde{z}^L	0.0668	μ_L/μ_E	1.028	1.025
\tilde{z}^{I}	0.0999	μ_I/μ_E	0.997	0.997
B^L	0.0947	σ_L/σ_E	2.576	2.580
B^{I}	0.5071	σ_I/σ_E	0.758	0.758
r	0.0416	Crises Probability	0.088	0.080

Notes: For the period 1991-2017, the liquid asset here is defined as the sum of currencies and deposits collected from the IMF IIP data. The illiquid asset is the sum of direct investment and the gold holdings recorded by IMP IIP, as well as loans from Bankscope. Loans include corporate commercial loans, residential mortgage loans, loans and advances to banks, and other commercial or retail loans. Column "Data" lists the empirical moments and column "CE" indicates the moments of the competitive equilibrium in the model. Moments in the model are derived from 7,500 data points after dropping the first quarter of 10,000 times simulation.

arises from the financial friction.

Regarding the liability side, Figure 9 plots the policy function of the deposit in the CE under different equity shocks. The horizontal axis represents the initial holding of the deposit, and the vertical axis denotes the deposit selected in the current period. The next period deposit is monotonically increasing in the initial deposit during nonbinding states. This can be represented by the unshaded area, in which higher initial borrowing provides an incentive for agents to borrow more as long as the collateral constraint does not bind.

However, the entire policy function of the deposit is not monotonically increasing in the existing deposit. During the binding region, a high initial deposit implies low initial wealth, and thus a low level of the nominal value of asset holdings due to the wealth effect. Low values of asset holdings then support low borrowing, given that the collateral constraint is binding. The downward-sloping binding region can be observed from the shaded area, which indicates the binding region when the equity capital shock is -2.145σ .²⁴

Next, to analyze overborrowing, Figure 10 plots policy functions of the deposit in the CE

 $^{^{24}}$ To understand the relevance of the financial friction, I show the complete allocation of binding and nonbinding states in Figure A.7. The collateral constraint binds under low initial wealth, which results from either a high amount of the deposit or low levels of asset holdings.



Figure 9: Policy function of deposit in equilibrium Notes: This figure plots the policy function of the deposit when both the existing liquid and the illiquid asset are at the 24th percentile of grids.

and the equilibrium of the SP. The deposit decision in the CE is always higher than the level in the equilibrium of the SP, reflecting the agents' ignorance of the pecuniary externality in which the current accumulation of the deposit increases the risk of hitting a binding collateral constraint. These patterns are in line with numerical results commonly found in the literature, which quantifies the degree of the pecuniary externality and overborrowing.

The gap in the selected deposits between the solutions of the CE and the SP is not constant across states and is wider when the existing deposit is higher. The reason is that the future probability of binding is decreasing in the existing wealth (that is, increasing in the existing deposit) and the deviation widens as the collateral effect increases. The region in which two solutions are parallel indicates states in which the collateral constraint never binds. Within this interval, the gap only captures a nonzero trading effect, as shown by the optimality condition (21).

Regarding the asset side, quantities and prices of assets reveal the severity of the fire sale in the financial crisis. Figure 11 plots the policy functions of asset quantities and prices with respect to the existing deposit. As the deposit increases, prices and the quantities fall due to the wealth effect. The collateral constraint binds under negative capital shocks when the existing deposit is high, as shown by the sharp decreases in the slopes of the solid lines.

Note that the relative magnitude of changes in the quantity to changes in the price depends on asset liquidity. Specifically, the liquid asset features a more sensitive asset



Figure 10: Deposit policy function under -1.285σ capital shock Notes: This figure plots the policy function of the deposit when both the existing liquid and the illiquid asset are at the 80th percentile of grids.

quantity compared with the asset price, whereas changes in the volume of the illiquid asset mainly result from changes in the price. The key reason that drives different sensitivities of quantities and prices across assets is market liquidity B^{j} . If the asset is highly illiquid, the price has to decline drastically to trade a given amount of quantity. Comparing two assets, the quantity of the liquid asset is more sensitive than the illiquid asset. This feature supports the argument that the liquid asset should be sold more during adverse shocks due to market liquidity.

4.3 Optimal Macroprudential Policy

This subsection describes one of the main focuses of the paper: what is the optimal macroprudential policy to jointly manage asset composition and the size of borrowing? Figure 12 plots optimal macroprudential policies on asset purchases. The left (right) panel plots the optimal liquid (illiquid) tax as a function of the existing holding of the liquid (illiquid) asset. Optimal policies are similar to the derived policy in the stripped-down model, as shown in Figure 6. The taxes are positive in high-wealth states, since in those states agents tend to be future asset buyers, and thus they benefit from low asset holdings which can be achieved by positive taxes. The taxes become negative when the trading effect of selling assets dominates the effect of buying assets. Optimal policies become zero during binding states, in which the SP faces the same prices and borrowing constraint as agents.

Asset taxes are not monotone, even within the nonbinding region in which existing assets holdings are high. The concavity of taxes comes from nonlinearity of asset prices, as shown in the functional forms (9) and (10). As the existing asset level increases, the total expense of maintaining prices by purchasing assets becomes more expensive. To equate the marginal



Figure 11: Policy functions of asset prices and quantities

Notes: This figure plots the policy function of the asset prices and quantities when both the existing liquid and the illiquid asset are at the 24th percentile of grids. Shaded areas indicate binding states.

benefit of consumption and asset investment, agents then tend to allocate their available funding to consumption. The fact that the incentive to purchase assets declines lowers the positive taxes that aim to reduce future asset prices.

It is worth noting that taxes or subsidies implemented on purchases of the illiquid asset are larger than the liquid asset. In line with the stripped-down model, the intuition is that the illiquid asset provides a stronger wealth effect via a higher dividend. Moreover, the optimal macroprudential policy of one asset will switch from a tax to a subsidy at a higher level of existing holdings when the existing holding quantile of the other asset is lower. The reason is that lower existing holdings of the other asset implies lower initial wealth, and therefore agents tend to be future asset sellers, who benefit from current asset subsidies that eventually promote future asset prices.

Regarding the policy on the liability side, Figure 13 plots the optimal macroprudential policy of the deposit under different percentiles of existing assets. The tax is increasing in the deposit level, because the probability of crises is increasing in the existing deposit. When the pressure of a future crisis rises, the government provides an incentive for agents to borrow less by raising the deposit tax. Similarly, the fact that the probability of crises is decreasing in the existing asset holdings explains the pattern whereby the optimal deposit tax is decreasing in existing assets. When the collateral constraint binds within states that feature low wealth (which is often due to high existing deposit or low existing assets), the



Figure 12: Optimal macroprudential policies on assets

Notes: The left (right) panel plots the policy functions of the asset taxes when the deposit is at the 80th (70th) percentile of grids. The curves indicate the percentile of the existing level of the other asset. The shock on equity capital is assumed to be -1.285σ .

optimal policy is zero, as the SP faces the same borrowing capacity as the decentralized agents.

4.4 Impulse Responses: Binding and Nonbinding Shocks

This section highlights the crucial effect of the financial friction in the model: The fire sale results from a binding collateral constraint. I run and compare impulse responses of asset holdings and the deposit under two negative shocks on equity capital, in which one leads to a binding collateral constraint and the other does not. I show that adjustments of asset allocation and the size of borrowing are quite different under the binding and nonbinding shocks.

Figure 14 demonstrates impulse responses under exogenous shocks on equity capital. The solid line represents the case under a shock that triggers a financial crisis (i.e., hitting the collateral constraint) in period 7, which is labeled by vertical dashed lines. Here, agents hit the collateral constraint if they receive a -3σ shock for 4 consecutive periods. The dotted line represents the nonbinding case in which agents suffer from a -0.425σ shock for 4 periods. Several features distinguish the two scenarios.

First, the deposit rises and decreases smoothly following the end of the shock if the collateral constraint does not bind. Under nonbinding shocks, the impulse response of the



Figure 13: Optimal macroprudential policy on deposit

Notes: This figure plots the policy function of the deposit tax in which different curves indicate the tax rate when both existing assets are at a specific percentile. The shock on equity capital is assumed to be -1.285σ .

deposit increases during periods 4 and 7 to smooth consumption. However, if the collateral constraint binds, agents hit the maximum capacity of borrowing and subsequently trigger the deleveraging spiral.

Second, asset sales are more significant under the binding shock. Taking both assets into account, the total market value of assets drops by about 10% in the binding case, whereas the market value in the nonbinding case barely changes.

Third, I observe that in the binding case, the liquid asset features a more significant asset sale that lowers the liquid share during the crisis, as shown in the last panel. The value of the liquid asset declines by more than 10%, while the drop in illiquid holding is less than 7%. As a result, the liquid share drops by more than 1.5%, which roughly equals a 3% drop from the initial level. The model supports the empirical finding that the liquid share tends to decrease during financial crises. The result implies that the effect of market liquidity dominates funding liquidity, and therefore the liquid asset serves as a better buffer against crises.

In sum, I show that the model generates a reversal of borrowing, fire sales of assets, and a decline in the liquid share during the financial crisis. The remaining questions are related to the distribution of states. How often does the crisis occur? Is the financial friction economically significant? How large is the difference between the unregulated equilibrium and the constrained-efficient equilibrium? How much should the government tax or tighten the margin to achieve the constrained-efficient equilibrium?



Figure 14: Impulse responses under binding and nonbinding shocks Notes: The starting values of the initial state variables at the beginning of period 0 are averages of 7,500 times simulation by dropping the first quarter of the sequences of 10,000 times simulation. The vertical axis presents values normalized by numbers in period 0. Equity capital is assumed to be 1.0093 (i.e., $+0.425\sigma$) during the first four and the last four periods. The non-binding shock refers to a -0.425σ shock while the binding shock refers to a -3σ shock.

4.5 Simulated Results

This subsection first highlights the effect, values, and cyclicality of the optimal macroprudential policies, and quantitatively studies the relevance of the multi-asset structure. By comparing simulated moments under the CE and the equilibrium of the SP, I show that the model generates large overborrowing. Optimal macroprudential policies guide agents toward a more liquid portfolio and lower the probability of crises. Quantitatively, illiquid taxes tend to be higher than liquid taxes, which are often subsidies. Regarding cyclicality, I show that optimal macroprudential policies are procyclical. Finally, I find that the policy switch in this two-asset model occurs with a high probability.

4.5.1 Ergodic Distribution of Agents' Decisions

The importance of the macroprudential policy depends on the difference between the simulated results of the regulated and the unregulated equilibrium. This subsection compares simulated balance sheets under the two cases. To highlight the improvement the optimal macroprudential policy can achieve, I compare consumption and the probability of crises between the solutions of the CE and the SP.²⁵ Figure 15 presents simulated distributions of the CE and the equilibrium of the SP, in which simulated moments are listed in Table 4. Several findings emerge from the simulation results.



Figure 15: Simulated distribution in equilibria

First, the deposit distribution of the SP is to the left of the distribution in the CE because the SP internalizes the pecuniary externality, and thus decreases borrowing. Quantitatively, the degree of overborrowing equals 9.43% in terms of leverage or 2.24% in terms of the deposit-to-GDP ratio; both are more significant than the numbers quantified in the literature.

Second, agents can consume more on average in the constrained-efficient equilibrium. The optimal macroprudential policy raises the equilibrium consumption by 0.98%. Specifically, in the CE, consumption is lower during financial crises ($\mu_c = 1.0009$) compared with normal times ($\mu_c = 1.0121$).

Third, the distribution of the illiquid holding is similar between two cases, while the SP tends to accumulate more liquid assets, as shown in the bottom panels of Figure 15 and Table 4. The SP on average holds 3.73% more liquid assets, which results in a 5.1% increase in the liquid share.

Finally, the SP can completely eliminate the risk of financial crises by holding a portfolio that is both less leveraged and more liquid.

4.5.2 Implied Optimal Macroprudential Taxes

This subsection quantifies the distributions of optimal macroprudential policies. I particularly highlight results of policies on the two assets, as the main focus is to understand the

 $^{^{25}\}mathrm{See}$ Subsection 5.1 for a more detailed welfare analysis.

	$\mu_{D/GDP}$	μ_C	$\mu_{L/GDP}$	$\mu_{I/GDP}$	Leverage	P_{crisis}	Liquid Share
SP	0.3689	1.0193	0.2375	0.2080	0.7969	0.0000	0.5325
CE	0.3913	1.0095	0.2138	0.2081	0.8912	0.0080	0.5065
SP - CE	-0.0224	0.0098	0.0238	-0.0001	-0.0943	-0.0800	0.0260

Notes: This table plots the ergodic mean of consumption and ratios of balance sheet items to GDP, which is produced according to equation (5). The level of leverage is defined as the nominal value of deposits divided by the sum of nominal values of assets. The probability of crises is defined as the frequency at which the collateral constraint binds. The liquid share is defined as the nominal value of the liquid assets divided by the total assets.

regulations that guide asset composition.

Figure 16 plots the implied density of the macroprudential taxes on asset purchases (hereafter, asset taxes) and dividends (hereafter, dividend taxes).²⁶ To encourage agents to hold more liquid assets, the macroprudential tax on the liquid asset tends to be smaller than the illiquid asset. Some states even require a subsidy on liquid asset investment. The result shows that the optimal macroprudential policy should favor the purchase of the liquid asset.

It is worth noting that average asset taxes are quantitatively small.²⁷ The reason is that asset taxes create a first-order effect on the relative price of assets, as shown by equation (24). Therefore, asset taxes generate a strong substitution effect that distorts the portfolio toward the liquid asset. Moreover, asset taxes also create a wealth effect via nominal values of existing asset holdings. For example, with a subsidy on the liquid asset, agents buy the liquid asset and boost its price, which subsequently raises agents' wealth via the existing holding of the liquid asset. With the substitution and the wealth effect, minor asset taxes can significantly change asset composition.

The magnitude of the dividend taxes, on the other hand, is larger than the asset tax.²⁸ The reason is that dividend taxes only generate a wealth effect, and therefore the SP requires a stronger dividend tax to achieve the constrained efficient equilibrium.

²⁶Regarding the liability side, Figure A.8 plots the implied distribution of the deposit tax and the tightening of margins. The deposit tax tends to be positive (the mean equals 7.24×10^{-4}), and therefore provides less incentive for agents to borrow. The example of the margin requirement (27), which is more widely used, suggests an average tightening of the margin θ equals 1.5%.

²⁷The ergodic mean of the liquid asset tax is -3.33×10^{-4} , while the mean of the illiquid asset tax is 1.34×10^{-4} .

²⁸The ergodic mean of the liquid dividend tax is -2.14×10^{-3} while the mean of the illiquid dividend tax is 1.73×10^{-4} . The magnitude of the liquid dividend tax is consistent with the value calculated by Bianchi and Mendoza (2011), in which the average dividend tax equals -4.6×10^{-3} .



Figure 16: Density of implied taxes on assets

4.5.3 Dynamics in Financial Crises

This subsection highlights the average dynamics of financial crises and the cyclicality of optimal macroprudential policies. Figure 17 plots these dynamics.

The binding period is triggered by a consecutive decline in equity capital. To accommodate the negative growth of the equity, agents raise the deposit and lower consumption, as well as asset holdings, prior to financial crises. When the collateral constraint binds, the deposit suddenly shrinks, leading to lower borrowing capacity and asset holdings. As agents approach the limit of the borrowing capacity, the implied optimal macroprudential taxes on the deposit and the illiquid asset rise, and the tax on the liquid asset drops. This implies that the SP incentivizes agents to be more cautious by borrowing less and holing more liquid assets as the risk of crises increases.

Note that the optimal taxes are procyclical; that is, the SP mostly implements taxes during states with low equity. This result is in line with the work of Schmitt-Grohe and Uribe (2017), in which they find that capital controls are procyclical in models, which justify macroprudential policies via the pecuniary externality. The reason behind the procyclicality is that the probability of financial crises is high precisely during downturns.

4.5.4 Relevance of the Multi-asset structure

This subsection shows the relevance of the key ingredient of the model: the multi-asset structure. Table 5 lists the frequencies of scenarios, in which incorporating a second asset mitigates, amplifies, or switches the sign of the optimal macroprudential policy on the initial



Figure 17: Dynamics during financial crises

Notes: This figure plots the dynamics of aggregate variables and implied taxes from simulation for 100,000 periods. Period 0 indicates the period of financial crises, in which the collateral constraint binds.

asset. Specifically, a policy switch occurs when externalities of the one-asset model and the two-asset model are of different signs.²⁹ Simulated results show that the probability of the policy switch on the deposit is 18.9% and the probability of policy switch on the liquid asset is 5.5%. The probability in which at least one balance sheet item encounters a policy switch is 23.8%. These numbers imply that the policy switch, which results from the multiple asset structure, is not a knife-edge scenario.³⁰

²⁹Specifically, to eliminate the wealth effect, the one-asset model here features equity capital $\tilde{\omega} = \omega + q^I(\{a^j\}, d, \omega) \times (a^{I'}(\{a^j\}, d, \omega) - a^I) + z^I a^I$, where ω is equity capital of the two-asset model. The probability of the policy switch is indeed affected by the value of equity capital.

³⁰Note that the pecuniary externality of the illiquid asset is of different signs compared with the pecuniary externality of the liquid asset in the "Switch" and the "Offset" scenarios. The reason the sums of the probability of "Switch" and "Offset" are different between the deposit tax and the liquid asset tax is that the price elasticity of assets can be either positive or negative, depending on the existing assets the agent holds.

	Switch	Offset	Magnify
Deposit	0.189	0.769	0.042
Liquid asset	0.055	0.043	0.903

Table 5: Macroprudential policies and the two-asset structure

Notes: "Switch" is defined as the case in which the pecuniary externalities defined in equations (21) and (22) are of opposite signs compared with the pecuniary externality associated with only the liquid asset. "Offset" is defined as the case in which pecuniary externalities in the two scenarios are of the same sign, but the magnitude is smaller when considering two assets. "Magnify" is defined as the case in which pecuniary externalities are of the same sign, but the magnitude is smaller when considering two assets. "Magnify" is defined as the case in which pecuniary externalities in the two scenarios are of the same sign, but the magnitude is larger when considering two assets.

4.6 Event Study and Nontargeted Moments

This subsection validates the model by running an event study of the Argentine 2002 crisis and comparing simulated and empirical moments regarding the liquid share and leverage; both are key balance sheet ratios that measure either asset composition or the size of the portfolio.

4.6.1 Argentine 2002 Crisis

I use annual data of balance sheet items from Bankscope and the IMF IIP. The model is shocked by the exogenous process of equity capital that matches the empirical data. As Figure 18 shows, the main feature of the sudden stop is observed from the decrease in banks' borrowing during 2002 and can be replicated by the model. The model well matches empirical dynamics of the deposit, accumulation of liquid and illiquid assets, and liquid share, both quantitatively and qualitatively.

Several observations highlight the fact that the simulated series is consistent with the data. First, both the simulated and empirical trends show a decrease in deposits during crises, which can be explained by the collateral constraint that switches from being nonbinding to being binding. Second, the drop in asset accumulation captures the fire sale. The simulated trend of the total asset, as shown in the bottom left panel, explains 42.5% of the empirical asset decline during the 5-year windows. Third, both simulated series and data show a more sizable fire sale in the liquid asset compared with the illiquid asset. This again supports the argument that the liquid asset is a better buffer against crises. A more massive fire sale in the liquid asset then leads to a fall in the liquid share. The model can generate 73.9% of the drop in the liquid share during the crisis in 2002.

To further examine the credibility of the model via the event that triggered the recent reform of liquidity management, I also run an event study of the US Great Recession during



Figure 18: Event study of the Argentine 2002 crisis

Notes: Equity capital is portfolio investment of the equity by deposit-taking corporations (excludes the central bank) from IMF IIP. The debt level is the sum of debt instruments and debt securities. The empirical liquid asset is calculated as the sum of the currency deposit, the debt securities, and derivatives from IMF IIP. The empirical illiquid asset is defined as the sum of the equity asset, the gold holdings collected by IMF IIP, and the aggregate loan from the Bankscope database. The aggregate loan is defined as the sum of individual banks' residential mortgage loans, other consumer or retail loans, and corporate and commercial loans, as well as loans and advances to banks. Values are normalized by the base year 2000. The starting values of the initial state variables at the beginning of 2000 are averages of 7,500 times simulation by dropping the first quarter of the sequences of 10,000 times simulation. The black dashed line indicates the period in which collateral constraint binds.

2007Q3 to 2009Q4, as shown in Figure A.3. One advantage of this exercise is the frequency of the data with which the model can be tested using quarterly samples. Similar to the above example, the model generates a drop in the liquid share whose magnitude is similar to the decline in data. Moreover, the nominal value of the illiquid asset in both data and the model are less sensitive. In addition, the model can again capture the decrease in total asset holdings.

The event study demonstrates that the model can replicate the movement of the balance sheet, especially the relative trend of liquid and illiquid assets, in both emerging and advanced economies.

4.6.2 Nontargeted Moments: the Liquid Share and Leverage

To highlight the performance of the model in explaining the dynamics of key balance sheet ratios regarding asset composition and the portfolio size, this subsection compares empirical and simulated second moments of the liquid share and leverage. Table 6 lists the second moments of the two ratios of all financial institutions and depository institutions. A depository institution is a narrower and widely-used definition of banks that involve financial intermediation. The model generates standard deviations of the liquid share and leverage, which are smaller than empirical values. Importantly, the model replicates the correlation between the liquid share and leverage, in which the simulated value is in between the empirical value when considering all financial institutions and depository institutions.

Table 6:	Nontargeted	Moments
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	All financial institutions	Depository institutions	Model
σ_{LS}	0.0661	0.0657	0.0065
$\sigma_{Leverage}$	0.0556	0.0541	0.0346
Corr(LS, Leverage)	-0.9033	-0.5159	-0.7857

Notes: This table lists standard deviations of the liquid share and the leverage, and the correlation between the two ratios. Balance sheet ratios are calculated as the mean of bank-level ratios from 2005 to 2015. Depository institutions include bank holding companies, commercial banks, and saving banks. The share of depository institutions of all financial institutions is 0.76. Source: Bankscope.

Figure 19 further compares the scales of the two ratios. The negative correlation can be observed in both panels, in which values of liquid shares tend to be lower than leverage. Note that leverage is higher in binding periods because deposits equal the sum of collateralized values of assets, whereas in non-binding periods deposits are less than the sum of collateralized values of assets.

5 Welfare and Analyses of the Basel III Reform

This section studies welfare improvement generated by optimal macroprudential policies and optimality of the Basel III regulations as counterfactual. This counterfactual is important because there is a debate on whether liquidity regulation in Basel III is welfare improving or not. Regulated financial institutions face a trade-off between portfolio safety and profitability. It is, therefore, crucial to quantify welfare using a calibrated model. The Basel III reform provides simple rules to manage composition of banks' balance sheets: the Liquidity coverage ratio (LCR) and the Net stable funding ratio (NSFR).³¹ Here, I provide theoretical analyses and quantitative results concerning each policy and the complete Basel III reform, where both policies are considered.

The LCR is given by

 $^{^{31}}$ See BIS (2013) and BIS (2014) for detailed description and definitions.



Figure 19: Relations between leverage and the liquid share

 $\frac{\text{Stock of high-quality liquid assets}}{\text{Total net cash outflows over the next 30 calender days}} \ge 100\%,$

which guarantees banks' ability to meet their liquidity needs over the next 30 days. Highquality liquid assets are calculated as the sum of the product of assets and their liquidity factors, which is assigned based on the ease of liquidating for cash. Assets that are viewed as more liquid will be assigned higher factors. For example, the factors of coins and bank notes, which are almost equivalent to cash, are 100%, while the factor for qualifying common equity shares is 50%.

The denominator of the LCR adds up the expected cash outflows over the next 30 days by computing the sum of the product of funding sources and their cash-outflow factors. Funding that is viewed as more "stable" will be assigned lower cash-outflow factors. For example, stable deposits will be assigned factors from 3% to 5%, depending on their deposit insurance scheme. Such factors can be considered to be the probability that deposits are withdrawn by depositors.

Another regulation that manages balance sheet composition is the NSFR, which aims to lower the illiquid asset holding. The NSFR is defined as:

$$\frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \ge 100\%$$

The numerator is constructed by adding the product of funding and their availablestable-funding factors, which are determined by the stability of the funding sources and the probability that funding providers will withdraw the credit. Stable funding, such as regulatory capital or stable demand deposits, will be assigned factors typically higher than 95%. Similarly, the required amount of stable measures the sum of the product of findings and their required-stable-funding factors.

5.1 Models with Basel III Reform

To analyze and evaluate the two policies, I incorporate policies by mapping balance sheet items and set values of factors equal to the official number of the Basel III reform. The agents' optimization problem is as follows:

$$\max_{\{\pi_{it}, \{a_{i,t+1}^j\}, d_{i,t+1}\}} U_{i,t} = E_t [\sum_{s=1}^{\infty} \beta^s u(c_{i,t+s})]$$

subject to

$$\pi_{i,t} + \sum_{j=I,L} q_t^j a_{i,t+1}^j - \frac{d_{i,t+1}}{1+r_t} = \omega_t + \sum_{j=I,L} [(q_t^j + z^j) a_{i,t}^j] - d_{i,t},$$
(40)

$$c_{i,t+s} = \pi_{i,t+s},\tag{41}$$

$$\frac{d_{i,t+1}}{1+r_t} \le \sum_{j=I,L} \kappa^j a_{i,t+1}^j q_t^j, \tag{CC}$$

$$\phi_t^L a_{t+1}^{L'} q_t^L \ge \phi_t^d \frac{d_{t+1}}{(1+r_t)} \times r_{LCR}, \qquad (LCR) \qquad (43)$$

$$\omega_t + \delta_t^d \frac{d_{t+1}}{(1+r_t)} \ge \delta_t^I a_{t+1}^{I'} q_t^I \times r_{NSFR}, \qquad (NSFR) \qquad (44)$$

and foreign investors' optimality condition (9). Equation (43) and (44) are Basel III policies whereby ϕ_t^d is the cash-outflow factor and δ_t^d is the available-stable-funding factor of the deposit. δ_t^I is the required-stable-funding factor of the illiquid asset, and ϕ_t^L is the factor of high-quality liquid assets. To map the balance sheet items in the model with the policies, I assume that the liquid asset in the model is cash with factor ϕ_t^L equals 100% such that the stock of HQLA equals the nominal value of the liquid asset. According to BIS (2013), stable deposits (without deposit insurance) are assigned cash-outflow factor ϕ_t^d equals 5%.

Regarding the NSFR, two stable funding sources are available: equity capital and the deposit. Equity capital is assumed to be highly stable, with factor equal to 100%, while the available stable funding factor of the deposit, following the official policy scheme, equals 95%. I further assume that the illiquid asset in the model is similar to non-performing loans or loans with residual maturity of 1 year or more, which is viewed to be highly illiquid with a corresponding factor δ_t^I equal to 100%. Following the Basel III reform, factors are assumed

to be time-invariant. The lower bounds of the LCR and the NSFR, r_{LCR} and r_{NSFR} , are 100%.

The existence of additional constraints provides different incentives to hold assets and raise deposits. The equilibrium can be characterized by

$$q_t^L = \frac{E_{w_{t+1}|w_t}[\beta u'(c_{t+1})(q_{t+1}^{L'} + z^L)]}{u'(c_t) - \kappa^L \lambda_t - \phi_t^L \lambda_t^{LCR}},$$
(45)

$$q_t^I = \frac{E_{w_{t+1}|w_t}[\beta u'(c_{t+1})(q_{t+1}^{I'} + z^I)]}{u'(c_t) - \kappa^I \lambda_t + \delta_t^I \lambda_t^{NSFR} r_{NSFR}},\tag{46}$$

$$u'(c_t) = \beta(1+r)E_{w_{t+1}|w_t}[\beta u'(c_{t+1})] + \lambda_t + \phi_t^d \lambda_t^{LCR} r_{LCR} - \delta_t^d \lambda_t^{NSFR},$$
(47)

$$0 = \lambda_t^{LCR} [\phi_t^L a_{t+1}^{L'} q_t^L - \phi_t^d r_{LCR} \frac{d_{t+1}}{(1+r_t)}],$$
(48)

$$0 = \lambda_t^{NSFR} [\omega_t + \delta_t^d \frac{d_{t+1}}{(1+r_t)} - \delta_t^I a_{t+1}^{I'} q_t^I r_{NSFR}],$$
(49)

and conditions (9), (12), and (13). Compared with the optimality condition of prices (10), the LCR provides additional incentive for agents to hold liquid assets, since λ_t^{LCR} is nonnegative. In equilibrium, the price of the liquid asset is higher than the case without the LCR, and subsequently leads to higher liquid holdings due to lower demand from foreign investors. The NSFR, on the other hand, discourages domestic agents from holding the illiquid asset, as such holding requires a higher amount of stable funding. Equation (46) shows that the equilibrium price of the illiquid asset should be lower compared with the case without the NSFR, and therefore leads to lower illiquid accumulation as a result of higher foreign demand.

However, the level of the equilibrium deposit compared with the case without policies is ambiguous and affected by both policies. The LCR encourages domestic agents to hold less deposit to dampen future cash outflow, yet it also provides an incentive to borrow more deposit to finance purchase of the liquid asset. The other regulation, the NSFR, boosts the deposit level to meet the need of the available amount of stable funding. To understand the overall effect of the Basel reform, I will next provide numerical results regarding the distributions of the balance sheet items.

Figure 20 plots the distributions of consumption and investment in terms of ratios relative to GDP in regulated and unregulated cases. Compared with the competitive equilibrium, the Basel reform further raises the debt-to-GDP ratio. Consumption under the Basel reform features a more dispersed distribution and narrows the gap in average consumption between the solutions of the CE and SP.³² Regarding asset allocation, all three solutions feature

³²The average consumption of the CE with the Basel reform ($\mu_c^{CE} = 1.0150$) is in between the CE ($\mu_c^{CE} = 1.0095$) and the SP ($\mu_c^{SP} = 1.0193$). I will later compare the changes in welfare that result from the

similar distributions of the illiquid asset. The main difference on the asset side can be observed from the high liquid holdings when the government implements regulations. To further analyze the influence of each policy and the overall effects of the Basel reform, I will move on to some numerical results on the balance sheets in cases that include each policy individually, as well as the complete reform that incorporates both policies.



Figure 20: Simulated distributions in SP, CE, and CE with Basel reform *Notes*: "CE with Basel reform" indicates the scenario in which both the LCR and the NSFR are implemented. Distributions are simulated from 7,500 data points after dropping the first quarter of 10,000 times simulation. See Appendix 7.5.3 for the solution algorithm.

Table 7 compares the average holding of balance sheet items and the probability of crises. To examine the effects of policies on the holding of each asset, I focus on the level of holdings instead of the ratio relative to GDP, as reported in Table 4. Five observations emerge from the analysis. First, compared with the CE, all equilibria with regulations borrow more to finance the extra purchase of both kinds of assets. Second, the relative holdings of the liquid asset in regulated cases are higher than the CE, as shown in the last column.³³ Third, cases with the NSFR and the complete Basel III reform lead to overaccumulation of the liquid asset as liquid shares exceed the level of the solution of the SP. Fourth, the magnitude of the extra holdings of assets is larger than the rise in the deposit such that the leverage soars. Fifth, the probabilities of crises decrease under regulations. Specifically, the LCR cuts the probability by half while the NSFR and the Basel III reform entirely eliminate the risk of

differences in consumption.

 $^{^{33}}$ In equilibrium, agents must use additional funding to invest not only in the liquid asset but also the illiquid asset to ensure that expected rates of return (45) and (46) are the same across assets.

crises.

	μ_D	μ_L	μ_I	Leverage	P_{crisis}	Liquid Share
SP	1.834	0.738	0.432	0.797	0.000	0.532
CE	1.875	0.639	0.416	0.891	0.080	0.506
LCR ($\phi^d = 46\%$)	1.927	0.686	0.433	0.866	0.038	0.514
NSFR ($\delta^d = 95\%$)	1.949	0.813	0.429	0.808	0.000	0.560
Basel III reform	1.950	0.785	0.419	0.832	0.000	0.557

Table 7: Simulated moments: Basel III analysis

Notes: "Basel III reform" indicates the case in which both the LCR and the NSFR are implemented. In cases "LCR," "NSFR," and "Basel III reform," ϕ^L and δ^I are assumed to be one. All factors are time-invariant. μ_x represents the mean of variable x.

However, the elimination of crises comes at a cost, as agents under regulations tend to hold a liquid portfolio that has a low rate of return. The low-yield portfolio generates less utility, although the financial crisis can be fully prevented. Figure 21 compares the consumption-equivalent welfare loss from the four scenarios other than the level of the SP considered in Table 7. The welfare improvement achieved by the SP is roughly 1% higher than the CE in terms of the percentage of consumption. Basel policies can partially eliminate welfare loss.

An NSFR with standard rate $r_{NSFR} = 1$ cuts the welfare loss of the CE by 60%. The resulting welfare improvement is stable across different r_{NSFR} . The LCR, on the other hand, mitigates 23% of the welfare loss under standard $r_{LCR} = 1$. The improvement is similar when lowering the r_{LCR} , yet it shrinks and even becomes harmful as r_{LCR} rises, leading to a more stringent LCR. Reasons for the nonlinear welfare change are a higher binding probability of the LCR and the higher incentive to hold a more liquid portfolio when r_{LCR} soars.

The Basel III reform with standard $\{r_{NSFR}, r_{LCR}\}$, however, may lead to welfare deterioration. The fact that both the LCR and the NSFR provide incentives for agents to hold a liquid portfolio leads to overaccumulation of the liquid asset. It is also worth noting that the leverage and the liquid share under the Basel III reform, as a combination of the LCR and the NSFR, are precisely between the values under the two policies. Comparing the results in Table 7 and Figure 21, we observe that the utility is decreasing in both leverage and the liquid share. The Basel III reform, which features medium levels of the two indicators, achieves the lowest utility (which is 0.5% lower than in the CE) of the four suboptimal scenarios.

One of the critical factors that affect welfare is the probability of financial crises. When



Figure 21: Average consumption-equivalent welfare loss

Notes: This figure plots the mean of consumption-equivalent welfare loss in four cases. "CE with LCR" and "CE with NSFR" indicate the consumption equivalent utility when the policy thresholds r_{LCR} and r_{NSFR} equal [0.8, 0.9, 1, 1.1, 1.2]. Averages are calculated from 10,000 sequences; each starts from a random draw of a three-dimensional Gaussian mixture distribution from ergodic distributions of the endogenous state variables $\{a^L, a^I, d\}$. The exogenous state variable ω is independently sampled from the discrete distribution using the method proposed by Tauchen (1986). Each welfare sample is equal to the discounted sum of utility over the last 200 periods from 250 periods of simulation. The vertical axis plots welfare loss η_k such that $(1 + \eta_k)^{(1-\sigma)} \bar{u}_k = \bar{u}_{SP}$. k indicates all other suboptimal cases, and \bar{u} is the average utility from the simulation.

the economy faces a binding collateral constraint, consumption declines as borrowing collapses. Figure 22 plots the probability of crises under the above five scenarios. The dashed line represents the probability in the CE and is calibrated to the empirical frequency, which equals 8%. As also shown in Table (7), the NSFR, regardless of the policy rate r_{NSFR} within the selected range, can fully prevent financial crises, and so does the complete Basel III reform.

However, the effect of the LCR alone on the crises probability is nonmonotonic. The government can cut the probability of crises by half when applying the standard LCR, but can potentially increase the rates of crisis when the LCR becomes more stringent. Moreover, the probability of crises is increasing in the probability of hitting the LCR. The intuition is that the elasticity of the deposit with respect to the liquid asset holdings is decreasing in the policy rate r_{LCR} ,³⁴ and therefore the relative decline of liquid holdings to deposits when the LCR binds is larger when r_{LCR} is higher. Consequently, the adjustment of the balance sheet tightens the collateral constraint (CC), and thus raises the probability of financial crises.

³⁴Specifically, the elasticity of the deposit with respect to the liquid holding $\partial (d_{t+1}/(1+r))/\partial (a_{t+1}^L q_t^L)$ equals $\phi^L/(\phi^d r_{LCR})$ according to the binding condition (43).



Figure 22: Probabilities of financial crises and binding policies Notes: The left panel demonstrates the probability of hitting the collateral constraint (42). The solid line in the right panel plots the probability of hitting the LCR constraint (43), and the dotted line shows the probability of hitting the NSFR constraint (44).

To summarize the welfare analysis, policies proposed by the Basel Committee may either improve or deteriorate the welfare. With only the NSFR or the LCR, the government can shrink the welfare loss of the CE by 60% or 23%. Both policies lead to portfolios with higher liquid shares, lower leverages, and lower probabilities of crises. The complete set of Basel III reform that includes both policies, however, deteriorates welfare as agents overaccumulate the liquid asset. The Basel III reform, although it may not be optimal, provides a simple rule that can be easily implemented. In the next subsection, I move on to the design of a set of suboptimal Basel rates, which on average replicate the solution of the SP. The policy rates can be a compromise between implementability and optimality.

5.2 Optimality of the Basel III Reform

The Basel III reform fails to be optimal because it is not state-dependent. Here, I first analyze the Basel III reform with optimal factors on assets and the liability. I will then derive the suboptimal factors by taking averages of the optimal factors weighted by the frequency of states.

The optimal factors can be derived by comparing first-order conditions (45), (46), and (47) with (10) and (11). The optimal factors are state-dependent and correlated with the optimal macroprudential taxes, as shown in the following conditions:

$$\phi_t^L = \frac{-\tau_t^L u'(c_t)}{\lambda_t^{LCR}},$$

$$\delta_t^I = \frac{\tau_t^I u'(c_t)}{r_{NSFR} \lambda_t^{NSFR}},$$
$$-\tau_t^d u'(c_t) = \phi_t^d \lambda_t^{LCR} r_{LCR} - \delta_t^d \lambda_t^{NSFR}.$$

The above equations can be simplified such that the SP can be replicated by applying factors that satisfy the following condition:

$$\tau_t^d = \frac{\phi_t^d}{\phi_t^L} r_{LCR} \tau_t^L + \frac{\delta_t^d}{\delta_t^I} \frac{1}{r_{NSFR}} \tau_t^I \tag{50}$$

and replacing the taxes with the values in (24) and (25). Condition (50) implies that only the relative factors of debt to assets (e.g., ϕ_t^d/ϕ_t^L and δ_t^d/δ_t^I) matter as they control the tightness of the LCR and the NSFR. Intuitively, within states in which the liquid asset is less needed, and therefore τ_t^L is higher, the LCR becomes slacker as ϕ_t^d/ϕ_t^L decreases. Similarly, when τ_t^I is higher, agents are encouraged to hold fewer illiquid assets. This can also be achieved by imposing a more stringent NSFR with a lower δ_t^d/δ_t^I .

The Basel III reform with fixed factors cannot be optimal as long as there exist three states that generate nonlinear taxes $\{\tau_t^d, \tau_t^L, \tau_t^I\}$. During nonbinding states, the nonlinear patterns can be observed from Figures 12 and 13, where optimal macroprudential taxes are either strictly convex or concave.

A suboptimal yet simple policy is a set of rates $\{r_{LCR}, r_{NSFR}\}$ that on average satisfy (50), as shown by the following equation:

$$\bar{\tau_t^d} = \frac{\bar{\phi_t^d}}{\bar{\phi_t^L}} r_{LCR} \bar{\tau_t^L} + \frac{\bar{\delta_t^d}}{\bar{\delta_t^I}} \frac{1}{r_{NSFR}} \bar{\tau_t^I}, \tag{51}$$

where $\bar{\tau}_t^i$ is the mean of the ergodic distribution of the competitive equilibrium. Factors $\{\bar{\phi}, \bar{\delta}\}$ are the weighted mean of balance sheet items weighted by their share according to either the empirical portfolio or the hypothetical portfolio with only two assets and one deposit. Figure 23 plots the curves (51) and reveals two findings. First, r_{NSFR} should be much lower than the standard rate of 100% when r_{LCR} is 100%, implying a slacker NSFR to provide less incentive to accumulate a high deposit-illiquid asset ratio. Alternatively, a low r_{NSFR} encourages agents to hold a more illiquid asset for a given level of the liability to pursue higher consumption. Such incentive will correct overaccumulation of the liquid asset in the regulated "Basel" case, as highlighted in the last column of Table 7. Second, the suboptimal rate of r_{NSFR} should be higher when adopting the empirical portfolio. The reason is that factors calculated from empirical balance sheets lead to a more stringent LCR, and yet a looser NSFR. To achieve the hypothetical portfolio, the case with empirical factors

requires a higher r_{NSFR} under the same r_{LCR} .



Figure 23: Optimal rates of Basel III reform

Notes: Mean tax rates are calculated by the last 7,500 times from 10,000 times simulation. The portfolio in the model features stable demand-deposit (this implies $\bar{\delta^d} = 0.95$ and $\bar{\phi^d} = 0.05$) and equity capital on the liability side. The asset side includes the stable deposit without deposit insurance ($\bar{\phi^L} = 1$) as well as nonperforming loans ($\bar{\delta^I} = 1$). The factors of the empirical portfolio are the weighted number of various assets and liabilities recorded in BIS (2019), which records data as of June 30 2018. The factor $\bar{\phi^L}$ equals 0.9669 (calculated from BIS (2019) Table C.75); $\bar{\phi^d}$ equals 0.240 (calculated from BIS (2019) Table 10 and Table C.77); $\bar{\delta^d}$ equals 0.6177 and $\bar{\delta^I}$ equals 0.5148 (calculated from BIS (2019) Table C.77).

6 CONCLUSION

This paper develops a dynamic framework that allows us to derive and evaluate liquidity regulation that can be implemented on different assets. I show that optimal macroprudential policies are affected by heterogeneous liquidity across assets and the multi-asset structure. Asset illiquidity increases the price elasticity of the asset and magnifies the price drop during crises. The social planner, who takes into account the effect of current asset allocation on future prices, will thus have an incentive to hold less debt and more liquid assets. The multiasset structure affects the optimal policy of an asset through cross-price elasticity, which affects its marginal benefit of holding via the future trading position of the other asset, and the future tightness of the collateral constraint. A policy switch may frequently occur when agents are expected to be buyers of one asset and sellers of the other asset.

I show that market and funding liquidity determines the adjustment of the balance sheet. Market liquidity encourages agents to sell because of its low transaction cost, whereas funding liquidity suggests that agents maintain their asset holdings, since asset sales may significantly shrink agents' borrowing capacity. The relative strength of the two forces guides the direction of the macroprudential policies. When market liquidity dominates the movement of asset sales, the liquid asset should be sold more during financial distress, and therefore is a better buffer against shocks. The government should then implement a policy to guide agents toward a more liquid portfolio.

Empirically, I document drops in the liquid share during systemic sudden stop events. This finding implies that the effect of market liquidity dominates funding liquidity. The phenomenon is more evident in emerging countries. I validate the model by running event studies of the Argentine 2002 crisis and the US Great Recession and show that the model can quantitatively and qualitatively explain the dynamics of the liability, asset holdings, and balance sheet ratios, such as leverage and the liquid share. The model also generates moments of the liquid share and leverage, which are similar to empirical values.

Under calibrated simulation, the government should impose macroprudential policies that encourage agents to borrow less and invest more in liquid assets. The resulting overborrowing equals 9.43% in terms of leverage and 2.24% in terms of the debt-to-GDP ratio, both of which are larger than values in literature. Quantitatively, optimal macroprudential policies lower the probability of crises and increase welfare.

Finally, I evaluate the efficacy of the Basel III reform and show that both the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR) increase agents' liquid shares and decrease the probability of crises. However, the complete Basel policy leads to welfare deterioration due to overaccumulation of the liquid asset. To achieve, on average, the constrained-efficient equilibrium, the government should decrease the lower bound of the NSFR, given the standard LCR. The new policy rates correct overaccumulation of the liquid asset by relaxing the constraint on investing in the illiquid asset.

7 APPENDIX

7.1 Analytic details on the four-period model

For tractability, I assume that (1) the asset market is closed after period 2 (i.e., $a_3^j = a_4^j$), and (2) All deposits should be paid back at the end of period 2 such that $d_3 = d_4 = 0$. The remaining profit from asset investment and the interest payment will be consumed by agents such that $\pi_t = c_t$. The representative bank's problem in each period is as follows:

Period 3

By construction, assets and prices are $a_4^j = a_3^j$ with $q_3^j = z^j/[(1 - \beta)(1 + B^j\theta^j)]$. Since $d_3 = d_4 = 0$, consumption can be obtained by the budget constraint:

$$c_3 = \omega_3 + \sum_{j=I,L} (q_3^j + z^j) a_3^j - \sum_{j=I,L} q_3^j a_3^j.$$

Period 2

The optimization problem of the agent is given by

$$\max_{c_{2},\{a_{3}^{j}\},d_{3}}(lnc_{2}+\beta c_{3})$$

subject to

$$d_3 = 0,$$

$$c_2 = \omega_2 + \sum_{j=I,L} (q_2^j + z^j) a_2^j - d_2 - \sum_{j=I,L} q_2^j a_3^j.$$
(52)

From the asset demand of the foreign investor, the price of asset j in period 2 is given by

$$q_2^j = \frac{z^j}{1 - \beta} \frac{1}{1 - B^j (a_3^j - a_2^j - \theta^j)}.$$
(53)

Alternatively, the asset price, according to the asset demand of the domestic agent, can also be represented as

$$q_2^j = \frac{\beta u'(c_3)(\frac{z^j}{1-\beta}\frac{1}{1+B^j\theta^j}+z^j)}{u'(c_2)}$$

$$=\beta\left(\frac{z^{j}}{1-\beta}\frac{1}{1+B^{j}\theta^{j}}+z^{j}\right)c_{2}\triangleq\Phi^{j}c_{2},$$
(54)

where $\Phi^j = \beta (z^j / [(1 - \beta)(1 + B^j \theta^j)] + z^j)$. According to (54), the asset price is proportional to consumption as they are substitutes. The substitutability is controlled by the parameter Φ^j where a larger asymmetric effect θ^j and larger market illiquidity B^j indicates more consumption in exchange for a given unit of the asset. After plugging the price function into the budget constraint (52), the consumption and the asset investment are given by

$$c_2 = \omega_2 + \sum_{j=I,L} a_2^j z^j + \sum_{j=I,L} (a_2^j \Phi^j) c_2 - d_2 - \sum_{j=I,L} (a_3^j \Phi^j) c_2,$$
(55)

$$a_3^j = \left[1 - \frac{z^j}{1 - \beta} \frac{1}{\Phi_j c_2}\right] \frac{1}{B^j} + a_2^j + \theta^j, \tag{56}$$

where (56) is derived from (53) and (54). Consumption can then be solved by using the budget constraint (55), as shown in the following equation:

$$c_2 = \left[1 + \sum_{j=I,L} \left(\frac{\Phi^j}{B^j} + \Phi^j \theta^j\right)\right]^{-1} \left[\omega_2 + \sum_{j=I,L} a_2^j z^j - d_2 + \frac{1}{1-\beta} \sum_{j=I,L} \left(\frac{z^j}{B^j}\right)\right].$$
 (57)

The closed-form solution of c_2 is increasing in the initial wealth level in period 2 and therefore is increasing in equity capital ω_2 , state variables a_2^j and $-d_2$.

Period 1

For simplicity, I assume that the utility function in period 1 is linear. The optimization problem of the agent is given by

$$\max_{c_1, \{a_2^j\}, d_2} (lnc_1 + \beta lnc_2 + \beta^2 c_3)$$

subject to

$$c_1 = \omega_1 + \sum_{j=I,L} (q_1^j + z^j) a_1^j - d_1 - \sum_{j=I,L} q_1^j a_2^j + \frac{d_2}{1+r},$$
(58)

$$\frac{d_2}{1+r} \le \sum_{j=I,L} \kappa^j q_1^j a_2^j,\tag{59}$$

$$q_1^j = \frac{\beta[u'(c_2)(q_2^j + z^j)]}{u'(c_1) - \lambda_1 \kappa^j},\tag{60}$$

$$u'(c_1) = \beta(1+r)[u'(c_2)] + \lambda_1, \tag{61}$$

$$q_1^j = \frac{z^j}{1 - \beta} \frac{1}{1 - B^j (a_2^j - a_1^j - \theta^j)},\tag{62}$$

where states in period 1 can be separated into the binding and non-binding cases. The corresponding one-asset model, which is used to compare with the solution in the two-asset model, has the endowment $\tilde{\omega}_t = \omega_t + q_t^I(\{a_t^j\}, d_t, \omega_t) \times (a_{t+1}^I(\{a_t^j\}, d_t, \omega_t) - a_t^I) + z^I a_t^I$ where ω_t is the endowment of the two-asset model. This adjustment of endowments ensures that the solution of the liquid asset is not affected by the wealth effect where wealth only has to be allocated to consumption and investment of the liquid asset in the one-asset model. The assumption that the maximization problem of the SP has the same borrowing capacity as the equilibrium implies that the solution of the CE and the SP should be the same in binding states. Therefore, macroprudential policies are only meaningful and non-zero when the collateral constraint is non-binding. To study the sign and the size of the optimal macroprudential policy, I will first focus on the case where the collateral constraint does not bind.

(i) Non-binding states

The closed-form consumption can be solved by (58), (59), (60), (61), and (61). Since I am interested in the macroprudential policies, the main focus should be the sign of the price elasticity that governs the value of the pecuniary externality. The sign of the elasticity is discussed in Proposition 2.

(ii) Binding states

The binding states are characterized by (57), (58), (60), (61), (62), and (59) with equality under given states $\{\{a_1^s\}, d_1\}$ as well as endowments ω_1 and ω_2 .

Period 0

The optimal policy that regulates agents' decisions in period 0 reflects the sum of the trading and collateral effect. Assume that the utility function in period 0 is linear where u(c) = c, the optimal macroprudential tax in period 0 is given by

$$\tau_0^j = \frac{-\beta \left[-\frac{1}{c_1} \sum_s \left(\frac{\partial q_1^s}{\partial a_1^j} \Delta a_2^s\right) + \lambda_1 \sum_s \left(a_2^s \kappa^s \frac{\partial q_1^s}{\partial a_1^j}\right)\right]}{z^j / \left[(1-\beta)(1-B^j(a_1^j-a_0^j-\theta^j))\right]},$$

$$\tau_0^d = \beta(1+r) \left[\frac{1}{c_1} \sum_{s=I,L} \left(\frac{\partial q_1^s}{\partial d_1} \Delta a_2^s\right) - \lambda_1 \sum_{s=I,L} \kappa^s a_2^s \frac{\partial q_1^s}{\partial d_1}\right].$$

To simplify the problem and focus on the effect of the pecuniary externality on the macroprudential policy, I select the initial level of assets $\{a_0^j\}$ such that the asset decision $\{a_1^j\}$ equals initial assets $\{a_0^j\}$ in equilibrium.

The framework of a four-period structure is necessary to obtain endogenous price q_1^s and macroprudential policies, which can only be justified by a non-zero pecuniary externality that results from non-zero price elasticities.³⁵ The following observations can establish the endogenous relationship between q_1 and initial wealth in period 1. First, the price q_2^s is increasing in the initial wealth in period 2 because of the wealth effect. Second, q_1^s is increasing in future price q_2^s due to the forward-looking property. Third, the initial wealth in period 1 indirectly boosts up the initial wealth in period 2. Consequentially, factors that change the initial wealth in period 1, such as changes in d_1 and a_1 , affects present prices by changing the wealth in period 2 that results in a variation of future prices, which then shifts the forward-looking prices.

7.2 **Proofs of Propositions**

7.2.1 Proof of Proposition 1

The liquid share is defined as $a'_L q_L / (a'_L q_L + a'_I q_I)$. To compare the liquid share of the CE and the SP, we must examine the magnitude of $a^{L,SP}_{t+1} q^{L,SP}_{t+1} q^{I,SP}_{t+1}$ and $a^{L,CE}_{t+1} q^{L,CE}_{t+1} / a^{I,CE}_{t+1} q^{I,CE}_{t}$ where X^{SP} denotes the variables in the solution of the SP and X^{CE} denotes variables in the CE.

The asset price, as the discounted sum of future liquidity, is given by

$$q_t^j = E_t \left[\sum_{k=0}^{\infty} \prod_{i=0}^k m_{t+1+i} z^j\right]$$
$$m_{t+1+i} = \frac{\beta u'(c_{t+1+i})}{u'(c_{t+i}) - \lambda_{t+i} \kappa^j},$$

where m_{t+1} denotes the per-period stochastic discount factor that will discount the future dividend more if the state t is binding. The asset-specific price is further given by

³⁵If the model features less than four periods, say a three-period model (i.e., ends in period 2) with the ending conditions imposed in period 2, q_2 will be exogenous. Since the model is deterministic, agents will be able to entirely smooth consumption in period 1 under non-binding states. This then leads to exogenous prices q_1^s and fixed asset allocation, that can not be altered by policies.

$$\begin{split} q_t^{j,i} &= M_t^{j,i} \tilde{z}^j \quad \text{for } i \in \{CE, SP\} \text{ and } j \in \{I, L\}, \\ M_t^{j,i} &= E_t [\sum_{k=0}^{\infty} \prod_{s=0}^k m_{t+1+s}^{j,i}] \\ &= E[\beta \frac{u'(c_{t+1}^i)}{u'(c_t^i) - \lambda_t^i \kappa^j} + \beta^2 \frac{u'(c_{t+1}^i)}{u'(c_t^i) - \lambda_t^i \kappa^j} \frac{u'(c_{t+2}^i)}{u'(c_{t+1}^i) - \lambda_{t+1}^i \kappa^j} + \ldots], \end{split}$$

where $M_t^{j,SP}$ and $M_t^{j,CE}$ denote the stochastic discount factor (SDF) of the asset j in the CE and the equilibrium of the SP. The price in the CE and the equilibrium of the SP can be represented as the discounted sum of a stream of dividends $M_t^{j,SP}\tilde{z}^j$ and $M_t^{j,CE}\tilde{z}^j$.

When there exists overborrowing, the following condition holds in equilibrium:

$$\begin{split} u'(c_t^{SP}) &\geq u'(c_t^{CE}), \\ E[u'(c_{t+1}^{SP})] &\leq E[u'(c_{t+1}^{CE})], \\ \lambda_t^{SP} &\leq \lambda_t^{CE}. \end{split}$$

These conditions imply that the price in SP must be greater than CE, that is $M_t^{j,SP} \leq M_t^{j,CE}$. To see how this relates to the liquid share, we first examine the functional form of the market value of the asset j where

$$a_{t+1}^{j}q_{t}^{j} = \frac{-\tilde{z}^{j}}{B^{j}}\frac{1}{1-\beta} + (\frac{1}{B^{j}} + a_{t}^{j} + \theta^{j})q^{j},$$

$$= \frac{-\tilde{z}^{j}}{B^{j}}\frac{1}{1-\beta} + (\frac{1}{B^{j}} + a_{t}^{j} + \theta^{j})M_{t}^{j}\tilde{z}^{j}.$$

The liquid share can, therefore, be given by

$$\frac{a_{t+1}^L q_t^L}{a_{t+1}^I q_t^I} = \frac{\frac{-\tilde{z}^L}{B^L} \frac{1}{1-\beta} + (\frac{1}{B^L} + a_t^L + \theta^L) M_t^L \tilde{z}^L}{\frac{-\tilde{z}^I}{B^I} \frac{1}{1-\beta} + (\frac{1}{B^I} + a_t^I + \theta^I) M_t^I \tilde{z}^I}.$$

The argument that the liquid share in the solution of the SP is higher than that in the CE is true if the following condition holds:

$$\frac{\frac{-\tilde{z}^{L}}{B^{L}}\frac{1}{1-\beta} + (\frac{1}{B^{L}} + a^{L}_{t} + \theta^{L})M^{L,CE}_{t}\tilde{z}^{L}\Omega^{L}_{t}}{\frac{-\tilde{z}^{I}}{B^{I}}\frac{1}{1-\beta} + (\frac{1}{B^{L}} + a^{L}_{t} + \theta^{L})M^{L,CE}_{t}\tilde{z}^{L}}_{t} \geq \frac{\frac{-\tilde{z}^{L}}{B^{L}}\frac{1}{1-\beta} + (\frac{1}{B^{L}} + a^{L}_{t} + \theta^{L})M^{L,CE}_{t}\tilde{z}^{L}}{\frac{-\tilde{z}^{I}}{B^{I}}\frac{1}{1-\beta} + (\frac{1}{B^{I}} + a^{I}_{t} + \theta^{I})M^{I,CE}_{t}\tilde{z}^{I}},$$
(63)

where $\Omega_t^j = M_t^{j,SP}/M_t^{j,CE} \in [0,1]$ is the price ratio of asset j between the solutions of the CE and the SP. The condition (63) is equivalent to

$$0 \ge -(1 - \Omega_t^I)G_{2,t}^I G_1^L + (1 - \Omega_t^L)G_1^I G_{2,t}^L + (\Omega_t^I - \Omega_t^L)G_{2,t}^L G_{2,t}^I,$$
(64)

where

$$\begin{split} G_{1}^{j} &= -\frac{\tilde{z}^{j}}{B^{j}}\frac{1}{1-\beta} < 0, \\ G_{2,t}^{j} &= (\frac{1}{B^{j}} + a_{t}^{j} + \theta^{j})\tilde{z}^{j}M_{t}^{j,CE} \geq 0. \end{split}$$

Whether condition (64) holds depends on the relative size of the price ratio Ω_t^j between assets. Note that the relative size of Ω_t^j between assets with different liquidity is ambiguous due to the non-linearity of asset prices with respect to funding liquidity κ^j . There are only two possible cases:

(i) $0 \leq \Omega_t^I \leq \Omega_t^L \leq 1$

In this scenario, (64) can be guaranteed by the following condition:

$$(G_{2,t}^{I}G_{1}^{L} - G_{1}^{I}G_{2,t}) \ge 0.$$

The fact that the SDF is increasing in funding liquidity κ (i.e., $\partial M^{j,i}/\partial \kappa^j > 0$ for $j \in \{I, L\}$ and $i \in \{CE, SP\}$) further simplifies the sufficient condition into (65):

$$B^{L}(a_{t}^{L} + \theta^{L}) \ge B^{I}(a_{t}^{I} + \theta^{I}).$$

$$(65)$$

I will next consider the case where Ω_t^L is larger. (ii) $0 \le \Omega_t^L \le \Omega_t^I \le 1$

With the given assumption, condition (64) can be guaranteed by the following equation

$$(1 - \Omega_t^L)(G_1^I G_{2,t}^L - G_{2,t}^I G_1^L) + (\Omega_t^I - \Omega_t^L)G_{2,t}^I G_{2,t}^L \le 0.$$
(66)

The necessary condition to satisfy (66) is

$$G_1^I G_{2,t}^L - G_{2,t}^I G_1^L \le 0.$$

With the necessary condition, (66) can be guaranteed by

$$(\Omega_t^I - \Omega_t^L)(G_1^I G_{2,t}^L - G_{2,t}^I G_1^L + G_{2,t}^I G_{2,t}^L) \le 0.$$

By using equation (18), the SDFs are within the bound [0, 1/r] and in the order where $0 \le M_t^{I,CE} \le M_t^{L,CE} \le 1/r$. With the above order, a sufficient condition to guarantee (66) is

$$1 - (1 - \frac{1 - \beta}{r})B^{L}(a_{t}^{L} + \theta^{L}) + (1 + \frac{1 - \beta}{r})B^{I}(a_{t}^{I} + \theta^{I}) + B^{L}(a_{t}^{L} + \theta^{L})B^{I}(a_{t}^{I} + \theta^{I}) \le 0.$$
(67)

Since (65) can be guaranteed by (67), (67) is a sufficient condition where overborrowing implies a lower liquid share in *CE* than *SP*. Note that (67) requires $(1 - (1 - \beta)/r) > 0$ which holds under standard assumption $\beta(1+r) \leq 1$. (67) can be further given by

$$[B^{L}(a_{t}^{L}+\theta^{L})+(1+\frac{1-\beta}{r})][B^{I}(a_{t}^{I}+\theta^{I})-(1-\frac{1-\beta}{r})] \leq -(1+\frac{1-\beta}{r})(1-\frac{1-\beta}{r})-1, \quad (68)$$

which is equivalent to the following condition

$$B^{I}(a_{t}^{I} + \theta^{I}) \leq \frac{(1 - \frac{1 - \beta}{r})B^{L}(a_{t}^{L} + \theta^{L}) - 1}{1 + \frac{1 - \beta}{r} + B^{L}(a_{t}^{L} + \theta^{L})}.$$
(69)

Note that the condition (69) implies (65). *QED*

7.2.2 Proof of Proposition 2

Before determining the sign of the price elasticities, I first solve the closed-form consumption and prices in the equilibrium. In a non-binding state where $\lambda_1 = 0$, agents' optimality conditions are as follows:

$$u'(c_1) = \beta(1+r)u'(c_2),$$

$$q_1^j = (q_2^j + z^j)/(1+r)$$

$$= (\Phi^j c_2 + z^j)/(1+r)$$
(71)

$$= \Phi^{j}\beta c_{1} + \frac{z^{j}}{(1+r)},$$
(72)

where equation (71) incorporates the price in period 2. The relationship between asset prices can then be derived as

$$q_1^i = q_1^j \frac{\Phi^i}{\Phi^j} + \frac{1}{(1+r)} [z^i - \frac{\Phi^i}{\Phi^j} z^j].$$
(73)

Therefore, deposit d_2 can be expressed as follows:

$$d_{2} = \omega_{2} + \sum_{s} (q_{2}^{s} + z^{s}) a_{2}^{s} - \sum_{s} q_{2}^{s} a_{3}^{s} - \frac{d_{3}}{1+r} - c_{2}$$

= $\omega_{2} + (1+r) \sum_{s} q_{1}^{s} a_{2}^{s} - \sum_{s} q_{2}^{s} a_{3}^{s} - 0 - \beta (1+r) c_{1}$ (74)

$$=\omega_2 - \sum_s \{ [\frac{1}{B^s} (1 + B^s \theta^s) \Phi^s \beta (1 + r) + \beta (1 + r)] c_1$$
(75)

$$+ z^{s} \left[\frac{1}{B^{s}} \left(1 + \frac{1}{1 - \beta} - \frac{1}{\beta^{2} c_{1} + (1 - \beta)/(1 + r)} \right) + a_{1}^{s} + \theta^{s} \right] \right\}.$$
(76)

Equation (74) is derived from equation (70) and the terminal assumption that $d_3 = 0$. By plugging the functional forms of prices (72) and debt (76) into the budget constraint (58), we obtain the closed-form solution of the consumption c_1 as follows:

$$c_1 = \frac{-\xi_2 + \sqrt{\xi_2^2 - 4\xi_1\xi_3}}{2\xi_1},\tag{77}$$

where

$$\begin{split} \xi_1 &= \beta^2 (1+r) \eta_1, \\ \xi_2 &= (1-\beta) \eta_1 + \beta^2 (1+r) \eta_2, \\ \xi_3 &= (1-\beta) \eta_2 + \eta_3, \\ \eta_1 &= 1 + \beta + 2 \sum_s \left[\frac{\beta}{B^s} (1+B^s \theta^s) \Phi^s \right] > 0, \\ \eta_2 &= -\omega_1 - \sum_s (1 + \frac{1}{1+r}) z^s a_1^s + d_1 - \sum_s (1 + \frac{1}{1+r}) \frac{z^s}{B^s (1-\beta)} - \frac{\omega_2}{1+r}, \end{split}$$

$$\eta_3 = \sum_s \frac{z^s}{B^s} > 0.$$

The sign of η_2 and subsequently, ξ_2 and ξ_3 are determined by the values of portfolio states $\{\{a_1^s\}, d_1\}$. Equity capital ω_1 and ω_2 are set such that the non-negativity condition of consumption holds within portfolio states $\{\{a_1^s\}, d_1\}$. The closed-form consumption can then solve the entire system, according to equation (62), (72) and (74). Having solved the equilibrium, I move on to discuss the signs of the price elasticity and the consumption elasticity.

The sign of price elasticities can be obtained by using the chain rule, the closed-form solution (77) as well as equation (72). The price elasticities can be decomposed as follows:

$$\frac{\partial q_1^j}{\partial X} = \frac{\partial q_1^j}{\partial c_1} \frac{\partial c_1}{\partial X} \quad for \ X \in \{\{a_1^s\}, d_1\}.$$

The effect of the existing portfolio X on current consumption c_1 is

$$\begin{split} &\frac{\partial c_1}{\partial X} = \frac{1}{2\xi_1} \{ -\frac{\partial \xi_2}{\partial X} + (\xi_2^2 - 4\xi_1\xi_3)^{-1/2} [\xi_2 \frac{\partial \xi_2}{\partial X} - 2\xi_1(1-\beta)\frac{\partial \eta_2}{\partial X}] \},\\ &\frac{\partial \xi_2}{\partial X} = \beta^2 (1+r)\frac{\partial \eta_2}{\partial X},\\ &\frac{\partial \eta_2}{\partial X} = \begin{cases} -(1/(1+r)+1)z^s & if \ X = a_1^s\\ 1 & if \ X = d_1 \end{cases}. \end{split}$$

If $\xi_2 < 0$, the signs of consumption derivatives with respect to portfolio items are the following:

$$\frac{\partial c_1}{\partial a_1^s} > 0, \ and \ \frac{\partial c_1}{\partial d_1} < 0.$$

Note that the condition $\xi_2 < 0$ can be written as $\tilde{\omega} > \omega^*$ where

$$\tilde{\omega} = \omega_1 + \sum_{s=I,L} (1 + \frac{1}{1+r}) z^s a_1^s - d_1 + \frac{\omega_2}{(1+r)},$$
$$\omega^* = \frac{1-\beta}{1+r} \eta_1 - \sum_{s=I,L} (1 + \frac{1}{(1+r)}) \frac{z^s}{B^s(1-\beta)}.$$

Next, the relationship between asset price and consumption (72) implies that $\partial q_1^s / \partial c_1 =$

 $\Phi^s \beta > 0$. Finally, the chain rule implies that

$$\begin{aligned} \frac{\partial q_1^j}{\partial a_1^s} &> 0 \quad \forall s \in \{I, L\},\\ \frac{\partial q_1^j}{\partial d_1} &< 0 \end{aligned}$$

QED

7.2.3 Proof of Proposition 3

To compare the solutions in the one-asset model and the two-asset model, the asset that exists in both models is denoted as the asset j, whereas the additional asset in the twoasset model is denoted as the asset i. In a non-binding case, the only relevant factor that decides the policy is the trading effect (TE). The functional form of the trading effect of the two-asset model TE_j^T is the following:

$$\begin{split} TE_{j}^{T} &= TE_{j}^{S} - u'(c_{1})\frac{\partial q_{1}^{i}}{\partial a_{1}^{j}}\Delta a_{2}^{i} \\ &= -u'(c_{1})\frac{\partial q_{1}^{j}}{\partial a_{1}^{j}}[\Delta a_{2}^{j} + \frac{\Phi^{i}}{\Phi^{j}}\Delta a_{2}^{j}] \\ &= -u'(c_{1})\frac{\partial q_{1}^{j}}{\partial a_{1}^{j}}[((\frac{1}{B^{j}} + \theta^{j}) - \frac{z^{j}}{B^{j}(1 - \beta)q_{1}^{j}}) + \frac{\Phi^{i}}{\Phi^{j}}((\frac{1}{B^{i}} + \theta^{i}) - \frac{z^{i}}{B^{i}(1 - \beta)q_{1}^{i}})], \end{split}$$

where TE_j^s is the trading effect of the one-asset model. Next, I define two thresholds of q_1^j as

$$\begin{split} q^{j*} &= \frac{z^j}{1-\beta} \frac{1}{(1+B^j\theta^j)}, \\ f(q^{j**}) &= [((\frac{1}{B^j} + \theta^j) - \frac{z^j}{B^j(1-\beta)q^{j**}}) + \frac{\Phi^i}{\Phi^j}((\frac{1}{B^i} + \theta^i) - \frac{z^i}{B^i(1-\beta)[q^{j**}\frac{\Phi^i}{\Phi^j} + \frac{1}{1+r}(z^i - \frac{\Phi^i}{\Phi^j}z^j)]})] = 0, \\ q^{j^{***}} &= \frac{\Phi^j}{\Phi^i} \frac{z^i}{(1+\theta^i B^i)(1-\beta)} + \frac{1}{1+r}(z^j - \frac{\Phi^j}{\Phi^i}z^i), \end{split}$$

where q^{j*} is the threshold where $\Delta a_2^j > 0$ if $q^j > q^{j*}$; $q^{j^{***}}$ is the threshold where $\Delta a_2^i > 0$ if $q^j > q^{j***}$; q^{j***} ; q^{j**} ; q^{j**} is the threshold where $\Delta a_2^j + \Delta a_2^i > 0$ if $q^j > q^{j**}$. The signs of TE_j^T and TE_j^S are governed by the value of q_1^j according to the following criterion:

$$\begin{cases} (i) \ TE_{j}^{T} < TE_{j}^{S} < 0 & if \ q^{j} > max(q^{j*}, q^{j***}) \\ (ii) \ TE_{j}^{T} > TE_{j}^{S} > 0 & if \ q^{j} < min(q^{j*}, q^{j***}) \\ (iii) \ 0 < TE_{j}^{T} < TE_{j}^{S} & if \ q^{j} \in (q^{j***}, q^{j**}) \ and \ q^{j*} > q^{j***} \\ (iv) \ TE_{j}^{T} < 0 < TE_{j}^{S} & if \ q^{j} \in (q^{j**}, q^{j*}) \ and \ q^{j*} > q^{j***} \\ (v) \ 0 < TE_{j}^{T} < TE_{j}^{S} & if \ q^{j} \in (q^{j*}, q^{j**}) \ and \ q^{j*} < q^{j***} \\ (vi) \ TE_{j}^{T} < 0 < TE_{j}^{S} & if \ q^{j} \in (q^{j**}, q^{j**}) \ and \ q^{j*} < q^{j***} \\ (vi) \ TE_{j}^{T} < 0 < TE_{j}^{S} & if \ q^{j} \in (q^{j**}, q^{j***}) \ and \ q^{j*} < q^{j***} \end{cases}$$

Case (*iii*), (*iv*) and case (*v*), (*vi*) are mutually exclusive. Here I analyze the case where $q^{j*} > q^{j***}$, which can be guaranteed by condition (39). According to (62), (72), and the non-negativity of consumption, the minimum and maximum of the price q_1^j are

$$\begin{split} q^{j}_{min} &= max[\frac{z^{j}}{1-\beta}\frac{1}{1-B^{j}(\underline{a}^{j}-\bar{a}^{j})+B^{j}\theta^{j}}, \frac{z^{j}}{1+r}], \\ q^{j}_{max} &= \frac{z^{j}}{1-\beta}\frac{1}{1-B^{j}(\bar{a}^{j}-\underline{a}^{j})+B^{j}\theta^{j}} > q^{j*}. \end{split}$$

The fact that asset prices are increasing in the initial wealth $n_1 = \omega_1 - d_1$ (as shown in Proposition 2) suggests price thresholds as functions of the wealth where $q^* = q_1^j(n^*)$, $q^{**} = q_1^j(n^{**})$, and $q^{***} = q_1^j(n^{***})$ where $n^{***} \leq n^{**} \leq n^*$. Equivalently, wealth is a function of current prices such that $n^* = \omega_1 - d_1 + \sum_j (z^j + q^{j*})a_1^j$, and $n^{**} = \omega_1 - d_1 + \sum_j (z^j + q^{j**})a_1^j$. Under continuous n, the condition that $q_{max}^j > q^{j*}$ implies that (i) will be visited for sure. However, the existence of solutions that lies in case (ii), (iii) and (iv) depends on the size of q_{min}^j .

If $q_{min}^j < q^{j*}$, a solution must exist in case (iv) where the optimal policy on asset j is a subsidy in a single-asset model (with only a liquid asset) since the agent now sells the liquid asset, which generates a positive trading effect. More importantly, in the region where $q^j > q^{j**}$, the agent buys the illiquid asset, which gives a negative trading effect that outweighs the positive trading effect and thus sums up to a negative TE_j^T . In such a region, the policies on the liquid asset are in a different direction where the single-asset model suggests a subsidy and the two-asset model features a tax.

If $q_{min}^j \in (q^{j***}, q^{j**})$ as in case (*iii*), the agent in the two-asset model still buys the illiquid asset, but the negative trading effect is now dominated by the positive trading effect from the sale of the liquid asset. The positive TE in the single-asset model is, however, larger than that in the two-asset model because the former does not encounter an offsetting factor.

Finally, if $q_{min}^j < q^{j***}$, the agent becomes the seller of both the liquid and illiquid asset,
and therefore the positive TE in the two-asset model is larger than that in the single-asset model, that is, the subsidy on the liquid asset is reinforced by the existence of the illiquid asset.

To prove the sign and the relative size of trading effect of the deposit, I will first examine the definition of the trading effect as follows:

$$TE_d^T = TE_d^S + \frac{\partial q_1^i}{\partial d_1} \Delta a_2^i$$

= $u'(c_1) \frac{\partial q_1^j}{\partial d_1} [\Delta a_2^j + \frac{\Phi^i}{\Phi^j} \Delta a_2^j]$
= $u'(c_1) \frac{\partial q_1^j}{\partial d_1} [(\frac{1}{B^j} - \frac{z^j}{B^j(1-\beta)q_1^j}) + (\frac{\Phi^i}{\Phi^j} \frac{1}{B^i} q_1^j - \frac{z^i}{B^i} \frac{1}{1-\beta})]$

The partial derivative of price q_1^j with respect to the deposit d_1 can be derived from

$$\begin{split} \eta_0 \frac{\partial q_1^j}{\partial d_1} + \frac{\partial \eta_1}{\partial d_1} + \eta_2 (-1) \frac{1}{(q_1^j)^2} \frac{\partial q_1^j}{\partial d_1} &= 0, \\ \Rightarrow \frac{\partial q_1^j}{\partial d_1} = \frac{q_j^2}{\eta_0^j q_1^2 - 1}. \end{split}$$

The derivative is negative under the assumption that $\eta_0^j < 0$. The signs of Δa_2^i and Δa_2^j follow the analysis above, and therefore the sign of the trading effect of the deposit is also characterized by (78). In sum, the sign of the trading effect within different initial wealth can be completely described via the following five points:

(0) Definitions:

$$\begin{split} TE_j^S &= -u'(c_1) \frac{\partial q_1^j}{\partial a_1^j} \Delta a_2^j \\ TE_j^T &= -u'(c_1) \sum_{s=i,j} \frac{\partial q_1^s}{\partial a_1^j} \Delta a_2^s \\ TE_d^S &= u'(c_1) \frac{\partial q_1^j}{\partial d_1} \Delta a_2^j \\ TE_d^T &= u'(c_1) \sum_{s=i,j} \frac{\partial q_1^s}{\partial d_1} \Delta a_2^s \\ q_{max}^j &= \frac{z^j}{1-\beta} \frac{1}{1-B^j(\bar{a}^j-\underline{a}^j-\theta^j)} \end{split}$$

- (1) If $q_{min}^j > q^{j*}$, $TE_j^T < TE_j^S < 0$ for all possible states.
- (2) If $q^{j**} < q^j_{min} \le q^{j*}$, there exists a wealth threshold $n^* = \omega_1^* d_1^*$ such that

$$\begin{cases} TE_{j}^{S} > 0 > TE_{j}^{T} & if \ n_{1} \le n^{*} \\ TE_{j}^{T} < TE_{j}^{S} < 0 & if \ n_{1} > n^{*} \end{cases}$$

(3) If $q^{j***} < q^{j}_{min} \le q^{j**}$, there exists other thresholds $n^{***} < n^{**} < n^{*}$ such that

$$\begin{cases} TE_j^T < TE_j^S < 0 & if \ n_1 > n^* \\ TE_j^S > 0 > TE_j^T & if \ n^{**} < n_1 \le n^* \\ TE_j^S > TE_j^T > 0 & if \ n_1 < n^{**} \end{cases}$$

(4) If $q^{j***} > q^j_{min}$

- $\begin{cases} TE_j^T < TE_j^S < 0 & if \ n_1 > n^* \\ TE_j^S > 0 > TE_j^T & if \ n^{**} < n_1 \le n^* \\ TE_j^S > TE_j^T > 0 & if \ n^{***} < n_1 \le n^{**} \\ TE_j^T > TE_j^S > 0 & if \ n_1 \le n^{***} \end{cases}$
- (5) The relative size and sign of the trading effect TE_d^T and TE_d^S follow the same threshold criterion as TE_j^S and TE_j^T described in (1), (2), (3), and (4).

QED

7.3 Example: the US Great Recession

To analyze the role of liquidity during the US Great Recession, I will first start from examining the dynamics of liquid share. Using the Flow of Funds data, I show in Figure A.1 that financial institutions tend to sell liquid assets 1.5 years earlier than the illiquid assets following the Great Recession. Moreover, the decline of the liquid holdings from the peak to the trough is five times larger than the magnitude of the illiquid asset.³⁶ The relative magnitude of asset sales leads to a 10% drop in the liquid share. The trading pattern, again, implies that liquidity affects banks trading decision and that the liquid asset serves as a better buffer against crises due to its high market liquidity. The defined liquid asset excludes stocks, which are fundamentally different from bonds, but results are robust when including stocks as liquid assets.



Figure A.1: Asset growth of non-commercial banks

Notes: The left vertical axis represents the level of the asset holdings. Items except for the mutual fund shares, which are in terms of the market value, are recorded in book values. The solid line represents the illiquid asset, which is defined as the sum of loans, miscellaneous assets, and foreign direct investment. The dashed line indicates the liquid asset, which contains cash, money market fund shares, federal funds and security repos, mutual fund shares, treasury security, agency securities, municipal bonds, corporate securities, and the open market paper. The right vertical axis, along with the short-dashed line, describes the liquid share as the ratio of the nominal value of liquid holdings divided by the nominal total assets. The data of the Non-commercial Banks is constructed by calculating items in the balance sheet of the domestic financial sector (L.108) minus monetary authority (L.109) minus private depository institution (L.110). Source: Flow of Funds.

Table A.1 lists the values of parameters and the targeted moments. Equity capital is

 $^{^{36}}$ The categorization by liquidity reflects the order of the assets' liquidity weights calculated in Bai et al. (2018) where the weight is decreasing in asset's haircuts and the absolute value of risk exposure from the first principal component of haircuts across all assets.

proxied by the variable "Total bank equity capital" listed in FFIEC balance sheet data, which is a quarterly series ranges from 2000 Q1 to 2016 Q4. Using Flow of Funds data, dividends \tilde{z}^{j} are calibrated to the empirical median of the ratio of the nominal value of assets to equity capital. The coefficients of market liquidity B^{j} reflect the frequency of asset transactions and therefore, is calibrated to the median ratio of the relative second moment of the holding of asset j to the total asset. Similar to the example of the Argentina crisis, values of the asymmetric factors θ^{j} are set to ensure a unique steady state. The asset margin κ^{j} , estimated from the NY Fed Tri-Party/GCF Repo data, follows the same value as in the example of Argentina.

The model is solved and simulated for 10,000 times, according to the same algorithm adopted in the example of Argentina. Each state variable (i.e., $\{a^L, a^I, d\}$) is discretized into 10 grids, and equity capital is discretized into 4 grids. The number of the total states is thus 4,000.

β	0.9600	Standard value	\tilde{z}^L	0.0558	$median(q^L a^L/\omega) = 1.277$
β^f	0.9600	$\beta^f=\beta$	\tilde{z}^{I}	0.1012	$median(q^{I}a^{I}/\omega) = 5.747$
r	0.0417	$\beta(1+r) = 1$	B^L	0.0792	$std(q^La^L)/std(asset) = 0.4929$
κ^L	0.9720	Tri-repo data	B^{I}	9.8550	$std(q^{I}a^{I})/std(asset) = 0.7348$
κ^{I}	0.9000	Tri-repo data	θ^L	2.8539	$(1/\beta - 1)/B^L$
σ	2	Standard value	θ^{I}	7.3099	$(1/\beta - 1)/B^I$

Table A.1: Parameters

Figure A.2 presents the simulated distribution of the CE and the equilibrium of the SP. The distribution of the deposit again features overborrowing, which can also be supported by averages of the deposit μ_d and the leverage in Table A.2. Overborrowing equals 7% in terms of leverage.

Regarding consumption, a fraction of consumption distribution in the CE is smaller than the minimum of the consumption in the equilibrium of the SP due to the binding constraint. This can be supported by the simulated probability of financial crises listed in Table A.2 where no financial crisis occurs in the solution of the SP, whereas the probability is positive in the CE. According to the comparison of μ_c , the optimal macroprudential policy raises the equilibrium consumption by 0.81%. The low consumption average in the CE results from low equilibrium consumption during crises (0.9955), albeit that equilibrium consumption during normal times (1.0493) is higher than the average of the solution of the SP. The consumption distribution in the CE features several humps that correspond to the realization of equity capital discretized into four grids, whereas the distribution of the solution of the SP tends to be smoother.



Figure A.2: Simulated distribution in equilibria

As shown in the bottom panels of Figure A.2, the distribution of the illiquid holding is similar between two cases, while the social planner tends to accumulate more liquid asset. The observation is also supported by the mean statistics presented in Table A.2. The asset composition implies that the social planner tends to hold a more liquid balance sheet that allocates on average 3.73% more liquid assets, as shown in the last column.

The optimal macroprudential policy lowers the probability of crises by 21%. The probability is higher than the empirical values. Reinhart and Rogoff (2009) construct the series of financial crises where the probability of crises in the US is 13% (defined as the number of years in crises divided by the total number of years). However, if we narrow the definition of the financial crises as events where the collateral constraint binds, and consumption drops by 5% (which roughly equals to the empirical consumption documented by Nardi et al. (2012) and Petev et al. (2011)), the probability of crises in the CE becomes 5.3%, which is not far from the probability in the literature.

Finally, the event study is done by fitting the exogenous shocks on equity capital. I use quarterly data of the non-commercial financial sector from Flow of Funds and normalize data by values in 2007Q3. As shown in Figure A.3, the model matches the empirical movements of the leverage, the accumulation of liquid and illiquid assets, and liquid share both

	μ_d	μ_c	μ_L	μ_I	Leverage	P_{crisis}	LiquidShare
SP	2.2286	1.0310	0.6250	1.9904	0.8198	0.0000	0.2384
CE	2.3026	1.0227	0.5000	1.9841	0.8909	0.2112	0.2011
SP - CE	-0.0740	0.0083	0.1250	0.0063	-0.0711	-0.2112	0.0373

Table A.2: Simulated moments

quantitatively and qualitatively well. The simulated trend of the total asset explains 13% of the overall decline in empirical data.



Figure A.3: Event study of the Great Recession

Notes: Balance sheet items are collected from Flow of Funds. The liquid and illiquid assets are defined as in Figure A.1. Leverage is defined as the deposit divided by the nominal value of the total asset.

7.4 Systemic Sudden Stop Events

Country	Year	Group	Country	Year	Group
Australia	2008	ADV	United States	1991	ADV
Belgium	2010	ADV	United States	2009	ADV
Canada	1996	ADV	Argentina	1995	EM
Finland	1986	ADV	Argentina	2002	EM
Finland	1995	ADV	Brazil	2003	EM
Germany	2006	ADV	Colombia	1986	EM
Germany	2009	ADV	Colombia	1989	EM
Greece	2012	ADV	Colombia	1999	EM
Iceland	2001	ADV	Croatia	2010	$\mathbf{E}\mathbf{M}$
Iceland	2009	ADV	Czech Republic	2009	$\mathbf{E}\mathbf{M}$
Ireland	2012	ADV	Dominican Republic	2009	EM
Italy	1983	ADV	Ecuador	1999	EM
Italy	1993	ADV	Ecuador	2011	$\mathbf{E}\mathbf{M}$
Italy	2012	ADV	El Salvador	2010	$\mathbf{E}\mathbf{M}$
Netherlands	2003	ADV	Korea, Republic of	1998	EM
Netherlands	2009	ADV	Malaysia	1994	EM
New Zealand	2009	ADV	Pakistan	2009	$\mathbf{E}\mathbf{M}$
Norway	1990	ADV	Panama	2010	$\mathbf{E}\mathbf{M}$
Norway	2008	ADV	Peru	1989	EM
Portugal	2012	ADV	Peru	1998	EM
Spain	1984	ADV	Poland	2009	EM
Spain	2009	ADV	South Africa	1983	EM
Spain	2012	ADV	South Africa	2009	EM
Sweden	2003	ADV	Tunisia	2007	EM
Sweden	2006	ADV	Turkey	2009	$\mathbf{E}\mathbf{M}$
United Kingdom	1991	ADV	Ukraine	2012	$\mathbf{E}\mathbf{M}$
United Kingdom	2008	ADV	Venezuela	1990	EM

Table A.3: The list of systemic sudden stops

ADV: Advanced market; EM: Emerging market.

7.5 Computational Algorithm

This subsection illustrates the numerical solution method for the competitive and constrainedefficient equilibria. The model contains three endogenous state variables: two assets and the deposit, and is solved by the endogenous grid method. Within each iteration, I apply the Delaunay interpolation to obtain updates that are not on the exogenous grids. Following subsections illustrate the detailed algorithm.

7.5.1 Unregulated Competitive Equilibrium

The competitive equilibrium is characterized and solved by functions $C(x;\omega)$, $\{Q^j(x;\omega)\}$, $\{\mathcal{A}^j(x;\omega)\}$, $\mathcal{D}(x;\omega)$, $\lambda(x;\omega)$, and the following equations:

$$\mathcal{C}(x;\omega) = \omega + \sum_{j=I,L} (\mathcal{Q}^j(x;\omega) + \tilde{z}^j)a^j - d - \sum_{j=I,L} [\mathcal{Q}^j(x;\omega)a^{j'}] + \frac{d'}{1+r},$$
(79)

$$\frac{d'}{1+r} \le \sum_{j} \kappa^{j} \mathcal{Q}^{j}(x;\omega) a^{j'},\tag{80}$$

$$u_{\mathcal{C}}(\mathcal{C}(x;\omega)) = \beta(1+r)E_{\omega'|\omega}[u_{\mathcal{C}}(\mathcal{C}(x'(x;\omega);\omega')] + \lambda(x;\omega),$$
(81)

$$\mathcal{Q}^{j}(x;\omega) = E_{\omega'|\omega} \left[\frac{\beta u_{\mathcal{C}}(\mathcal{C}(x'(x;\omega);\omega'))}{u_{\mathcal{C}}(\mathcal{C}(x;\omega)) - \lambda(x;\omega)\kappa^{j}} (\mathcal{Q}^{j}(x'(x;\omega);\omega') + \tilde{z}^{j}) \right], \tag{82}$$

$$Q^{j}(x;\omega) = \frac{\tilde{z}^{j}}{1 - \beta^{f}} \frac{1}{1 - B^{j}(a^{j'} - a^{j} + \theta^{j})},$$
(83)

$$a^{j'} = \mathcal{A}^j(x;\omega),\tag{84}$$

$$d' = \mathcal{D}(x;\omega),\tag{85}$$

for $j \in \{I, L\}$. $x = \{a^L, a^I, d\}$ is a set of endogenous state variables, where

$$x'(x;\omega) = \{ \mathcal{A}^L(x;\omega), \mathcal{A}^I(x;\omega), \mathcal{D}(x;\omega) \}.$$

Equation (83) is derived by plugging the market clearing condition of assets into the agents' first-order conditions with respect to assets. I then proceed with the following steps:

1. Construct the following exogenous grids

$$G_{x'} = \{x'_{\{a_1^{L'}, a_1^{I'}, d_1'\}}, \dots x'_{\{a_N^{L'}, a_1^{I'}, d_1'\}}, x'_{\{a_1^{L'}, a_2^{I'}, d_1'\}}, \dots, x'_{\{a_N^{L'}, a_2^{I'}, d_1'\}}, \dots, x'_{\{a_N^{L'}, a_N^{I'}, d_1'\}}, \dots, x'_{\{a_N^{L'}, a_N^{I'}, d_1'\}}\}, \dots, x'_{\{a_N^{L'}, a_1^{I'}, d_1'\}}\}$$

where the current endogenous state variables $x' = \{a^{L'}, a^{I'}, d'\}$. I assume N = 50 grids for each endogenous variables and the equity ω is discretized into the shock state space

 $G_{\omega} = \{\omega_1, ..., \omega_M\}$ where M = 8. The number of the total states is therefore 1 million.

- 2. Conjecture policy functions $\mathcal{C}(x;\omega)$ and $\{\mathcal{Q}^j(x;\omega)\}\$ for each states $\{x;\omega\}$. Pick a tolerance level ϵ .
- 3. Assume that the collateral constraint under every state $\{x; \omega\}$ is non-binding, that is, $\lambda(x; \omega)$ equals 0. Obtain the current consumption c using equation (81) and the policy guesses $\mathcal{C}(x'; \omega'(\omega))$ and $\{\mathcal{Q}^j(x'; \omega'(\omega))\}$.
- 4. Obtain the current price $\{q^j\}$ using (82), current consumption, and policy guesses $\mathcal{C}(x';\omega'(\omega))$ and $\{\mathcal{Q}^j(x';\omega'(\omega))\}$.
- 5. For each grid on $\{x', \omega\}$, obtain the $\{a^j\}$ and d using (83) and (79). For a given ω , the resulting $\{a^j\}$ and d form a three-dimensional space S_N^{ω} , which is constructed under the assumption of a non-binding collateral constraint. Construct a set of tetrahedra T_N^{ω} via the Delaunay triangulation.
- 6. Assume that the collateral constraint under every state $\{x; \omega\}$ is binding. Plug (81) into (82) by replacing the non-zero $\lambda(x; \omega)$, we then get the following equation:

$$q^{j} = \frac{E_{\omega'|\omega}[\beta u_{\mathcal{C}}(\mathcal{C}(x';\omega'))(\mathcal{Q}^{j}(x';\omega') + \tilde{z}^{j})]}{(1-\kappa^{j})u_{c}(c) + \kappa^{j}\beta(1+r)E_{\omega'|\omega}[u_{\mathcal{C}}(\mathcal{C}(x';\omega'))]}.$$
(86)

7. Plug (86) into the binding collateral constraint (80) such that

$$\frac{d'}{1+r} = \sum_{j=I,L} \{ [\frac{E_{\omega'|\omega} [\beta u_{\mathcal{C}}(\mathcal{C}(x';\omega'))(\mathcal{Q}^j(x';\omega') + \tilde{z}^j)]}{(1-\kappa^j)u_c(c) + \kappa^j\beta(1+r)E_{\omega'|\omega} [u_{\mathcal{C}}(\mathcal{C}(x';\omega'))]}]a^{j'} \}.$$

We can then obtain consumption c with policy guesses and given grids x'. Note that with a CRRA utility function and the assumption that the risk aversion $\sigma = 2$, c has a closed-form solution and does not require a root-finding algorithm.

- 8. Compute prices $q^{j'}$ and existing state variables $\{a^j\}$ and d, as demonstrated in step 4 and 5. For a given ω , construct the three-dimensional space S_B^{ω} where all states x are assumed to encounter a binding collateral constraint. Construct a set of of tetrahedra T_B^{ω} via the Delaunay triangulation.
- 9. For each ω' , check whether each endogenous grid x' is inside a tetrahedron $t_n^{\omega'}$ that belongs to $T_N^{\omega'}$. If it does, compute the updates of $\mathcal{C}(x';\omega')$ and $\{\mathcal{Q}^j(x';\omega')\}$ as the weighted sum of the corresponding cand q^j of the four points $\{x_{[1]}, x_{[2]}, x_{[3]}, x_{[4]}\}$ that forms $t_n^{\omega'}$, weighted by the barycentric coordinates. If not, proceed to step 10.

- 10. Check if the state $\{x', \omega'\}$ is within a tetrahedron $t_b^{\omega'}$ that belongs to $T_B^{\omega'}$. If it does, compute the updates as demonstrated in step 9. If not, proceed to step 11.
- 11. For the rest of the states $\{x', \omega'\}$, find the closest $\{x, \omega'\}$ and replace $\mathcal{C}(x'; \omega')$ and $\{\mathcal{Q}^j(x'; \omega')\}$ with $c(x; \omega')$ and $\{q^j(x'; \omega)\}$.
- 12. Calculate the norm between the updated policy $\tilde{\mathcal{C}}(x';\omega'(\omega))$, $\{\tilde{\mathcal{Q}}^j(x';\omega'(\omega))\}$ and the initial guesses. If $norm(\tilde{\mathcal{C}}-\mathcal{C},\tilde{\mathcal{Q}}^L-\mathcal{Q}^L,\tilde{\mathcal{Q}}^I-\mathcal{Q}^I) < \epsilon$, we have obtained the equilibrium solution. If not, set the new guesses $\mathcal{X}^{new} = \alpha \mathcal{X} + (1-\alpha)\tilde{\mathcal{X}}$ where

$$\alpha = \begin{cases} 0.99 & \text{for the first 200 iteration} \\ max(0.99, 1 - norm(\tilde{\mathcal{C}} - \mathcal{C}, \tilde{\mathcal{Q}}^L - \mathcal{Q}^L, \tilde{\mathcal{Q}}^I - \mathcal{Q}^I)) & \text{otherwise} \end{cases}$$

7.5.2 Constrained-Efficient Equilibrium

The constrained-efficient equilibrium is characterized and solved by functions $C(x; \omega)$, $\{Q^j(x; \omega)\}$, $\{\mathcal{A}^j(x; \omega)\}$, $\mathcal{D}(x; \omega)$, $\lambda(x; \omega)$, (79), (80), (83), (84), (85) and the following equations:

$$u_{\mathcal{C}}(\mathcal{C}(x;\omega)) = \lambda(x;\omega) + \beta(1+r)E_{\omega'|\omega}[u_{\mathcal{C}}(\mathcal{C}(x';\omega'))] + E_{\omega'|\omega}[\sum_{s=I,L} [u_{\mathcal{C}}(\mathcal{C}(x';\omega'))\frac{\partial \mathcal{Q}^{s}(x';\omega')}{\partial d'}\Delta\mathcal{A}^{s}(x';\omega') - \lambda'(x';\omega')\kappa^{s}\mathcal{A}^{s}(x';\omega')\frac{\partial \mathcal{Q}^{s}(x';\omega')}{\partial d'}]\}],$$
(87)

$$\mathcal{Q}^{j}(x;\omega) = \frac{\beta}{u_{\mathcal{C}}(\mathcal{C}(x;\omega)) - \kappa^{j}\lambda(x;\omega)} \times E_{\omega'|\omega} \{ u_{\mathcal{C}}(\mathcal{C}(x';\omega')(\mathcal{Q}^{j}(x'(x;\omega);\omega') + \tilde{z}^{j}) + \sum_{s=I,L} [-u_{\mathcal{C}}(\mathcal{C}(x';\omega'))\frac{\partial \mathcal{Q}^{s}(x';\omega')}{\partial a^{j'}} \Delta \mathcal{A}^{s}(x';\omega') + \lambda'(x';\omega')\kappa^{s}\mathcal{A}^{s}(x';\omega')\frac{\partial \mathcal{Q}^{s}(x';\omega')}{\partial a^{j'}}] \}.$$

$$(88)$$

The algorithm is the same as the competitive equilibrium after replacing (81) and (82) with (87) and (88). Note that the future asset decisions $\mathcal{A}^s(x';\omega')$ can be obtained by (83) with the initial grids on x' and the price guesses $\{\mathcal{Q}^s(x';\omega')\}$, as shown by the following equation:

$$\mathcal{A}^s(x';\omega') = (1 - \frac{z^s}{1-\beta}\frac{1}{\mathcal{Q}^s(x';\omega')})\frac{1}{B^s} + \theta^s + a^{s'}.$$

Note that with $\sigma = 2$, consumption again has a closed-form solution, which significantly speeds up the algorithm.

7.5.3 Competitive Equilibrium with Basel III Reform

This subsection describes the solution steps to solve the competitive equilibrium taking into account the LCR and the NSFR. I will lay out the algorithm where two policies coexist. For special cases where agents are only constrained by one policy, the algorithm can be simplified from the general version by dropping the redundant complementary slackness condition and set the corresponding shadow value to zero. With two policies and the collateral constraint, states can be categorized into the following eight cases:

	CC	LCR	NSFR
<i>(i)</i>	Ν	Ν	Ν
(ii)	В	Ν	Ν
(iii)	В	Ν	В
(iv)	В	В	Ν
(v)	Ν	В	Ν
(vi)	Ν	Ν	В
(vii)	Ν	В	В
(viii)	В	В	В

Table A.4: Categories of states

Notes: CC: Collateral constraint; LCR: Liquidity coverage ratio; NSFR: Net stable funding ratio. N represents the case where the constraint is non-binding whereas B indicates a binding condition.

Similar to the algorithm of the competitive equilibrium, I use the endogenous grid method and construct grids $G_{x'}$. I will proceed with the following steps:

- 1. By guessing policy functions $C(x; \omega)$ and $\{Q^j(x; \omega)\}$ for each states $\{x; \omega\}$, control variables $\{q^j\}$ and c, and endogenous grids $\{a^j\}$ and d in cases (i) and (ii) can be obtained, as shown in Subsection 7.5.1.
- 2. Calculate the solution of the rest of the cases.
 - (a) Case (*iii*): Derive q^I the binding NSFR where q^I = [ω + δ^dd'/(1+r)]/(δ^Ia^{I'}) and get q^L from the binding CC where q^L = [d'/(1+r) κ^Ia^{I'}q^I]/(κ^La^{L'}). {a^j} can then be solved from (9). Since λ^{LCR} equals zero, we obtain the value of λ via (45). Next, calculate the value of λ^{NSFR} from (46). Finally, solve c through (47) and d through the budget constraint (40) for a given capital ω.
 - (b) Case (*iv*): Derive q^L from the binding LCR where $q^L = [(\phi^d d')/(1+r)]/a^{L'}$ and get q^I from the binding CC where $q^I = [d'/(1+r) \kappa^L a^{L'} q^L]/(\kappa^I a^{I'})$. Since λ^{NSFR} equals zero, we obtain the value of λ via (46). Next, calculate the value of λ^{LCR} from (45). We then solve c, $\{a^j\}$ and d as in step (a).

- (c) Case (v): Derive q^L from binding (43). Next, solve λ^{LCR} and c jointly from (45) and (47) as λ^{NSFR} and λ are zero. We then solve $\{a^j\}$ and d as in step (a).
- (d) Case (vi): Derive q^{I} from binding (44). Next, solve λ^{NSFR} and c jointly from (46) and (47) as λ^{LCR} and λ are zero. We then solve $\{a^{j}\}$ and d as in step (a).
- (e) Case (*vii*): Derive q^L and q^I from binding (43) and (44). Next, solve λ^{LCR} , λ^{NSFR} and c jointly from (45), (46) and (47). We then solve $\{a^j\}$ and d as in step (a).
- (f) Case (*viii*): Under current parameterization, it is not possible to encounter a situation where all three constraints bind. Specifically, suppose asset factors follow the official value where $\delta^I = \phi^L = 1$, binding Basel policies implies that the sum of the collaterals $\sum (\kappa q a) = (\kappa^L \phi^d + \kappa^I \delta^d) d'/(1+r) + \omega$ can easily exceed the deposit when ω is large enough and the asset margins $\{\kappa^j\}$ are close to one.³⁷
- 3. For each above case z, construct a set of tetrahedra T_z^{ω} by connecting points in the $d a^L a^I$ space for each ω .
- 4. Identify the case where each exogenous grids $G_{x'}$ belongs and update the guesses as in 7.5.1 until the solution converges.

To solve the equilibrium with only one policy, we can simply drop cases where the other policy binds and then follow the same steps.

To numerically highlight effects and the interaction between constraints on the equilibrium solution, Figure A.9 plot the binding region under the model with the CC and the LCR with calibrated parameters. When the liquid asset is relatively high, only CC is binding, as marked by the dark gray area. Both constraints bind when the total asset holdings and the liquid asset are scarce, as highlighted by the light gray area.

Table A.7 presents the frequencies of all four possible states that the equilibrium with only the LCR may feature. With the standard cash-outflow factor that equals 46%, the probability of the financial crisis is 0.038. The probability of financial crisis lowers to 3.2% when ϕ_t^d is 50%. The reason for this decline is a more liquid portfolio that is driven by the tightening LCR. It is also worth noting that under a high cash-outflow factor, agents may experience states in which only the LCR binds.

³⁷Assumptions that prevent case (viii) are consistent with Cecchetti and Kashyap (2018) and Kashyap et al. (2017) where the LCR and the NSFR will almost not be binding simultaneously.





Figure A.4: Changes in liquid share of emerging countries



Figure A.5: Liquid share in sudden stop events

Notes: This figure plots countries' liquid share using same data as in Figure 1. Note that the liquid share of emerging economies is higher than advanced economies in both data. This can be explained by the fact that emerging markets tend to be more volatile and less capable of avoiding bank runs, and therefore they require a more liquid portfolio as a precautionary motive. It is also worth noting that the liquid share calculated in the FSIs is significantly smaller than numbers from the IIP. This is due to the narrower definition of liquid assets the FSIs adopts. Other than currencies and deposits, the FSIs only include financial assets that are available within 3 months and securities that are traded in liquid markets, whereas the liquid share I calculate from the IIP is a broad measure of debt securities and financial derivatives.



Figure A.6: Asset transactions during crises: Emerging countries



Figure A.7: Binding and non-binding states under -1.285σ shock



Figure A.8: Implied optimal macroprudential policies on deposit



Figure A.9: Policy function of deposit in equilibrium with the LCR *Notes*: The cash-outflow factor ϕ^d here is assumed to be 0.50. The deposit is at its 80th percentile to demonstrate states in multiple binding situations.

Table A.5: Parameters for the single-asset model

σ	κ	a'	d'	ω_L	ω_F	β	z
2	0.9	1	4	0.9	1	0.96	0.18

Table A.6: Parameters for the two-asset model with finite periods

σ	β	r	σ	y_2	z^{I}	z^L	B^{I}	B^L	κ^{I}	κ^L	θ^{I}	θ^L	a_1^L	a_1^I	d_1
2	0.96	0.04	1	1.5	0.10	0.05	10	5	0.900	0.972	0.1	0.1	1	1	2

Table A.7: Probability of binding and nonbinding states

$\{CC, LCR\}$	$\{N, N\}$	$\{B, N\}$	$\{N, B\}$	$\{B, B\}$
$\phi^d = 46\%$	0.944	0.038	0.000	0.000
$\phi^d=50\%$	0.953	0.003	0.013	0.029

Notes: This table reports the probability of encountering nonbinding (N) and binding (B) constraints over 7,500 times simulation (from running 10,000 times simulation and dropping the first quarter of the samples). The first and second elements inside the bracket in the first row indicate whether CC and LCR are binding. Probability of financial crisis is defined as $Prob(\{B, N\} \cup \{B, B\})$.

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