Emerging Market Business Cycles with Heterogeneous Agents*

Seungki Hong†

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Abstract

A central question in open economy macroeconomics is how to explain excess consumption volatility in emerging economies. This paper argues that to understand this phenomenon, it is important to take into account households' idiosyncratic income risk, precautionary saving, and marginal propensities to consume (MPCs). The paper makes a first attempt to study emerging market business cycles in a heterogeneous-agent open economy model. Financial frictions determining asset liquidity in the model are calibrated such that MPCs are as high as empirical estimates from emerging market micro data, which are substantially greater than the U.S. MPC estimates. I then estimate the model using macro data and Bayesian methods. The model captures the observed excess consumption volatility well. To highlight the importance of high-MPC households in driving this result, I show that excess consumption volatility disappears when households are counterfactually replaced with those exhibiting U.S. MPCs. High-MPC households contribute to consumption volatility through i) their strong consumption response to resource fluctuations and ii) large consumption reduction when assets become more illiquid. The transmission mechanisms of trend shocks and interest rate variations that previous studies use to explain excess consumption volatility are dampened because households significantly deviate from the permanent income hypothesis, on which these mechanisms crucially depend.

***JEL classification*:** E21, E32, F41, D31

***Keywords*:** emerging economy, business cycle, heterogeneous agents, MPC

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†Department of Economics, Columbia University. E-mail: sh3454@columbia.edu

I Introduction

One of the most salient patterns of emerging market business cycles is the phenomenon of 'excess consumption volatility': consumption is more volatile than output in emerging economies, while it is not in developed economies. An extensive literature is devoted to explaining excess consumption volatility, and the dominant modeling framework is representative-agent small open economy models. At the heart of these models, representative households optimize according to the permanent income hypothesis (PIH) because they can frictionlessly borrow from the international financial market. More importantly, the widely accepted mechanisms for excess consumption volatility in the literature crucially depend on the PIH behavior of households. However, micro data suggest that the PIH is not a good description of the consumption behavior of households in emerging economies. Hong (2020) estimates the marginal propensity to consume (MPC) out of transitory income shocks by applying a standard estimation method devised by Blundell, Pistaferri, and Preston (2008) to a Peruvian household survey and finds that the MPC estimates of Peru are substantially greater than those of the U.S. obtained by the same method. Given that the MPC out of transitory income shocks is close to zero under the PIH, this micro evidence suggests that the consumption behavior of households deviates significantly from the PIH in emerging economies.

Motivated by this observation, this paper revisits the driving mechanisms of emerging market business cycles through the lens of a heterogeneous-agent small open economy model in which households' MPCs are as high as the empirical estimates from the Peruvian data. To the best of my knowledge, this is a first attempt to study emerging market business cycles using a heterogeneous-agent model. To achieve empirically realistic MPCs in the model, I introduce Kaplan, Moll, and Violante (2018)'s two-asset environment over different degrees of liquidity into the model and calibrate the parameters governing financial frictions for illiquid assets (which are incorporated as adjustment costs for these assets in the model) jointly with the time discount factor by targeting the empirical MPC estimates.[1] I then take this model to Peruvian macro data through Bayesian estimation to explain emerging market business cycles.

After the Bayesian estimation, the model successfully generates the stylized patterns of emerging market business cycles, including excess consumption volatility. To evalu-

[1] I use a two-asset model because it can successfully target both the high MPCs and the correct amount of aggregate wealth: households can exhibit high MPCs by being liquidity poor while they hold a large amount of illiquid assets. In a one-asset model, on the other hand, households have to hold a small amount of assets to yield high MPCs. This leads to an insufficient amount of aggregate capital, which is problematic for a business cycle analysis.

ate the role of high-MPC households (or, more precisely, the role of the environment that makes households exhibit high MPCs) in emerging market business cycles, I run a counterfactual experiment in which Peruvian households are replaced with those exhibiting U.S. MPCs, which are substantially lower than Peruvian MPCs. Specifically, I recalibrate a subset of parameters including the time discount factor and the parameters governing financial frictions for illiquid assets by targeting the U.S. MPCs. After the recalibration, I find that aggregate consumption volatility declines by 26%, and as a consequence, the phenomenon of excess consumption volatility disappears. This result suggests that high-MPC households play an important role in generating excess consumption volatility.

To examine the mechanisms through which high-MPC households contribute to the high consumption volatility of emerging economies, I conduct three decomposition exercises: variance decomposition, variance change decomposition, and consumption response decomposition. I begin by decomposing the variances into the components generated by each aggregate shock in the baseline Peruvian economy. This variance decomposition shows that consumption variations are mostly driven by two aggregate shocks: i) stationary productivity shocks, which are the usual technology shocks in real business cycle models, and ii) illiquidity shocks, which change the degree of illiquidity of illiquid assets by shifting their adjustment costs.[2]

Once I implement the same variance decomposition for the counterfactual economy in which households are replaced with those exhibiting U.S. MPCs, I can decompose the variance changes between the baseline economy and the counterfactual economy into the changes generated by each shock. This variance change decomposition shows that consumption volatility substantially decreases in the counterfactual economy because both stationary productivity shocks and illiquidity shocks generate significantly less consumption variation.

Ultimately, consumption is determined by households after they observe the variations in variables that are relevant for their optimization, including prices and the degree of illiquidity. I name such variables 'drivers'. To see why stationary productivity shocks and illiquidity shocks generate substantially less consumption variation in the counterfactual economy, I decompose households' total consumption response with respect to these shocks into the consumption responses to each driver.

In response to a stationary productivity shock, I find that households' total consumption response is mostly driven by two drivers in the baseline economy: labor income per

[2] As we shall see later, Bayesian estimation assigns a sizable explanatory power for consumption variations to illiquidity shocks because of their crucial role in explaining a low correlation between consumption growth and investment growth, which is commonly observed in Peru and other emerging economies.

idiosyncratic labor productivity and illiquid asset returns. In the counterfactual economy, the total consumption response is substantially weaker because the responses to both drivers are substantially weaker. Importantly, the responses to both drivers are weaker in the counterfactual economy despite the fact that the equilibrium paths of the two drivers after the shock are similar in the two economies. These observations reveal the first main channel through which high-MPC households contribute to aggregate consumption volatility: the consumption of high-MPC households in emerging economies responds to individual resource fluctuations (mainly generated by the two drivers) far more strongly than that of the counterfactual households exhibiting U.S. MPCs.

In response to an illiquidity shock, households' total consumption response is mostly driven by the direct effect of the shock rather than by indirect effects through other drivers in the baseline economy. The direct effect changes households' consumption as follows. In my model, households allocate the vast majority of their savings to illiquid assets because liquid asset returns are too low compared to illiquid asset returns.[3] Because households face large financial frictions in trading illiquid assets in the baseline economy, it is expensive for them to cash out their illiquid assets when they need to do so by facing bad idiosyncratic income shocks. When an illiquidity shock hits the economy and the degree of illiquidity increases, it becomes more expensive for households to cash out their illiquid assets. In response to this shock, both households facing bad idiosyncratic income shocks and those facing good idiosyncratic income shocks reduce their consumption substantially. For households who face bad idiosyncratic income shocks at the moment of the illiquidity shock, they need to cash out their illiquid assets to smooth their consumption, but it is more difficult to do so because their assets are now more illiquid. As a consequence, they fail to smooth consumption more significantly, and their consumption plunges. For households who face good idiosyncratic income shocks at the moment of the illiquidity shock, they recognize that it will be more expensive to cash out their illiquid assets for a while. Therefore, they prepare themselves for situations in which bad idiosyncratic income shocks are realized in a near future by reducing consumption and accumulating more buffer stocks. In the counterfactual economy, this direct effect is substantially weaker because households face much weaker financial frictions in the first place. Therefore, even if the degree of illiquidity increases, it distorts households' consumption-saving decisions far more mildly in the counterfactual economy. These observations reveal the second main channel through which high-MPC households con-

[3]The liquid and illiquid asset returns on the balanced growth path are calibrated to match the long-run average values of deposit rates and lending rates in the data. On the balanced growth path, the aggregate amount of illiquid assets is 51 times greater than that of liquid assets.

tribute to aggregate consumption volatility: their consumption plunges when assets become more illiquid because some of them experience aggravated consumption smoothing failure and others come to have an enhanced precautionary-saving motive.

There are existing theories for the excess consumption volatility of emerging economies based on representative-agent small open economy models. I find that the driving mechanisms of excess consumption volatility in these conventional theories do not play an important role in my model. The key reason is that these conventional mechanisms require the PIH behavior by households, while the high-MPC households in my model significantly deviate from it.

The first well-accepted theory in the literature is the one developed by Aguiar and Gopinath (2007). The main mechanism of this theory operates through households' strong consumption response to a trend shock (or, equivalently, a shock to the growth of technology) as follows. When a positive trend shock hits the economy, output not only increases today but also grows substantially in the future. Representative households who follow the PIH increase their current consumption substantially more than the current increase in output because their decisions reflect the large increase in their permanent income due to the future income growth. Similarly, when a negative trend shock hits the economy, households decrease their current consumption substantially more than the current decrease in output because they recognize the large decrease in their permanent income due to the negative future income growth. This mechanism enables representative-agent models to generate excess consumption volatility.

In my model, the future output growth in response to positive trend shocks enters into households' budget constraints through two channels. First, the future growth of aggregate labor income enters into households' budget constraints as the future growth of labor income per idiosyncratic labor productivity. Second, the future growth of aggregate capital income is reflected in the asset price of illiquid assets and thus enters into households' budget constraints as a jump in illiquid asset returns on impact. Both channels make households' idiosyncratic income profiles either more increasing or less decreasing, and by this positive wealth effect, households would want to consume more. However, there is a strong counteracting force in my model. The first of these two channels also increases future idiosyncratic income risk (as labor income per idiosyncratic labor productivity grows in the future), and households' precautionary-saving motive becomes stronger. In particular, because households allocate the vast majority of their savings to illiquid assets and it is expensive to cash out their illiquid assets when they need to do so by facing bad idiosyncratic income shocks, the amount of additional precautionary saving in response to the increased idiosyncratic risk is large. This enhanced precautionary-saving motive offsets most of the positive wealth effect, and as a result, the consumption response is substantially subdued. When negative trend shocks hit the economy, the mechanism operates in the opposite way. The shocks make households' idiosyncratic income profiles either less increasing or more decreasing and generate a negative wealth effect. However, a weakened precautionary-saving motive offsets most of this negative wealth effect. As a result, the consumption decrease in response to negative trend shocks is substantially dampened.

The second well-accepted theory in the literature is the one developed by Neumeyer and Perri (2005). The main mechanism of this theory operates through households' intertemporal substitution of consumption in response to interest rate variations. Emerging economies face volatile interest rate fluctuations, induced either by domestic economic conditions or purely external factors. These interest rate fluctuations change the relative prices of consumption between different time periods. When representative households who follow the PIH face these interest rate fluctuations, they intertemporally substitute their consumption. This intertemporal substitution can generate large consumption variations without any significant variations in output. Representative-agent models can explain excess consumption volatility by using this mechanism.

In my model, households cannot incorporate this mechanism well for two reasons. First, unlike representative households whose consumption is solely determined by their lifetime wealth and the degree of intertemporal substitution, the consumption-saving behavior of households in my model is also substantially affected by the precautionary-saving motive. Second, the fact that households in my model allocate most of their savings to illiquid assets makes it even more difficult for them to shift resources across time.

This paper is closely related to two strands of literature. The first is a recently growing literature devoted to understanding how microlevel household behavior shapes macroeconomic dynamics or the transmission mechanisms of economic policies. Well-known works in this literature include Kaplan et al. (2018), Auclert (2019), Krueger, Mitman, and Perri (2016), McKay, Nakamura, and Steinsson (2016), Auclert, Rognlie, and Straub (2018), Bayer, Luetticke, Pham-Dao, and Tjaden (2019), and Oh and Reis (2012), among many others. Many studies in this literature focus on the fact that even in advanced economies such as the U.S., a sizable fraction of households exhibit MPCs that are significantly higher than the MPCs predicted by the PIH. They examine how the model prediction changes once this fact is realistically incorporated into the model. This paper contributes to this literature by exploiting a different margin: the MPCs of emerging economies are substantially greater than those of developed economies. It finds that once this margin is incorporated, microlevel household behavior matters for aggregate dynamics to the extent that it can actually explain one of the most salient patterns of emerging market business cycles, namely excess consumption volatility.

The second literature is the one devoted to explaining the stylized patterns of emerging market business cycles using macroeconomic models. Well-known works in this literature include Neumeyer and Perri (2005), Aguiar and Gopinath (2007), Uribe and Yue (2006), Garcia-Cicco, Pancrazi, and Uribe (2010), Chang and Fernández (2013), and Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2011), among many others. All these studies are based on representative-agent models. My paper contributes to this literature by bringing new intuitions and tools regarding how household heterogeneity and microlevel behavior affect aggregate dynamics from the first related literature, applying them in the context of emerging market business cycles, and deriving new explanations.

In addition to these two most closely related strands of literature, this paper is also related to multiple other lines of research. In terms of methodology, this paper has a commonality with Bayer, Born, and Luetticke (2020) and Auclert, Rognlie, and Straub (2020) in that Bayesian methods are applied to estimate a heterogeneous-agent model. Bayesian estimation requires a model to be solved a large number of times. It only recently became possible to solve heterogeneous-agent models fast enough to conduct Bayesian estimation thanks to the development of new computational methods. The main contributors to this recent computational development include Auclert, Bardóczy, Rognlie, and Straub (2019), Boppart, Krusell, and Mitman (2018), Ahn, Kaplan, Moll, Winberry, and Wolf (2018), Bayer and Luetticke (2020), Winberry (2018), and Reiter (2009). Among the new methods, I use the one developed by Auclert et al. (2019).

My paper is also related to studies that incorporate household heterogeneity into an open economy model. In this line of research, the most closely related to my work is Guntin, Ottonello, and Perez (2020). Guntin et al. (2020) compute the elasticity of group-average consumption to group-average income for each of income deciles during crises characterized by a large consumption decline using micro data and use the empirical results to identify the driving mechanism of the crises in a heterogeneous-agent small open economy model. There are also papers that study monetary phenomena through the lens of a heterogeneous-agent small open economy model with nominal frictions. De Ferra, Mitman, and Romei (2020) study how exchange rate fluctuations induced by a large current-account reversal affect the economy through the revaluation of households' foreign-currency-denominated debt. Sunel (2018) examines the welfare implication of a large and gradual disinflation that emerging economies experienced over the past two decades.[4]

The remainder of this paper is organized as follows. Section II provides micro evidence on differences in MPCs between emerging and developed economies. After specifying the model in Section III, I take the model to the data in Section IV through a two-step procedure that includes calibration and Bayesian estimation. In section V, I run a counterfactual experiment in which households are replaced with those exhibiting U.S. MPCs. I then examine the underlying mechanisms through decomposition exercises in section VI. Section VII examines the extent to which the mechanisms of conventional theories are dampened in my model and discusses the economic reasons. Section VIII concludes the paper.

II Micro Evidence

This paper starts from the empirical finding of Hong (2020) that MPCs out of transitory income shocks in emerging economies are substantially greater than those in developed economies. To obtain this finding, Hong (2020) employs the method devised by Blundell et al. (2008), which is one of the widely accepted MPC estimation methods in the literature. This method imposes a theory-guided covariance structure on the joint dynamics of income and consumption and estimates the MPCs from this structure. Specifically, Hong (2020) applies this method to a nationally representative Peruvian household survey, Encuesta Nacional de Hogares (ENAHO), which is one of the rare emerging market micro datasets that satisfy the data requirements of the method.[5] Then, the Peruvian MPC estimates are compared with the U.S. MPC estimates obtained by the same method using the Panel Study of Income Dynamics (PSID).[6]

In this section, I make a few revisions to Hong (2020)'s procedure that are necessary to use the MPC estimates in disciplining the model presented in this paper. These revisions include i) a change in the consumption measure from nondurable consumption to total consumption (including both nondurable and durable consumption) to be consistent with the aggregate consumption measure, ii) a change in the sample periods necessary for the

[4]These studies and my paper incorporate household heterogeneity in asset positions and labor productivity as in Aiyagari (1994) and Kaplan et al. (2018). There are also studies that incorporate household heterogeneity in open economies by introducing a finite number of different types of households, such as Cugat (2019) and Iyer (2015).

[5]Blundell et al. (2008)'s method requires household surveys to include both income and consumption. It also requires a panel structure such that households have to appear at least three consecutive times.

[6]Specifically, Hong (2020) uses the replication dataset of Kaplan, Violante, and Weidner (2014), which the authors construct from the PSID to estimate Blundell et al. (2008)'s partial insurance parameter upon which Hong (2020)'s MPC estimates are also based. In this paper, instead of reusing Kaplan et al. (2014)'s dataset, I reconstruct the U.S. data from the PSID to incorporate the revisions discussed below.

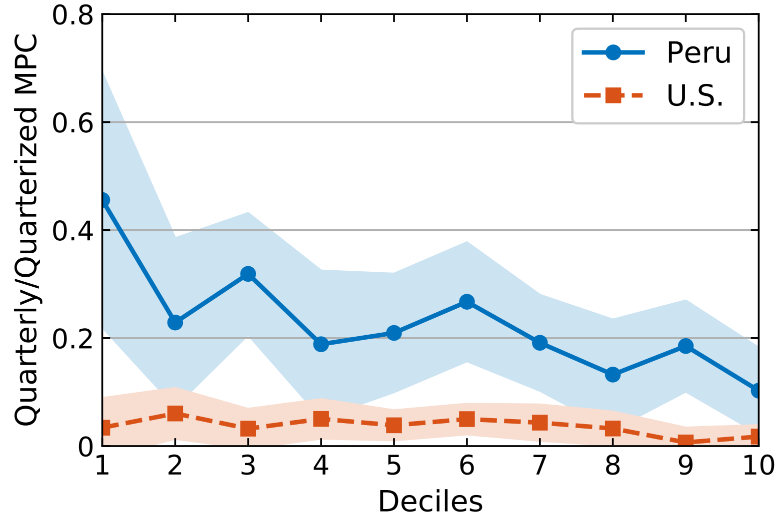


Figure 1: Quarterly/Quarterized MPC Estimates in Peru and U.S.

Notes: Figure 1 plots the quarterly MPC estimates of Peru and the quarterized annual MPC estimates of the U.S. according to Auclert (2019)'s model-free conversion formula: $1 - MPC_G^Q = (1 - MPC_G^A)^{0.25}$. Shaded areas represent 95% confidence intervals.

availability of some key durable expenses regarding the first change, and iii) a change in the income process specification to be consistent with the model specification in this paper. I provide details of the MPC estimation procedure, the three revisions, and data processing in Appendix A.

After reflecting these changes, the final Peruvian sample used in this paper comes from the 2011-2018 waves of ENAHO. Because ENAHO is conducted annually and provides one quarterly income and consumption per annual survey, we can obtain seven years of year-over-year growth of quarterly income and consumption from this sample. From this data structure, we can estimate quarterly MPCs of Peruvian households by applying Blundell et al. (2008)'s method. The blue solid line with circle markers in Figure 1 plots the MPC estimates over the labor income deciles of Peru.[7] The shaded area around the line represents the 95% confidence intervals.

The final U.S. sample used in this paper comes from the 2005-2017 waves of the PSID. The PSID is conducted biannually during the sample period and provides one annual income and consumption per biannual survey. Therefore, the PSID sample gives six years

[7] Admittedly, I do not use wealth grouping, which is a more common grouping strategy in the literature, because ENAHO does not include wealth data. Instead, I use labor income grouping because it can effectively detect the different degrees of liquidity constraint effects. (For example, Zeldes (1989) detects the presence of liquidity constraints by using lagged incomes as instruments.) See Appendix A.6 for details on how I construct the labor income deciles.

of two-year-over-two-year growth of annual income and consumption. From this data structure, we can obtain annual MPC estimates of U.S. households using Blundell et al. (2008)'s method. To compare the annual MPC estimates of the U.S. and the quarterly MPC estimates of Peru without resorting to a model, here I adopt the frequency-conversion formula that Auclert (2019) uses for the same purpose of comparing quarterly and annual MPC estimates:

$$1 - MPC_G^Q = (1 - MPC_G^A)^{0.25} \tag{1}$$

in which MPC_G^Q denotes the quarterly MPC and MPC_G^A denotes the annual MPC of group G.[8] The red dashed line with square markers in Figure 1 plots the quarterized U.S. MPC estimates according to this model-free frequency conversion (1) over the U.S. labor income deciles. The shaded area around the line represents the 95% confidence intervals.[9]

Figure 1 compares the quarterly Peruvian MPC estimates and the quarterized U.S. MPC estimates. This figure reaffirms Hong (2020)'s finding that the MPCs of Peruvian labor income deciles are substantially higher than those of U.S. labor income deciles.

One important disadvantage in this comparison is that the model-free frequency conversion (1) cannot address the problem that the time frame applied to the Peruvian data is different from that applied to the U.S. data: Peruvian households are assumed to be subject to quarterly income processes and make quarterly consumption decisions, while U.S. households are assumed to be subject to annual income processes and make annual consumption decisions. This discrepancy can be directly taken into account once we have a model. I revisit this issue in section V.

III The Model

I construct a heterogeneous-agent small open economy model by combining i) the two-asset household heterogeneity over liquid and illiquid assets of Kaplan et al. (2018)[10] and ii) the standard emerging market, small open economy features of Garcia-Cicco et al. (2010).[11] One difficulty in combining these two structures is that conventional representative-

[8] Auclert (2019) derives this formula by assuming that the response of quarterly consumption in period $t+j$ to a shock in period t decreases exponentially in j, and the interest rate is close to zero. He argues that this formula is a good approximation in partial equilibrium Bewley models.

[9] When constructing the confidence intervals, the standard errors are also converted using equation (1) and the Delta method.

[10] Note, however, that I do not incorporate the nominal rigidity of Kaplan et al. (2018)'s Heterogeneous-Agent New Keynesian (HANK) model because the model is intended to be as close as possible to the conventional real models of emerging economies except for household heterogeneity.

[11] The emerging market, small open economy features of Garcia-Cicco et al. (2010) include free access to the international financial market through which external debt is borrowed, the open goods market in which the gap between domestic output and demand is offset by net exports, and capital adjustment costs

agent small open economy models already have their own two-asset environment between international debt and capital in households' optimization. Since there is no obvious one-to-one mapping between the former two assets (liquid assets and illiquid assets) and the latter two assets (international debt and capital), letting households decide all of them requires more than two assets and thus is subject to the curse of dimensionality. I circumvent this problem by exploiting the following feature of conventional small open economy models: there are multiple ways to decentralize the economy (while maintaining the same set of equilibrium conditions), and one of them decentralizes it such that firms face the two-asset problem between international debt and capital, and households only deal with one asset, namely firm shares. I start from this decentralized version of the conventional small open economy model and incorporate the two-asset environment of Kaplan et al. (2018) into it by revising the firm shares into illiquid assets (by introducing adjustment costs in trading the shares) and additionally introducing liquid assets.

A Households

A continuum of households live in this economy. Each household i is heterogeneous in its illiquid asset position $a_{i,t-1}$, liquid asset position $b_{i,t-1}$, and idiosyncratic labor productivity $e_{i,t}$ in each period t. The illiquid assets are the shares of firms that households hold, and the liquid assets are households' bank deposits. Households face the following tradeoff between illiquid assets and liquid assets: illiquid assets pay higher returns than liquid assets on the balanced growth path, but households have to pay adjustment costs when accumulating or running down illiquid assets (while liquid assets can be adjusted costlessly). Households cannot take short positions in both illiquid assets and liquid assets. In each period, household i solves the following optimization problem.

$$\max_{\{c_{i,t},b_{i,t},a_{i,t},v_{i,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{i,t}^{1-\gamma}}{1-\gamma} \tag{2}$$

$$s.t.$$

$$c_{i,t} + b_{i,t} + v_{i,t} + \eta_t \chi_t(v_{i,t}, a_{i,t-1}) = w_t e_{i,t} \bar{l}_t + (1-\xi)(1+r_t^b) b_{i,t-1},$$

$$v_{i,t} = a_{i,t} - (1+r_t^a) a_{i,t-1}, \quad \text{and}$$

$$b_{i,t} \geq 0, \quad a_{i,t} \geq 0.$$

In the households' budget constraint, $(1+r_t^a)$ is the gross return rate on illiquid assets,

that are necessary for small open economy models to yield a realistic degree of investment fluctuations (because interest rates are directly affected by exogenous shocks and thus highly volatile in these models).

$(1-\xi)(1+r_t^b)$ is the gross return rate on liquid assets, and $\eta_t\chi_t(v_{i,t}, a_{i,t-1})$ is the adjustment cost for the illiquid asset positions. Parameter ξ is strictly positive and $r^a = r^b$ on the balanced growth path.[12] The term $w_t e_{i,t}\bar{l}_t$ is the labor income of household i, in which w_t is the wage rate per efficiency unit of labor and $\bar{l}_t$ is the labor supply. The labor supply is common to all households and determined by the labor union of the economy, which will be specified in the following subsection.[13]

The adjustment cost for illiquid assets, $\eta_t\chi_t(v_{i,t}, a_{i,t-1})$, is the product of two components: an aggregate shock to the degree of illiquidity η_t and the adjustment cost on the balanced growth path $\chi_t(v_{i,t}, a_{i,t-1})$. For the functional form of $\chi_t(v_{i,t}, a_{i,t-1})$, I closely follow Auclert et al. (2019)'s discrete-time version of Kaplan et al. (2018)'s model as follows.

$$\chi_t(v_{i,t}, a_{i,t-1}) = \chi_1 \left| \frac{v_{i,t}}{(1+r_t^a)a_{i,t-1} + \chi_0 X_{t-1}} \right|^{\chi_2} \left((1+r_t^a)a_{i,t-1} + \chi_0 X_{t-1}\right)$$

in which $\chi_0 > 0, \chi_1 > 0$, and $\chi_2 > 1$, and X_{t-1} is the stochastic trend of the economy.

Parameter χ_1 is the scaling factor for the adjustment cost, and it determines the overall importance of the adjustment cost term in households' optimization. As parameter χ_1 increases, it becomes more expensive to trade illiquid assets. Importantly, when parameter

[12] As we shall see in subsection III.C, $(1+r_t^b)$ is banks' gross financing cost when they finance through intermediating household deposits. This financing cost consists of an intermediation cost $\xi(1+r_t^b)$ that banks incur and a gross return on household deposits $(1-\xi)(1+r_t^b)$.

[13] When individual households determine their labor supply under widely used preference specifications, the model exhibits counterfactual patterns in important dimensions. When the labor supply is determined by individual households under separable labor disutility, the aggregate labor supply declines substantially during booms because of the wealth effect. This phenomenon is common in macroeconomic models of emerging economies (including those with representative households) because they are designed to exhibit large consumption fluctuations, which also generate large fluctuations in the wealth effect. For this reason, macroeconomic models of emerging economies usually impose the preferences introduced by Greenwood, Hercowitz, and Huffman (1988) (GHH preferences hereafter) instead of separable labor disutility because the wealth effect disappears under GHH preferences. In my model, however, when individual households determine their labor supply under GHH preferences, another counterfactual pattern emerges: the MPC estimates from the model become abnormally high compared to the estimates from micro data. This is because under GHH preferences, the per-period utility over consumption c and labor supply l is given by $\frac{(c-g(l))^{1-\gamma}}{1-\gamma}$, and thus households tend to smooth $(c-g(l))$ rather than c. As a consequence, consumption comoves too strongly with income.

Note that in the HANK literature, researchers also find that models exhibit counterfactual patterns when individual households determine their labor supply, although on different aspects than the counterfactual patterns that my model exhibits under the individual labor supply decisions. Some researchers prefer to circumvent this problem by introducing a labor union to which the labor supply decision is delegated. (See Auclert and Rognlie (2017) for a detailed discussion of this issue in the context of the HANK literature.) In the same spirit, I introduce a labor union to circumvent the problem caused by the individual labor supply decisions in my model. In particular, I write the objective function of the labor union such that the aggregate labor supply equation is identical to that in a typical representative-agent model with GHH preferences. See subsection III.B for details on the optimization problem of the labor union.

χ_1 is higher, households i) save more and ii) exhibit higher MPCs. For households facing bad idiosyncratic income shocks, they need to cash out their illiquid assets to smooth their consumption. However, it is more difficult to do so when χ_1 is higher, and therefore, they have to save more. Moreover, these households exhibit higher MPCs when χ_1 is higher because of the aggravated consumption smoothing failure they experience. For households facing good idiosyncratic income shocks, their precautionary-saving motive is stronger when χ_1 is higher because they recognize that it will be more expensive to cash out their illiquid assets when they need to do so by facing bad idiosyncratic income shocks in the future. Therefore, they prepare themselves for such cases by accumulating more buffer stocks. Moreover, these households exhibit higher MPCs when χ_1 is higher because of their enhanced precautionary-saving motive.

When parameter χ_2 is equal to one, the adjustment cost becomes proportional to the absolute amount of illiquid asset position adjustment. As χ_2 increases above one, the adjustment cost becomes less costly for rich households (who have higher values of $(1+r_t^a)a_{i,t-1}$). Therefore, parameter χ_2 captures how less costly it is for wealthier households to adjust their illiquid asset positions. For this reason, parameter χ_2 is useful to make rich and poor households face different degrees of financial frictions and thus have different MPCs. Later, I calibrate χ_1 and χ_2 (jointly with β) to match the MPC estimates and aggregate wealth. I find that calibrating this small number of parameters can effectively match the ten MPC moments over the labor income deciles and the correct amount of aggregate wealth in the economy.

I assume that idiosyncratic labor productivity $\log e_{i,t}$ is composed of a persistent component that follows an $AR(1)$ process and a transitory component that follows an $I.I.D.$ process as follows.[14]

$$\log e_{i,t} = \log e_{1,i,t} + \log e_{2,i,t},$$

$$\log e_{1,i,t} = \rho_{e_1} \log e_{1,i,t-1} + \epsilon_{1,i,t}, \quad \epsilon_{1,i,t} \sim N(0, \sigma^2_{\epsilon_1}),$$

$$\log e_{2,i,t} = \epsilon_{2,i,t}, \quad \epsilon_{2,i,t} \sim N(0, \sigma^2_{\epsilon_2}).$$

Let $\Psi_t(e_1, e_2, b_-, a_-)$ denote the economy's cumulative distribution function of (e_1, e_2, b_-, a_-)

[14]The labor productivity process specification in the model is consistent with the income process specification imposed in the MPC estimation. When we generate individual households' log labor income $\log w_t e_{i,t} \bar{l}_t$ from the model and control for the time fixed effect as in the empirical MPC estimation, we obtain $\log e_{i,t}$ as the residual. Therefore, $\log e_{i,t}$ is the model counterpart of the residual of log income, $y_{i,t}$, in the empirical MPC estimation. As seen in Appendix A.1, the specification of the $\log e_{i,t}$-process in the model is exactly equal to the specification of the $y_{i,t}$-process in the MPC estimation.

in period t:

$$\Psi_t(e_1, e_2, b_-, a_-) := P(e_{1,t} \le e_1,\ e_{2,t} \le e_2,\ b_{t-1} \le b_-,\ a_{t-1} \le a_-).$$

Moreover, let $c_t(e_1, e_2, b_-, a_-)$, $b_t(e_1, e_2, b_-, a_-)$, and $a_t(e_1, e_2, b_-, a_-)$ denote the policy functions of households in period t.[15] The law of motion for the distribution can be described as follows.

$$\begin{aligned}\Psi_{t+1}(e_1', e_2', b, a) = \int_{e_1, e_2, b_-, a_-} & P(e_{1,t+1} \le e_1' | e_{1,t} = e_1) P(e_{2,t+1} \le e_2') \\ & I_{\{b_t(e_1,e_2,b_-,a_-) \le b,\ a_t(e_1,e_2,b_-,a_-) \le a\}}(e_1, e_2, b_-, a_-)\, d\Psi_t\end{aligned} \tag{3}$$

in which $I_{\{X\}}(x)$ is an indicator function (*i.e.*, $I_{\{X\}}(x) = 1$ if $x \in X$, 0 otherwise).

Let C_t, B_t, A_t, and χ_t^{agg} be the aggregate quantities that sum up the corresponding individual variables as follows.

$$\begin{aligned} C_t &= \int_{e_1,e_2,b_-,a_-} c_t(e_1, e_2, b_-, a_-)\, d\Psi_t, \\ B_t &= \int_{e_1,e_2,b_-,a_-} b_t(e_1, e_2, b_-, a_-)\, d\Psi_t, \\ A_t &= \int_{e_1,e_2,b_-,a_-} a_t(e_1, e_2, b_-, a_-)\, d\Psi_t, \quad \text{and} \\ \chi_t^{agg} &= \int_{e_1,e_2,b_-,a_-} \eta_t \chi_t(a_t(e_1, e_2, b_-, a_-) - (1 + r_t^a) a_-, a_-)\, d\Psi_t. \end{aligned} \tag{4}$$

By aggregating the individual households' budget constraints, we can obtain

$$C_t + B_t + A_t + \chi_t^{agg} = w_t \bar{e} \bar{l}_t + (1 - \xi)(1 + r_t^b) B_{t-1} + (1 + r_t^a) A_{t-1} \tag{5}$$

in which $\bar{e} := E[e_{i,t}]$ is the cross-sectional average of idiosyncratic labor productivity.

B Labor Union

The labor supply decision is made collectively by the labor union. The labor union linearly weights the cross-sectional average of labor income $w_t \bar{e} \bar{l}_t$ and labor disutility

[15] I attach the time subscript to the policy functions because they depend on the state vector $\mathcal{S}_t$, which includes the distribution function $\Psi_t(e_1, e_2, b_-, a_-)$, the stochastic trend X_{t-1}, other predetermined variables and exogenous variables in the economy. I specify which objects constitute the state vector $\mathcal{S}_t$ in footnote 18 after I complete the model specification. Using the state vector $\mathcal{S}_t$, one can alternatively denote the policy functions as time-invariant functions $c(e_1, e_2, b_-, a_-; \mathcal{S}_t)$, $b(e_1, e_2, b_-, a_-; \mathcal{S}_t)$, and $a(e_1, e_2, b_-, a_-; \mathcal{S}_t)$.

$X_{t-1}\frac{1}{1+\omega}\bar{l}_t^{1+\omega}$ in making the decision as follows.[16]

$$\max_{\bar{l}_t} \; w_t\bar{e}\bar{l}_t - \kappa\left(X_{t-1}\frac{1}{1+\omega}\bar{l}_t^{1+\omega} \right)$$

in which $\kappa > 0$. As a result of the labor union's optimization, the labor supply is determined by the following equation.

$$w_t\bar{e}^{1+\omega} = \kappa X_{t-1}(\bar{e}\bar{l}_t)^{\omega}. \tag{6}$$

Note that this aggregate labor supply equation is equal to that in conventional representative-agent small open economy models with GHH preferences. In this sense, my model does not deviate from the conventional models in the dimension of the aggregate labor supply.

C Domestic Banks

There are an infinite number of representative and competitive domestic banks that finance funds either by intermediating household deposits or by borrowing from the international financial market and then lend the funds to firms. As introduced above, B_t is the aggregate amount of household deposits. Let D_t be the banks' debt from the international financial market and F_t be the amount of funds that the banks lend to firms. By construction, we have

$$F_t = B_t + D_t.$$

$(1+r_t^b)$ is banks' gross financing cost when they finance through intermediating household deposits B_{t-1}. This financing cost consists of an intermediation cost $\xi(1+r_t^b)$ that banks incur and a gross return on household deposits $(1-\xi)(1+r_t^b)$. The banks can frictionlessly adjust their sources of financing, and therefore, the financing cost is equalized between the two sources. In other words, we have

$$1 + r_t^b = 1 + r_{t-1}, \quad t \geq 0 \tag{7}$$

in which r_{t-1} is the interest rate on the international debt D_{t-1}. Because banks are competitive and there is no additional cost in lending funds to firms, banks lend funds F_t to firms at interest rates r_t.

[16] In a model with stochastic growth, it is a common practice that labor disutility is augmented with stochastic growth X_{t-1}. As explained in Uribe and Schmitt-Grohé (2017), this practice technically enables the quantity variables of the model to grow along the balanced growth path while labor supply does not in the long run. The augmentation of X_{t-1} can be economically interpreted as an advancement of home-production technology, such as the popularization of dishwashers and microwave ovens.

D Firms

There are an infinite number of representative and competitive firms that produce outputs Y_t using capital K_{t-1} and labor L_t, make investment I_t to accumulate capital, and borrow funds F_t from domestic banks. Moreover, they facilitate trade in their own shares (which are illiquid assets from the perspective of households) and earn facilitation fees χ_t^{agg}.[17] Specifically, they solve the following optimization problem.

$$\max_{\{K_t,F_t,L_t,Y_t,I_t,\Pi_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} Q_{o,t}\Pi_t \tag{8}$$

$$s.t.$$

$$\Pi_t = Y_t - w_t L_t - I_t - \Phi(K_t, K_{t-1}) + F_t - (1 + r_{t-1})F_{t-1} + \chi_t^{agg},$$

$$Y_t = z_t K_{t-1}^{\alpha}(X_t L_t)^{1-\alpha},$$

$$\nu_t I_t = K_t - (1-\delta)K_{t-1},$$

$$\nu_t \Phi(K_t, K_{t-1}) = \frac{\phi}{2}\left(\frac{K_t}{K_{t-1}} - g^*\right)^2 K_{t-1},$$

$$Q_{0,t} = \begin{cases} 1 & \text{if} \quad t = 0, \\ \frac{1}{\prod_{s=1}^{t}(1+r_s^a)} & \text{otherwise,} \end{cases} \quad \text{and}$$

$$\lim_{j\to\infty} \frac{F_{t+j}}{\prod_{s=1}^{j}(1+r_{t+s}^a)} \leq 0$$

in which Π_t is the per-period profit, $\Phi(K_t, K_{t-1})$ is an adjustment cost for the accumulation of capital, z_t is the stationary component of firms' productivity, and X_t is the nonstationary component (or stochastic trend) of firms' productivity. The variable ν_t is an aggregate shock to capital accumulation: when ν_t is higher, firms need to spend a smaller amount of resources as investment and capital adjustment costs to achieve the same amount of capital K_t given the same amount of previous capital K_{t-1}. This shock is often called an investment shock in the literature, and Justiniano, Primiceri, and Tambalotti (2010) study its role in U.S. business cycles. Firms discount profit flows using return rates on illiquid assets. As we shall see in the following subsection, this objective function is the total value of the firms because illiquid assets are the shares of the firms

[17]This is one way to return χ_t^{agg} to households. Alternatively, one could assume that the trade in firms' shares is facilitated by banks, illiquid assets are composed of banks' shares and firms' shares, and households can frictionlessly and instantaneously adjust the proportion of the two shares within the illiquid asset portfolio. This alternative specification yields the same equilibrium conditions.

that households hold.

E Illiquid Asset Return and Price

Households' illiquid assets are the shares of firms they hold. Let $s_{i,t}$ be the share that individual household i holds when the total shares are normalized to 1. Let q_t be the price of the illiquid assets after the current profits are distributed as dividends. Since total shares are normalized to 1, q_t also represents the total value of the firms after distributing current profits. By construction, we have the following equations.

$$a_{i,t-1} = q_{t-1}s_{i,t-1}, \quad \text{and}$$

$$(1+r_t^a)a_{i,t-1} = \Pi_t s_{i,t-1} + q_t s_{i,t-1}.$$

From these two equations, we can obtain

$$1 + r_t^a = \frac{\Pi_t + q_t}{q_{t-1}}, \quad t \geq 0. \tag{9}$$

By solving equation (9) forward and taking an expectation, we can obtain

$$q_t = E_t \sum_{j=1}^{\infty} \frac{\Pi_{t+j}}{\prod_{s=1}^{j}(1+r_{t+s}^a)}.$$

Therefore, the objective function of the firms' optimization problem $E_0 \sum_{t=0}^{\infty} Q_{0,t}\Pi_t$ is equal to $\Pi_0 + q_0$. In other words, firms maximize their total value before distributing current profits. This explains why firms discount profit flows with illiquid asset returns in their optimization.

It is worth noting how $\{r_t^a\}_{t=0}^{\infty}$ are determined in equilibrium. The illiquid asset returns from period 1 onward, $\{r_t^a\}_{t=1}^{\infty}$, are subject to the following optimimality condition for firms with respect to determining F_t.

$$E_t\left[\frac{1+r_t}{1+r_{t+1}^a}\right] = 1, \quad t \geq 0. \tag{10}$$

When we consider the impulse responses after an MIT shock (*i.e.*, without aggregate uncertainty), this equation becomes $r_{t+1}^a = r_t,\ t \geq 0$. On the other hand, the illiquid asset return in period 0, r_0^a, is not determined by equation (10). Instead, r_0^a is solely determined by Π_0, q_0, and q_{-1} through equation (9).

F Interest Rates in the International Financial Market

The interest rates in the international financial market, r_t, are specified in a standard way as follows.

$$r_t = r^* + \psi\left\{\exp\left(\frac{\hat{D}_t/X_t - \tilde{D}^*}{\tilde{Y}^*}\right) - 1\right\} - \theta_z(z_t - 1) - \theta_g\left(\frac{g_t}{g^*} - 1\right) + \mu_t - 1 \tag{11}$$

in which $\psi > 0$, $\theta_z > 0$, and $\theta_g > 0$. r^* is the long-run average of the interest rate, and $\hat{D}_t$ is the cross-sectional average of firms' international debt. Individual firms regard $\hat{D}_t$ as exogenously given, but in equilibrium, individual firms' international debt D_t is equal to their cross-sectional average $\hat{D}_t$:

$$\hat{D}_t = D_t. \tag{12}$$

$\tilde{D}^*$ is the long-run average of $\hat{D}_t/X_t$, $\tilde{Y}^*$ is the long-run average of Y_t/X_{t-1}, and g^* is the long-run average of the gross growth rate of the stochastic trend, $g_t := X_t/X_{t-1}$. μ_t is an aggregate shock to interest rates.

A reduced-form specification of the interest rates in the international financial market, as in equation (11), is widely used in small open economy models for business cycle studies, particularly when the models are intended to be first-order approximated with respect to aggregate shocks. (See Neumeyer and Perri (2005), Garcia-Cicco et al. (2010), and Chang and Fernández (2013), for example.) In equation (11), interest rates are determined to be higher when the economy's international debt is larger and the productivities of the economy are lower. In this respect, equation (11) reflects the theoretical implication of sovereign default models such as Arellano (2008) and Mendoza and Yue (2012) in a reduced-form manner.

G Aggregate Shock Processes

The model economy is hit by five aggregate shocks: a stationary productivity shock z_t, a trend shock g_t, an interest rate shock μ_t, an illiquidity shock η_t, and an investment shock ν_t. I assume that each aggregate shock follows an AR(1) process as follows.

$$\begin{aligned}
\log z_t &= \rho_z \log z_{t-1} + \epsilon_t^z, \quad \epsilon_t^z \sim N(0, \sigma_z^2), \\
\log(g_t/g^*) &= \rho_g \log(g_{t-1}/g^*) + \epsilon_t^g, \quad \epsilon_t^g \sim N(0, \sigma_g^2), \\
\log \mu_t &= \rho_\mu \log \mu_{t-1} + \epsilon_t^\mu, \quad \epsilon_t^\mu \sim N(0, \sigma_\mu^2), \\
\log \eta_t &= \rho_\eta \log \eta_{t-1} + \epsilon_t^\eta, \quad \epsilon_t^\eta \sim N(0, \sigma_\eta^2), \quad \text{and} \\
\log \nu_t &= \rho_\nu \log \nu_{t-1} + \epsilon_t^\nu, \quad \epsilon_t^\nu \sim N(0, \sigma_\nu^2).
\end{aligned} \tag{13}$$

H Market Clearing and Trade Balance

The market clearing conditions are specified as follows.

$$L_t = \bar{e}\bar{l}_t \quad \text{(labor market)}, \tag{14}$$

$$F_t - D_t = B_t \quad \text{(liquid asset market), and} \tag{15}$$

$$q_t = A_t \quad \text{(illiquid asset market)}. \tag{16}$$

By Walras' law, we can derive the following resource constraint (or, equivalently, the goods market clearing condition in the open economy) using equations (5), (7), (9), (14), (15), and (16).

$$C_t + I_t + \Phi(K_t, K_{t-1}) + \xi(1 + r_{t-1})B_{t-1} = Y_t + D_t - (1 + r_{t-1})D_{t-1}. \tag{17}$$

The trade balance of the economy TB_t is determined as follows.

$$\begin{aligned} TB_t &= Y_t - C_t - I_t - \Phi(K_t, K_{t-1}) - \xi(1 + r_{t-1})B_{t-1} \\ &= -D_t + (1 + r_{t-1})D_{t-1}. \end{aligned} \tag{18}$$

I Equilibrium

Given the initial conditions on $\Psi_0(e_1, e_2, b_-, a_-)$, X_{-1}, A_{-1}, K_{-1}, D_{-1}, B_{-1}, F_{-1}, and r_{-1},[18]

i) individual households' policy functions $\{c_t(e_1, e_2, b_-, a_-), b_t(e_1, e_2, b_-, a_-), a_t(e_1, e_2, b_-, a_-)\}_{t=0}^{\infty}$ that solve the households' optimization problem (2),

ii) cross-sectional cumulative distributions $\{\Psi_t(e_1, e_2, b_-, a_-)\}_{t=1}^{\infty}$ that evolve over time according to equation (3),

iii) aggregate variables $\{C_t, B_t, A_t, \chi_t^{agg}\}_{t=0}^{\infty}$ constructed by aggregating corresponding individual variables according to equation (4),

iv) prices and aggregate variables $\{r_t^b, r_t^a, r_t, w_t, q_t, \bar{l}_t, L_t, \Pi_t, Y_t, I_t, K_t, F_t, D_t, \hat{D}_t, TB_t\}_{t=0}^{\infty}$ satisfying firms' optimality conditions (including constraints) for their optimization problem (8) and other equilibrium conditions (6), (7), (9), (11), (12), (14), (15), (16), and (18), and

[18] Referring back to footnote 15, state vector $\mathcal{S}_t$ is composed of predetermined objects $\Psi_t(e_1, e_2, b_-, a_-)$, X_{t-1}, A_{t-1}, K_{t-1}, D_{t-1}, B_{t-1}, F_{t-1}, and r_{t-1} and aggregate exogenous variables z_t, g_t, μ_t, η_t, and ν_t. The initial conditions in this subsection specify the predetermined objects of $\mathcal{S}_0$.

v) aggregate shocks $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$, which follow the processes specified in (13)

constitute the equilibrium of the economy.

J Solving the Model

To study business cycles through Bayesian estimation, we need to solve a model a large number of times. To this end, a model needs to be solved quickly (at least within a second). My model is a heterogeneous-agent model with aggregate uncertainty, and it only recently became possible to solve this class of models within a second thanks to the development of new computational methods.[19]

Among the new computational methods developed, I adopt Auclert et al. (2019)'s method, which computes the linearized dynamics of the macroeconomic variables (including aggregate quantities and prices) based on Boppart et al. (2018)'s finding that the impulse responses after an MIT shock are equivalent to the MA(∞) representation of the first-order-approximated model with aggregate uncertainty. Under this method, deviations of the macroeconomic variables from the balanced growth path caused by aggregate uncertainty are linearized, while the nonlinearity of individual households' decisions with respect to idiosyncratic uncertainty on the balanced growth path is still preserved. Since this method uses the impulse responses after an MIT shock (*i.e.*, no aggregate uncertainty after a one-time shock) to recover the linearized dynamics of the original economy with aggregate uncertainty, in Appendix B.1, I recharacterize the equilibrium under the circumstance in which the economy is subject to deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$.

Another important aspect in solving the model is that the quantity variables in the economy inherit the stochastic trend, and thus, we need to detrend the equilibrium to make it stationary. In Appendix B.2, I detrend the quantity variables and define a stationary detrended equilibrium under deterministic paths of $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$. I then solve the detrended equilibrium using Auclert et al. (2019)'s method. Appendix B.3 briefly describes how the method works to solve the equilibrium. Once the detrended equilibrium is solved, we can recover the original equilibrium. Appendix B.4 discusses how to recover the statistics of the original equilibrium from the detrended equilibrium.

[19] The main contributors to the recent development include Auclert et al. (2019), Boppart et al. (2018), Ahn et al. (2018), Bayer and Luetticke (2020), Winberry (2018), and Reiter (2009).

IV Taking the Model to the Data

The goal of this paper is to study the stylized patterns of emerging market business cycles appearing in macro data through the lens of a model that incorporates the degree of household heterogeneity and consumption responses appearing in micro data. To achieve this goal, I employ both calibration and Bayesian estimation. First, I calibrate a subset of parameters to match the key empirical moments from the micro data. These micro moments include i) the estimates of the labor income process, which is the ultimate source of household heterogeneity in the model, and ii) the MPC estimates over the labor income deciles, which capture the degree of households' consumption responses.[20] Then, I estimate the rest of the parameters using Bayesian methods and macro data. The parameters governing the exogenous shock processes are estimated in this step. Through this Bayesian estimation, I run a horse race among different aggregate shocks to identify the main drivers of emerging market business cycles. Conventional candidates in the literature, including trend shocks and interest rate shocks, are included in the race. Note that this two-step estimation procedure (calibration in the first step, Bayesian estimation in the second step) is possible because the calibration in the first step is conducted by targeting moments on the balanced growth path, and the Bayesian-estimated parameters in the second step do not affect the balanced growth path of the economy.

A Calibration

The time unit in the model is meant to be one quarter. Table 1 reports the calibrated parameters, calibrated values, and a brief description of the target moments or sources of information used for the calibration. The parameters governing the labor income process are ρ_{e_1}, σ_{e_1}, and σ_{e_2}. These parameters are calibrated by applying Floden and Lindé (2001)'s method to the labor income data from ENAHO, as discussed in Appendix A.1 .

On the balanced growth path, r_t^a, r_t^b, and r_t are all equal to r^*. (See Appendix B.2.3 for details.) I calibrate r^* by matching the long-run average of the real lending rates in the data, 0.022. The real lending rate series are constructed by deflating the data series on quarterly nominal lending rates for foreign-currency-denominated assets in 1992Q1-2017Q1 from International Financial Statistics (IFS, hereafter) with the expected inflation

[20]There are also parameters that I calibrate either by matching the long-run average statistics from the macro data or by adopting commonly used values in the literature on emerging market business cycles, as we shall see in the following subsection.

Table 1: Calibrated Parameters for the Peruvian Economy

	Description	Value	Target / source
labor income process			
ρ_{e_1}	persistence of the AR(1) component	0.968	ENAHO
σ_{e_1}	S.D. of shocks to the AR(1) component	0.128	ENAHO
σ_{e_2}	S.D. of shocks to the *i.i.d.* component	0.470	ENAHO
long-run averages			
g^*	long-run average gross growth rate	1.004	$E[Y_t/Y_{t-1}]$
r^*	long-run average lending rate	0.022	IFS, U.S. CPI
ξ	long-run average spread	0.020	IFS, U.S. CPI
α	capital income share	0.385	$(K/Y)(r^*+\delta)/g^*$
δ	depreciation rate	0.014	$g^*(I/Y)/(K/Y)-(g^*-1)$
parameters from the literature			
γ	inverse of IES	2.000	Garcia-Cicco et al. (2010)
ω	inverse of labor supply elasticity	0.600	Garcia-Cicco et al. (2010)
κ	scale parameter of labor disutility	4.038	$L=1$ on the b.g.p
targeting MPCs over the labor income deciles & Aggregate Wealth			
β	discount factor	0.948	MPC estimates (from ENAHO) and TB/Y
χ_1	scale parameter of illiquid adj. cost	1.347	MPC estimates (from ENAHO) and TB/Y
χ_2	convexity parameter of illiquid adj. cost	1.496	MPC estimates (from ENAHO) and TB/Y
χ_0	non-zero denom. in illiquid adj. cost	0.010	

Notes: The time unit is one quarter. The abbreviation 'b.g.p' in the 'Target/source' column of parameter κ represents the balanced growth path of the equilibrium.

on U.S. CPIs.[21] [22]

[21]The expected inflation is constructed by taking the average of inflation rates in the current and past three quarters, following Neumeyer and Perri (2005) and Uribe and Yue (2006). Atkeson and Ohanian (2001) provide empirical evidence supporting this practice.

[22]In the literature on emerging market business cycles, interest rate series are often constructed by adding J.P. Morgan's EMBIG spreads of sovereign bonds with real interest rates of U.S. 3-month Treasury Bills. ('EMBIG interest rates' in this footnote). Instead, I construct real interest rates based on IFS data series ('IFS interest rates' in this footnote). I find that the EMBIG interest rates and the IFS interest rates are highly correlated (correlation 0.843), but their means are substantially different. In terms of the (nonannualized) quarterly rate, the mean of the EMBIG interest rates is 0.007, while the mean of the IFS interest rates is 0.022. Given that the long-run average trade-balance-to-output ratio in the model is targeted to its data counterpart, there is a one-to-one relationship between r^* and the ratio of net foreign asset position (NFA, hereafter) to output, $-D_t/Y_t$ on the balanced growth path through the following equation: $r^* = \frac{TB_t/Y_t}{D_t/Y_t} g^* + (g^*-1)$. (This equation comes from equation (B.65) in Appendix B.2.3.) Using this equation, I recover the value of r^* that gives the exact long-run average value of the Peruvian NFA-to-output-ratio in Milesi-Ferretti and Lane (2017)'s dataset. The value of this r^* is 0.025, which is much closer to the mean of the IFS rates than to the mean of the EMBIG rates. Based on this observation, I use IFS interest rates instead of EMBIG interest rates so that the model generates (D_t/Y_t) close to Milesi-Ferretti and Lane (2017)'s debt data on the balanced

Parameter ξ is calibrated such that the liquid asset return $(1-\xi)(1+r^*)$ is matched with the long-run average value of the real deposit rates in the data, 0.001. The real deposit rate series are constructed by deflating the data series on the quarterly nominal deposit rates for foreign-currency-denominated assets in 1992Q1-2017Q1 from IFS with the expected inflation on U.S. CPIs.

There are other parameters that I calibrate by matching the long-run average statistics from macro data. Parameter g^* is calibrated by matching the long-run average value of (Y_t/Y_{t-1}) in the quarterly national accounts in 1980-2018 from Banco Central de Reserva del Perú (BCRP, hereafter). Parameter α is calibrated by using the following equation on the balanced growth path: $r^* + \delta = \alpha g^*(Y_t/K_t)$.[23] Specifically, I compute the long-run average value of the capital-to-output ratio using the annual capital stock and output series in 1980-2017 from Feenstra, Inklaar, and Timmer (2015)'s Penn World Table (version 9.1). I transform the average annual capital-to-output ratio to a quarterly ratio by multiplying by four and obtain a value of 10.906. Parameter δ is calibrated using another equation on the balanced growth path: $\delta + g^* - 1 = g^* \frac{I_t/Y_t}{K_t/Y_t}$.[24] The long-run average investment-to-output ratio is computed using the quarterly national accounts in 1980-2018 from BCRP, and I obtain a value of 0.191.

Parameters γ and ω are assigned the values used in Garcia-Cicco et al. (2010), which are common in related business cycle studies. Parameter κ is calibrated such that aggregate labor supply is normalized to be one on the balanced growth path.

Given the parameter values assigned above, parameters β, χ_1, and χ_2 are calibrated by targeting the ten MPC estimates over the labor income deciles and the aggregate wealth of the economy (or, equivalently, the aggregate amount of households' asset holdings $A_t + B_t$) on the balanced growth path.[25] [26] Specifically, I implement this calibration by minimizing the following objective function J:

$$\begin{aligned} J =& w_{TB/Y}\left\{(TB/Y)_{model} - (TB/Y)_{data}\right\}^2 \\ &+ (1-w_{TB/Y})\left\{\sum_{j=1}^{10} w_{d_j}^{LY}(MPC_{d_j,model} - MPC_{d_j,data})^2\right\} \end{aligned}$$

in which $w_{TB/Y}$ denotes the weight on the first target (TB/Y), $(TB/Y)_{model}$ and $(TB/Y)_{data}$

growth path.

[23]This equation comes from (B.53) in Appendix B.2.3.

[24]This equation comes from (B.52) in Appendix B.2.3.

[25]As discussed in subsection III.A, χ_1 and χ_2 affect both MPCs and aggregate wealth.

[26]For χ_0, I assign an arbitrary small number, 0.01, as the sole purpose of including the term $\chi_0 X_{t-1}$ in the functional form of $\chi_t(v_{i,t}, a_{i,t-1})$ is to ensure that the denominator of $\left(\frac{v_{i,t}}{(1+r_t^a)a_{i,t-1}+\chi_0 X_{t-1}}\right)$ is nonzero.

denote the trade-balance-to-output ratio on the balanced growth path of the model and its long-run average value in the data, respectively, $w_{d_j}^{LY}$ denotes the share of labor income in the j-th labor income decile d_j in the model, $MPC_{d_j,model}$ is the model-generated MPC in decile d_j, and $MPC_{d_j,data}$ is the MPC estimate of decile d_j in the data.

In constructing the objective function J, I target the trade-balance-to-output ratio instead of the wealth-to-output ratio $(A_t + B_t)/Y_t$. I do so because there are no direct data on aggregate wealth, $A_t + B_t$. Given that there exist data on the capital-to-output ratio K_t/Y_t and trade-balance-to-output ratio TB_t/Y_t, however, the following long-run relationship among stock variables of the model disciplines what the correct amount of aggregate wealth is.

$$(K_t/X_t) + \left(\frac{\chi_t^{agg}}{1+r_{ss}-g_{ss}}\frac{1}{X_t}\right) = (A_t/X_t) + (B_t/X_t) + (D_t/X_t) \quad \text{on the b.g.p}$$

$$\Leftrightarrow (K_t/Y_t) + \left(\frac{\chi_t^{agg}}{1+r^*-g^*}\frac{1}{Y_t}\right) = (A_t/Y_t) + (B_t/Y_t) + (D_t/Y_t) \quad \text{on the b.g.p}$$

$$\Rightarrow (K_t/Y_t) - \frac{g^*}{1+r^*-g^*}(TB_t/Y_t) = (A_t/Y_t) + (B_t/Y_t) - \left(\frac{\chi_t^{agg}}{1+r^*-g^*}\frac{1}{Y_t}\right) \quad \text{on the b.g.p}$$

in which b.g.p denotes the balanced growth path.[27] In the last equation, I target the model-generated value of the right-hand-side toward the data counterpart on the left-hand-side. Such calibration can be achieved by targeting TB_t/Y_t only because in the step of calibrating α, the model's long-run average value of K_t/Y_t is already matched with the data. I compute the long-run average value of TB_t/Y_t using the quarterly national accounts in 1980-2018 from BCRP and obtain a value of 0.043.

After the calibration of β, χ_1, and χ_2, the model generates both a trade-balance-to-output ratio and an MPC graph over the labor income deciles that are quite similar to their data counterparts, despite the fact that I only use three parameters to target eleven moments. First, the model-generated trade-balance-to-output ratio on the balanced growth path is 0.042, and its data counterpart is 0.043. Second, the model-generated MPCs over the labor income deciles are plotted as a thick black solid line in Figure 2.[28] In this figure, the blue solid line with circle markers and the shaded area around the line represent the quarterly MPC estimates from ENAHO and their 95% confidence intervals, respec-

[27] The first line comes from equation (B.68), and the derivation of the third line from the second line comes from equation (B.65) in Appendix B.2.3.

[28] The model-generated MPCs of the labor income deciles are computed by simulating the consumption and income of 1,000,000 households over nine quarters, constructing year-over-year growth of consumption and income over two consecutive years, and applying to the simulated data the exactly same MPC estimation procedure applied to the actual data (ENAHO).

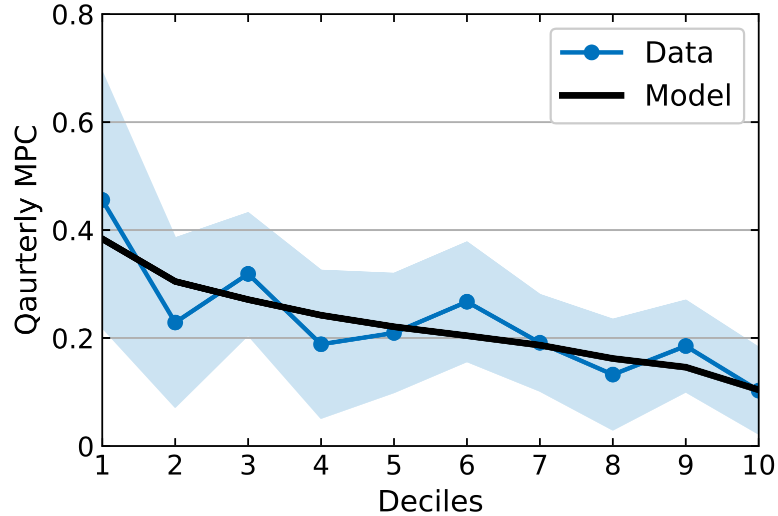


Figure 2: Quarterly MPCs in Peru: Data vs Model

Notes: Figure 2 plots the quarterly MPC estimates from ENAHO and their model counterparts under the calibration targeting the estimated Peruvian MPC graph.

tively.[29] As this figure shows, the MPC graph generated from the model closely tracks the MPC estimates from the data.

There is one additional noteworthy observation here. After the calibration, the aggregate wealth is saved almost entirely in the form of illiquid assets. On the balanced growth path, the aggregate amount of illiquid assets is 51 times greater than that of liquid assets. This outcome reflects how households optimize when they face a large spread between liquid asset and illiquid asset return rates: the latter (0.022) is approximately 22 times greater than the former (0.001) on the balanced growth path.

B Bayesian Estimation

I estimate model parameters ψ (the debt elasticity of interest rates in the international financial market), ϕ (the parameter governing the capital adjustment cost), θ_z and θ_g (the sensitivities of interest rates in the international financial market to stationary and nonstationary productivity shocks) as well as parameters governing aggregate shock processes ρ_z, σ_z, ρ_g, σ_g, ρ_μ, σ_μ, ρ_η, σ_η, ρ_ν, and σ_ν using Bayesian methods. Following previous studies that apply Bayesian methods to estimate representative-agent small open economy models using emerging market macro data, such as Garcia-Cicco et al. (2010) and Chang

[29]The blue solid line with circle markers and the shaded area around the line in Figure 2 are identical to those in Figure 1.

and Fernández (2013), I employ aggregate output, consumption, investment, and trade-balance-to-output ratio series for the Bayesian estimation. These macro data series are from the quarterly national accounts of BCRP in the period 1980-2018.

Two specific details are worth noting in determining the counterparts between the model and the data for the estimation. First, financial intermediation services $\zeta(1 + r_{t-1})B_{t-1}$ in the model must be designated either as final consumption or as intermediate consumption in national accounting. I assume that they are designated as final consumption. Under this assumption, the model counterpart of the final consumption in the national accounts, which I denote by C_t^{msd}, is determined as follows. [30]

$$C_t^{msd} := C_t + \xi(1 + r_{t-1})B_{t-1}.$$

If the financial intermediation services are instead designated as intermediate consumption, the model counterpart of output in the national accounts becomes $Y_t - \zeta(1 + r_{t-1})B_{t-1}$ (in which Y_t is gross value-added and $\zeta(1 + r_{t-1})B_{t-1}$ is intermediate consumption). However, because $\zeta(1 + r_{t-1})B_{t-1}$ is very small relative to C_t and Y_t in the model and does not fluctuate substantially in equilibrium, whether $\zeta(1 + r_{t-1})B_{t-1}$ is designated as final consumption or intermediate consumption has no meaningful effect on the results.[31]

Second, I use log output growth ($\Delta \log Y_t$), log consumption growth ($\Delta \log C_t^{msd}$), log investment growth ($\Delta \log I_t$), and the first difference of the trade-balance-to-output ratio ($\Delta TB_t/Y_t$) in taking the model to the data, as in Chang and Fernández (2013). Garcia-Cicco et al. (2010) use the same set of statistics except for using TB_t/Y_t instead of $\Delta TB_t/Y_t$. Both choices are acceptable from a statistical perspective, as neither of them inherits a trend in the data and the model. I choose $\Delta TB_t/Y_t$ over TB_t/Y_t because the countercyclicality of the trade balance, which is a common pattern for both emerging and developed economies, is better reflected in the estimation when $\Delta \log Y_t$ is correlated with $\Delta TB_t/Y_t$ rather than with TB_t/Y_t.

For the Bayesian estimation, I implement the Random Walk Metropolis-Hastings (RWMH) algorithm described in Herbst and Schorfheide (2015). Specifically, I construct the poste-

[30] Superscript *msd* abbreviates 'measured'.

[31] In real-world national accounting, financial intermediation services are labeled Financial Intermediation Services Indirectly Measured (FISIM) and measured as follows. First, 'reference rates' are determined according to specific rules such that they are located between lending rates and deposit rates. Second, FISIM is defined as the sum of the following two components: the amount of deposits multiplied by the spread between reference rates and deposit rates and the amount of loans multiplied by the spread between reference rates and lending rates. Third, the former component is designated as the final consumption of depositors, while the latter component is designated as the intermediate consumption of borrowers. Therefore, my assumption of assigning $\zeta(1 + r_{t-1})B_{t-1}$ to final consumption is equivalent to assuming that the statistical agency determines the reference rates as r_t.

rior distribution by sampling 500,000 draws through the RWMH algorithm and burning the initial 100,000 draws. A successful implementation of the algorithm requires i) a good variance-covariance matrix of the proposal distribution, which should be close to the variance-covariance matrix of the posterior distribution itself after scaling, and ii) the correct scaling factor for the matrix, which achieves an acceptance rate in the range 0.2-0.4. To this end, I run multiple preliminary stages of the RWMH algorithm and its variant before the main RWMH algorithm through which i) the draws of the chain move closer to the posterior mode, ii) the variance-covariance matrix of the proposal distribution is updated to become closer to the variance-covariance matrix of the posterior distribution after scaling, and iii) the scaling factor is updated to achieve the target acceptance rate of 0.27.[32]

I impose a fairly flat prior distribution, as reported in the left vertical panel of Table 2. Regarding the autocorrelation coefficients of the exogenous shock processes ρ_z, ρ_g, ρ_μ, ρ_η, and ρ_ν, I assume that they follow a beta distribution after being scaled by (1/0.99) in the prior. The scaling is to ensure that the autocorrelation coefficients do not exceed 0.99 under any posterior draw, as the precision of Auclert et al. (2019)'s computation method becomes compromised when the economy becomes too persistent.[33] I set the mean and standard deviation of the beta distribution as 0.5 and 0.2, respectively, for all the autocorrelation coefficients except ρ_g. For ρ_g, I use 0.3 as the mean of the beta distribution, reflecting the conventional view that trend shocks are transitory.[34]

The standard deviations of the exogenous shock processes, σ_z, σ_g, σ_μ, σ_η, and σ_ν, follow an inverse gamma distribution with a mean of 0.01 and a standard deviation of 0.02 in the prior distribution. For the parameter governing the capital adjustment cost, ϕ, I impose a gamma distribution with a mean of 15.0 and a standard deviation of 15.0 in the prior distribution. For the rest of the parameters, I impose uniform distributions.

The parameters in the last four rows of Table 2, σ_y^{me}, σ_c^{me}, σ_i^{me}, and σ_{tby}^{me} represent the standard deviations of the measurement errors for $\Delta \log Y_t$, $\Delta \log C_t^{msd}$, ΔI_t, and $\Delta TB_t/Y_t$, respectively. I allow these measurement errors to explain up to 6.25% of the variances of the observed variables.

The right vertical panel of Table 2 reports key statistics of the posterior distribution, including the mean, standard deviation, 5th percentile, and 95th percentile of the marginal

[32]The acceptance rate of the main RWMH algorithm is 0.271, which is close to the target.

[33]An acceptable degree of persistence depends on the length of the sequence used in Auclert et al. (2019)'s sequence-space approach. I set the sequence length as $T = 300$. (See Appendix B.3 for a brief description of how their sequence space approach works.) Auclert et al. (2019) report that their two-asset HANK model, which has an almost identical household block to my model, can be solved within an acceptable range of precision under an autocorrelation coeffieicnt of 0.99 for the stationary productivity shock.

[34]See Aguiar and Gopinath (2007) and Garcia-Cicco et al. (2010), for example.

Table 2: Prior and Posterior Distributions of the Bayesian Estimation

	Prior Distribution		Posterior Distribution		
	density	[meta1, meta2]	mean	S.D.	[0.05, 0.95]
ψ	Uniform	[0.000,2.000]	1.623	0.294	[1.051,1.975]
ϕ	Gamma	[15.000,15.000]	11.863	2.365	[8.073,15.854]
θ_z	Uniform	[0.000,2.000]	0.374	0.181	[0.091,0.692]
θ_g	Uniform	[0.000,2.000]	0.622	0.504	[0.035,1.651]
ρ_z	0.99·Beta	[0.500,0.200]	0.849	0.034	[0.794,0.905]
σ_z	Invgamma	[0.010,0.020]	0.017	0.001	[0.015,0.019]
ρ_g	0.99·Beta	[0.300,0.200]	0.880	0.105	[0.725,0.960]
σ_g	Invgamma	[0.010,0.020]	0.003	0.001	[0.002,0.004]
ρ_μ	0.99·Beta	[0.500,0.200]	0.484	0.198	[0.160,0.809]
σ_μ	Invgamma	[0.010,0.020]	0.005	0.002	[0.002,0.009]
ρ_η	0.99·Beta	[0.500,0.200]	0.835	0.147	[0.527,0.960]
σ_η	Invgamma	[0.010,0.020]	0.329	0.053	[0.243,0.418]
ρ_ν	0.99·Beta	[0.500,0.200]	0.451	0.129	[0.237,0.663]
σ_ν	Invgamma	[0.010,0.020]	0.036	0.008	[0.025,0.050]
σ_y^{me}	Uniform	[0.000,0.007]	0.006	0.000	[0.006,0.007]
σ_c^{me}	Uniform	[0.000,0.009]	0.009	0.000	[0.008,0.009]
σ_i^{me}	Uniform	[0.000,0.045]	0.044	0.001	[0.043,0.045]
σ_{tby}^{me}	Uniform	[0.000,0.004]	0.004	0.000	[0.004,0.004]

Notes: Estimation is based on the quarterly national accounts of Peru in the period 1980-2018. In the prior density column, '0.99 · Beta' means that the corresponding parameter multiplied by (1/0.99) follows a beta distribution. The column labeled '[meta1,meta2]' reports the meta parameters of the prior distributions. For a uniform distribution, [meta1,meta2] is [lower bound, upper bound]. For inverse gamma distribution and gamma distribution, [meta1,meta2] is [mean, standard deviation]. For '0.99 · Beta', [meta1,meta2] is [mean, standard deviation] of the beta distribution part. Posterior statistics are based on 500,000 posterior draws from the RWMH algorithm, of which the initial 100,000 draws are burned.

posterior distribution of each parameter. I highlight three notable features. First, the posterior means of σ_g and σ_μ are very small, implying that trend shocks and interest rate shocks might not play an important role in explaining emerging market business cycles in my model. Second, θ_g is weakly identified, which is understandable given that I do not employ interest rate series for the estimation.[35] As we shall see in sections V and VI, however, the key model statistics of this paper do not inherit the weak identification of θ_g because σ_g is very small. Third, the posterior mean of ρ_η is 0.835, which is as large as that of ρ_z. Moreover, the posterior mean of σ_η is 0.329, which is markedly large. The

[35] As in Aguiar and Gopinath (2007), Garcia-Cicco et al. (2010), and Chang and Fernández (2013), interest rate series are not used for the estimation because they are available only for substantially shorter time periods than other observable variables.

Table 3: Standard Deviations and Correlations: Model vs Data

		$\Delta \log Y_t$	$\Delta \log C_t^{msd}$	$\Delta \log I_t$	$\Delta(TB_t/Y_t)$
standard deviation					
	model	0.029	0.037	0.140	0.017
	data	0.027	0.036	0.179	0.017
contemporaneous correlation					
with $\Delta \log Y_t$	model		0.668	0.472	-0.229
	data		0.682	0.437	-0.346
with $\Delta(TB_t/Y_t)$	model		-0.325	-0.539	
	data		-0.318	-0.460	
with $\Delta \log C_t^{msd}$	model			-0.155	
	data			-0.158	
autocorrelation					
with lag 1	model	-0.035	-0.035	-0.153	0.010
	data	0.404	0.078	-0.304	0.023
with lag 2	model	-0.027	-0.052	-0.078	-0.090
	data	0.009	0.036	-0.094	-0.077
with lag 3	model	-0.020	-0.049	-0.046	-0.099
	data	-0.090	-0.112	0.026	-0.061

Notes: The model statistics are computed under each posterior draw, and the means over the posterior distribution are reported in this table.

combination of the high degree of persistence and the large standard deviation suggests that illiquidity shocks (η_t) might play an important role in accounting for the business cycles. Of course, we cannot compare which shocks are more important by simply comparing the autocorrelation coefficients and standard deviations because they hit different objects of different sizes in the economy. The variance decomposition in subsection VI.A provides a formal comparison of the relative importance of the shocks in explaining the business cycles.

C Model Performance

Table 3 compares key business cycle moments between the model and the data after the Bayesian estimation. This table shows that i) the Peruvian macro data exhibit the stylized patterns of emerging market business cycles, and ii) the model simulates these patterns quite well.

First, the standard deviation of output growth is 0.027 in the Peruvian data. This number is far beyond the average standard deviation of output growth in rich countries, 0.008, reported in Table 1.6 of Uribe and Schmitt-Grohé (2017). In other words, the Peruvian data exhibit the stylized pattern of emerging economies whereby output volatility is substantially greater than that of developed economies. The model simulates this pattern well by generating an output growth volatility of 0.029, which is similar to the data counterpart.

Second, the Peruvian data exhibit the stylized pattern of emerging economies whereby consumption is more volatile than output (excess consumption volatility): the standard deviation of consumption growth (0.036) is markedly greater than the standard deviation of output growth (0.027) in the Peruvian data. The model again simulates this stylized pattern well by generating a standard deviation of consumption growth (0.037) that is substantially greater than the standard deviation of output growth (0.029).

In addition to these well-known stylized patterns of emerging market business cycles, the model also closely matches other business cycle moments reported in Table 3, including the standard deviations of other variables, contemporaneous correlations, and autocorrelations.[36] In Appendix C, I further compare the cross-autocorrelograms between the model and the data and find that the model again closely mimics the data.

I highlight one more moment in Table 3 for later discussion, although it has received less attention in the literature to date. The correlation between consumption growth and investment growth is substantially lower than one in the Peruvian data.[37] Moreover, low correlation between consumption growth and investment growth is not an abnormal phenomenon of the Peruvian data: I find that the correlation of emerging countries is 0.189 on average and that of developed countries is 0.278 on average.[38] The model successfully simulates this pattern by generating a correlation (-0.158) that is substantially lower than one.

[36] One exception is the autocorrelation of $\Delta \log Y_t$ with a one-quarter lag: the model yields -0.035, which is noticeably smaller than the data counterpart, 0.404. However, this discrepancy quickly dissipates from a two-quarter lag forward. Given that this discrepancy survives only one quarter and that output growth variations almost entirely come from stationary productivity shocks, as we shall see in subsection VI.A, it is likely that replacing the conventional AR(1) process of stationary productivity shocks with an ARMA(1,1) process can fix this discrepancy. I do not impose this unconventional assumption, however, because the model aims to minimize changes from the conventional representative-agent models other than the heterogeneous household block with high MPCs.

[37] It is indeed negative in the Peruvian data, but as we shall see in the later discussion, what matters in this paper is not its being negative but being substantially less than one.

[38] In computing the average correlation for emerging countries and developed countries, I use the quarterly macro data series and country categorization used for the business cycle statistics in the first chapter of Uribe and Schmitt-Grohé (2017). From the data set, sample countries are selected if all five data series of output, investment, exports, imports, and consumption are available for at least twenty years. After the sample selection, 16 emerging countries and 17 rich countries remain in the sample. In averaging the correlation over multiple countries, I use population weights.

V Counterfactual Experiment

To what extent do high-MPC households (or, more precisely, the environment that makes households exhibit high MPCs) contribute to the large consumption volatility of emerging economies? To answer this question, I conduct a counterfactual experiment in which I replace the Peruvian households with those exhibiting the U.S. MPCs. I then examine whether the phenomenon of excess consumption volatility survives.

To this end, I recalibrate the parameters that I use to target the Peruvian MPCs in the baseline calibration, including the parameters governing the adjustment cost for illiquid assets (χ_1 and χ_2) and the time discount factor (β). These parameters are recalibrated by targeting the ten MPC estimates of the U.S. labor income deciles from the PSID and the Peruvian trade-balance-to-output ratio.[39] As discussed in subsection IV.A, targeting the trade-balance-to-output ratio disciplines the model to have the correct amount of aggregate wealth on the balanced growth path. I target the Peruvian (not U.S.) trade-balance-to-output ratio because the recalibration aims to minimize changes in the economy other than households' MPCs. In targeting the U.S. MPC estimates from the PSID, I face a frequency mismatch problem between the model and the data: the PSID provides annual income and consumption, while one period is set equal to one quarter in the model. This frequency mismatch problem is addressed as follows: I simulate annual consumption and income series by aggregating model-generated quarterly consumption and income series over every four quarters and then apply to the simulated annual data the same MPC estimation procedure applied to the PSID data.[40] By doing so, I directly target the annual MPC estimates from the PSID rather than targeting the quarterized estimates according to Auclert (2019)'s model-free frequency conversion formula presented in section II.

Table 4 reports the values of the recalibrated parameters. The value of χ_1 (0.246) under the recalibration is markedly lower than the value (1.347) under the baseline calibration

[39]Although χ_1, χ_2, and β are important determinants of the MPCs in the model, there are other parameters that also affect MPCs. Such parameters include the difference in return rates between liquid and illiquid assets ξ and the parameters governing the labor productivity process ρ_{e_1}, σ_{e_1}, and σ_{e_2}. In Appendix D, I run an alternative counterfactual experiment in which these parameters are also recalibrated. Specifically, I recalibrate ξ, ρ_{e_1}, σ_{e_1}, and σ_{e_2} using relevant U.S. data first and then recalibrate χ_1, χ_2, and β by targeting the U.S. MPC estimates and the Peruvian trade-balance-to-output ratio. I find that the results do not change in any meaningful way.

[40]Specifically, the PSID provides two-year-over-two-year growth of annual income and consumption because the survey is conducted biannually, and each wave provides only one annual consumption and income. To create the same data structure, I simulate the consumption and income of 1,000,000 households over twenty quarters, convert the twenty-quarter quarterly series into five-year annual series by aggregating them over every four quarters, and construct two consecutive two-year-over-two-year growth rates of annual income and consumption. I then apply the same estimation procedure used in the PSID to the simulated data.

Table 4: Recalibrated Parameters for the Counterfactual Economy

	Description	Value	Target / source
targeting MPCs over the labor income deciles & Aggregate Wealth			
β	discount factor	0.968	MPC estimates (from PSID) and Peruvian TB/Y
χ_1	scale parameter of illiquid adj. cost	0.246	
χ_2	convexity parameter of illiquid adj. cost	1.366	

in Table 1. This is because the U.S. MPC estimates discipline the model to exhibit lower MPCs than the Peruvian MPC estimates do. As discussed in subsection III.A, a lower value of χ_1 weakens financial frictions and thus decreases the MPCs of households. On the other hand, the value of β (0.968) under the recalibration is noticeably greater than the value (0.948) under the baseline calibration. This is because households save less under the lower value of χ_1 as discussed in subsection III.A, and this weaker saving due to the lower value of χ_1 needs to be compensated by a higher value of β to match the correct amount of aggregate wealth in the counterfactual economy.

After the recalibration, both the model-generated trade-balance-to-output ratio and the MPC graph over the labor income deciles closely match their data counterparts in the counterfactual economy. The model-generated trade-balance-to-output ratio on the balanced growth path is 0.041, and its data counterpart is 0.043. The model counterparts of the U.S. MPC estimates are plotted as a thick black dashed line in Figure 3. In this figure, the red dashed line with square markers and the shaded area around the line represent the the annual MPC estimates from the PSID and their 95% confidence intervals, respectively.[41] As this figure shows, the model-generated MPC graph closely tracks the estimates from the PSID.

Referring back to the problem of time-frame inconsistency under the model-free frequency conversion (1) in section II, we can now compare Peruvian and U.S. MPCs without this problem by using the model. Figure 4 compares the model-predicted quarterly MPCs between the baseline economy, which is calibrated by targeting the quarterly Peruvian MPC estimates, and the counterfactual economy, which is calibrated by targeting the annual U.S. MPC estimates. Two important observations are made from the comparison between Figure 4 and Figure 1. First, the MPC gap between Peru and the U.S. predicted by the model in Figure 4 is narrower than that predicted by the model-free frequency conversion (1) in Figure 1. Second, despite this tendency, the model still predicts a substantial MPC gap between Peru and the U.S. In Figure 4, the population-weighted average of the

[41] The red dashed line with square markers and the shaded area around the line in Figure 3 are different from those in Figure 1 because the latter is the quarterized version of the former according to equation (1).

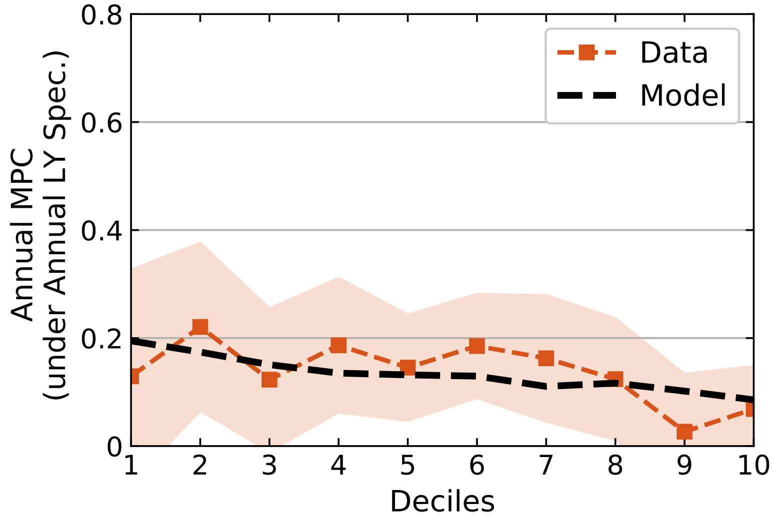


Figure 3: Annual MPCs in U.S.: Data vs Model

Notes: Figure 3 plots the annual MPC estimates from the PSID and their model counterparts under the calibration targeting the estimated U.S. MPC graph. In computing the model counterparts of the estimates, I simulate annual consumption and income series by aggregating model-generated quarterly consumption and income series over every four quarters and then apply to the simulated annual data the same MPC estimation procedure applied to the PSID data.

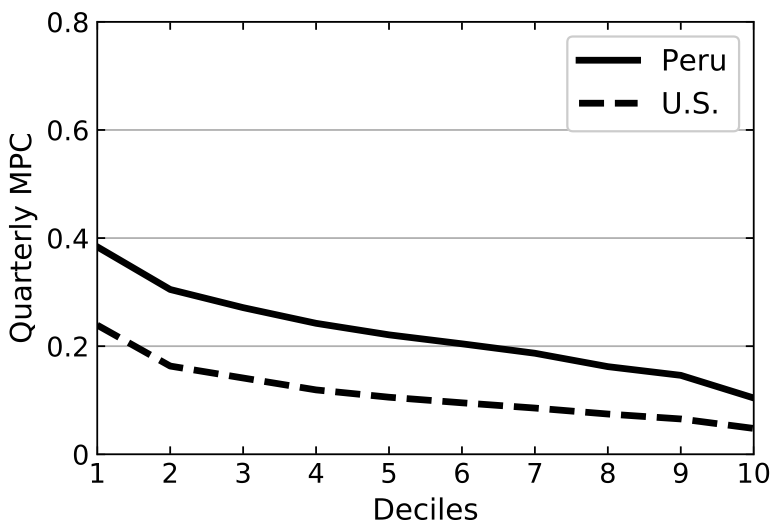


Figure 4: Model-Predicted Quarterly MPCs: Peru and U.S.

Notes: Figure 4 compares the model-predicted quarterly MPCs between the baseline economy, which is calibrated by targeting the quarterly Peruvian MPC estimates, and the counterfactual economy, which is calibrated by targeting the annual U.S. MPC estimates.

Table 5: The Absence of Excess Consumption Volatility in the Counterfactual Economy

	$\sigma(\Delta \log Y_t)$	$\sigma(\Delta \log C_t^{msd})$	$\frac{\sigma(\Delta \log C_t^{msd})}{\sigma(\Delta \log Y_t)}$
Baseline	0.029	0.037	1.283
	(0.002)	(0.002)	(0.075)
Counterfactual	0.028	0.027	0.963
	(0.002)	(0.002)	(0.069)

Notes: The statistics are computed under each posterior draw, and their means and standard deviations over the posterior distribution are reported in this table. The numbers in parentheses are the posterior standard deviations.

Peruvian quarterly MPCs is 0.223, which is 2.0 times greater than that of the U.S. quarterly MPCs, 0.114. In terms of income-weighted averages, the average Peruvian quarterly MPC is 0.175, which is 2.1 times greater than the average U.S. MPC, 0.084.

Figure 4 suggests that the MPCs of Peruvian households are substantially higher than the MPCs of U.S. households when we interpret the MPC estimation results through the lens of the model. Now, we are ready to examine the business cycle implications of this MPC gap.

Table 5 compares the output growth volatility, consumption growth volatility, and the ratio of the two between the baseline economy and the counterfactual economy. As the first column shows, the output growth volatility of the counterfactual economy is similar to that of the baseline economy. In the model, output is solely determined by the firms' Cobb-Douglas production, $Y_t = z_t K_{t-1}^{\alpha} (X_t L_t)^{1-\alpha}$. Because K_{t-1} is a slow-moving variable and L_t is determined by z_t, X_t, and K_{t-1} (through the labor union's labor supply decision (6) and firms' hiring decision (B.14)), the processes of z_t and g_t almost entirely determine output volatility. Given this feature of the model, it is not surprising that output volatility is similar between the two economies.

Regarding consumption volatility, on the other hand, there is a substantial difference between the two economies. As reported in the second column of Table 5, the standard deviation of consumption growth is 0.027 in the counterfactual economy, which is 25.8% lower than that of the baseline economy, 0.037. As a consequence, the ratio between the consumption volatility and output volatility, $\frac{\sigma(\Delta \log C_t^{msd})}{\sigma(\Delta \log Y_t)}$, falls from 1.283 in the baseline economy to 0.963 in the counterfactual economy. In other words, the stylized pattern of emerging economies whereby consumption is more volatile than output, namely, excess consumption volatility, disappears once Peruvian households are counterfactually replaced with those with U.S. MPCs. This result strongly suggests that the high-MPC households in the Peruvian economy generate excess consumption volatility.

VI Driving Mechanisms

Through which mechanisms do the high-MPC households in emerging economies contribute to the large aggregate consumption volatility? To answer this question, this section conducts three decomposition exercises: variance decomposition, variance change decomposition, and consumption response decomposition.

A Variance Decomposition

I begin by decomposing the variances of observable variables into the variances generated by each aggregate shock. Table 6 reports the result of this variance decomposition. To understand the variance decomposition result, in Figure 5, I plot the impulse responses of output (Y_t), consumption (C_t), investment (I_t), and the trade-balance-to-output ratio (TB_t/Y_t) in terms of their deviations from the balanced growth path after each one-standard-deviation aggregate shock.[42] [43]

Table 6 shows that output growth variations are almost entirely driven by stationary productivity shocks. To generate large variations in output growth (or equivalently, the first difference in log output), a shock should generate a large response on impact because an abrupt output change is needed. As the (1,3)-th, (1,4)-th, and (1,5)-th panels of Figure 5 suggest, interest rate shocks, illiquidity shocks, and investment shocks cannot generate any large output response on impact. Given this feature of the model, the actual candidates for output growth variations are stationary productivity shocks and trend shocks.

In the model of Aguiar and Gopinath (2007), representative households' strong consumption response to a trend shock on impact that is substantially stronger than the output response on impact is the key feature of the model that enables it to generate the stylized patterns of emerging market business cycles, including excess consumption volatility and a countercyclical trade balance. Unlike this model, however, my model does not exhibit this feature. As seen from the (2,2)-th panel of Figure 5, the consumption response to a trend shock is not much greater than the output response on impact. Instead, the consumption response grows gradually in the subsequent periods as the output response does.[44] As a result, the ability of trend shocks to account for emerging market business

[42] In the figures plotting the impulse responses of consumption, such as Figure 5, I plot the impulse responses of C, not C^{msd}. The differences between the impulse responses of C and those of C^{msd} are negligibly small.

[43] Appendix E presents impulse responses for a more comprehensive set of model variables. In particular, Appendix E.1 presents the impulse responses in the baseline economy with 90-percent credible bands over the posterior distribution, and Appendix E.2 compares the impulse responses between the baseline economy and the counterfactual economy.

[44] In the next section, I examine why trend shocks do not generate a strong consumption response in my

Table 6: Variance Decomposition

	$\Delta \log Y_t$	$\Delta \log C_t^{msd}$	$\Delta \log I_t$	$\Delta(TB_t/Y_t)$
stationary productivity shock (z_t)	0.961	0.423	0.228	0.075
	(0.019)	(0.051)	(0.048)	(0.040)
trend shock (g_t)	0.034	0.023	0.091	0.230
	(0.019)	(0.011)	(0.041)	(0.103)
interest rate shock (μ_t)	0.000	0.002	0.006	0.034
	(0.000)	(0.002)	(0.006)	(0.032)
illiquidity shock (η_t)	0.003	0.423	0.310	0.144
	(0.001)	(0.054)	(0.050)	(0.040)
investment shock (ν_t)	0.002	0.130	0.365	0.518
	(0.001)	(0.046)	(0.066)	(0.091)
total	1.000	1.000	1.000	1.000

Notes: The statistics of decomposed shares are computed under each posterior draw, and their means and standard deviations over the posterior distribution are reported in this table. The numbers in parentheses are the posterior standard deviations.

cycles is significantly limited in my model, and the model assigns all the explanatory power for the output growth variations to stationary productivity shocks.

Regarding consumption growth variations, Table 6 shows that most of the variations are explained in equal parts by stationary productivity shocks and illiquidity shocks. To understand why illiquidity shocks are assigned a sizable explanatory power for consumption growth variations, we need to focus on the correlation between consumption growth and investment growth. As the (2,1)-th, (3,1)-th, (2,2)-th, (3,2)-th, (2,3)-th, and (3,3)-th panels of Figure 5 suggest, stationary productivity shocks, trend shocks, and interest rate shocks generate a strongly positive correlation between consumption growth and investment growth because the impact effects of these shocks on consumption and investment are in the same direction. However, as Table 3 shows, the correlation is substantially lower than one in the data. Provided that stationary productivity shocks play a very large role in generating output growth variations and thus also generate a sizable amount of consumption growth variations that are strongly positively correlated with investment growth variations, the model seeks a shock that can generate a negative correlation between consumption growth and investment growth. As the (2,4)-th, (3,4)-th, (2,5)-th, and (3,5)-th panels of Figure 5 suggest, illiquidity shocks and investment shocks can generate a strong negative correlation. Between the two shocks, the model assigns a

model.

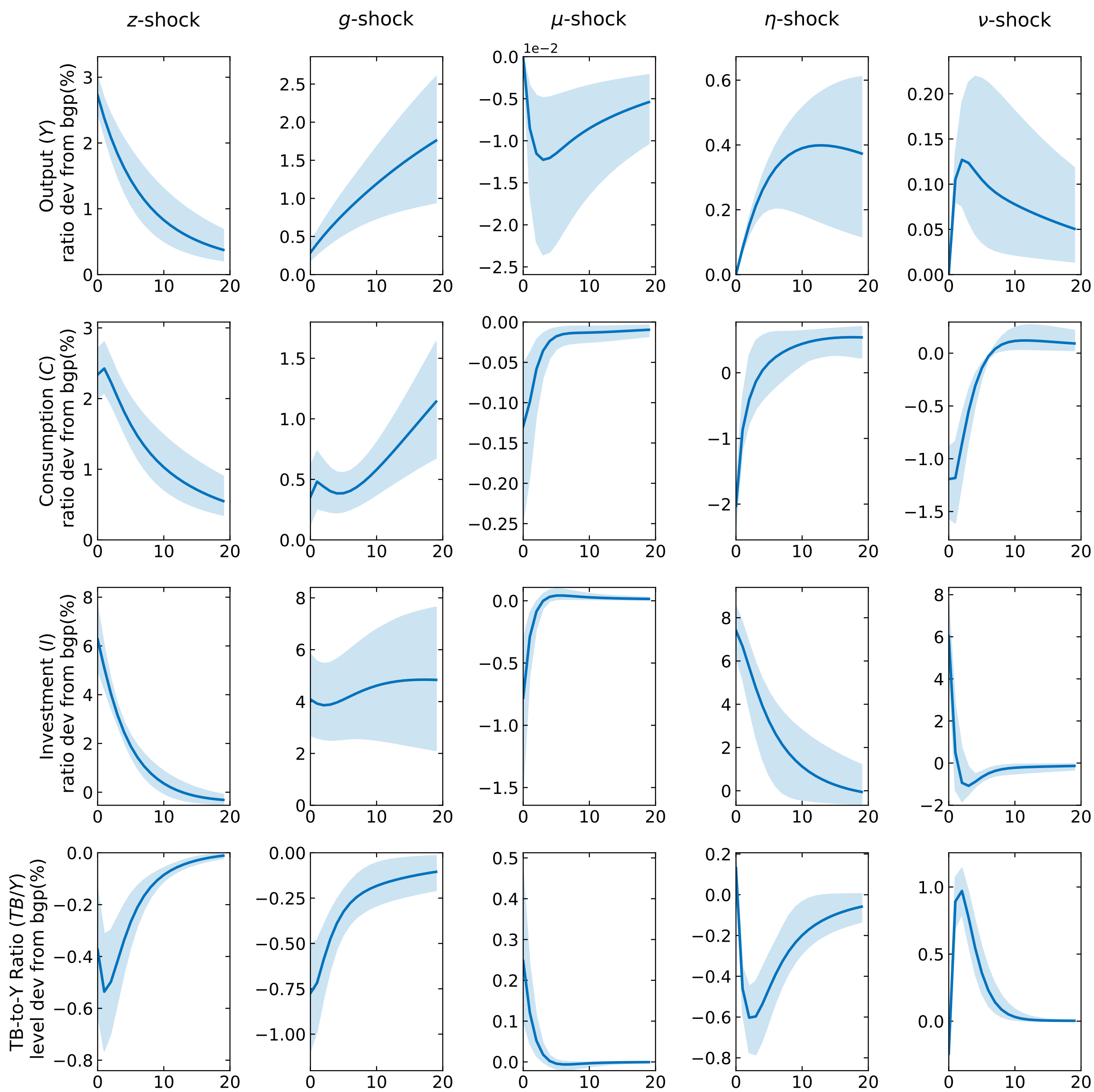


Figure 5: Impulse Responses of Output (Y), Consumption (C), Investment (I), and the Trade-Balance-to-Output Ratio (TB/Y) to 1 S.D. Shocks

Notes: In each plot, the blue solid line represents the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock. The unit on the y-axis in the first three rows is 'ratio dev from bgp(%)', which represents the deviation divided by the value on the balanced growth path, expressed in percent. The unit on the y-axis in the last row is 'level dev from bgp(%)', which represents the deviation itself, expressed in percent. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure. The shaded area in each plot represents 90% credible bands over the posterior distribution.

greater explanatory power for the consumption growth variations to illiquidity shocks.[45]

Illiquidity shocks generate a strong negative correlation between consumption growth and investment growth for the following reasons. When the degree of illiquidity increases, households increase saving, as discussed in III.A. As a result, aggregate consumption decreases while aggregate wealth increases. Since the aggregate wealth is almost entirely saved in illiquid assets, which are firms' shares, the greater amount of illiquid assets should be explained by a greater amount of investment from the perspective of firms.[46] As a consequence, aggregate consumption plunges while investment jumps in response to a positive illiquidity shock.

Table 6 also reports the variance decomposition of investment growth and the first difference in the trade-banace-to-output ratio. More than 65 percent of the investment growth variations are explained in equal parts by investment shocks and illiquidity shocks, and stationary productivity shocks also contribute to the variations to a lesser extent. More than half of the variations of the first difference in the trade-balanace-to-output-ratio are explained by investment shocks, and trend shocks and illiquidity shocks also contribute to the variations to a lesser extent.

B Variance Change Decomposition

In subsection VI.A, I conduct the variance decomposition for the baseline economy. Once I implement the same variance decomposition for the counterfactual economy calibrated to the U.S. MPCs in section V, I can examine which shock is responsible for the variance change between the two economies. Specifically, I decompose the variance changes from the baseline economy to the counterfactual economy into the changes generated by each shock as follows. Let $V(\Delta \log C_t^{msd})^{Base}$ be the variance of consumption

[45] Between the two shocks that generate a negative correlation between consumption growth and investment growth, illiquidity shocks generate consumption growth variations (relative to investment growth variations) more intensely than investment shocks. In terms of the ratio between the absolute size of the impact effect on consumption and that on investment, illiquidity shocks yield approximately 1/3.5, while investment shocks yield approximately 1/5. Given this feature of the model, if we Bayesian estimate an advanced economy that features low consumption volatility and a low correlation between consumption growth and investment growth in its macro data, a sizable fraction of consumption variations might be captured by investment shocks instead of illiquidity shocks. (As discussed in footnote 38, advanced economies also tend to have a low correlation between consumption growth and investment growth.)

[46] The economic mechanism behind this investment increase is as follows. As a result of households' increased demand for illiquid assets in response to illiquidity shocks, illiquid asset price jumps on impact, and then gradually returns to its value on the balanced growth path. Therefore, illiquid asset return jumps on impact, plunges to a negative value in period 1 (due to the asset price jump in period 0), and then gradually returns to its value on the balanced growth path. Because firms discount profits with illiquid asset returns, and their investment decision in period t is directly affected by the illiquid asset return in period $t+1$, investment jumps on impact, and gradually returns to its value on the balanced growth path. See impulse responses to illiquidity shocks in Figure E.4 for the graphical illustration of the mechanism.

Table 7: Variance Change Decomposition
(from Baseline to Counterfactual)

	$\Delta \log Y_t$	$\Delta \log C_t^{msd}$
stationary productivity shock (z_t)	-0.012	-0.207
	(0.000)	(0.022)
trend shock (g_t)	-0.011	0.053
	(0.008)	(0.027)
interest rate shock (μ_t)	0.000	-0.001
	(0.000)	(0.001)
illiquidity shock (η_t)	-0.001	-0.245
	(0.001)	(0.039)
investment shock (ν_t)	-0.000	-0.049
	(0.000)	(0.020)
variance change (in ratio)	-0.024	-0.449
	(0.008)	(0.042)

Notes: The last row reports the fraction of [(variance change from the baseline economy to the counterfactual economy) / (variance in the baseline economy)]. The first five rows report the fraction of [(variance change generated by each shock) / (variance in the baseline economy)], in which the denominator is the variance generated by all shocks (*i.e.*, the same denominator used in the fraction reported in the last row.) By construction, the last row is the sum of the first five rows. The statistics are computed under each posterior draw, and their means and standard deviations over the posterior distribution are reported in this table. The numbers in parentheses are the posterior standard deviations.

growth and $V(\Delta \log C_t^{msd})^{Base}_{shock}$, $shock \in \{z, g, \mu, \eta, \nu\}$ be the variances decomposed by each shock in the baseline economy. Similarly, let $V(\Delta \log C_t^{msd})^{Counter}$ be the variance of consumption growth, and $V(\Delta \log C_t^{msd})^{Counter}_{shock}$, $shock \in \{z, g, \mu, \eta, \nu\}$ be the variances decomposed by each shock in the counterfactual economy. The variance change (in ratio terms) is decomposed according to the following equation.

$$\frac{V(\Delta \log C_t^{msd})^{Counter} - V(\Delta \log C_t^{msd})^{Base}}{V(\Delta \log C_t^{msd})^{Base}} = \sum_{shock \in \{z,g,\mu,\eta,\nu\}} \frac{V(\Delta \log C_t^{msd})^{Counter}_{shock} - V(\Delta \log C_t^{msd})^{Base}_{shock}}{V(\Delta \log C_t^{msd})^{Base}}.$$

In the same way, I also decompose the variance change of output growth.

Table 7 reports the result of this variance change decomposition. As reported in the bottom row of the table, the consumption growth variance decreases by 44.9 percent when Peruvian households are replaced with those exhibiting U.S. MPCs. Of the 44.9

percent decrease in the consumption growth variance, a 20.7 percent decrease comes from the variance change generated by stationary productivity shocks, and a 24.5 percent decrease comes from the variance change generated by illiquidity shocks. This result shows that consumption volatility substantially decreases in the counterfactual economy because both stationary productivity shocks and illiquidity shocks generate substantially less consumption variation.

Table 7 also reports the variance change decomposition of output growth. The output growth variance decreases only by 2.5 percent from the baseline economy to the counterfactual economy. This small variance change comes from the variance changes caused by stationary productivity shocks and trend shocks.

C Consumption Response Decomposition

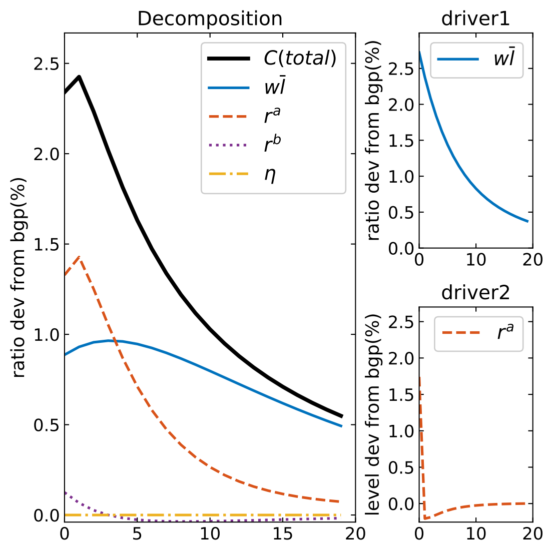
Ultimately, consumption is determined by households after they observe the paths of variables that are relevant for their optimization, $\{w_t\bar{l}_t, r_t^a, r_t^b, \eta_t\}_{t=0}^{\infty}$.[47] I name these variables 'drivers'. To understand economic reasons why both stationary productivity shocks and illiquidity shocks generate substantially more consumption variation in the baseline economy than they do in the counterfactual economy, I decompose households' consumption response to these shocks into the responses to each driver.

Figure 6a presents the decomposition of the consumption response with respect to stationary productivity shocks in the baseline economy. This figure consists of three subplots. The largest subplot on the left side labeled 'Decomposition' presents the total consumption response to a stationary productivity shock in the baseline economy, as well as the decomposed consumption responses to each driver. This subplot shows that the consumption response is mainly driven by two drivers: $w_t\bar{l}_t$ and r_t^a. The other small subplots on the right side, which are labeled 'driver1' and 'driver2', plot the equilibrium paths of the two drivers after the shock. The first driver, the labor income per idiosyncratic labor productivity $w_t\bar{l}_t$, jumps on impact and then gradually returns to zero. The second driver, the return rate on illiquid assets r_t^a, also jumps on impact, but then it suddenly falls below zero in period 1 and gradually returns to zero.[48]

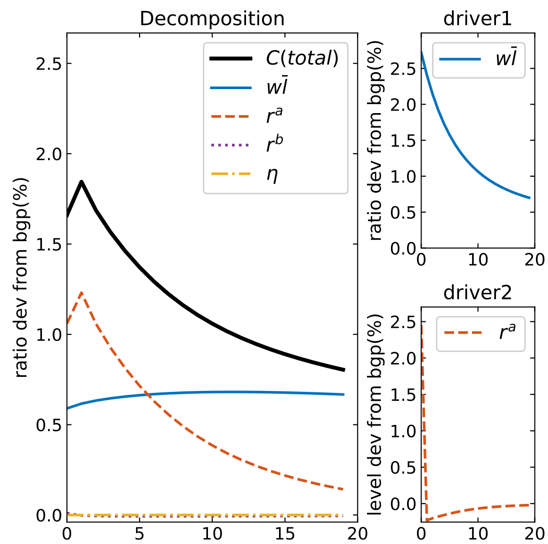
Figure 6b presents the same consumption response decomposition with respect to stationary productivity shocks but in the counterfactual economy. Three important observations are made from the comparison between Figure 6a and Figure 6b. First, the total

[47] In other words, $\{w_t\bar{l}_t, r_t^a, r_t^b, \eta_t\}_{t=0}^{\infty}$ are exogenous variables in the partial equilibrium of the households' optimization, although $\{w_t\bar{l}_t, r_t^a, r_t^b\}_{t=0}^{\infty}$ are endogenous in general equilibrium.

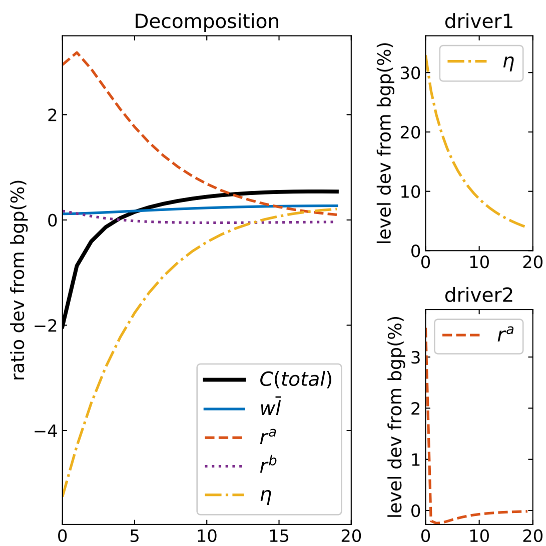
[48] The jump of the illiquid asset return on impact is due to the jump in the illiquid asset price, which reflects the high capital returns in the future. From period 1 onward, the illiquid asset return r_t^a is equalized with the interest rate in the international financial market r_{t-1} by firms' optimality condition (10).



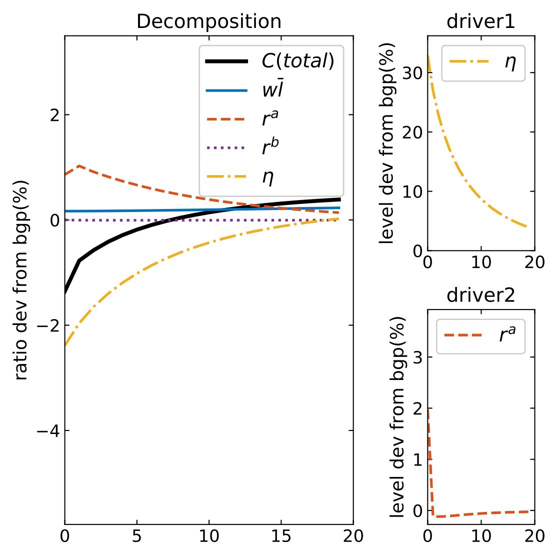
(a) Baseline, z-shock



(b) Counterfactual, z-shock



(c) Baseline, η-shock



(d) Counterfactual, η-shock

Figure 6: Decomposition of the Consumption Responses to the z-shock and η-shock

Notes: Panels 6a, 6b, 6c, and 6d present the consumption response decomposition with respect to stationary productivity shocks (z) and illiquidity shocks (η) in the baseline economy and the counterfactual economy, respectively. Each panel consists of three subplots, where the large subplot on the left shows the total consumption response as well as decomposed consumption responses to each driver of $\{w_t\bar{l}_t, r_t^a, r_t^b, \eta_t\}_{t=0}^{\infty}$, and the other two small subplots on the right show the equilibrium paths of the two main drivers after the shock. The consumption responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.

consumption response in the counterfactual economy is substantially weaker than that in the baseline economy. Second, the weaker total consumption response in the counterfactual economy is due to the weaker consumption responses to both $w_t\bar{l}_t$ and r_t^a. Third, the consumption responses to these drivers are weaker in the counterfactual economy despite the fact that the equilibrium paths of the drivers after the shock are similar in the two economies.[49] These observations reveal the first main mechanism through which high-MPC households in the baseline economy contribute to the large consumption volatility: their consumption responds to individual resource fluctuations (mainly generated by the two drivers, $w_t\bar{l}_t$ and r_t^a) far more strongly than the consumption of the counterfactual households exhibiting U.S. MPCs.[50]

Figure 6c shows the result of the consumption response decomposition with respect to illiquidity shocks in the baseline economy. In response to a positive illiquidity shock, total consumption plunges on impact and then gradually recovers. The plunge in total consumption is mainly driven by the direct effect of illiquidity shocks rather than by indirect effects through other drivers. In the model, the direct effect of illiquidity shocks is realized as follows. When the degree of illiquidity increases, it becomes more expensive for households to liquidate their illiquid assets. For households facing bad idiosyncratic income shocks at the moment of the illiquidity shock, they need to cash out their illiquid assets to smooth their consumption, but it is more difficult to do so because the assets are more illiquid. Therefore, these households fail to smooth consumption more seriously, and as a result, their consumption plunges. For households facing good idiosyncratic income shocks at the moment of the illiquidity shock, they recognize that it will be more difficult to cash out their illiquid assets for a while. Therefore, they prepare themselves for situations in which bad idiosyncratic income shocks are realized in a near future by reducing consumption substantially and accumulating more buffer stocks. In other words, these households' precautionary-saving motive is significantly enhanced.

There is one more noteworthy observation in Figure 6c. The illiquid asset return significantly jumps on impact in response to a positive illiquidity shock and generates a

[49]The initial jump of r_t^a is rather larger in the counterfactual economy, which works against the weaker consumption response.

[50]Households in the baseline economy exhibit stronger consumption responses to individual resource fluctuations than those in the counterfactual economy for the same reason why the former households exhibit higher MPCs than the latter. Households exhibit higher MPCs in the baseline economy because both the degree of consumption smoothing failure of households who face bad idiosyncratic income shocks and the precautionary-saving motive of households who face good idiosyncratic income shocks are substantially stronger, and therefore, a positive transitory income shock relaxes them substantially more in the baseline economy than in the counterfactual economy. For the same reason, households' resource fluctuations in response to a positive transitory productivity shock relax the degree of consumption smoothing failure and the precautionary-saving motive substantially more in the baseline economy than in the counterfactual economy (despite the fact that households face similar degrees of resource fluctuations in the two economies), and therefore, consumption responds more strongly in the baseline economy.

substantial positive consumption response.[51] However, the consumption plunge due to the direct effect of the illiquidity shock outweighs the consumption increase in response to illiquid asset returns by a sizable margin. As a result, the total consumption response is negative on impact and over several subsequent periods.

Figure 6d presents the same consumption response decomposition with respect to illiquidity shocks but in the counterfactual economy. Two important observations are made from the comparison between Figure 6c and Figure 6d. First, the total consumption response with respect to illiquidity shocks is significantly weaker in the counterfactual economy. Second, the weaker total consumption response is driven by the weaker direct effect of illiquidity shocks on consumption.[52] In the counterfactual economy, households face a much smaller adjustment cost for illiquid assets. Therefore, the same degree of increase in η_t distorts households' consumption-saving decisions far more mildly in the counterfactual economy than in the baseline economy. These observations reveal the second main channel through which high-MPC households in the baseline economy contribute to aggregate consumption volatility: high-MPC households' consumption plunges when assets become more illiquid because some of them experience aggravated consumption smoothing failure and others come to have an enhanced precautionary-saving motive.

VII Inspecting Conventional Theories

In section VI, I examine how excess consumption volatility is realized in the model through the consumption-saving behavior of high-MPC households in emerging economies. I find that the main mechanisms of this model are quite different from the existing theories on excess consumption volatility based on representative-agent models. In this section, I examine the extent to which the driving mechanisms of the conventional theories are dampened in my model and discuss the economic reasons.

A Strong Consumption Response to Trend Shocks

The first well-accepted theory in the literature is the one developed by Aguiar and Gopinath (2007). They argue that households' consumption responds strongly to trend

[51] The initial jump of the illiquid asset return is due to the jump of the illiquid asset price. The illiquid asset price jumps because the stronger degree of illiquidity increases the aggregate adjustment cost χ_t^{agg}, which appears as a part of firms' profit.

[52] The competing force generated by the response to illiquid asset returns is also weaker in the counterfactual economy. However, the direct effect of illiquidity shocks is more significantly weakened, and as a consequence, the total consumption response is weakened in the counterfactual economy.

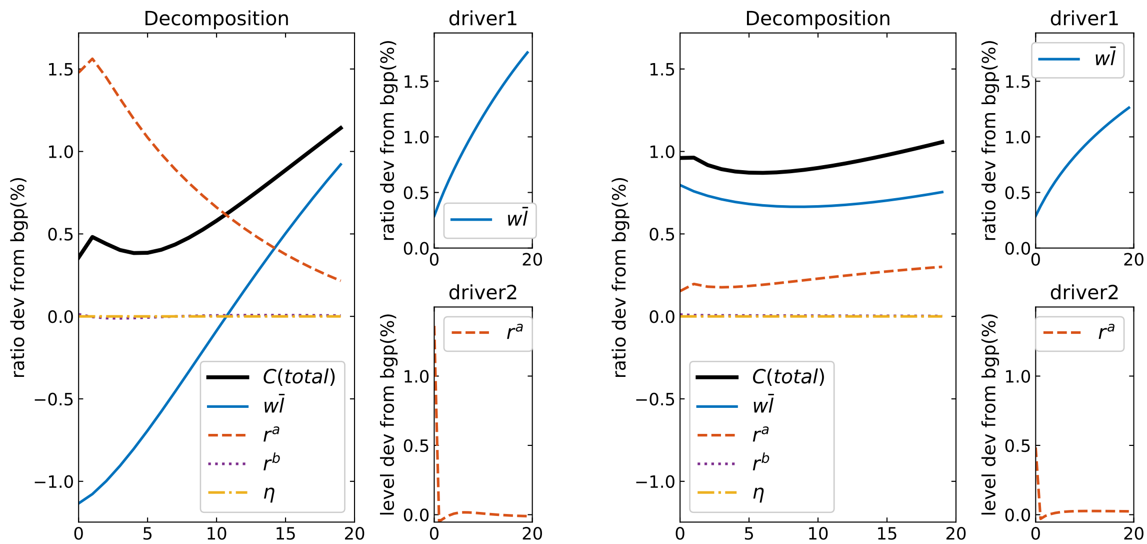
shocks, and such consumption behavior can generate excess consumption volatility. When a positive trend shock hits the economy, output not only jumps on impact but also grows further in the subsequent periods. Reflecting the future output growth, households' permanent income increases significantly. Since representative households follow the PIH, they increase their consumption accordingly. As a result, their consumption jumps far more than their output on impact. Through this mechanism, Aguiar and Gopinath (2007)'s representative-agent model successfully generates excess consumption volatility.

Unlike representative households, the heterogeneous households in my model significantly deviate from the PIH because they face both a large amount of idiosyncratic income risk and financial frictions. In particular, their consumption-saving behavior is different from that of representative households such that Aguiar and Gopinath (2007)'s mechanism cannot be accommodated well.

Figure 7a presents the result of the consumption response decomposition with respect to trend shocks in the baseline economy. The total consumption response is mainly driven by consumption responses to two drivers: $w_t\bar{l}_t$ and r_t^a. After a positive trend shock, these drivers move as follows. Driver $w_t\bar{l}_t$ jumps on impact and then increases further in the subsequent periods. Driver r_t^a, on the other hand, jumps substantially on impact, plunges to a negative value in the next period, and then gradually returns to zero.

These observations show how the future output growth in response to a positive trend shock enters into households' budget constraints through two channels. First, the future growth of aggregate labor income enters into the households' budget constraints as the future growth of labor income per idiosyncratic labor productivity ($w_t\bar{l}_t$). Second, the future growth of aggregate capital income is reflected in the asset price of illiquid assets and thus enters into the households' budget constraints as a jump in illiquid asset returns (r_t^a) on impact. Both channels make heterogeneous households' idiosyncratic income profiles either more increasing or less decreasing and thus create a positive wealth effect. Households would want to increase their consumption in response to this positive wealth effect.

As the large panel on the left of Figure 7a shows, households exhibit a strong positive consumption response to driver r_t^a, reflecting the positive wealth effect. However, households substantially decrease their consumption in response to driver $w_t\bar{l}_t$, despite the positive wealth effect it creates. The economic reason for this consumption plunge is an enhanced precautionary-saving motive. In the model, the future growth of $w_t\bar{l}_t$ means that idiosyncratic income risk will grow in the future as the variance of $w_t\bar{l}_te_{i,t}$ increases. Households prepare themselves for this greater income risk by accumulating additional buffer stocks. Since households allocate most of their savings to illiquid assets, they have to pay a large adjustment cost when they need to cash out illiquid assets by



(a) Baseline, g-shock

(b) Counterfactual, g-shock

Figure 7: Decomposition of the Consumption Responses to the g-shock

Notes: Panels 7a and 7b present the consumption response decomposition with respect to trend shocks (g) in the baseline economy and the counterfactual economy, respectively. Each panel consists of three subplots, where the large subplot on the left shows the total consumption response as well as decomposed consumption responses to each driver of $\{w_t\bar{l}_t, r^a_t, r^b_t, \eta_t\}_{t=0}^{\infty}$, and the other two small subplots on the right show the equilibrium paths of the two main drivers after the shock. The consumption responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.

facing bad idiosyncratic income shocks. In this environment, the amount of additional buffer stocks that households accumulate in response to increased income risk is also large. Because of this greatly enhanced precautionary-saving motive, households substantially decrease their consumption. In terms of the total consumption response, this enhanced precautionary-saving motive offsets most of the positive wealth effect, and as a consequence, the total consumption response is significantly subdued.

Figure 7b presents the same consumption response decomposition with respect to trend shocks but in the counterfactual economy. Two important observations are made from the comparison between Figure 7a and Figure 7b. First, the total consumption response to a trend shock in the counterfactual economy is substantially stronger than the response in the baseline economy on impact. Second, the stronger initial jump of total consumption in the counterfactual economy is driven by the strong and positive consumption response to $w_t\bar{l}_t$, which is in the opposite direction from the strong and negative response to it in the baseline economy. The sign of the consumption response to driver $w_t\bar{l}_t$ is flipped because the precautionary-saving motive is enhanced far less intensely in the counterfactual economy (since households face much smaller illiquid asset adjustment costs). As a consequence, the positive wealth effect dominates the enhanced precautionary-saving motive in the counterfactual economy.[53]

The observations from Figure 7a and Figure 7b show that Aguiar and Gopinath (2007)'s mechanism does not drive the consumption response in the baseline economy because households' consumption-saving behavior is very different from that of representative households, particularly due to the precautionary-saving motive.

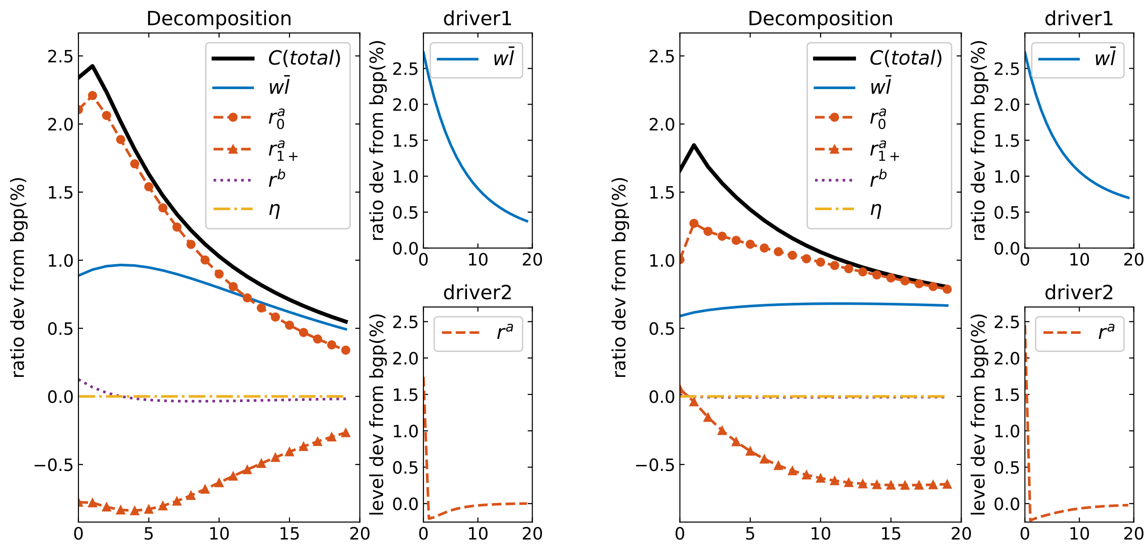
B Intertemporal Substitution in Response to Interest Rate Variations

The second well-accepted theory in the literature is the one developed by Neumeyer and Perri (2005). Emerging economies face volatile interest rate variations, induced either by domestic economic conditions or purely external factors. Neumeyer and Perri (2005) argue that households intertemporally substitute their consumption in response to interest rate variations, and such consumption behavior can generate the excess consumption volatility of emerging economies.

In my model, the interest rates in the international financial market are determined by equation (11). Provided that interest rate shocks (μ_t) and trend shocks (g_t) both have small volatility as a result of the Bayesian estimation, a large fraction of interest rate fluctuations are induced by stationary productivity shocks.

To see how the stationary-productivity-shock-induced interest rate fluctuations affect households' consumption, in Figure 8a, I report the same consumption response decomposition with respect to stationary productivity shocks as in Figure 6a except for one difference: the consumption response to driver r^a is further decomposed into the consumption responses to r^a_0 and to $r^a_t,\ t \geq 1$. Note that $r^a_t = r_{t-1},\ t \geq 1$, as equation (10) dictates. Importantly, only $r^a_t,\ t \geq 1$ (but not r^a_0) can generate the intertemporal substitution of consumption in households' optimization. Specifically, the consumption response to r^a_0 purely comes from a positive wealth effect, while the consumption response to $\{r^a_t\}_{t\geq 1}$ comes from both a negative wealth effect and a positive intertemporal substitution effect. As Figure 8a shows, the consumption response to $\{r^a_t\}_{t\geq 1}$ is negative on impact and in the subsequent periods, meaning that the negative wealth effect dominates the positive in-

[53]The positive consumption response to driver r^a_t is also weakened in the counterfactual economy. There are two economic reasons. First, the initial jump of r^a_t is far weaker in the counterfactual economy because the aggregate adjustment cost for illiquid assets χ^{agg}_t increases much less than in the baseline economy. Second, even if we control for the dynamics of driver r^a_t, households in the counterfactual economy respond much more mildly than those in the baseline economy because of the MPC difference. Despite all the reasons for the weakened consumption response to driver r^a_t, the total consumption response is still substantially stronger in the counterfactual economy than in the baseline economy because the effect of the flipped consumption response to driver $w_t\bar{l}_t$ dominates.



(a) Baseline, z-shock

(b) Counterfactual, z-shock

Figure 8: Decomposition of the Consumption Responses to the z-shock, $\{r_0^a\}$ and $\{r_t^a\}_{t\geq 1}$ split

Notes: Panels 8a and 8b present the same consumption response decomposition with respect to stationary productivity shocks (z) as panels 6a and 6b except for one difference: the consumption response to $\{r_t^a\}_{t\geq 0}$ is further decomposed into the responses to $\{r_0^a\}$ and $\{r_t^a\}_{t\geq 1}$. The consumption responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.

tertemporal substitution effect. Regarding the total consumption response, however, the positive wealth effect in the consumption response to r_0^a dominates the negative wealth effect in the consumption response to $r_t^a,\ t \geq 1$.

Figure 8b plots the same consumption response decomposition as Figure 8a but in the counterfactual economy. In the consumption response to $r_t^a,\ t \geq 1$, the positive intertemporal substitution effect is no longer dominated by the negative wealth effect on impact in the counterfactual economy. This change occurs because i) the intertemporal substitution effect recovers as the illiquid asset adjustment cost decreases, and ii) the wealth effect is weakened due to the lower MPCs. Despite this less negative consumption response to $r_t^a,\ t \geq 1$, however, the total consumption response is significantly weaker in the counterfactual economy because the positive wealth effect in the consumption response to r_0^a is substantially subdued due to the lower MPCs.

The observations from Figure 8a and Figure 8b show that unlike representative-agent models, the intertemporal substitution of consumption does not drive the consumption response in the baseline economy of my model. The economic reasons why this mechanism is dampened in my model are as follows. First, unlike representative households whose consumption is solely determined by their lifetime wealth and the degree of intertemporal substitution, the consumption-saving behavior of households in my model is also greatly affected by the precautionary-saving motive, as we see in the previous discussions. Second, the fact that households in my model allocate most of their savings to illiquid assets makes it even more difficult for them to shift resources across time.

VIII Conclusion

This paper studies the role of high-MPC households in emerging market business cycles through the lens of a heterogeneous-agent small open economy model. To this end, I discipline the model using both micro moments (MPC estimates from micro data) and macro moments (covariances of macro data) from Peru. Through the counterfactual experiment in which I replace Peruvian households with those exhibiting U.S. MPCs, which are substantially lower than the Peruvian MPCs, I find that the high-MPC households play a key role in generating the phenomenon of excess consumption volatility. Three decomposition exercises (variance decomposition, variance change decomposition, and consumption response decomposition) reveal that high-MPC households contribute to the large aggregate consumption volatility of emerging economies through i) their strong consumption response to individual resource fluctuations and ii) large consumption reduction when illiquid assets become more illiquid (because of some households' aggravated consumption smoothing failure and other households' enhanced precautionary saving). The driving mechanisms in conventional theories do not play an important role in my model because the high-MPC households, who significantly deviate from the PIH, cannot accommodate them well.

The scope of this paper is confined to the business cycle implications of high MPC households in emerging economies. However, it is likely that the presence of high-MPC households in emerging economies has many other interesting macroeconomic implications, as previous studies focused on developed economies suggest. Revisiting key macroeconomic issues of emerging economies – such as aggregate dynamics during financial crises and the transmission mechanisms of macroeconomic policies – through the lens of a heterogeneous-agent model in which households exhibit MPCs as high as the empirical estimates from micro data would be an important future avenue in the field of international macroeconomics.

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[Online Appendix]

Emerging Market Business Cycles
with Heterogeneous Agents

Seungki Hong

A Details on MPC Estimation

A.1 Method

The method I use in this paper to estimate MPCs out of transitory income shocks is an extended version of Blundell et al. (2008). Let the individual labor income $Y_{i,t}$ be specified as follows.

$$\log Y_{i,t} = Z'_{i,t}\varphi_t + P_{i,t} + \epsilon_{i,t},$$

$$P_{i,t} = \rho P_{i,t-1} + \zeta_{i,t},$$

$$\zeta_{i,t} \sim_{iid} (0, \sigma^2_{ps}), \quad \epsilon_{i,t} \sim_{iid} (0, \sigma^2_{tr}), \quad \text{and} \quad (\zeta_{i,t})_t \perp (\epsilon_{i,t})_t$$

in which $(x_t)_t$ represents time series $(\cdots, x_{t-2}, x_{t-1}, x_t, x_{t+1}, x_{t+2}, \cdots)$. $Z_{i,t}$ denotes a vector of dummy variables for observable characteristics of household i.[54]

Let ψ_G be Blundell et al. (2008)'s partial insurance parameter to transitory income shocks for group G, which is defined as follows.

$$\psi_G = \frac{cov[\Delta c_{i,t}, \epsilon_{i,t} | (i,t) \in G]}{cov[\Delta y_{i,t}, \epsilon_{i,t} | (i,t) \in G]}$$

in which $c_{i,t}$ and $y_{i,t}$ are the log consumption and log income after controlling for the observable characteristics. (By definition, $y_{i,t} = \log Y_{i,t} - Z'_{i,t}\varphi_t = P_{i,t} + \epsilon_{i,t}$.) In other words, parameter ψ_G is the elasticity of consumption with respect to income when the income change is caused by a transitory income shock. We can obtain the estimate of ψ_G following Kaplan and Violante (2010)'s identification strategy for Blundell et al. (2008)'s partial insurance parameters under the 'AR(1)+I.I.D.' specification of the income process

[54] The observable characteristics of households include education, ethnicity, employment status, region, cohort, household size, number of children, urban area, the existence of members other than heads and spouses earning income, and the existence of persons who do not live with but are financially supported by the household. Among these characteristics, education, ethnicity, employment status, and region are allowed to have time-varying effects.

as follows. Let $\tilde{\Delta}^K y_{i,t}$ and $\Delta^K c_{i,t}$ be

$$\tilde{\Delta}^K y_{i,t} := y_{i,t} - \rho^K y_{i,t-K}, \quad K \geq 1, \quad \text{and}$$

$$\Delta^K c_{i,t} := c_{i,t} - c_{i,t-K}, \quad K \geq 1.$$

Then, we have

$$\tilde{\Delta}^K y_{i,t} = \sum_{s=0}^{K-1} \rho^s \zeta_{i,t-s} + \epsilon_{i,t} - \rho^K \epsilon_{i,t-K}.$$

When the grouping of observation (i,t) is independent of $(\zeta_{i,t+j}, \epsilon_{i,t+j})_{j\geq 0}$ and $\Delta c_{i,t}$ is independent of $(\zeta_{i,t+j}, \epsilon_{i,t+j})_{j\geq 1}$, we can derive

$$\psi_G = \frac{cov[\Delta^K c_{i,t}, \tilde{\Delta}^K y_{i,t+K} | (i,t) \in G]}{cov[\tilde{\Delta}^K y_{i,t}, \tilde{\Delta}^K y_{i,t+K} | (i,t) \in G]}. \tag{A.1}$$

To identify ψ_G using equation (A.1), we need the value of ρ. Adopting Floden and Lindé (2001)'s identification strategy, parameter ρ is estimated using the autocovariance moments of $y_{i,t}$ as follows.[55]

$$E[y_{i,t}^2] = \frac{\sigma_{ps}^2}{1-\rho^2} + \sigma_{tr}^2,$$

$$E[y_{i,t}, y_{i,t+nK}] = \frac{\sigma_{ps}^2}{1-\rho^2} \rho^{nK}, \quad n \geq 1.$$

Once ρ is estimated, I estimate ψ_G using equation (A.1). Since ψ_G is an elasticity, I obtain the MPC estimate by multiplying ψ_G with the consumption-to-income ratio as follows.

$$MPC_G = \psi_G \frac{E[C_{i,t-K} | (i,t) \in G]}{E[Y_{i,t-K} | (i,t) \in G]}.$$

As discussed in section II, ENAHO provides year-over-year growth of quarterly income and consumption. Therefore, I set one period as a quarter and $K = 4$ for the Peruvian sample. As a result of estimation, I obtain the quarterly MPCs of Peruvian households. On the other hand, the PSID provides two-year-over-two-year growth of annual income and consumption. Thus, I set one period as a year and $K = 2$ for the U.S. sample. As a result of estimation, I obtain the annual MPCs of U.S. households.

[55] In this estimation, I obtain the estimates of ρ, σ_{ps}, and σ_{tr}. I also use these estimates in calibrating the idiosyncratic labor productivity process in the model of this paper in subsection IV.A.

A.2 Revisions on Hong (2020)

In section II, I make three revisions to the MPC estimation procedure of Hong (2020) that are necessary to use the MPC estimates in disciplining the model presented in this paper. First, I change the consumption measure from non-durable consumption to total consumption (including both non-durable consumption and durable consumption). When taking the model to data in section IV, after the model is calibrated by targeting the MPC estimates, aggregate shock processes (together with a few other model parameters) are Bayesian-estimated using macro data. In this step, I use total consumption series (as most studies in the literature of emerging market business cycles do) because non-durable consumption series is not available in Peruvian national accounts. To make the consumption concept consistent between micro and macro data, I use the total consumption measure when analyzing the micro data, too. Second, I change the sample periods for both ENAHO and the PSID because some of the key durable expenses are available only after certain years in both surveys. Specifically, I use the 2011-2018 waves of ENAHO and the 2005-2017 waves of the PSID.[56] Third, the income process specification is revised to be consistent with the model in this paper. Blundell et al. (2008)'s estimation method requires the structural specification of the income process. In the baseline estimation of Hong (2020), the income process is specified as the sum of a permanent component and a transitory component in which the permanent component follows a random walk, as in the original specification of Blundell et al. (2008). In this paper, I instead specify the income process as the sum of a persistent (but not permanent) component and a transitory component by replacing the random walk component with an AR(1) process so that the income process used in the empirical estimation is consistent with the model in this paper.[57]

[56] My ENAHO sample starts from 2011 because of the following reason. ENAHO is conducted continuously (*i.e.*, households are interviewed in different months) and the reference periods of income and expense items are usually in the format of a specified period before the interview (such as 'previous n months') rather than a fixed calender period (such as 'during 2014'). Naturally, I set the reference periods of the consumption and income measures in the same format (*i.e.*, a specified period before the interview such as 'previous n months'). One exception is Questionnaire 612. This questionnaire collects information on household furnishings, equipment, and vehicles, which take a sizable portion of durable goods. Until 2010, this questionnaire asks which calendar year each item is acquired, and thus it is not possible to aggregate this questionnaire's expense items with other expense items under a consistent reference period format. From 2011 onward, Questionnaire 612 asks the acquisition month instead of the acquisition year, which makes it possible to recover this questionnaire's expense items during a specified period before the interview (such as 'previous n months') and to aggregate these expense items with other expense items under a consistent reference period format. My PSID sample starts from 2005 because the survey began to collect expenses on household furnishings and equipment since then. Moreover, some non-durable items including clothing and recreation are also collected from 2005 onward.

[57] Hong (2020) also uses this income process specification in one of the robustness checks in his Appendix D.1.5.

A.3 Reference Periods

In ENAHO, the reference periods are not equal across income and expense items. Typically, expenses or incomes that occur less frequently tend to have longer reference periods. More importantly, households report most items (97.7% of income items, 92.9% of expense items, on average) with reference periods no longer than the previous three months. Given this feature of the data, I set the reference period of consumption and income measures as the previous three months. In aggregating items to construct income and consumption, items with different reference periods than the previous three months are scaled to three-month expenses or incomes. (For example, monthly tobacco expenses are scaled up to three-month expenses by multiplying by three.) Moreover, in order to remove any comovement between income and consumption generated by income shocks prior to the previous three months, I exclude income items with reference periods longer than the previous three months in constructing income.

In the PSID, the reference periods of incomes are firmly fixed to a calendar year. However, the reference periods of expense items can depend on interpretation, as Crawley (2019) points out. For example, food expenses in the PSID can be interpreted either as the last week's expense or the average weekly expense during the calendar year. I adopt the latter interpretation as many other studies do, and treat the reference periods of expense items as being synchronized with those of income items. Naturally, I set the reference period of consumption and income measures as the corresponding calender year.

As discussed in section II, ENAHO is conducted annually, and I use the 2011-2018 waves. This ENAHO sample provides seven years of year-over-year growth of quarterly income and consumption. For the PSID, I use the 2005-2017 waves, and the survey is conducted biannually during the sample period. This PSID sample provides six years of two-year-over-two-year growth of annual income and consumption.

A.4 Variable Construction

The consumption measure used in the MPC estimation is total consumption, which includes both non-durable consumption and durable consumption. To this end, I aggregate the following expenses in each of ENAHO and the PSID: non-durable expenses including 1) food, 2) clothing (including clothing services, footwear, watches and jewelry), 3) housing rent, rental equivalence of owned or donated housing, 4) utilities (heat, electricity, water, etc.), 5) telephone and cable, 6) vehicle repairs and maintenance, 7) gasoline and oil, 8) parking, 9) public transportation, 10) household repairs and maintenance, 11) recreation, 12) insurance (home insurance, car insurance, health insurance, etc.), 13) childcare, 14) domestic services and other home services, 15) personal care, 16) alcohol, 17) tobacco, and 18) daily non-durables (laundry items, bathroom items, matches, candle, stationeries, etc.), and durable expenses including 19) vehicles, 20) furnishings and equipment (textiles, furniture, floor coverings, appliances, housewares, etc.), 21) health, and 22) education.[58] Among the listed expenses, ENAHO does not have expenses on 13) childcare, and the PSID does not have expenses on 14) domestic services and other home services, 15) personal care, 16) alcohol, 17) tobacco, and 18) daily non-durables (laundry items, bathroom items, matches, candle, stationeries, etc.).

For both ENAHO and the PSID, the income measure used in the MPC estimation is the sum of disposable labor income and transfers, as in Blundell et al. (2008). Capital income is excluded in order not to falsely attribute endogenous capital income changes as income shocks. In ENAHO, capital income and labor income are not distinguishable in self-employment income. As in Diaz-Gimenez, Quadrini, and Rios-Rull (1997), Krueger and Perri (2006), and Hong (2020), I split the self-employment income into labor income part and capital income part using the ratio between unambiguous capital income and unambiguous labor income in the sample.[59] In ENAHO, there are a small fraction of income items that have reference periods longer than the previous three months. As discussed in subsection A.3, I exclude them from the income measure. In the PSID, I closely follow Kaplan et al. (2014) in constructing disposable labor income and transfers. Specifically, disposable labor income and transfers are constructed by i) estimating federal income taxes for total income (including both the component of labor income and transfers and the component of capital income) by TAXSIM program, ii) allocating federal taxes for the component of labor income and transfers using the ratio between this component and the component of capital income in the total income, and iii) subtracting the estimated federal taxes for labor income and transfers from gross labor income and transfers.

For both ENAHO and the PSID, consumption and income are deflated using CPI series. Unlike the reference periods in the PSID sample, the reference periods in the ENAHO sample are not fixed to a calendar period. For example, the three-month window of the reference periods for households surveyed in January, 2015 is one-month earlier than the three-month window for households surveyed in February, 2015. Fortunately, this data

[58]In listing the expenses, I categorize expenses on 21) health and 22) education as durable expenses because of their durable nature. In national accounts, however, they are categorized as non-durable consumption. Since I use total consumption, whether these expenses are categorized as durable expenses or non-durable expenses do not have any effect.

[59]In the ENAHO sample, the ratio of $\frac{\text{(unambiguous labor income)}}{\text{(unambiguous labor income)+(unambiguous capital income)}}$ is 0.817. This ratio is close to the ratio that Diaz-Gimenez et al. (1997) and Krueger and Perri (2006) use for their U.S. sample, 0.864.

feature does not complicate the deflation procedure because ENAHO provides within-year-deflated values for income and expense items. For example, the within-year-deflated values of the food expenses spent on July 2018 are expressed in terms of the 2018 price level. Using these within-year-deflated values, I construct real income and consumption of the ENAHO sample as follows. I first aggregate the within-year-deflated values of income and expense items to construct the within-year-deflated income and consumption measures. Then, I deflate these within-year-deflated income and consumption measures using annual CPI series.

A.5 Sample Selection

The sample selection procedure closely follows Hong (2020). To describe the procedure, it is convenient to distinguish different units of observations. Let type-n observation be an observation of a household over n consecutive periods. For example, if a household appears three consecutive times in the sample, we obtain three type-1 observations, two type-2 observations, and one type-3 observation associated with this household. Note that observation (i, t) in the MPC estimation procedure described in Appendix A.1 represents a type-3 observation, as it is composed of observations on income and consumption of household i in period $t - K$, t, and $t + K$.

Each of the sample selection criteria is applied to either type-1 or type-2 observations. When a type-1 observation is dropped, any type-2 and type-3 observations containing the dropped type-1 observation are dropped. When a type-2 observation is dropped, any type-1 observations that do not belong to any other type-2 observation are dropped, and any type-3 observations that contain the dropped type-2 observations are dropped.

In ENAHO, the sample selection proceeds as follows. First, type-1 observations are dropped if they do not belong to any type-2 observation. Second, type-2 observations that have at least one of the following problems are dropped: i) the interview months are not matched between the two consecutive surveys, ii) the observations are likely to falsely connect two different households[60], or iii) the head of the household changed between the two consecutive surveys. Third, type-1 observations are dropped if the interviews are categorized as 'incomplete' by pollsters. Fourth, type-1 observations are dropped if household heads are younger than 25 or older than 65. Fifth, type-1 observations are dropped

[60] In ENAHO, panel households are selected based on addresses. The risk of falsely connecting two different households exists when an old household moves out and a new household moves in to a selected address. Type-2 observations subject to this risk, which Hong (2020) define as 'potentially fake type-2 observations', can be effectively detected and dropped by checking the household-member-level match between the two consecutive periods. For detailed discussion on why this problem exists and how to detect the potentially fake type-2 observations, see Appendix B.3 of Hong (2020).

if any of the household characteristics in $Z_{i,t}$ are missing. Sixth, type-1 observations are dropped when consumption or income are non-positive. Seventh, type-1 observations are dropped when they have too much value in income items with reference periods longer than the previous three months, or more specifically, when (income items with reference periods longer than the previous three months) / ((baseline income measure)+(income items with reference periods longer than the previous three months)) is greater than 5%. Eighth, type-1 observations are dropped when households are categorized as income outliers. Households are categorized as income outliers if their income growth falls into the range of extreme 1% (0.5% at the top, 0.5% at the bottom) in calendar-year subsamples at least one time. As a result of the sample selection, I obtain 40,677 type-1 observations, 22,936 type-2 observations, and 9,906 type-3 observations. Panel A of Table A.1 reports the selected observations in each step of the sample selection in ENAHO.

The sample selection in the PSID proceeds similarly to the sample selection in ENAHO as follows. First, type-1 observations are dropped if they do not belong to any type-2 observation with a continued household head. Second, I drop type-1 observations if they

Table A.1: Sample Selection

	type-1	type-2	type-3
A. ENAHO			
type-1 obs. not belonging to any type-2 obs.	87,305	59,691	32,077
months not matched, fake type-2 obs., or head changed	73,248	47,950	22,652
incomplete survey	63,410	38,343	17,386
age restriction, 25-65	48,636	28,983	12,971
observable characteristics missing	48,403	28,904	12,955
non-positive Y and C	48,144	28,613	12,788
too much 3ml in Y	41,357	23,397	10,154
outliers on income growth	40,677	22,936	9,906
B. The PSID			
type-1 obs. not belonging to any type-2 obs. with a continued head	57,560	45,553	33,546
SEO sample 1968 and Latino sample 1990/1992	39,660	31,523	23,386
topcoded obs.	39,650	31,507	23,369
age restriction, 25-65	31,447	24,380	17,711
observable characteristics missing	30,225	23,277	16,805
non-positive Y and C	30,028	23,021	16570
outliers on income growth	29,145	22,345	16,092

Notes: In the penultimate line of panel A, '3ml' is an abbreviation for 'items with reference periods longer than the previous three months'.

belong to the sample from Survey of Economic Opportunities (SEO) (added to the PSID in 1968) or to the Latino sample (added to the PSID in 1990 and 1992). Third, I drop type-1 observations if they have topcoded values in income or expense items. Fourth, type-1 observations are dropped if household heads are younger than 25 or older than 65. Fifth, type-1 observations are dropped if any of the household characteristics in $Z_{i,t}$ are missing. Sixth, type-1 observations are dropped when income or consumption are non-positive. Seventh, type-1 observations are dropped when households are categorized as income outliers. The definition of income outliers is the same as in the ENAHO sample selection. Through this sample selection, I obtain 29,145 type-1 observations, 22,345 type-2 observations, and 16,092 type-3 observations. Panel B of Table A.1 reports the selected observations in each step of the sample selection in the PSID.

A.6 Labor Income Grouping

As discussed in section II, I group observation (i,t)s into labor income deciles and estimate the MPCs of each decile. The labor income deciles are constructed as follows. In the estimation procedure described in Appendix A.1, observation (i,t) is composed of household i's income and consumption in period $t-K$, t, and $t+K$. These observation (i,t)s are sorted with the unpredictable component of labor income in period $t-K$, $y_{i,t-K}$.[61] Specifically, in accordance with the time unit of each survey (a quarter for ENAHO, a year for the PSID), observations are sorted within a calender quarter subsample in ENAHO and within a calendar year subsample in the PSID. Survey weights are taken into account when computing the quantiles of the sorted observations.

As discussed in subsection A.4, the income measure of Peruvian households does not include income items with reference periods longer than the previous three months. Moreover, observations having too much value in this component are dropped in the sample selection, as discussed in subsection A.5. If the share of this component in the household income is correlated with the income level, this sample selection can create a selection bias. To resolve this concern, I follow Hong (2020)'s approach as follows. When sorting observations and computing their quantiles, I include dropped observations due to having too much value in income items with reference periods longer than the previous three months. When sorting the selected observations together with the dropped observations, I use the unpredictable component of a comprehensive income that includes not only the baseline income measure but also the income items with reference periods longer than the previous three months. The income items with reference periods longer than the

[61] Because observations are sorted by $y_{i,t-K}$, the grouping of observation (i,t)s is independent of $(\zeta_{i,t+j}, \epsilon_{i,t+j})_{j\geq 0}$, which is a necessary condition for the identification equation (A.1).

previous three months are bad because it generates comovement between income and consumption caused by income shocks prior to the previous three months. However, they are helpful in determining where the selected observations are located in the income distribution.

B Details on How to Solve the Model

B.1 Equilibrium under Deterministic Paths of Aggregate Exogenous Variables

In this subsection, I characterize the equilibrium when the economy is subject to deterministic paths of $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$.

B.1.1 Households

Under deterministic paths of $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$, households' optimization problem can be expressed as the following Bellman equation.

$$V_t(e_1, e_2, b_-, a_-) = \max_{c,b,a} \frac{c^{1-\gamma}}{1-\gamma} + \beta \sum_{e_1', e_2'} P(e_1', e_2' | e_1, e_2) V_{t+1}(e_1', e_2', b, a)$$

$$s.t.$$

$$c + b + a + \eta_t \chi_t(a - (1 + r_t^a)a_-, a_-) = w_t e \bar{l}_t + (1 - \xi)(1 + r_t^b) b_- + (1 + r_t^a) a_-,$$

$$a \geq 0, \quad b \geq 0, \quad \text{and}$$

$$\log e = \log e_1 + \log e_2.$$

On the balanced growth path in which $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$ are constant at their long-run average values, V_t, $t \geq 0$ grows with the rate of $(g^*)^{1-\gamma}$ (or equivalently, $V_{t+1} = (g^*)^{1-\gamma} V_t$).

Given the parametrization of $\chi_t(v, a_-)$ in the main text, its first-order derivatives are

$$\chi_{1,t}(v, a_-) = sign(v)\, \chi_1 \chi_2 \left| \frac{v}{(1 + r_t^a) a_- + \chi_0 X_{t-1}} \right|^{\chi_2 - 1} \quad \text{and}$$

$$\chi_{2,t}(v, a_-) = \chi_1 (1 - \chi_2) \left| \frac{v}{(1 + r_t^a) a_- + \chi_0 X_{t-1}} \right|^{\chi_2} (1 + r_t^a).$$

Both $\chi_{1,t}(v, a_-)$ and $\chi_{2,t}(v, a_-)$ are continuous everywhere, including the area around $v = 0$. Therefore, $\chi_t(v, a_-)$ is continuous and differentiable everywhere.

The optimality conditions of the households' problem can be derived as follows.

$$
\begin{aligned}
V_t(e_1, e_2, b_-, a_-) = \max_{c,b,a,\lambda,\varphi^b,\varphi^a} \ & \frac{c^{1-\gamma}}{1-\gamma} + \beta \sum_{e_1', e_2'} P(e_1', e_2' | e_1, e_2) V_{t+1}(e_1', e_2', b, a) \\
& + \lambda\{w_t e \bar{l}_t + (1-\xi)(1+r_t^b)b_- + (1+r_t^a)a_- \\
& \quad - c - b - a - \eta_t \chi_t(a - (1+r_t^a)a_-, a_-)\} + \varphi^b b + \varphi^a a.
\end{aligned}
$$

$$\lambda = c^{-\gamma}, \tag{B.1}$$

$$\lambda = \beta \sum_{e_1', e_2'} P(e_1', e_2' | e_1, e_2) V_{b,t+1}(e_1', e_2', b, a) + \varphi^b, \tag{B.2}$$

$$\lambda\{1 + \eta_t \chi_{1,t}(a - (1+r_t^a)a_-, a_-)\} = \beta \sum_{e_1', e_2'} P(e_1', e_2' | e_1, e_2) V_{a,t+1}(e_1', e_2', b, a) + \varphi^a, \tag{B.3}$$

$$V_{b,t}(e_1, e_2, b_-, a_-) = (1-\xi)(1+r_t^b)\lambda, \tag{B.4}$$

$$
\begin{aligned}
V_{a,t}(e_1, e_2, b_-, a_-) = \lambda\{& (1+r_t^a) + (1+r_t^a)\eta_t \chi_{1,t}(a - (1+r_t^a)a_-, a_-) \\
& - \eta_t \chi_{2,t}(a - (1+r_t^a)a_-, a_-)\},
\end{aligned} \tag{B.5}
$$

$$c + b + a + \eta_t \chi_t(a - (1+r_t^a)a_-, a_-) = w_t e \bar{l}_t + (1-\xi)(1+r_t^b)b_- + (1+r_t^a)a_-, \tag{B.6}$$

$$\varphi^b \geq 0, \quad b \geq 0, \quad \varphi^b b = 0, \quad \text{and} \tag{B.7}$$

$$\varphi^a \geq 0, \quad a \geq 0, \quad \varphi^a a = 0. \tag{B.8}$$

B.1.2 Firms

Under deterministic paths of $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$, firms solve the following optimization problem.

$$\max_{\{K_t, F_t, L_t, Y_t, I_t, \Pi_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} Q_{o,t} \Pi_t$$

$$s.t.$$

$$\Pi_t = Y_t - w_t L_t - I_t - \Phi(K_t, K_{t-1}) + F_t - (1 + r_{t-1})F_{t-1} + \chi_t^{agg}, \tag{B.9}$$

$$Y_t = z_t K_{t-1}^{\alpha} (X_t L_t)^{1-\alpha}, \tag{B.10}$$

$$\nu_t I_t = K_t - (1-\delta)K_{t-1}, \tag{B.11}$$

$$\nu_t \Phi(K_t, K_{t-1}) = \frac{\phi}{2}\left(\frac{K_t}{K_{t-1}} - g^*\right)^2 K_{t-1},$$

$$Q_{0,t} = \begin{cases} 1 & \text{if} \quad t = 0, \\ \frac{1}{\prod_{s=1}^{t}(1+r_s^a)} & \text{otherwise, and} \end{cases}$$

$$\lim_{j\to\infty} \frac{F_{t+j}}{\prod_{s=1}^{j}(1+r_{t+s}^a)} \leq 0.$$

The optimality conditions of the problem can be obtained as follows.

$$\max_{\{K_t,F_t,L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} Q_{0,t} \Bigg[z_t K_{t-1}^{\alpha}(X_t L_t)^{1-\alpha} - w_t L_t - \frac{1}{\nu_t}\left(K_t + (1-\delta)K_{t-1}\right) \\ - \frac{1}{\nu_t}\frac{\phi}{2}\left(\frac{K_t}{K_{t-1}} - g^*\right)^2 K_{t-1} + F_t - (1+r_{t-1})F_{t-1} + \chi_t^{agg} \Bigg].$$

$$(1+r_{t+1}^a)\frac{1}{\nu_t}\left\{1 + \phi\left(\frac{K_t}{K_{t-1}} - g^*\right)\right\} = \alpha z_{t+1}\left(\frac{K_t}{X_{t+1}L_{t+1}}\right)^{\alpha-1} \\ + \frac{1}{\nu_{t+1}}\left\{1 - \delta + \phi\left(\frac{K_{t+1}}{K_t} - g^*\right)\frac{K_{t+1}}{K_t} - \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - g^*\right)^2\right\}, \quad t \geq 0, \tag{B.12}$$

$$1 + r_{t+1}^a = 1 + r_t, \quad t \geq 0, \quad \text{and} \tag{B.13}$$

$$w_t = (1-\alpha) z_t X_t \left(\frac{K_{t-1}}{X_t L_t}\right)^{\alpha}. \tag{B.14}$$

B.1.3 Other Parts

The rest of the model (other than the households' problem in Appendix B.1.1 and the firms' problem in Appendix B.1.2) remains the same regardless of whether aggregate uncertainty is present or not. In other words, the law of motion for $\Psi_t(e_1, e_2, b_-, a_-)$ (equation (3)), aggregation of quantities (equation (4)) and households' budget constraints (equation (5)), the labor union's intratemporal labor supply decision (equation (6)), domestic banks' intratemporally equalized financing costs between the two sources (equation (7)), the relationship between illiquid asset returns and firms' profits and total values (equation (9)), the determination of the interest rates in the international financial market (equations (11) and (12)), the market clearing conditions (equations (14), (15), and (16)), the resource constraint (equation (17)), and the identity equation for the trade balance (equation (18)) hold true regardless of whether the economy faces aggregate uncertainty or it faces deterministic paths of $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$.

B.1.4 Equilibrium under Deterministic Paths of Aggregate Exogenous Variables

Given the initial conditions on $\Psi_0(e_1, e_2, b_-, a_-)$, X_{-1}, A_{-1}, K_{-1}, D_{-1}, B_{-1}, F_{-1}, r_{-1}, and deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$,

i) individual households' policy functions $\{c_t(e_1, e_2, b_-, a_-), b_t(e_1, e_2, b_-, a_-), a_t(e_1, e_2, b_-, a_-)\}_{t=0}^{\infty}$, first-order derivatives of the value functions $\{V_{b,t}(e_1, e_2, b_-, a_-), V_{a,t}(e_1, e_2, b_-, a_-)\}_{t=0}^{\infty}$, and Lagrangian multipliers $\{\lambda_t(e_1, e_2, b_-, a_-), \varphi_t^b(e_1, e_2, b_-, a_-), \varphi_t^a(e_1, e_2, b_-, a_-)\}_{t=0}^{\infty}$ that satisfy households' optimality conditions (B.1), (B.2), (B.3), (B.4), (B.5), (B.6), (B.7), and (B.8),

ii) cross-sectional cumulative distributions $\{\Psi_t(e_1, e_2, b_-, a_-)\}_{t=1}^{\infty}$ that evolve over time according to equation (3),

iii) aggregate variables $\{C_t, B_t, A_t, \chi_t^{agg}\}_{t=0}^{\infty}$ constructed by aggregating corresponding individual variables according to equation (4),

iv) prices and aggregate variables $\{r_t^b, r_t^a, r_t, w_t, q_t, \bar{l}_t, L_t, \Pi_t, Y_t, I_t, K_t, F_t, D_t, \hat{D}_t, TB_t\}_{t=0}^{\infty}$ satisfying firms' optimality conditions (B.9), (B.10), (B.11), (B.12), (B.13), and (B.14), and other equilibrium conditions (6), (7), (9), (11), (12), (14), (15), (16), and (18)

constitute the equilibrium of the economy.

B.2 Detrended Equilibrium under Deterministic Paths of Aggregate Exogenous Variables

B.2.1 Detrending

Since the equilibrium characterized in subsection B.1 exhibits nonstationarity inherited from the stochastic trend $\{X_t\}_{t=0}^{\infty}$, we need to detrend the equilibrium to make it stationary. To this end, I detrend the variables and functions as follows.[62]

$$\tilde{c}_{i,t} := c_{i,t}/X_{t-1}, \quad \tilde{b}_{i,t} := b_{i,t}/X_t, \quad \tilde{a}_{i,t} := a_{i,t}/X_t,$$

$$\tilde{\lambda}_{i,t} := \lambda_{i,t}/X_{t-1}^{-\gamma}, \quad \tilde{\varphi}_{i,t}^b := \varphi_{i,t}^b/X_{t-1}^{-\gamma}, \quad \tilde{\varphi}_{i,t}^a := \varphi_{i,t}^a/X_{t-1}^{-\gamma},$$

$$\tilde{Y}_t := Y_t/X_{t-1}, \quad \tilde{C}_t := C_t/X_{t-1}, \quad \tilde{I}_t := I_t/X_{t-1},$$

[62] I detrend flow variables with X_{t-1} and stock variables with X_t as I find the consequent detrended equilibrium conditions convenient to deal with. However, how the variables are detrended is immaterial to the equilibrium dynamics of the original equilibrium once recovered from the equilibrium dynamics of the detrended equilibrium.

$$\tilde{\chi}_t^{agg} := \chi_t^{agg}/X_{t-1}, \quad \tilde{w}_t := w_t/X_{t-1}, \quad \tilde{\Pi}_t := \Pi_t/X_{t-1}, \quad \tilde{TB}_t := TB_t/X_{t-1},$$

$$\tilde{B}_t := B_t/X_t, \quad \tilde{A}_t := A_t/X_t, \quad \tilde{q}_t := q_t/X_t,$$

$$\tilde{D}_t := D_t/X_t, \quad \tilde{\hat{D}}_t := \hat{D}_t/X_t, \quad \tilde{K}_t := K_t/X_t, \quad \text{and} \quad \tilde{F}_t := F_t/X_t.$$

$$\tilde{\Psi}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-) := \Psi_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1}),$$

$$\tilde{V}_{b,t}(e_1, e_2, \tilde{b}_-, \tilde{a}_-) := V_{b,t}(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1})/X_{t-1}^{-\gamma},$$

$$\tilde{V}_{a,t}(e_1, e_2, \tilde{b}_-, \tilde{a}_-) := V_{a,t}(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1})/X_{t-1}^{-\gamma},$$

$$\tilde{c}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = c_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1})/X_{t-1},$$

$$\tilde{b}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = b_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1})/X_t,$$

$$\tilde{a}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = a_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1})/X_t,$$

$$\tilde{\lambda}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = \lambda_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1})/X_{t-1}^{-\gamma},$$

$$\tilde{\varphi}_t^b(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = \varphi_t^b(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1})/X_{t-1}^{-\gamma},$$

$$\tilde{\varphi}_t^a(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = \varphi_t^a(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1})/X_{t-1}^{-\gamma}, \quad \text{and}$$

$$\tilde{\chi}_t(\tilde{v}, \tilde{a}_-) := \chi_1 \left| \frac{\tilde{v}}{(1 + r_t^a)\tilde{a}_- + \chi_0} \right|^{\chi_2} ((1 + r_t^a)\tilde{a}_- + \chi_0).$$

The first-order derivatives of $\tilde{\chi}_t(\tilde{v}, \tilde{a}_-)$ are

$$\tilde{\chi}_{1,t}(\tilde{v}, \tilde{a}_-) = sign(\tilde{v})\chi_1\chi_2 \left| \frac{\tilde{v}}{(1 + r_t^a)\tilde{a}_- + \chi_0} \right|^{\chi_2 - 1}, \quad \text{and}$$

$$\tilde{\chi}_{2,t}(\tilde{v}, \tilde{a}_-) = \chi_1(1 - \chi_2) \left| \frac{\tilde{v}}{(1 + r_t^a)\tilde{a}_- + \chi_0} \right|^{\chi_2} (1 + r_t^a).$$

Note that when $v_{i,t} = a_{i,t} - (1 + r_t^a)a_{i,t-1}$ and $\tilde{v}_{i,t} = v_{i,t}/X_{t-1} = g_t\tilde{a}_{i,t} - (1 + r_t^a)\tilde{a}_{i,t-1}$, the following relationships hold.

$$\tilde{\chi}_t(\tilde{v}_{i,t}, \tilde{a}_{i,t-1}) = \chi_t(v_{i,t}, a_{i,t-1})/X_{t-1},$$

$$\tilde{\chi}_{1,t}(\tilde{v}_{i,t}, \tilde{a}_{i,t-1}) = \chi_{1,t}(v_{i,t}, a_{i,t-1}), \quad \text{and}$$

$$\tilde{\chi}_{2,t}(\tilde{v}_{i,t}, \tilde{a}_{i,t-1}) = \chi_{2,t}(v_{i,t}, a_{i,t-1}).$$

The optimality conditions of households are detrended as follows.

$$\tilde{\lambda} = \tilde{c}^{-\gamma}, \tag{B.15}$$

$$\tilde{\lambda} = \beta g_t^{-\gamma} \sum_{e_1', e_2'} P(e_1', e_2' | e_1, e_2) \tilde{V}_{b,t+1}(e_1', e_2', \tilde{b}, \tilde{a}) + \tilde{\varphi}^b, \tag{B.16}$$

$$\begin{aligned} &\tilde{\lambda}\{1+\eta_t \tilde{\chi}_{1,t}(g_t \tilde{a} - (1+r_t^a)\tilde{a}_-, \tilde{a}_-)\} \\ &\qquad = \beta g_t^{-\gamma} \sum_{e_1', e_2'} P(e_1', e_2' | e_1, e_2) \tilde{V}_{a,t+1}(e_1', e_2', \tilde{b}, \tilde{a}) + \tilde{\varphi}^a, \end{aligned} \tag{B.17}$$

$$\tilde{V}_{b,t}(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = (1-\xi)(1+r_t^b)\tilde{\lambda}, \tag{B.18}$$

$$\begin{aligned} \tilde{V}_{a,t}(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = \tilde{\lambda}\{&(1+r_t^a) + (1+r_t^a)\eta_t \tilde{\chi}_{1,t}(g_t \tilde{a} - (1+r_t^a)\tilde{a}_-, \tilde{a}_-) \\ &- \eta_t \tilde{\chi}_{2,t}(g_t \tilde{a} - (1+r_t^a)\tilde{a}_-, \tilde{a}_-)\}, \end{aligned} \tag{B.19}$$

$$\tilde{c} + g_t \tilde{b} + g_t \tilde{a} + \eta_t \tilde{\chi}_t(g_t \tilde{a} - (1+r_t^a)\tilde{a}_-, \tilde{a}_-) = \tilde{w}_t e \bar{l}_t + (1-\xi)(1+r_t^b)\tilde{b}_- + (1+r_t^a)\tilde{a}_-, \tag{B.20}$$

$$\tilde{\varphi}^b \geq 0, \quad \tilde{b} \geq 0, \quad \tilde{\varphi}^b \tilde{b} = 0, \quad \text{and} \tag{B.21}$$

$$\tilde{\varphi}^a \geq 0, \quad \tilde{a} \geq 0, \quad \tilde{\varphi}^a \tilde{a} = 0. \tag{B.22}$$

The optimality conditions of firms are detrended as follows.

$$\tilde{\Pi}_t = \tilde{Y}_t - \tilde{w}_t L_t - \tilde{I}_t - \frac{1}{\nu_t}\frac{\phi}{2}\left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}} g_t - g^*\right)^2 \tilde{K}_{t-1} + g_t \tilde{F}_t - (1+r_{t-1})\tilde{F}_{t-1} + \tilde{\chi}_t^{agg}, \tag{B.23}$$

$$\tilde{Y}_t = z_t g_t^{1-\alpha} \tilde{K}_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{B.24}$$

$$\tilde{I}_t = \frac{1}{\nu_t}(g_t \tilde{K}_t - (1-\delta)\tilde{K}_{t-1}), \tag{B.25}$$

$$\begin{aligned} &(1+r_t)\frac{1}{\nu_t}\left\{1 + \phi\left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}} g_t - g^*\right)\right\} = \alpha z_{t+1} g_{t+1}^{1-\alpha}\left(\frac{\tilde{K}_t}{L_{t+1}}\right)^{\alpha-1} \\ &+ \frac{1}{\nu_{t+1}}\left\{1 - \delta + \phi\left(\frac{\tilde{K}_{t+1}}{\tilde{K}_t} g_{t+1} - g^*\right)\frac{\tilde{K}_{t+1}}{\tilde{K}_t} g_{t+1} - \frac{\phi}{2}\left(\frac{\tilde{K}_{t+1}}{\tilde{K}_t} g_{t+1} - g^*\right)^2\right\}, \quad t \geq 0, \end{aligned} \tag{B.26}$$

$$1 + r_{t+1}^a = 1 + r_t, \quad t \geq 0, \quad \text{and} \tag{B.27}$$

$$\tilde{w}_t = (1-\alpha) z_t g_t^{1-\alpha}\left(\frac{\tilde{K}_{t-1}}{L_t}\right)^{\alpha}. \tag{B.28}$$

The other equilibrium conditions (3), (4), (6), (7), (9), (11),(12), (14), (15), (16), and (18) are detrended as follows.

$$
\begin{aligned}
\tilde{\Psi}_{t+1}(e_1', e_2', \tilde{b}, \tilde{a}) = \int_{e_1, e_2, \tilde{b}_-, \tilde{a}_-} & P(e_{1,t+1} \le e_1' | e_{1,t} = e_1) P(e_{2,t+1} \le e_2') \\
& I_{\{\tilde{b}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-) \le \tilde{b},\ \tilde{a}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-) \le \tilde{a}\}}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\, d\tilde{\Psi}_t,
\end{aligned}
\tag{B.29}
$$

$$
\begin{aligned}
\tilde{C}_t &= \int_{e_1,e_2,\tilde{b}_-,\tilde{a}_-} \tilde{c}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\, d\tilde{\Psi}_t, \\
\tilde{B}_t &= \int_{e_1,e_2,\tilde{b}_-,\tilde{a}_-} \tilde{b}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\, d\tilde{\Psi}_t, \\
\tilde{A}_t &= \int_{e_1,e_2,\tilde{b}_-,\tilde{a}_-} \tilde{a}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\, d\tilde{\Psi}_t, \quad \text{and} \\
\tilde{\chi}_t^{agg} &= \int_{e_1,e_2,\tilde{b}_-,\tilde{a}_-} \eta_t \tilde{\chi}_t(g_t \tilde{a}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-) - (1 + r_t^a)\tilde{a}_-, \tilde{a}_-)\, d\tilde{\Psi}_t,
\end{aligned}
\tag{B.30}
$$

$$
\tilde{w}_t \bar{e}^{1+\omega} = \kappa L_t^{\omega}, \tag{B.31}
$$

$$
1 + r_t^b = 1 + r_{t-1}, \quad t \ge 0, \tag{B.32}
$$

$$
1 + r_t^a = \frac{\tilde{\Pi}_t + g_t \tilde{q}_t}{\tilde{q}_{t-1}}, \quad t \ge 0, \tag{B.33}
$$

$$
r_t = r^* + \psi \left\{ \exp\left(\frac{\tilde{D}_t - \tilde{D}^*}{\tilde{Y}^*} \right) - 1 \right\} - \theta_z (z_t - 1) - \theta_g \left(\frac{g_t}{g^*} - 1 \right) + \mu_t - 1, \tag{B.34}
$$

$$
\tilde{\tilde{D}}_t = \tilde{D}_t, \tag{B.35}
$$

$$
L_t = \bar{e} \bar{l}_t, \tag{B.36}
$$

$$
\tilde{F}_t - \tilde{D}_t = \tilde{B}_t, \tag{B.37}
$$

$$
\tilde{q}_t = \tilde{A}_t, \quad \text{and} \tag{B.38}
$$

$$
\tilde{TB}_t = -g_t \tilde{D}_t + (1 + r_{t-1}) \tilde{D}_{t-1}. \tag{B.39}
$$

In addition, the aggregated budget constraint of households (5) and the resource constraint (17) are detrended as follows.

$$
\tilde{C}_t + g_t \tilde{B}_t + g_t \tilde{A}_t + \tilde{\chi}_t^{agg} = \tilde{w}_t \bar{e} \bar{l}_t + (1 - \xi)(1 + r_t^b)\tilde{B}_{t-1} + (1 + r_t^a)\tilde{A}_{t-1}, \quad \text{and} \tag{B.40}
$$

$$
\begin{aligned}
\tilde{C}_t + \tilde{I}_t + \frac{1}{\nu_t} \frac{\phi}{2} \left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}} g_t - g^* \right)^2 \tilde{K}_{t-1} + \xi (1 + r_{t-1}) \tilde{B}_{t-1} \\
= \tilde{Y}_t + g_t \tilde{D}_t - (1 + r_{t-1}) \tilde{D}_{t-1}.
\end{aligned}
\tag{B.41}
$$

B.2.2 Detrended Equilibrium under Deterministic Paths of Aggregate Exogenous Variables

Given the initial conditions on $\tilde{\Psi}_0(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$, $\tilde{A}_{-1}$, $\tilde{K}_{-1}$, $\tilde{D}_{-1}$, $\tilde{B}_{-1}$, $\tilde{F}_{-1}$, r_{-1}, and deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$,

i) individual households' detrended policy functions $\{\tilde{c}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-), \tilde{b}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-), \tilde{a}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\}_{t=0}^{\infty}$, detrended first-order derivatives of the value functions $\{\tilde{V}_{b,t}(e_1, e_2, \tilde{b}_-, \tilde{a}_-), \tilde{V}_{a,t}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\}_{t=0}^{\infty}$, and detrended Lagrangian multipliers $\{\tilde{\lambda}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-), \tilde{\varphi}_t^b(e_1, e_2, \tilde{b}_-, \tilde{a}_-), \tilde{\varphi}_t^a(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\}_{t=0}^{\infty}$ that satisfy households' detrended optimality conditions (B.15), (B.16), (B.17), (B.18), (B.19), (B.20), (B.21), and (B.22),

ii) cross-sectional cumulative distributions $\{\tilde{\Psi}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\}_{t=1}^{\infty}$ that evolve over time according to equation (B.29),

iii) detrended aggregate variables $\{\tilde{C}_t, \tilde{B}_t, \tilde{A}_t, \tilde{\chi}_t^{agg}\}_{t=0}^{\infty}$ constructed by aggregating corresponding detrended individual variables according to equation (B.30),

iv) detrended prices and aggregate variables $\{r_t^b, r_t^a, r_t, \tilde{w}_t, \tilde{q}_t, \bar{l}_t, L_t, \tilde{\Pi}_t, \tilde{Y}_t, \tilde{I}_t, \tilde{K}_t, \tilde{F}_t, \tilde{D}_t, \tilde{T}B_t\}_{t=0}^{\infty}$ satisfying firms' optimality conditions (B.23), (B.24), (B.25), (B.26), (B.27), and (B.28), and other equilibrium conditions (B.31), (B.32), (B.33), (B.34), (B.36), (B.37), (B.38), and (B.39)

constitute the detrended equilibrium of the economy.

B.2.3 Steady State of the Detrended Equilibrium

I solve the detrended equilibrium using Auclert et al. (2019)'s method, and the first step is to solve its steady state. This subsection specifies the equilibrium conditions in the steady state of the detrended equilibrium. For any variable U_t and any function $F_t(\cdot)$, U_{ss} and $F_{ss}(\cdot)$ represent their steady state values, respectively.

The steady state values of the exogenous variables $\{z_t, g_t, \mu_t, \eta_t, \nu_t\}_{t=0}^{\infty}$ are determined as follows.

$$z_{ss} = 1, \quad g_{ss} = g^*, \quad \mu_{ss} = 1, \quad \eta_{ss} = 1, \quad \text{and} \quad \nu_{ss} = 1$$

By the definition of $\tilde{D}^*$ and $\tilde{Y}^*$, we have

$$\tilde{D}_{ss} = \tilde{D}^* \quad \text{and} \quad \tilde{Y}_{ss} = \tilde{Y}^*.$$

In the steady state, households' detrended policy functions $\tilde{c}_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$, $\tilde{b}_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$, and $\tilde{a}_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$, detrended first-order derivatives of the value functions $\tilde{V}_{b,ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$ and $\tilde{V}_{a,ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$, and detrended Lagrangian multipliers $\tilde{\lambda}_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$, $\tilde{\varphi}^b_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$, and $\tilde{\varphi}^a_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$ solve the following optimality conditions (B.42), (B.43), (B.44), (B.45), (B.46), (B.47), (B.48), and (B.49).

$$\tilde{\lambda} = \tilde{c}^{-\gamma}, \tag{B.42}$$

$$\tilde{\lambda} = \beta g_{ss}^{-\gamma} \sum_{e_1', e_2'} P(e_1', e_2' | e_1, e_2) \tilde{V}_{b,ss}(e_1', e_2', \tilde{b}, \tilde{a}) + \tilde{\varphi}^b, \tag{B.43}$$

$$\begin{aligned} \tilde{\lambda}\{1 + &\tilde{\chi}_{1,ss}(g_{ss}\tilde{a} - (1 + r^a_{ss})\tilde{a}_-, \tilde{a}_-)\} \\ &= \beta g_{ss}^{-\gamma} \sum_{e_1', e_2'} P(e_1', e_2' | e_1, e_2) \tilde{V}_{a,ss}(e_1', e_2', \tilde{b}, \tilde{a}) + \tilde{\varphi}^a, \end{aligned} \tag{B.44}$$

$$\tilde{V}_{b,ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = (1 - \xi)(1 + r^b_{ss})\tilde{\lambda}, \tag{B.45}$$

$$\begin{aligned} \tilde{V}_{a,ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-) = \tilde{\lambda}\{&(1 + r^a_{ss}) + (1 + r^a_{ss})\tilde{\chi}_{1,ss}(g_{ss}\tilde{a} - (1 + r^a_{ss})\tilde{a}_-, \tilde{a}_-) \\ &- \tilde{\chi}_{2,ss}(g_{ss}\tilde{a} - (1 + r^a_{ss})\tilde{a}_-, \tilde{a}_-)\}, \end{aligned} \tag{B.46}$$

$$\begin{aligned} \tilde{c} + g_{ss}\tilde{b} + g_{ss}\tilde{a} + &\tilde{\chi}_{ss}(g_{ss}\tilde{a} - (1 + r^a_{ss})\tilde{a}_-, \tilde{a}_-) \\ &= \tilde{w}_{ss} e \bar{l}_{ss} + (1 - \xi)(1 + r^b_{ss})\tilde{b}_- + (1 + r^a_{ss})\tilde{a}_-, \end{aligned} \tag{B.47}$$

$$\tilde{\varphi}^b \geq 0, \quad \tilde{b} \geq 0, \quad \tilde{\varphi}^b \tilde{b} = 0, \quad \text{and} \tag{B.48}$$

$$\tilde{\varphi}^a \geq 0, \quad \tilde{a} \geq 0, \quad \tilde{\varphi}^a \tilde{a} = 0. \tag{B.49}$$

The detrended optimality conditions of firms become the following equations in the steady state.

$$\tilde{\Pi}_{ss} = \tilde{Y}_{ss} - \tilde{w}_{ss} L_{ss} - \tilde{I}_{ss} - (1 + r_{ss} - g_{ss})\tilde{F}_{ss} + \tilde{\chi}^{agg}_{ss}, \tag{B.50}$$

$$\tilde{Y}_{ss} = z_{ss} g_{ss}^{1-\alpha} \tilde{K}_{ss}^{\alpha} L_{ss}^{1-\alpha}, \tag{B.51}$$

$$\tilde{I}_{ss} = (g_{ss} - 1 + \delta)\tilde{K}_{ss}, \tag{B.52}$$

$$r_{ss} + \delta = \alpha z_{ss} g_{ss}^{1-\alpha} \left(\frac{\tilde{K}_{ss}}{L_{ss}}\right)^{\alpha - 1} \quad \left(= \alpha \frac{\tilde{Y}_{ss}}{\tilde{K}_{ss}}\right), \tag{B.53}$$

$$r^a_{ss} = r_{ss}, \quad \text{and} \tag{B.54}$$

$$\tilde{w}_{ss} = (1 - \alpha) z_{ss} g_{ss}^{1-\alpha} \left(\frac{\tilde{K}_{ss}}{L_{ss}}\right)^{\alpha} \quad \left(= (1 - \alpha)\frac{\tilde{Y}_{ss}}{L_{ss}}\right). \tag{B.55}$$

The other detrended equilibrium conditions become the following equations in the steady state.

$$\tilde{\Psi}_{ss}(e_1', e_2', \tilde{b}, \tilde{a}) = \int_{e_1, e_2, \tilde{b}_-, \tilde{a}_-} P(e_{1,t+1} \le e_1' | e_{1,t} = e_1) P(e_{2,t+1} \le e_2') \\ I_{\{\tilde{b}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-) \le \tilde{b},\ \tilde{a}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-) \le \tilde{a}\}}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\, d\tilde{\Psi}_{ss}, \tag{B.56}$$

$$\begin{aligned} \tilde{C}_{ss} &= \int_{e_1, e_2, \tilde{b}_-, \tilde{a}_-} \tilde{c}_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\, d\tilde{\Psi}_{ss}, \\ \tilde{B}_{ss} &= \int_{e_1, e_2, \tilde{b}_-, \tilde{a}_-} \tilde{b}_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\, d\tilde{\Psi}_{ss}, \\ \tilde{A}_{ss} &= \int_{e_1, e_2, \tilde{b}_-, \tilde{a}_-} \tilde{a}_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-)\, d\tilde{\Psi}_{ss}, \quad \text{and} \\ \tilde{\chi}_{ss}^{agg} &= \int_{e_1, e_2, \tilde{b}_-, \tilde{a}_-} \tilde{\chi}_{ss}(g_{ss}\tilde{a}_{ss}(e_1, e_2, \tilde{b}_-, \tilde{a}_-) - (1 + r_{ss}^a)\tilde{a}_-, \tilde{a}_-)\, d\tilde{\Psi}_{ss}, \end{aligned} \tag{B.57}$$

$$\tilde{w}_{ss}\bar{e}^{1+\omega} = \kappa L_{ss}^{\omega}, \tag{B.58}$$

$$r_{ss}^b = r_{ss}, \tag{B.59}$$

$$\Pi_{ss} = (1 + r_{ss}^a - g_{ss})q_{ss}, \tag{B.60}$$

$$r_{ss} = r^*, \tag{B.61}$$

$$L_{ss} = \bar{e}\bar{l}_{ss}, \tag{B.62}$$

$$\tilde{F}_{ss} - \tilde{D}_{ss} = \tilde{B}_{ss}, \tag{B.63}$$

$$\tilde{q}_{ss} = \tilde{A}_{ss}, \quad \text{and} \tag{B.64}$$

$$T\tilde{B}_{ss} = (1 + r_{ss} - g_{ss})\tilde{D}_{ss}. \tag{B.65}$$

In addition, the detrended aggregated budget constraint of households (B.40) and the detrended resource constraint (B.41) become the following equations in the steady state.

$$\tilde{C}_{ss} + g_{ss}\tilde{B}_{ss} + g_{ss}\tilde{A}_{ss} + \tilde{\chi}_{ss}^{agg} = \tilde{w}_{ss}\bar{e}\bar{l}_t + (1 - \xi)(1 + r_{ss}^b)\tilde{B}_{ss} + (1 + r_{ss}^a)\tilde{A}_{ss}, \quad \text{and} \tag{B.66}$$

$$\tilde{C}_{ss} + \tilde{I}_{ss} + \xi(1 + r_{ss})\tilde{B}_{ss} = \tilde{Y}_{ss} - (1 + r_{ss} - g_{ss})\tilde{D}_{ss}. \tag{B.67}$$

By combining equations (B.60), (B.64), (B.50), (B.54), (B.55), (B.52), (B.53), and (B.63), we can obtain the following relationship among stock variables in the steady state.

$$\tilde{K}_{ss} + \frac{1}{1 + r_{ss} - g_{ss}} \tilde{\chi}_{ss}^{agg} = \tilde{A}_{ss} + \tilde{B}_{ss} + \tilde{D}_{ss} \tag{B.68}$$

B.3 Solving the Detrended Equilibrium using Auclert et al. (2019)'s Method

I solve the detrended equilibrium using Auclert et al. (2019)'s method. In this subsection, I briefly describe how this method solves the equilibrium. The first step is to solve heterogeneous households' policy functions and the stationary distribution over the household heterogeneity in the steady state. For this step, Auclert et al. (2019) develop a fast algorithm that extends Carroll (2006)'s method of endogenous gridpoints to the two-asset environment in order to solve the steady state of their two-asset HANK model. Since the household block of my model is almost identical to the household block of their model, I closely follow this algorithm to compute the steady state of my model. See Appendix B.1 of Auclert et al. (2019) for the details of this algorithm.[63]

Once the steady state is pinned down, Auclert et al. (2019)'s method computes the Jacobians of 'blocks'. Here, a 'block' is a function that maps the sequences of input variables $\{x_{1,t}, \cdots, x_{n_x,t}\}_{t=0}^{T}$ into the sequences of output variables $\{y_{1,t}, \cdots, y_{n_y,t}\}_{t=0}^{T}$ using a subset of equilibrium conditions. The Jacobian of the block is a matrix composed of the following elements. $\{\frac{\partial y_{j,s}}{\partial x_{i,t}}\}_{1\le i\le n_x,\ 1\le j\le n_y,\ 0\le s,t\le T}$. For example, the household block of my model maps the sequences of its input variables $\{\tilde{w}_t, r_t^a, r_t^b, g_t, \bar{l}_t, \eta_t\}_{t=0}^{T}$ into the sequences of its output variables $\{\tilde{C}_t, \tilde{B}_t, \tilde{A}_t\}_{t=0}^{T}$ using equilibrium conditions (B.15), (B.16), (B.17), (B.18), (B.19), (B.20), (B.21), and (B.22). The Jacobian of the household block is composed of $\left\{\frac{\partial y_s}{\partial x_t}\right\}_{x\in\{\tilde{w},r^a,r^b,g,\bar{l},\eta\},\ y\in\{\tilde{C},\tilde{B},\tilde{A}\},\ 0\le s,t\le T}$.

Figure B.1 is the directed acyclical graph (DAG) representation of the detrended equilibrium, in which blocks, input variables, and output variables are indicated. Each of the blue rectangles and red ellipses in the figure represent the blocks of the equilibrium. For each block, variables coming into the block and variables coming out of the block (indicated by arrows connecting blocks) are inputs and outputs of the block, respectively. Within each block, the bullet points and following parentheses indicate the names of the equilibrium conditions, corresponding equation numbers, and output variables pinned down by the equilibrium conditions.

Following the notations of Auclert et al. (2019), let Z be a stacked vector of the sequences of exogenous variables and U be a stacked vector of the sequences of unknown variables indicated in the black diamond box in Figure B.1. Moreover, let $H(U, Z)$ be a

[63]For grids, I use 9 gridpoints for e_1, 13 gridpoints for e_2, and 70 gridpoints for each of $\tilde{b}_-$ and $\tilde{a}_-$.

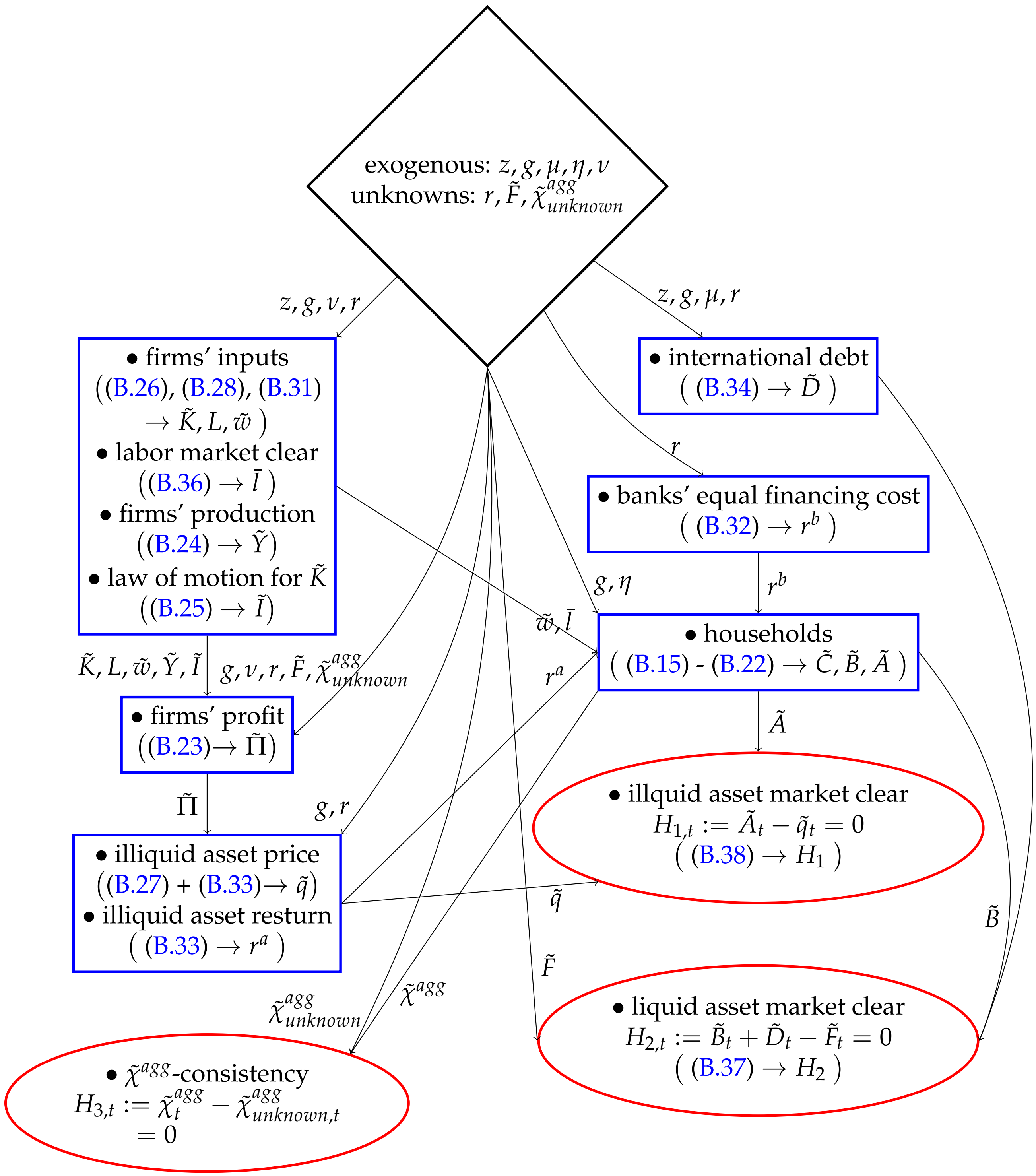


Figure B.1: Directed Acyclical Graph (DAG) representation of the detrended equilibrium function that maps U and Z into a stacked vector of $\{H_{1,t}, H_{2,t}, H_{3,t}\}_{t=0}^{T}$ in which $H_{1,t}$,$H_{2,t}$, and $H_{3,t}$ are defined as

$$H_{1,t} = \tilde{A}_t - \tilde{q}_t, \quad H_{2,t} = \tilde{B}_t + \tilde{D}_t - \tilde{F}_t, \quad \text{and} \quad H_{3,t} = \tilde{\chi}_t^{agg} - \tilde{\chi}_{unknown,t}^{agg},$$

as indicated in the red ellipses of Figure B.1.

Under this formulation, 'solving the model' boils down to finding U that satisfies

$$H(U, Z) = 0$$

for given Z. Under the first-order approximation, this equation becomes

$$H_U dU + H_Z dZ = 0$$

$$\Leftrightarrow dU = -H_U^{-1} H_Z dZ. \tag{B.69}$$

By combining the Jacobians of the blocks through the Chain Rule, Auclert et al. (2019)'s method computes H_U and H_Z. Then, the method solves dU using equation (B.69), and recovers the linearized dynamics of other variables by again combining the Jacobians through the Chain Rule along the directed acyclical graph in Figure B.1.

In the whole computation procedure, the most time-consuming steps are i) solving the steady state and ii) computing the Jacobian of the household block. In particular, calibrating β, χ_1, and χ_2 requires solving the steady state multiple times, and this step takes longer than a day. However, once these parameters are calibrated and I have both the computed steady state and the Jacobian of the household block in my hand, the rest of the computation steps are very quick (taking less than a second). This is why the Bayesian estimation of the model is possible as long as the parameters to be estimated are not inside the household block.

B.4 Recovering the Original Equilibrium

Once the detrended equilibrium is solved, we can recover the original equilibrium. In particular, the figures and tables in this paper report results based on the following statistics of the original equilibrium: i) the standard deviations and correlations of $\Delta \log Y_t$, $\Delta \log C_t$, $\Delta \log I_t$, and $\Delta(TB_t/Y_t)$, and ii) the impulse responses of the model variables in terms of their deviations from the balanced growth path.

B.4.1 $\Delta \log Y_t$, $\Delta \log C_t$, $\Delta \log I_t$, and $\Delta(TB_t/Y_t)$ of the original equilibrium

$\Delta \log Y_t$, $\Delta \log C_t$, $\Delta \log I_t$, and $\Delta(TB_t/Y_t)$ of the original equilibrium are recovered using the following equations.

$$\Delta \log Y_t = \Delta \log \tilde{Y}_t - \Delta \log \tilde{Y}_{t-1} + \log g_{t-1},$$

$$\Delta \log C_t = \Delta \log \tilde{C}_t - \Delta \log \tilde{C}_{t-1} + \log g_{t-1},$$

$$\Delta \log I_t = \Delta \log \tilde{I}_t - \Delta \log \tilde{I}_{t-1} + \log g_{t-1}, \quad \text{and}$$

$$\Delta(TB_t/Y_t) = \frac{\tilde{TB}_t}{\tilde{Y}_t} - \frac{\tilde{TB}_{t-1}}{\tilde{Y}_{t-1}}.$$

B.4.2 Impulse Responses in terms of Deviations from the Balanced Growth Path

The impulse responses of the model variables in terms of their deviations from the balanced growth path in the original equilibrium are recovered as follows. First, I define the 'constant-growth trend of the balanced growth path' X_t^* as follows. Given that a shock is realized in period 0,

$$X_t^* = (g^*)^{t+1} X_{-1}.$$

Let M_t^f be one of the flow variables in the original equilibrium, which is detrended with X_{t-1} in constructing the detrended equilibrium. (Variables Y_t, C_t, I_t, Π_t, and w_t belong to this category.) Let $\tilde{M}_t^f := M_t^f / X_{t-1}$ be the detrended variable, and $\tilde{M}_{ss}^f$ be the steady state value of $\tilde{M}_t^f$ in the detrended equilibrium. $\{M_t^f\}_{t=0}^{\infty}$ on the balanced growth path, which I denote as $\{M_t^{f*}\}_{t=0}^{\infty}$, is determined by

$$M_t^{f*} = \tilde{M}_{ss}^f X_{t-1}^*, \quad t \geq 0.$$

This is the path of $\{M_t^f\}_{t=0}^{\infty}$ when there is no shock in period 0. The impulse response of M_t^f in terms of their ratio deviations from the balanced growth path is constructed by

$$IRF_{M^f}(t) = \frac{M_t^f - M_t^{f*}}{M_t^{f*}} = \frac{M_t^f / X_{t-1}^* - \tilde{M}_{ss}^f}{\tilde{M}_{ss}^f}.$$

I compute this impulse response as follows. By solving the detrended equilibrium using Auclert et al. (2019)'s method, I obtain $d\tilde{M}_t^f = \tilde{M}_t^f - \tilde{M}_{ss}^f$, in which the d-operator on the left hand side means the level deviation from the steady state of the detrended equilibrium. Then, I use the following equation, which holds under the first-order approximation, to obtain $d(M_t^f/X_{t-1}^*) = M_t^f/X_{t-1}^* - \tilde{M}_{ss}^f$.

$$\begin{aligned}
d(M_t^f/X_{t-1}^*) &= d(\tilde{M}_t^f(X_{t-1}/X_{t-1}^*)) \\
&= d\tilde{M}_t^f + \tilde{M}_{ss}^f\, d(X_{t-1}/X_{t-1}^*) \\
&= d\tilde{M}_t^f + \tilde{M}_{ss}^f\, d\left(\frac{g_0}{g^*}\frac{g_1}{g^*}\cdots\frac{g_{t-1}}{g^*}\right) \\
&= d\tilde{M}_t^f + \frac{\tilde{M}_{ss}^f}{g^*}\left(\sum_{j=0}^{t-1} dg_j\right).
\end{aligned}$$

By dividing $d(M_t^f/X_{t-1}^*)$ with $\tilde{M}_{ss}^f$, I obtain $IRF_{M^f}(t)$.

Impulse responses for the stock variables can be computed in a similar way. Let M_t^s be one of the stock variables in the original equilibrium, which is detrended with X_t in the detrended equilibrium. (Variables K_t, A_t, B_t, D_t, and F_t belong to this category.) Let $\tilde{M}_t^s := M_t^s/X_t$ be the detrended variable, and $\tilde{M}_{ss}^s$ be the steady state value of $\tilde{M}_t^s$ in the detrended equilibrium. $\{M_t^s\}_{t=0}^\infty$ on the balanced growth path, which I denote as $\{M_t^{s*}\}_{t=0}^\infty$, is determined by

$$M_t^{s*} = \tilde{M}_{ss}^s X_t^*, \quad t \geq 0.$$

The impulse response of M_t^s in terms of their ratio deviations from the balanced growth path is constructed by

$$IRF_{M^s}(t) = \frac{M_t^s - M_t^{s*}}{M_t^{s*}} = \frac{M_t^s/X_t^* - \tilde{M}_{ss}^s}{\tilde{M}_{ss}^s}.$$

After obtaining $d\tilde{M}_t^s = \tilde{M}_t^s - \tilde{M}_{ss}^s$ by solving the detrended equilibrium with Auclert et al. (2019)'s method, I compute $d(M_t^s/X_t^*) = M_t^s/X_t^* - \tilde{M}_{ss}^s$ using the following first-order-approximated equation.

$$d(M_t^s/X_t^*) = d\tilde{M}_t^s + \frac{\tilde{M}_{ss}^s}{g^*}\left(\sum_{j=0}^{t} dg_j\right).$$

By dividing $d(M_t^s/X_t^*)$ with $\tilde{M}_{ss}^s$, I obtain $IRF_{M^s}(t)$.

There are variables in the original equilibrium that are not detrended in the detrended equilibrium. (Variables r_t^a, r_t^b, r_t, and L_t belong to this category). Let M_t^n be one of such variables. By construction, these variables have the same steady state values between the original equilibrium and the detrended equilibrium. For their impulse responses, I use their level deviations from their steady state values, $dM_t^n = M_t^n - M_{ss}^n$.[64]

[64]In all the impulse response plots reported in this paper, I indicate in the label of the y-axis whether the

C Cross-autocorrelogram

This section compares the cross-autocorrelograms between the model (the baseline economy) and the data.

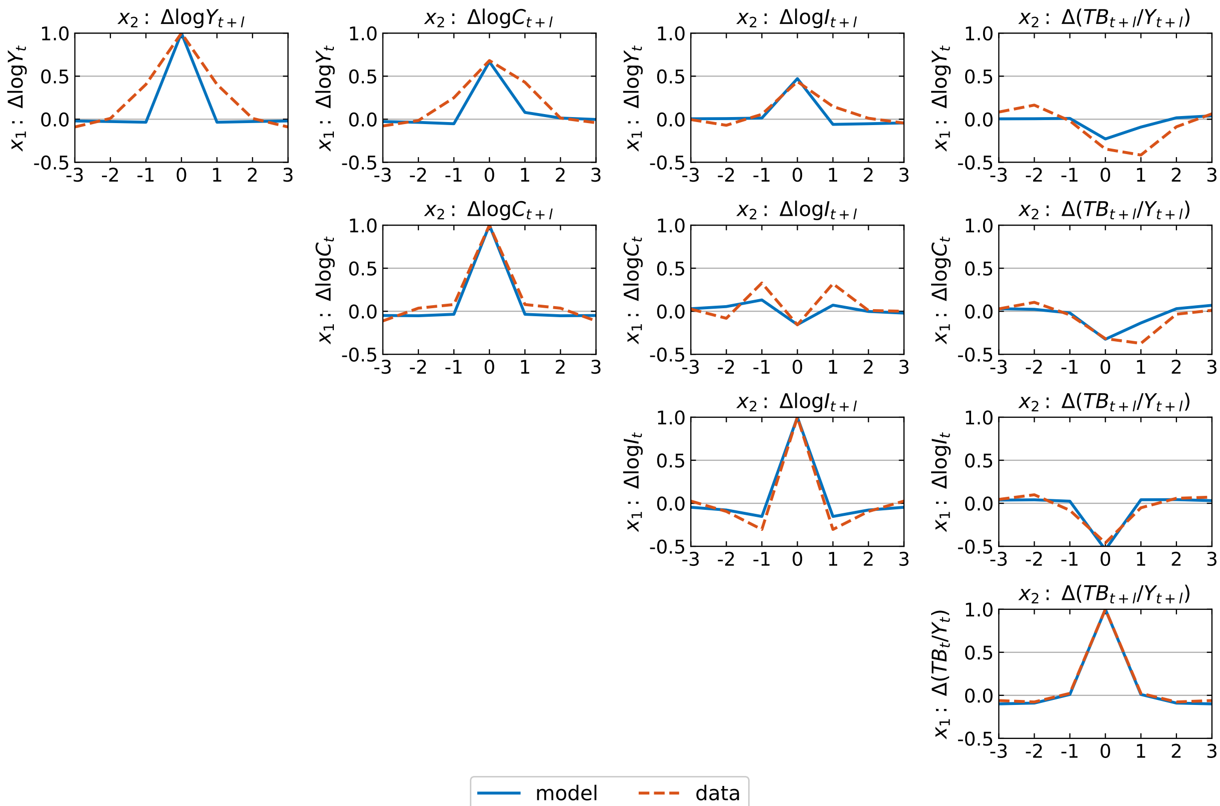


Figure C.1: cross-autocorrelogram $corr[x_{1,t}, x_{2,t+l}]$: model vs data

Notes: This figure plots the cross-autocorrelograms computed from the Peruvian macro data and those generated from the model after the calibration and Bayesian estimation discussed in section IV. The model statistics are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.

D Broader Recalibration for the Counterfactual Economy

I only recalibrate χ_1, χ_2, and β in the counterfactual experiment of the main text in section V. Although χ_1, χ_2, and β are key determinants of the MPCs in the model, there are other parameters that also affect MPCs. Such parameters include the difference in return rates between liquid and illiquid assets ξ, and the parameters governing the labor

ratio deviations are plotted or the level deviations are plotted.

Table D.1: Broader Recalibration for the Counterfactual Economy

	Description	Value	Target / source
labor income process			
ρ_{e_1}	persistence of the AR(1) component	0.988	PSID
σ_{e_1}	S.D. of shocks to the AR(1) component	0.073	
σ_{e_2}	S.D. of shocks to the *i.i.d.* component	0.605	
long-run averages			
ξ	long-run average spread	0.006	FRB, OECD, U.S. CPI
targeting MPCs over the labor income deciles & Aggregate Wealth			
β	discount factor	0.970	MPC estimates (from PSID) and Peruvian TB/Y
χ_1	scale parameter of illiquid adj. cost	0.204	
χ_2	convexity parameter of illiquid adj. cost	1.365	

productivity process ρ_{e_1}, σ_{e_1}, and σ_{e_2}. In this section, I run an alternative counterfactual experiment in which these parameters are also recalibrated. Specifically, I recalibrate ξ, ρ_{e_1}, σ_{e_1}, and σ_{e_2} using relevant U.S. data first and then recalibrate χ_1, χ_2, and β by targeting the U.S. MPC estimates and the Peruvian trade-balance-to-output ratio. Table D.1 reports the recalibrated values of the parameters.

Parameters ρ_{e_1}, σ_{e_1}, and σ_{e_2} are recalibrated using the PSID data. Note that these parameters specify the quarterly labor productivity process in the model, while the PSID provides annual data. Given the frequency mismatch between the model and the data, I estimate these parameters according to the following steps. First, I estimate the annual income process from the PSID sample using Floden and Lindé (2001)'s method. Then, I find parameters ρ_{e_1}, σ_{e_1}, and σ_{e_2} for the quarterly labor productivity process that yield the same estimation result when I simulate annual income series by aggregating the model-generated quarterly income series over every four quarters and apply to the simulated annual series the same estimation procedure applied to the PSID data.

Parameter ξ is recalibrated to match the gap between U.S. real lending rates and deposit rates. The real lending rates and deposit rates are constructed by subtracting the expected inflation of the U.S. CPIs from the nominal lending rates and deposit rates, respectively. For the nominal lending rates, bank prime loan rates from Board of Governors of the Federal Reserve System (retrieved from FRED, Federal Reserve Bank of St. Louis) are used. For the nominal deposit rates, 3-month rates on Certificates of Deposit from OECD (retrieved from FRED, Federal Reserve Bank of St. Louis) are used.[65]

[65] Although these two data series come from different sources, the real lending rates and real deposit rates constructed from each data series track each other very closely once the gross rates of the former are

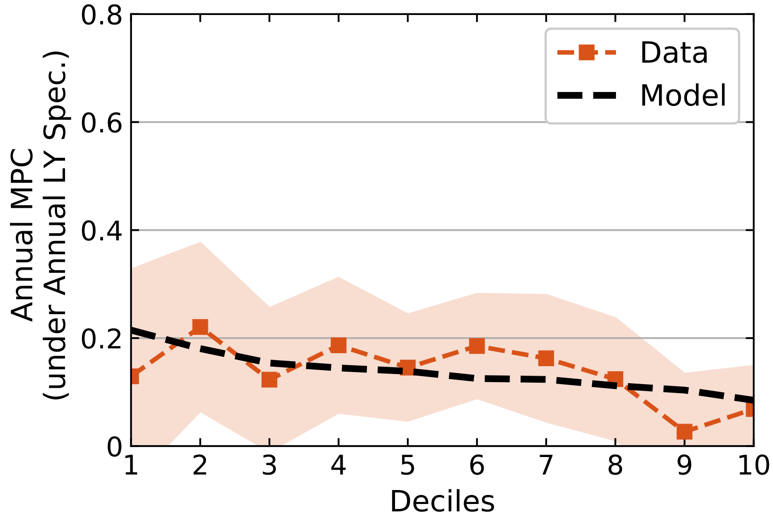


Figure D.1: Annual MPCs in U.S.: Data vs Model
under Broader Recalibration

Notes: Figure D.1 plots the annual MPC estimates from the PSID and their model counterparts under the broader recalibration in Appendix D. In computing the model counterparts of the estimates, I simulate annual consumption and income series by aggregating model-generated quarterly consumption and income series over every four quarters and then apply to the simulated annual data the same MPC estimation procedure applied to the PSID data.

Once parameters ρ_{e_1}, σ_{e_1}, σ_{e_2}, and ξ are recalibrated, I recalibrate parameters χ_1, χ_2, and β by targeting the U.S. MPC estimates of the labor income deciles and the Peruvian trade-balance-to-output ratio. Again, the calibration is successful despite the fact that I only use three parameters to target eleven moments. The model-generated trade-balance-to-output ratio on the balanced growth path is 0.042, and its data counterpart is 0.043. Moreover, Figure D.1 shows that the model-generated U.S. MPCs of the labor income deciles closely track the data counterparts.

Figure D.2 shows that under the broader recalibration of the counterfactual economy, the following patterns robustly appear: i) the MPC gap between Peru and the U.S. predicted by the model is narrower than that predicted by the model-free frequency conversion in Figure 1, and ii) despite this tendency, the model still predicts a substantial MPC gap between Peru and the U.S. In fact, the MPC gap under the broader recalibration in Figure D.2 is slightly larger than the gap under the benchmark recalibration in Figure 4.

scaled by $(1-\xi)$.

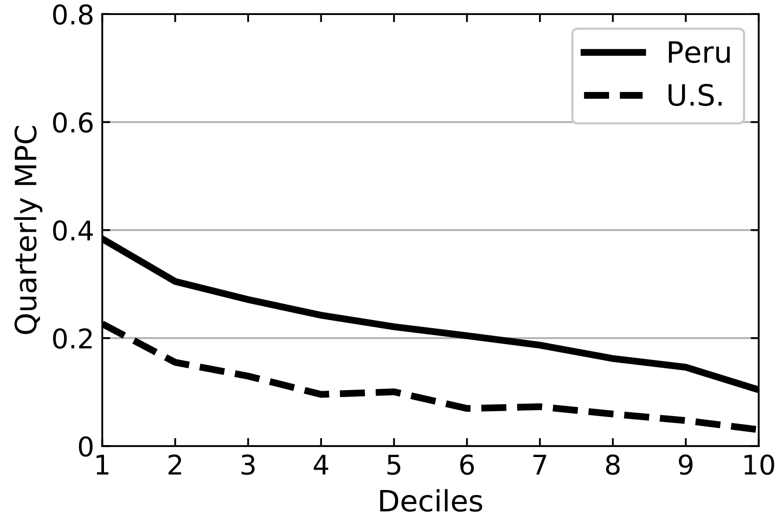


Figure D.2: Model-Predicted Quarterly MPCs: Peru and U.S.
under Broader Recalibration

Notes: Figure D.2 compares the model-predicted quarterly MPCs between the baseline economy and the counterfactual economy under the broader recalibration in Appendix D.

Table D.2 compares the standard deviations of output growth, consumption growth, and the ratio of the two between the baseline economy and the counterfactual economy under the broader recalibration. As we can see from this table, the phenomenon of excess consumption volatility robustly disappears in this alternative counterfactual experiment.

In the rest of this section, I also report the results of variance change decomposition and consumption response decomposition in this alternative counterfactual experiment under the broader recalibration. These results verify that all the main observations from

Table D.2: Absence of Excess Consumption Volatility in the Counterfactual Economy
under Broader Recalibration

	$\sigma(\Delta \log Y_t)$	$\sigma(\Delta \log C_t^{msd})$	$\frac{\sigma(\Delta \log C_t^{msd})}{\sigma(\Delta \log Y_t)}$
Baseline	0.029	0.037	1.283
	(0.002)	(0.002)	(0.075)
Counterfactual	0.028	0.026	0.914
	(0.002)	(0.002)	(0.071)

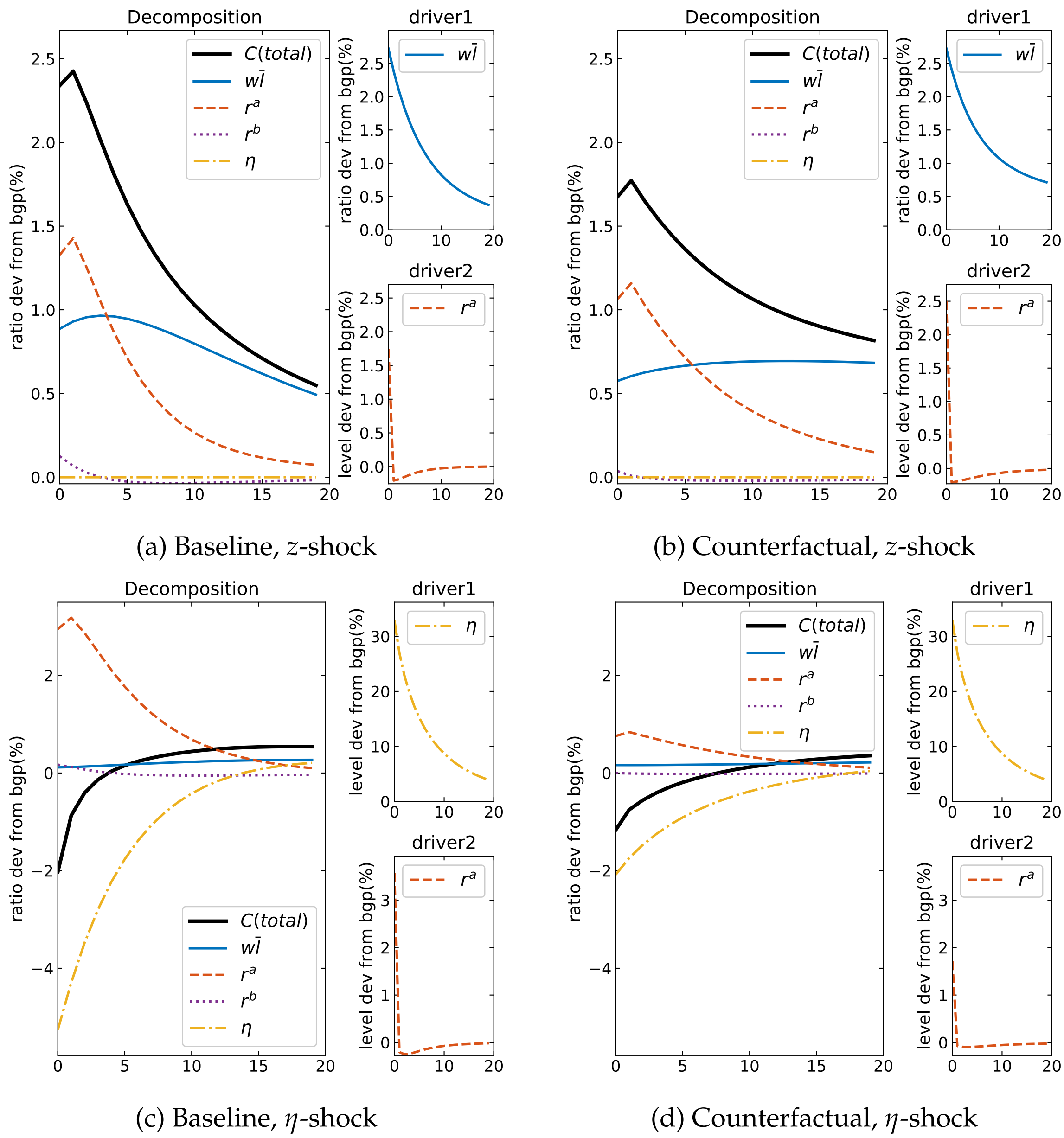
Notes: The statistics are computed under each posterior draw, and their means and standard deviations over the posterior distribution are reported in this table. The numbers in parentheses are the posterior standard deviations.

variance change decomposition and consumption response decomposition discussed in the main text remain unchanged.

Table D.3: Variance Change Decomposition under Broader Recalibration (from Baseline to Counterfactual)

	$\Delta \log Y_t$	$\Delta \log C_t^{msd}$
stationary productivity shock (z_t)	-0.012	-0.207
	(0.000)	(0.021)
trend shock (g_t)	-0.011	0.049
	(0.008)	(0.024)
interest rate shock (μ_t)	0.000	-0.001
	(0.000)	(0.001)
illiquidity shock (η_t)	-0.001	-0.296
	(0.001)	(0.050)
investment shock (ν_t)	-0.000	-0.049
	(0.000)	(0.020)
variance change (in ratio)	-0.025	-0.504
	(0.008)	(0.049)

Notes: The last row reports the fraction of [(variance change from the baseline economy to the counterfactual economy) / (variance in the baseline economy)]. The first five rows report the fraction of [(variance change generated by each shock) / (variance in the baseline economy)], in which the denominator is the variance generated by all shocks (*i.e.*, the same denominator used in the fraction reported in the last row.) By construction, the last row is the sum of the first five rows. The statistics are computed under each posterior draw, and their means and standard deviations over the posterior distribution are reported in this table. The numbers in parentheses are the posterior standard deviations.



(a) Baseline, z-shock

(b) Counterfactual, z-shock

(c) Baseline, η-shock

(d) Counterfactual, η-shock

Figure D.3: Decomposition of the Consumption Responses to the z-shock and η-shock under Broader Recalibration

Notes: Panels D.3a, D.3b, D.3c, and D.3d present the consumption response decomposition with respect to stationary productivity shocks (z) and illiquidity shocks (η) in the baseline economy and the counterfactual economy, respectively. Each panel consists of three subplots, where the large subplot on the left shows the total consumption response as well as decomposed consumption responses to each driver of $\{w_t \bar{l}_t, r_t^a, r_t^b, \eta_t\}_{t=0}^{\infty}$, and the other two small subplots on the right show the equilibrium paths of the two main drivers after the shock. The consumption responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.

E Impulse Response Functions (IRFs)

E.1 Baseline Economy

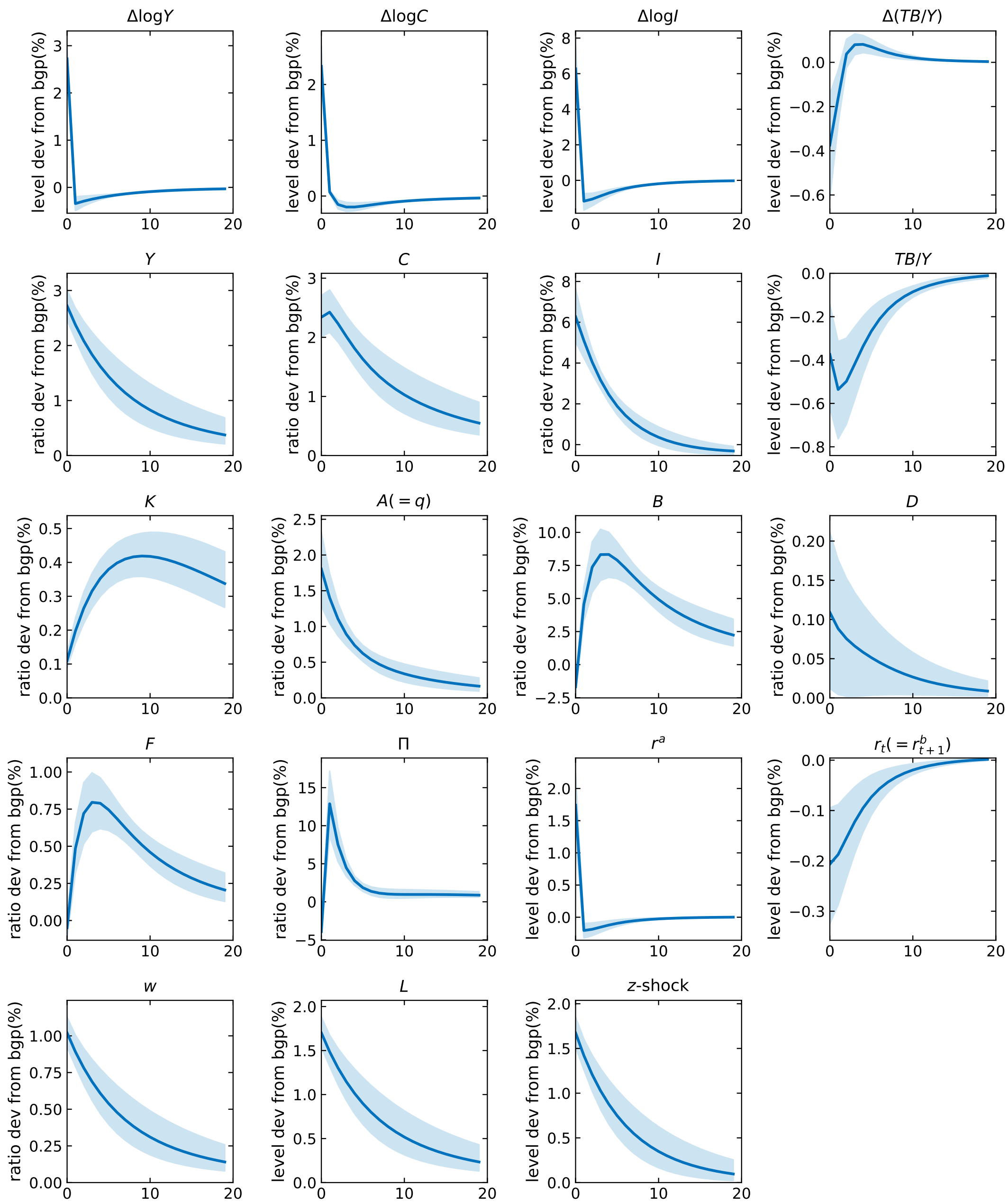


Figure E.1: IRFs to a 1 S.D. Stationary Productivity Shock (z): Baseline Economy

Notes: In each plot, the blue solid line represents the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure. The shaded area in each plot represents 90% credible bands over the posterior distribution.

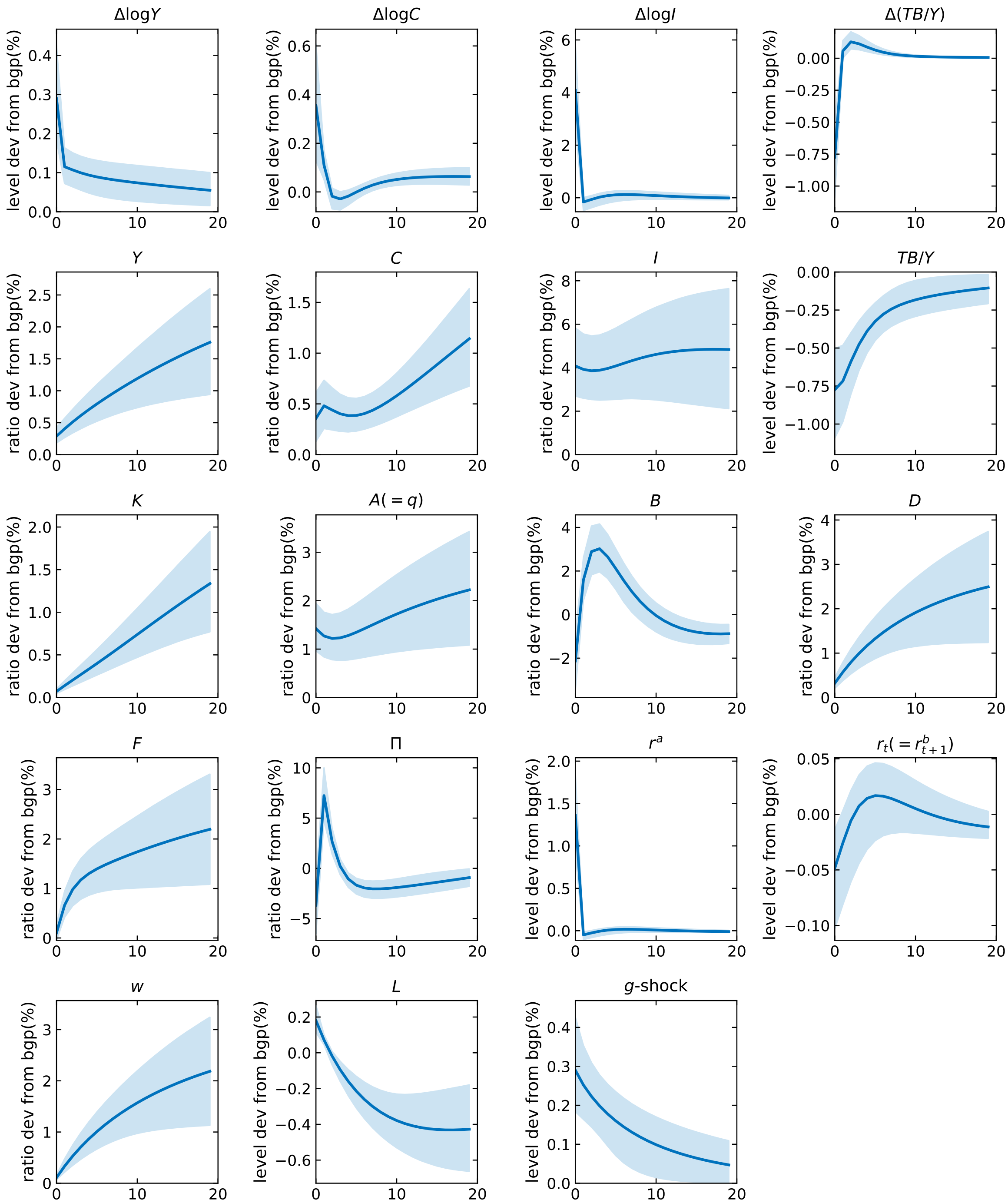


Figure E.2: IRFs to a 1 S.D. Trend Shock (g): Baseline Economy

Notes: In each plot, the blue solid line represents the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure. The shaded area in each plot represents 90% credible bands over the posterior distribution.

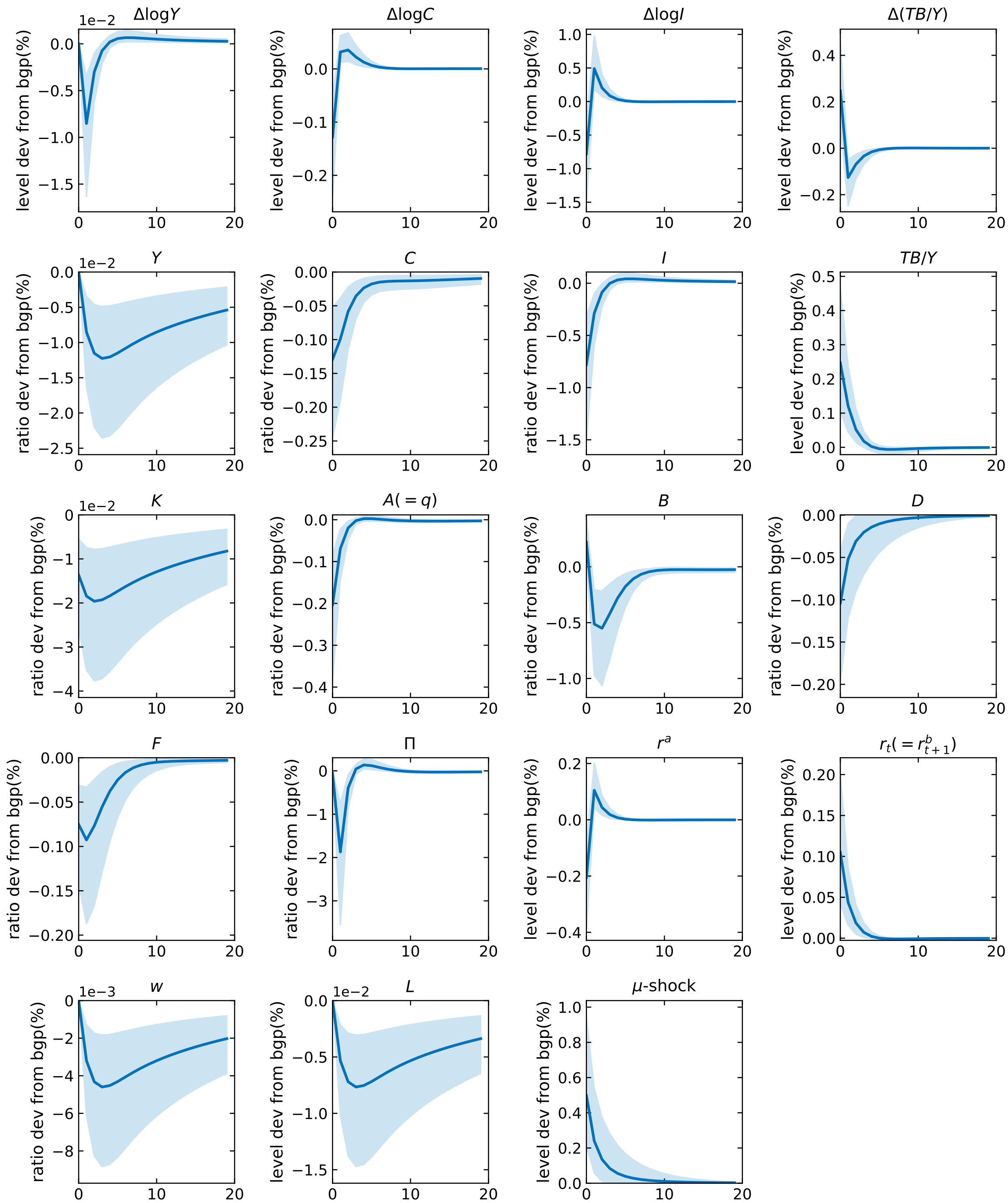


Figure E.3: IRFs to a 1 S.D. Interest Rate Shock (μ): Baseline Economy

Notes: In each plot, the blue solid line represents the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure. The shaded area in each plot represents 90% credible bands over the posterior distribution.

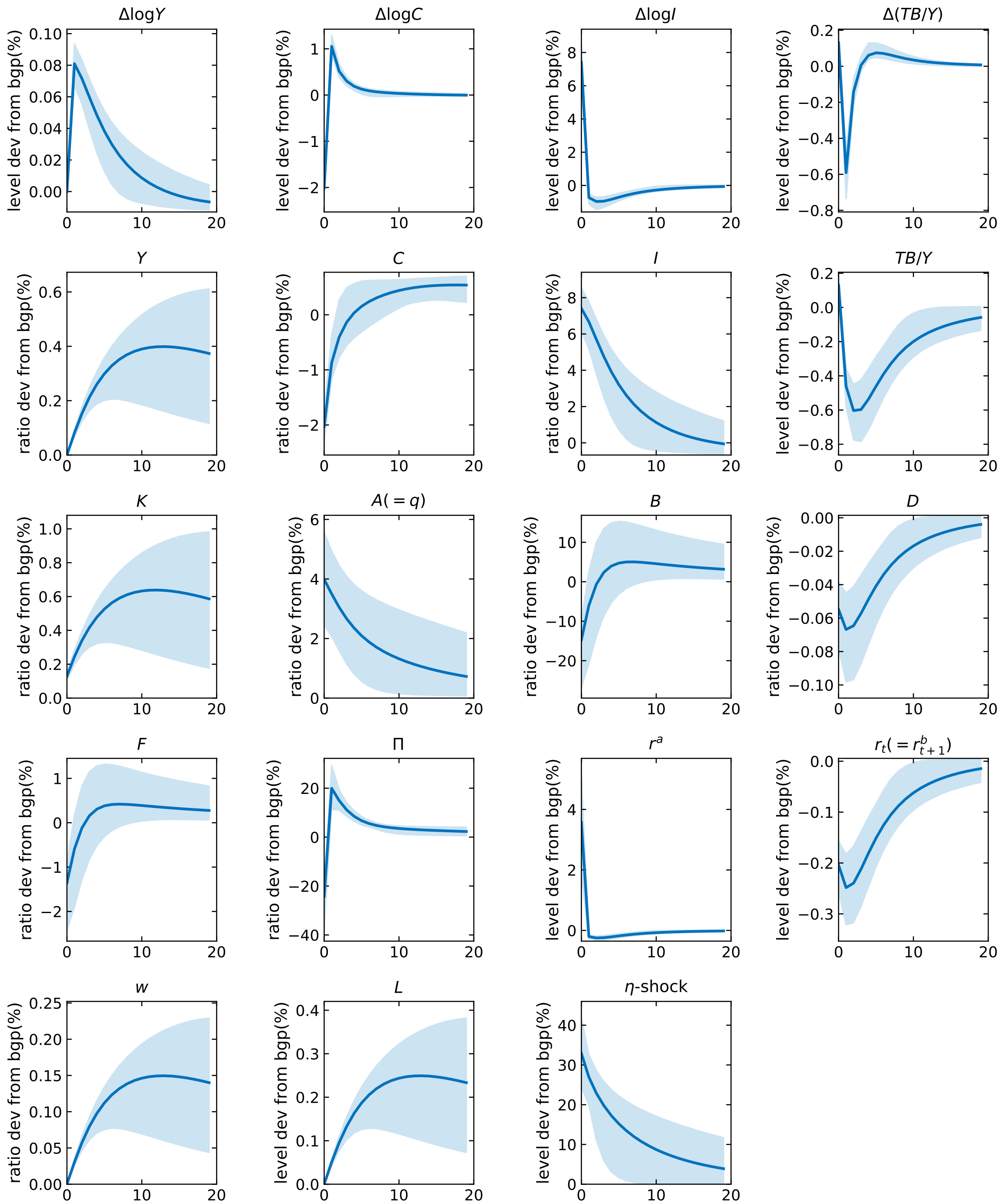


Figure E.4: IRFs to a 1 S.D. Illiquidity Shock (η): Baseline Economy

Notes: In each plot, the blue solid line represents the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure. The shaded area in each plot represents 90% credible bands over the posterior distribution.

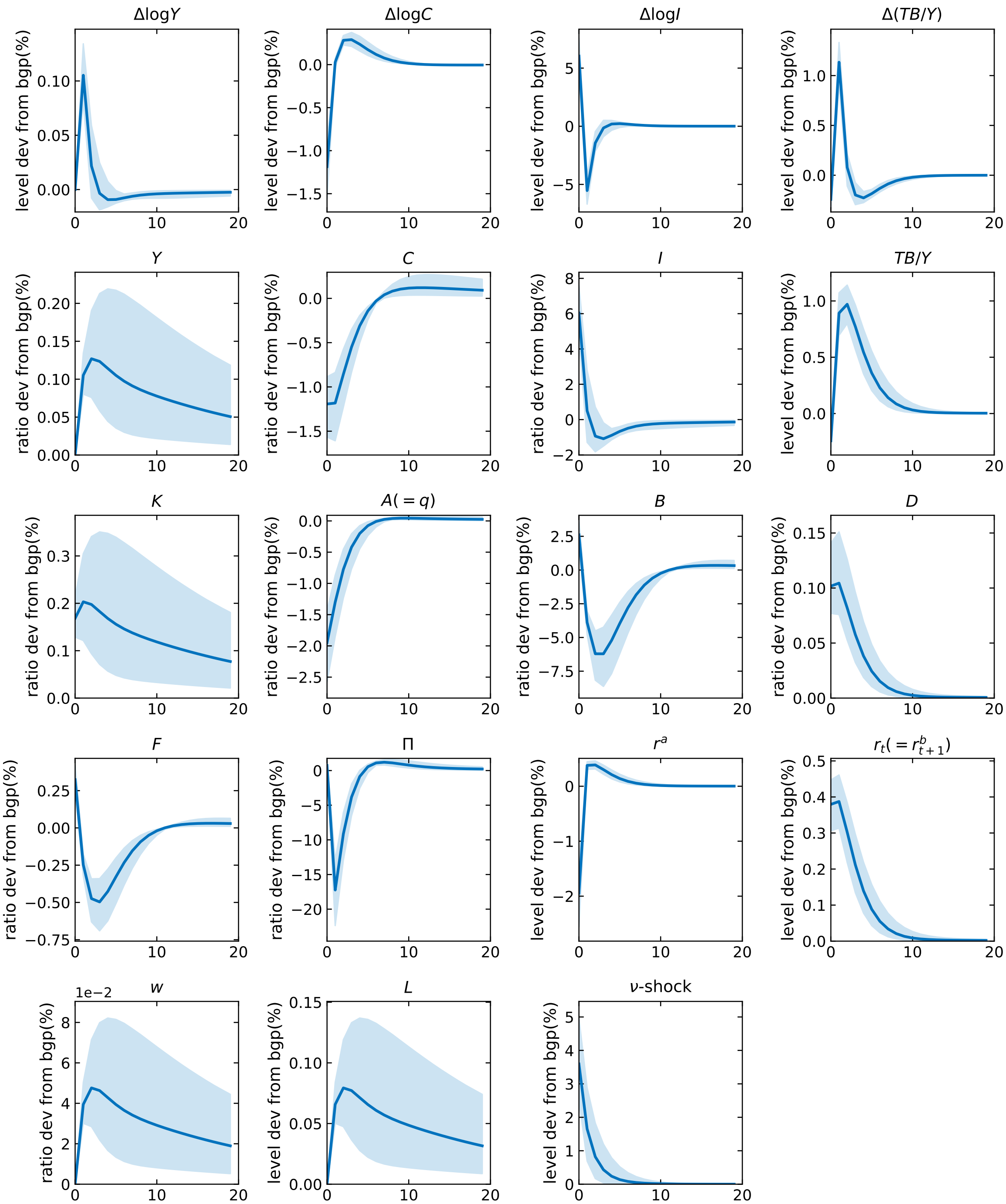


Figure E.5: IRFs to a 1 S.D. Investment Shock (ν): Baseline Economy

Notes: In each plot, the blue solid line represents the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure. The shaded area in each plot represents 90% credible bands over the posterior distribution.

E.2 Baseline vs Counterfactual

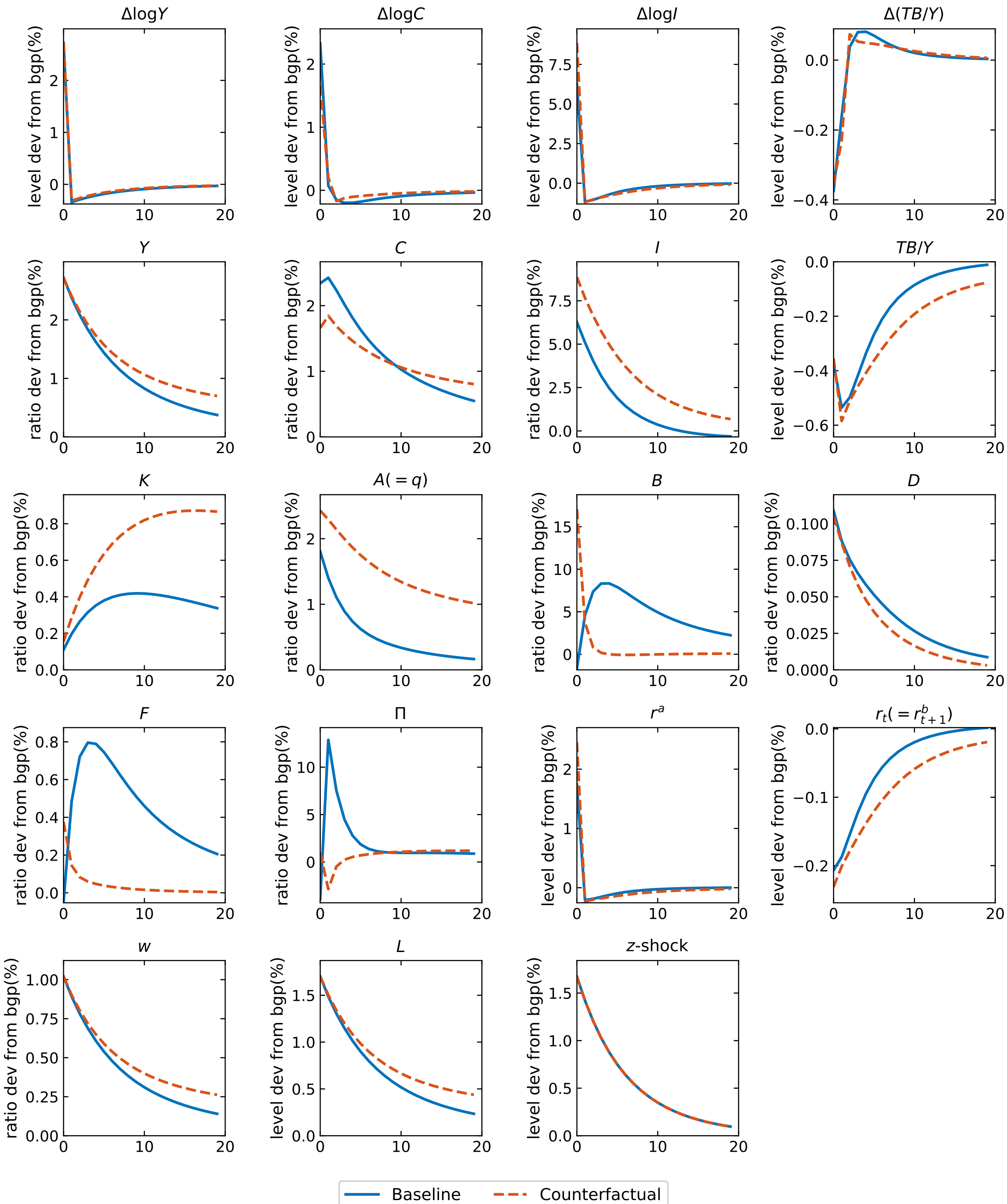


Figure E.6: IRFs to a 1 S.D. Stationary Productivity Shock (z): Baseline vs Counterfactual

Notes: In each plot, the blue solid line and the red dashed line represent the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock in the baseline economy and the counterfactual economy, respectively. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.

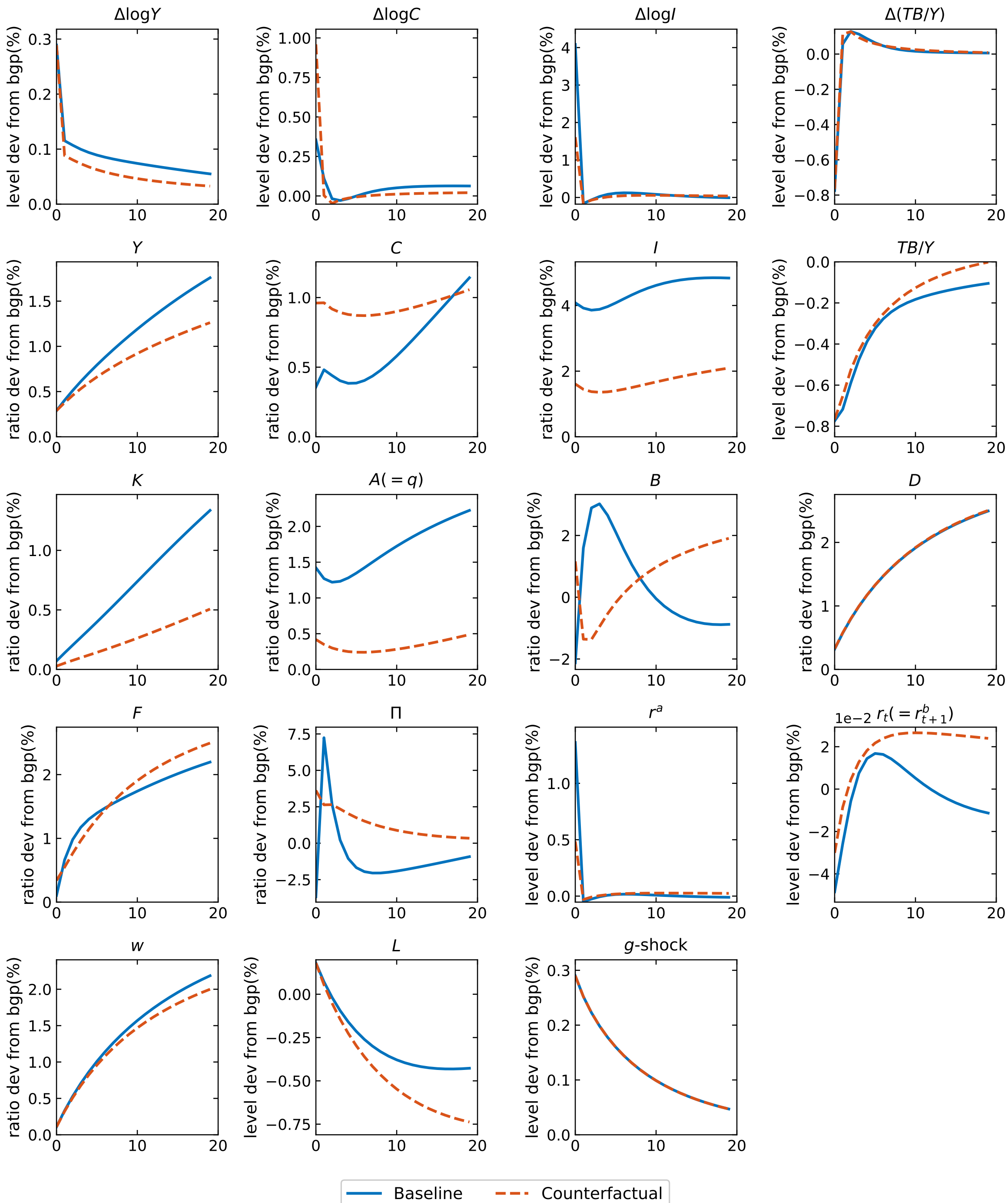


Figure E.7: IRFs to a 1 S.D. Trend Shock (g): Baseline vs Counterfactual

Notes: In each plot, the blue solid line and the red dashed line represent the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock in the baseline economy and the counterfactual economy, respectively. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.

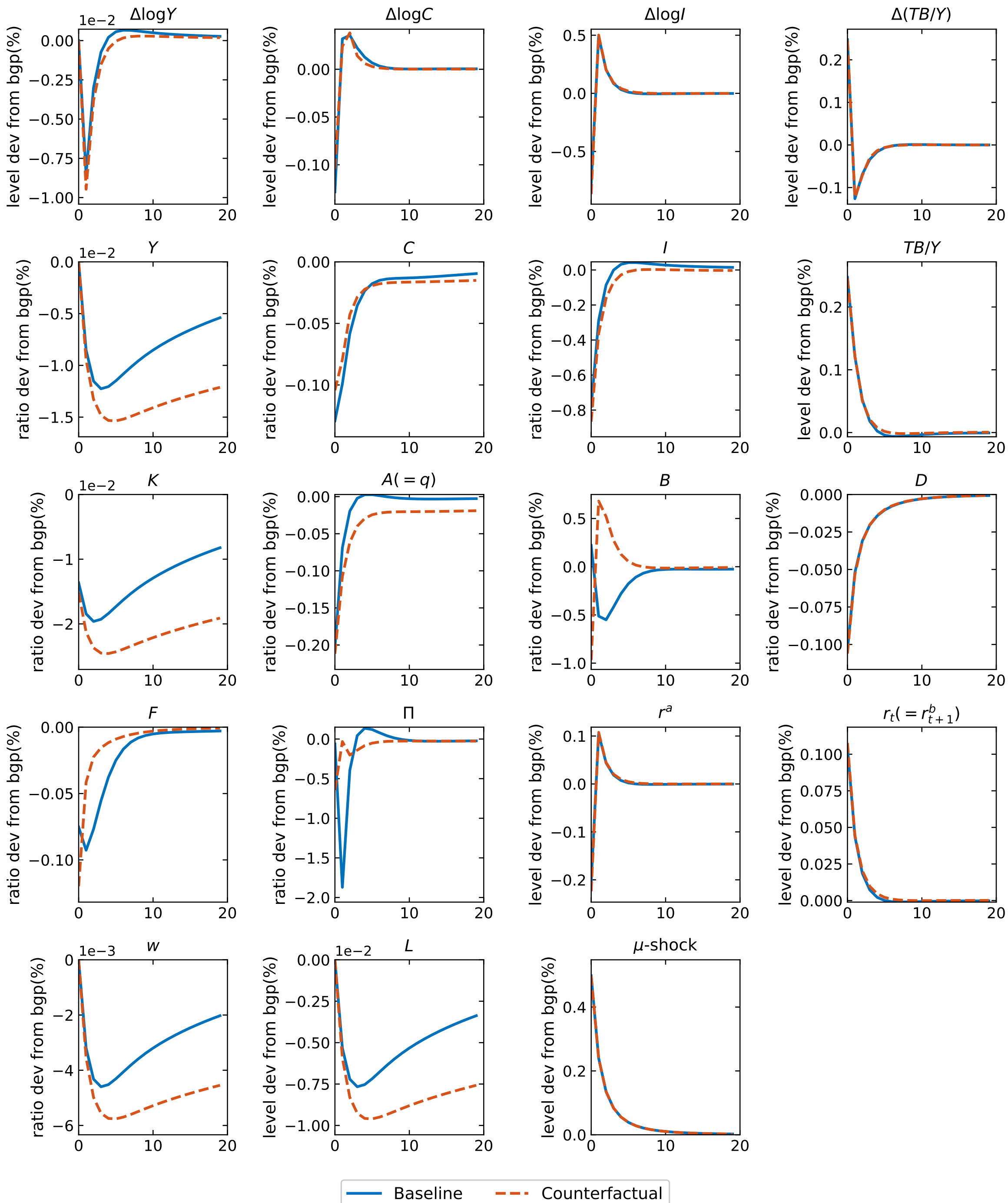


Figure E.8: IRFs to a 1 S.D. Interest Rate Shock (μ): Baseline vs Counterfactual

Notes: In each plot, the blue solid line and the red dashed line represent the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock in the baseline economy and the counterfactual economy, respectively. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.

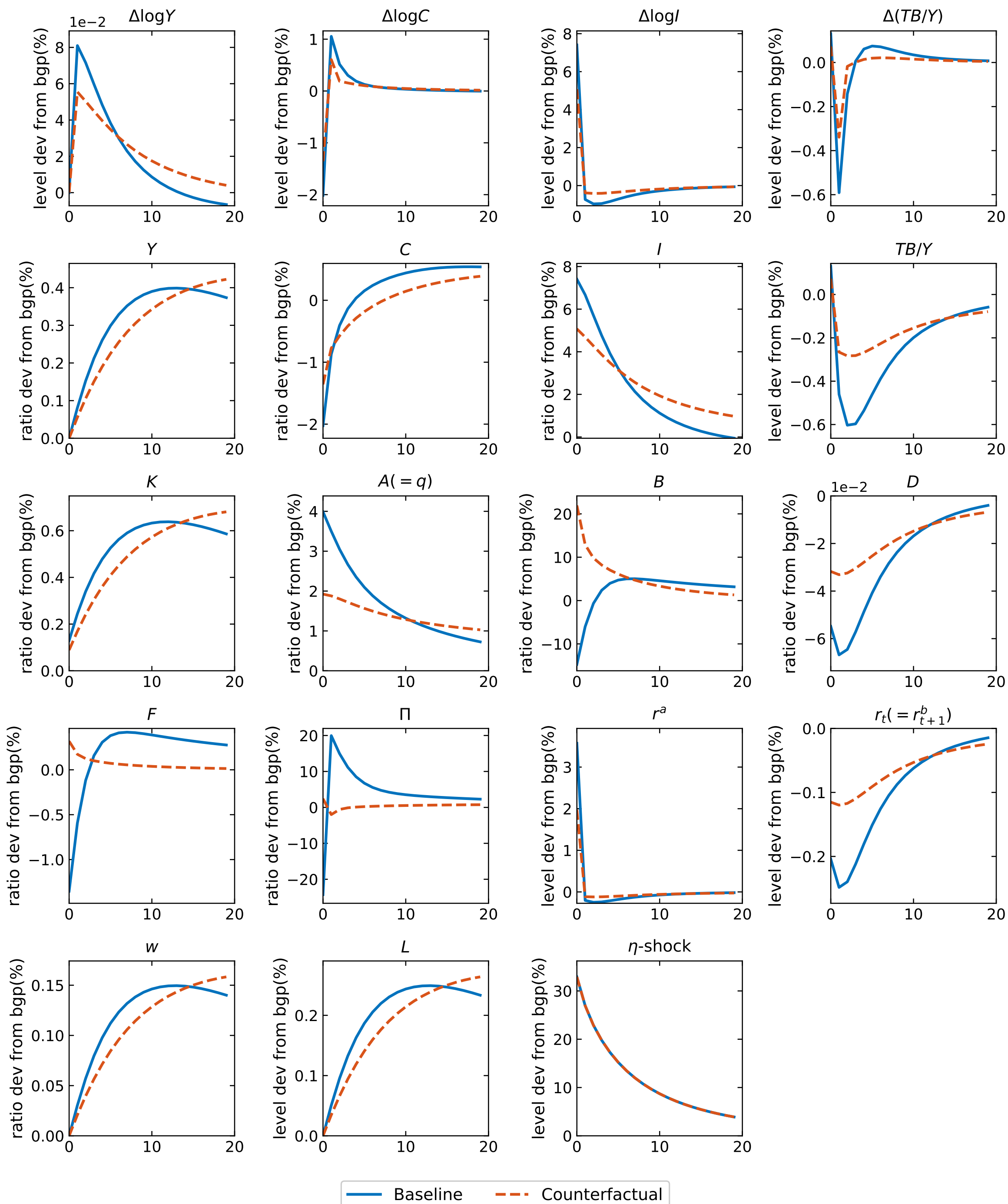


Figure E.9: IRFs to a 1 S.D. Illiquidity Shock (η): Baseline vs Counterfactual

Notes: In each plot, the blue solid line and the red dashed line represent the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock in the baseline economy and the counterfactual economy, respectively. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.

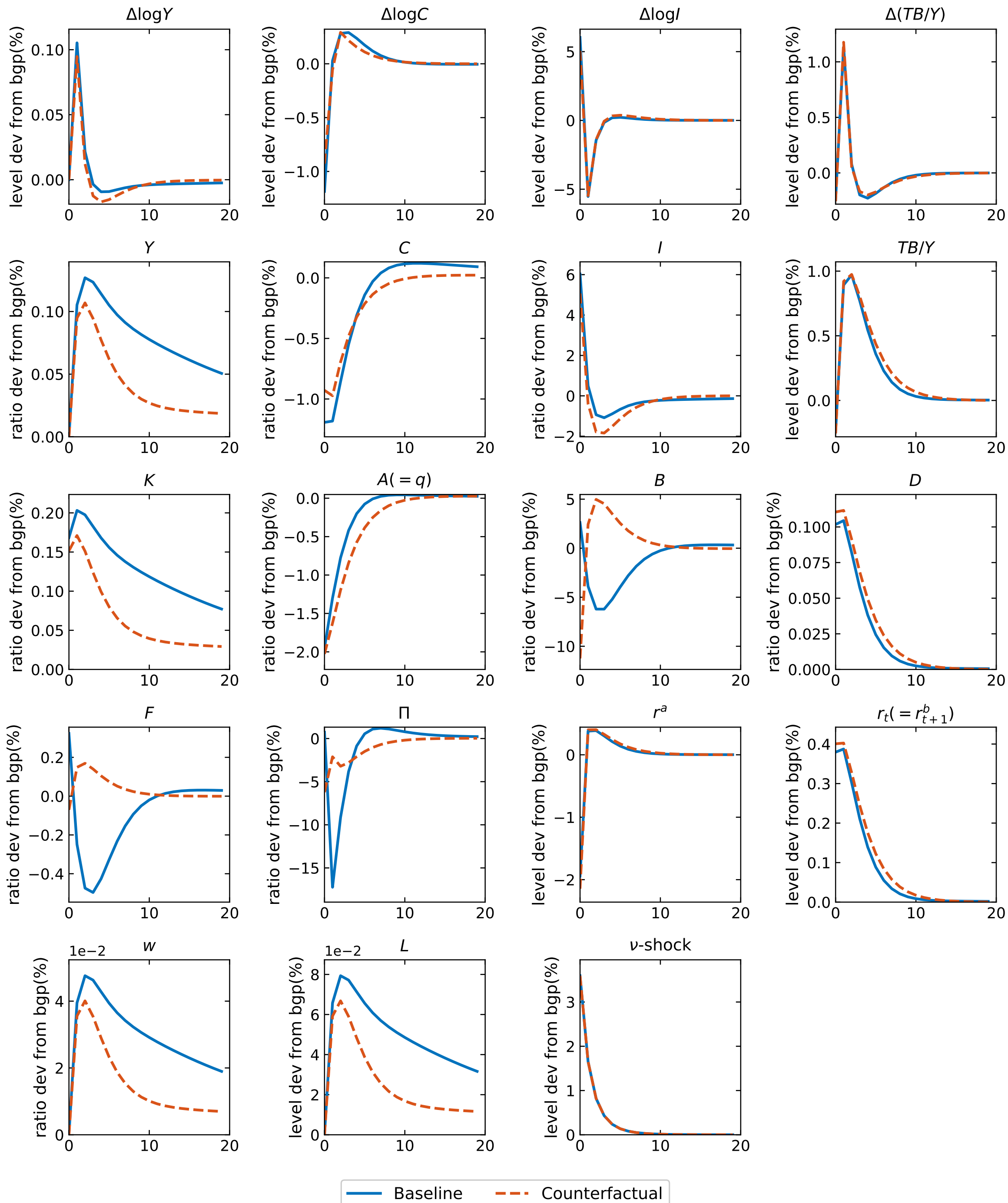


Figure E.10: IRFs to a 1 S.D. Investment Shock (ν): Baseline vs Counterfactual

Notes: In each plot, the blue solid line and the red dashed line represent the impulse responses in terms of the deviation from the balanced growth path to a one-standard-deviation shock in the baseline economy and the counterfactual economy, respectively. The impulse responses are computed under each posterior draw, and their means over the posterior distribution are plotted in this figure.