

# Comovements of the U.S. and Canadian Financial Markets

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## **Abstract**

I estimate indices of financial conditions for the U.S. and Canada from 1984 to the present using a dynamic single factor model with stochastic volatility. I use these indices to study the relationship between the U.S. and Canadian financial systems. The indices confirm that Canada, as a small open economy, has a more robust link between stock prices, commodity prices, the exchange rate, and interest rates than the U.S. market does, and that shocks in the U.S. market have a significant impact on the Canadian market. The correlation between the two indices increases during times of financial stress, as expected, but decreases afterward, suggesting divergent recoveries from recent recessions in the U.S. and Canada.

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# 1 Introduction

The financial crisis that began in 2007 resulted in the most severe recession since the Great Depression. Since then, policymakers and economists have become increasingly concerned with the monitoring and forecasting of the condition of the financial system, given its potential effects on the economy's output. A popular way to do this is by creating a financial conditions index (FCI) that can provide a single measure of the financial health of an economy and forecast a financial disruption that has the potential to cause a recession. The Bank of Canada was the first to introduce a monetary conditions index, composed of the exchange rate and refinancing rate (Freedman, 1996). These indices proliferated and have been used as a tool to gauge the transmission of monetary policy. Since the financial crisis, many indices of financial conditions have been developed for the purpose of monitoring fluctuations of the state of the overall financial system. There are currently several well established FCIs, including the Bloomberg FCI, the Citi FCI, the Deutsche Bank FCI, the Goldman Sachs FCI and the Kansas City Federal Reserve Financial Stress Index (Hatzius *et al.* , 2010). Most FCIs include a measure of short-term interest rates, long-term interest rates, risk premia, the equity market, and exchange rates (Hatzius *et al.* , 2010).

The financial markets in the U.S. and Canada are considered closely integrated - Canada's economy relies heavily on the U.S. export market, and Canadian financial markets are historically highly correlated with U.S. markets and responsive to U.S. shocks. However, there have been crises in the U.S. and Canada that have affected the two markets in different ways. The CCB and Northland Bank failures in the mid 1980s were not closely related to a period of American economic recession, the tech bubble collapse was limited in its effects on Canada's commodity-based economy, and the Canadian economy's recovery after the market turmoil beginning in 2007 was much faster than the American recovery. More recently, worries about a slowdown in China and a plummeting gold price have pressured the Canadian stock

and bond markets that are dominated by commodity producers, while the volatility of American financial markets has been affected more by uncertainty about Fed interest rate policy and quantitative easing. Estimating a financial conditions index for the U.S. and Canada allows changes in the relationship of the two markets over time to be examined and quantified.

For this paper, I created a dynamic factor model of financial conditions in the United States and Canada, with stochastic volatility, in order to study the comovements of financial conditions in the United States and Canada. Incorporating stochastic volatility in the index model, which is not done in most of the comparable literature, allows for modelling of fat-tailed returns and long-memory in volatility described by Mandelbrot (1963) as characteristic of financial markets. A stochastic volatility model allows volatility to fluctuate over time and current values of volatility to depend on past values. It gives parameter estimates that measure the kurtosis of the index and the persistence of its volatility. These parameters are relatively independent in the stochastic volatility framework. GARCH estimation treats the variance of the current error term as a function of past error terms. Stochastic volatility is considered more flexible than GARCH because parameters determining persistence and kurtosis are more independent.

Less comprehensive Canadian data and the much smaller size of the Canadian financial market has resulted in a lack of study of the Canadian financial markets, compared to the extensive study of the U.S. markets. My comparable Canadian and U.S. financial conditions indices provide a novel set of data for testing hypotheses about the relationship between the state of financial markets in the U.S. and Canada, which are each other's largest trading partners.

The index and volatility estimates that result from my model allow for several types of inquiry. I test the signalling capacity of the Canadian and U.S. FCIs, both for financial disruptions and recessions within each country and as a signal for periods when the two countries diverged. A measure that

is a good signal should indicate a high percentage of financially disruptive events, without giving many false signals. My indices signal many of the financially disruptive events, but they do give false signals, and are dominated by the 2008 recession. For the indication that the two financial markets have diverged, I find that many signals are difficult to explain, and are most likely false signals, but periods where there was obvious divergence, such as the 1980s bank collapse in Canada, do appear.

Changing the variables that are included in the indices and the time periods that they are estimated on also allows for inferences about the financial systems of the U.S. and Canada to be made. Since Canada is a small, open economy, the Canadian financial system is considered to be much more sensitive to commodity prices and exchange rates than the American system. I have found that the parameters of the Canadian financial conditions index are much more robust to changes in the input data, suggesting that there is more of a link between equity markets and commodity prices and exchange rates in Canada compared to the U.S. I have estimated a U.S. and Canadian index, Index 2, which had parameters estimated using only the data before the Great Recession, as well as an Index 3, which does not include an exchange rate measure as an input into the index. The parameters for the U.S. index become insignificant in both Index 2 and Index 3, but for Canada, the parameters that were significant in the original index remain significant. This suggests that the link between segments of the financial market is stronger for Canada than it is for the U.S. It is also interesting to note how much the year 2008 dominates the index. Index 2 is more sensitive to financial disruptions than the original index is. It signals the 1987 stock market crash and the early-1990s recession while the full index does not. The indices I created indicate that the financial stress that occurred during the Great Recession far exceeded any other disruption in the preceding 25 years.

I study the correlation of the two indices to test how closely the Canadian index follows the U.S. index and how this relationship changes over

time, especially during recessions. From 1984 to the present, the FCIs for the U.S. and Canada are closely correlated. I have looked at changes in the 12-month trailing correlation between the Canadian and U.S. Financial Conditions Index, and attempted to relate the shifts in this trailing correlation to financial crises in Canada and the U.S. For example, before 2007, Canadian financial conditions were considered very closely linked to American financial conditions. However, after the collapse of Lehman Brothers and the onset of the worst of the financial crisis, the Canadian financial system, and the Canadian economy, which was not tied as closely to the American housing market as the American system, was able to recover very quickly. The correlation plots show that correlation did break down after the recession ended, but not permanently. The trailing correlation of the indices shows increases in correlation during and immediately preceding financial crises, which is in line with existing literature. In addition, in all three U.S. recessions from 1984-2014, there are decreases in correlation immediately after the financial crises.

Financial markets are not considered very persistent. An up day in the stock market today gives little information on if there will be an up or a down day tomorrow. Cross-correlation gives an estimate of how correlated one series is with the lags of another index. As expected, the cross-correlation of the indices shows that significant positive correlation dies out after 1 lag. I also repeat the correlation analysis with the volatility estimates. The correlation of the volatility of the indices is also high, and shows little persistence in the cross-correlation plot. The lack of long-term cross correlation, which indicates low persistence of volatility of the indices, is in contrast to the financial literature and my own estimates that show the inputs to the FCIs as having very high persistence of volatility.

Lastly, using the two FCIs, I have created a VAR(1) model that estimates the impulse response of a shock to the American financial system in the Canadian financial system. I find that shocks to the U.S. do significantly

affect the Canadian market. Variance decomposition shows that the U.S. explains over 20% of variance in Canadian forecast error in a two year forecast, while Canada explains less than 5% of the U.S. variance. Granger causality tests show that the U.S. index does Granger-cause the Canadian index. Unexpectedly, the Canadian index also Granger-causes the U.S. index. This is likely because there is a third shock, some sort of world shock, that is affecting both of the FCIs and has been left out of the VAR.

My paper makes two main contributions. The first contribution is the addition of a stochastic volatility model with a persistence parameter to the dynamic factor model and the demonstration of the feasibility of estimating a common factor index on financial data using such a model. The second is demonstrating how to use comparable FCIs for two different countries to demonstrate relationships between financial conditions in the two countries, such as testing if one country's index leads the other, and quantifying the effect of shocks in one country on another.

This following sections describe the estimation of the financial conditions indices that I have modeled, and details the results of the analysis of the relationship between these indices. Section 2 reviews related work. Section 3 describes the data used in the index. Section 4 explains the model that is used to create the indices and gives a state space representation of the model that is used in estimation. Section 5 gives a basic introduction to estimation techniques and the estimation procedure used for the model. Section 6 gives the results of the estimation and the analysis of the relationship between the U.S. and Canada that rely on the estimated indices. Section 7 concludes the paper.

## 2 Related Work

The literature describes a variety of methods used to estimate a common factor in a set of financial and real economic time series. Engle & Watson (1981) developed a dynamic factor model for wages in Los Angeles, and showed how to use the Kalman filter to obtain maximum likelihood estimates of the parameters of dynamic factor models, using prediction error decomposition. I rely on this method of using the Kalman Filter to select the common factor in a dynamic factor model to estimate the FCI, given the other parameters in the model. Stock & Watson (2008) created a dynamic multiple factor model with stochastic volatility for the housing market and used the results to compare regional declines in volatility of building permits to the dates of the Great Moderation in U.S. economic activity. Stock & Watson (2008)'s paper provided the framework for adding stochastic volatility to a dynamic factor model. Stock & Watson (1988) used a dynamic single factor model without stochastic volatility and the Kalman Filter to create an index of real economic indicators, composed of industrial production, personal income less transfer payments, total manufacturing and trade sales, and employees on nonagricultural payrolls. They compare their index to the Department of Commerce's index that was created to summarize the state of macroeconomic activity, and find that the two indices are remarkably similar. Stock & Watson (1988)'s successful use of the dynamic factor model approach to create a real economic indicator motivates using a similar approach to create a financial indicator.

More recently, methods incorporating a large set of financial data, based on principal components analysis, have been developed. Hatzius *et al.* (2010) presents a financial conditions index based on a principal components approach, extracting a common factor from a large set of financial variables. Koop & Korobilis (2013) uses a factor augmented vector autoregressive model to develop an FCI that allows the set of variables that make up the FCI to change over time. Since I am only using a few series for each country, I do not

use the principal components method, but instead focus on the dynamic factor model with stochastic volatility, which combines the methods of Stock & Watson (2008) and Stock & Watson (1988). However, in my dynamic factor model, I use the univariate stochastic volatility model proposed by Taylor (1986), rather than the random walk stochastic volatility model found in Stock & Watson (2008). Taylor (1986)'s model includes a parameter that estimates the persistence of volatility and addresses some of the criticism of the volatility model that is contained in Stock & Watson (2008)'s dynamic factor model.

Many researchers have created Canada-specific FCIs. Gauthier *et al.* (2003) used monthly housing prices, equity prices, bond risk premia, short and long interest rates and the exchange rate to create FCIs using three different techniques, one of which was factor analysis, to predict output in Canada. I have relied on the extensive research in their paper regarding which variables to include in an FCI to decide how to choose and transform the inputs into my FCI. Illing & Liu (2003) created Canadian Financial Stress Indices, specifically defining stress as uncertainty and changing expectations of loss. They recognized the role of volatility in financial conditions and used GARCH modelling to model the volatility of the series that make up their indices. Rather than give their index itself volatility they included the volatility of financial variables, such as the volatility of an equity index, as an input into their Financial Stress Index. Illing & Liu (2003) also compiled a list of disruptive events to the Canadian financial system from the early 1980s to the early 2000s. Christensen & Li (2013) created a financial stress index for Canada, the United States, the United Kingdom, France and Germany. Christensen & Li (2013) used both macroeconomic indicators, such as real GDP growth, inflation, the short-term interest rate, and depreciation, as well as financial indicators, including measures of M2, private credit, bank reserve ratios, stock prices, house prices, and current account/GDP. They then evaluate the signaling capacity of combinations of individual indicators,

and find that these composite indicators do have use in predicting financial stress events.

Various researchers have shown that there are strong relationships between different segments of the financial market. Patro *et al.* (2002) found that exchange rate risk can explain significant portions of international equity returns. Fama & French (1993) found five common risk factors in the returns on stocks and bonds that are able to explain returns on stocks and bonds. Kilian & Park (2009) found that demand and supply shocks that drive the crude oil market explain 22% of the long-run variation in U.S. stock returns. Akram (2009) investigated the relationship between interest rates and the U.S. dollar and commodity prices, finding that shocks to interest rates and the dollar account for a significant share of fluctuations of commodity prices. The directional relationships between stocks, commodities, interest rates, and exchange rates are not understood completely; for example, researchers have not found a stable relationship between oil prices and stock prices, despite finding that oil price shocks do explain variation in stock returns in the long run. Though I do not attempt to analyze the directional relationships among the various inputs to the FCI, the numerous relationships between stocks, interest rates, commodities, and currencies that have been documented suggests a common factor in the four markets exists.

There is also a wealth of literature on financial contagion, defined as an increase cross-market links during a crisis, with a variety of techniques used to estimate the effects of shocks in one financial market on other financial markets. This is closely related to the study of how correlation in the FCIs changes, and how the Canadian financial system reacts to shocks in the American financial system. King & Wadhvani (1990) found that volatility correlation coefficients of stock markets between the U.S., the U.K. and Japan increased after the 1987 stock market crash. Normandin (2004) used a two factor model with GARCH variances to test the integration of U.S. and Canadian financial markets, finding that it depends on the risk prices of the

factors. Li (2009) developed a test for contagion to look at the effects of multiple recent financial crises on the Canadian banking system, including the 1987 U.S. stock market crash, the 1994 Mexican peso crisis, the 1997 East Asian crisis and the 2007 subprime mortgage crisis. He identified the mortgage crisis as having the strongest contagion impact on the Canadian banking system.

### 3 Data

For the U.S. financial conditions index, I will initially be using four monthly data series from October 1984 to January 2014 (see Figure 1). The first series is the Trade Weighted U.S. Dollar Index published by the Federal Reserve, which is made up of the exchanges rates of the U.S. Dollar with the Euro (57.6%), the Japanese yen (13.6%), the British pound sterling (11.9%), the Canadian dollar (9.1%), the Swedish krona (4.2%) and the Swiss franc (3.6%). The other three series are the S&P 500 Index, the West Texas Intermediate Crude Oil Price in Cushing, Oklahoma, and the TED spread, which is the spread between 3 month LIBOR and the 3 month T-Bill rate. The data are transformed by taking logs (for all series except the TED spread), then differencing, demeaning, and normalizing the standard deviation of the series to the S&P 500 standard deviation.

For the Canadian financial conditions index, I will be using four comparable monthly financial data series from October 1984 to January 2014 (see Figure 2). The first two series are the USD/CAD Exchange Rate and the S&P TSX Composite Index. The third series is the BCPI Commodity Price Index, which is a chain Fisher price index of the spot or transaction prices in U.S. dollars of 24 commodities produced in Canada and sold in world markets. Commodity weights are updated on an annual basis and the commodity prices that make up the index are WTI crude oil, Brent crude oil, Western Canada crude oil, natural gas, coal, potash, aluminum, gold, nickel,

iron, copper, silver, zinc, lead, lumber, pulp, newsprint, cattle, canola, wheat, hogs, corn, barley, potatoes, finfish and shellfish. The last series is a Canadian version of the TED spread, the spread between 3 month LIBOR and the 3 month Canadian T-Bill Rate. The data are transformed by taking logs (for all series except the Canadian TED spread), then differencing, demeaning, and normalizing the standard deviation of the series to the TSX standard deviation.

A correlation table for the ten series used as inputs into the FCIs is presented in Table 1. The corresponding series for the U.S. and Canada are highly correlated, as expected. The S&P 500 is highly correlated with the TSX, the BCPI follows the WTI oil price, the U.S. currency basket is correlated with the USD/CAD exchange rate, and the movements of the TED spreads are closely related. Within the Canadian series, the series are positively and reasonably highly correlated with each other, except for the Canadian TED spread. This result does not hold with the U.S. series, where the S&P 500 is negatively correlated with the currency basket and the TED spread, but not closely correlated with the oil price. Increases in the Canadian dollar are correlated with positive Canadian stock and commodity returns, but the opposite is true for the U.S. Though the Canadian and U.S. financial markets are closely related, the directional relationships among the U.S. series and the Canadian series are different. This reflects Canada's small, resource and export-based economy: for example, the Canadian stock market relies more on commodity producers, and is affected more by fluctuations in the exchange rate than the U.S. market.

The data necessary for building the financial conditions index is easily accessible through Yahoo! Finance and the Federal Reserve Bank of St. Louis. To obtain the data, I mostly used the free package for Matlab provided by Quandl (<http://www.quandl.com>). The BCPI series that is not available on Quandl can be accessed from the Bank of Canada website. Programming for the estimation of the model has been done entirely in Matlab. Due to the

computationally intensive nature of the model estimation, Columbia’s Hot-foot High Performance Computing Cluster was used to estimate the multiple variations of the FCI model described in this report. Both R and Matlab were used for the analysis of the resulting indices.

The C.D. Howe Institute publishes business cycle dates for Canada, and the NBER publishes business cycle dates for the United States. Table 2 gives the recession dates for the U.S. and Canada published by the NBER and the C.D. Howe Institute. The C.D. Howe Institute also rates each recession on a scale from 1 to 5 - all three recessions since 1984 have been given a rating of 4 out of 5. Table 3 includes a description of general financial disruptions to the Canadian and the U.S. financial markets from 1984-2013 that I reference in the interpretation of the financial indices. Except for the collapse of the CCB and Northland banks, which was a Canada specific event, the events affected both the U.S. and the Canadian financial system. I compiled the events given in Table 3 from Illing & Liu (2003), who surveyed economists at the Bank of Canada to determine the events that were most stressful to the Canadian financial system, and Christensen & Li (2013), who compiled a list of events that affected the global financial system.

As a way of describing the dynamics of the volatility of the input data, which affect the level and volatility of the financial conditions index, I have estimated the stochastic volatility of the input series. The model used for the volatility estimation is as follows, where  $y_t$  is the series for which volatility is being estimated.

$$y_t = e^{h_t/2}w_t, w_t \sim i.i.d.N(0, 1) \tag{3.1}$$

$$h_t = \mu + \beta h_{t-1} + \tau\eta_t, \quad \eta_t \sim i.i.d.N(0, 1) \tag{3.2}$$

The U.S. and Canadian estimates of the volatility of the input series are shown in Figures 3 and 4. The parameter estimates for the eight series are reported in Table 4 and Table 5.  $\tau^2$ , which is the variance of the volatility

series  $h_t$  and indicates how much stochastic volatility the series has, is significant for all the series except the currency basket, which does not appear to have stochastic volatility. The series with the highest volatility, as measured by  $\tau^2$ , are the TED spreads, followed by the stock markets, the commodity markets and the USD/CAD exchange rate. The period of very low volatility from 2003 until the mortgage crisis in 2007 is visible in the volatility plots of the stock markets and TED spreads. Periods of high volatility for the stock markets and crude oil are the Great Recession, the early 2000s, and the 1987 stock market crash. The commodity basket shows a trend of increasing volatility since 1984, and the USD/CAD plot shows generally low volatility until just before 2005. The estimates of the  $\beta$  parameter, the persistence of volatility, are high for the 7 series excluding the currency basket. For example, the TED Spread for the U.S. and Canada has a  $\beta$  close to 1. This parameter describes how the interest rate spread is generally stable and close to 0, but in times of financial distress, it becomes very volatile, and remains that way for more than one period. This confirms the literature that describe financial markets as having long memory in volatility.

## 4 Financial Conditions Index Model

The model that I am using is based on a combination of Stock & Watson (2008)'s model for U.S. Housing Construction factors, Stock & Watson (1988)'s model of real economic indicators, and Taylor (1986)'s stochastic volatility model (which replaces the random walk stochastic volatility representation in Stock & Watson (2008)).

The model for the Financial Conditions Index is presented below. For the U.S. index,  $y_{it}$  is the observations of the input series at month  $t$ , where  $i = 1, 2, 3, 4$  corresponds to the transformed S&P 500, WTI crude oil price, USD currency basket, and the TED spread, respectively. The model for the Canadian financial index is identical, with the four U.S. financial time series

replaced by four Canadian financial time series, given by Figure 2.

$$y_{it} = \lambda_i C_t + e_{it}, \quad i = 1, 2, 3, 4 \quad (4.1)$$

$\lambda_1$  is set to 1 to prevent identification issues. The common factor,  $C_t$ , and the error terms,  $e_{it}$  follow an AR(2) model:

$$C_t = \phi_1 C_{t-1} + \phi_2 C_{t-2} + e^{h_t/2} w_t, \quad w_t \sim i.i.d.N(0, 1) \quad (4.2)$$

The estimated  $C_t$  for  $t = 1, \dots, T$  is the financial conditions index.

$$e_{it} = \psi_{i1} e_{i,t-1} + \psi_{i2} e_{i,t-2} + \epsilon_{it}, \quad \epsilon_{it} \sim i.i.d.N(0, \sigma_i^2) \quad (4.3)$$

The error of the common factor has stochastic volatility.

$$h_t = \mu + \beta h_{t-1} + \tau \eta_t, \quad \eta_t \sim i.i.d.N(0, 1) \quad (4.4)$$

The unobserved process  $h_t$  is random and can be thought of as the uneven flow of new information in financial markets.  $e^{h_t}$  is the volatility series of the financial conditions index.  $\beta$  is the persistence in volatility.

For Index 3, the index is estimated without the exchange rate. The resulting model is identical to the above model, except  $y_i$  for  $i = 1, 2, 3$  corresponds to the transformed S&P 500, WTI crude oil price, and the TED spread for the U.S. index, and the transformed TSX, BCPI commodity basket, and the Canadian TED spread, for the Canadian index.

### State Space Representation of the Model

A state space model includes a measurement equation and transition equation. A variety of algorithms take a model in state space form as their input. The measurement equation, which defines the relationship between the vector

of observations  $Y_t$  and the unobserved state  $\alpha_t$  is

$$Y_t = Z_t \alpha_t + d_t + \epsilon_t \quad (4.5)$$

$\epsilon_t$  is mean zero and has covariance matrix  $H_t$ . The state vector  $\alpha_t$  is not directly observable. The transition equation is

$$\alpha_t = T_t \alpha_{t-1} + c_t + R_t \eta_t \quad (4.6)$$

where  $\eta_t$  is a vector of disturbances that is mean 0, with covariance matrix  $Q_t$ .

State space representations are not unique. A convenient state space representation of the model follows and is used as an input to the Kalman Filter to back out the index of financial conditions once the parameters are estimated.

Setting  $Y_t = [y_{1t} \ y_{2t} \ y_{3t} \ y_{4t}]'$ , the measurement equation is:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_t \\ C_{t-1} \\ e_{1t} \\ e_{1,t-1} \\ e_{2t} \\ e_{2,t-1} \\ e_{3t} \\ e_{3,t-1} \\ e_{4t} \\ e_{4,t-1} \end{bmatrix} \quad (4.7)$$

$$H = \text{diag}([0 \ 0 \ 0 \ 0])$$

The transition equation is:

$$\begin{bmatrix} C_t \\ C_{t-1} \\ e_{1t} \\ e_{1,t-1} \\ e_{2t} \\ e_{2,t-1} \\ e_{3t} \\ e_{3,t-1} \\ e_{4t} \\ e_{4,t-1} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \psi_{11} & \psi_{12} & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & \psi_{41} & \psi_{42} \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} C_{t-1} \\ C_{t-2} \\ e_{1,t-1} \\ e_{1,t-2} \\ e_{2,t-1} \\ e_{2,t-2} \\ e_{3,t-1} \\ e_{3,t-2} \\ e_{4,t-1} \\ e_{4,t-2} \end{bmatrix} + \begin{bmatrix} e^{ht/2} w_t \\ 0 \\ \epsilon_{1t} \\ 0 \\ \epsilon_{2t} \\ 0 \\ \epsilon_{3t} \\ 0 \\ \epsilon_{4t} \\ 0 \end{bmatrix} \quad (4.8)$$

$$Q = \text{diag}(e^{ht} \ 0 \ \sigma_1^2 \ 0 \ \sigma_2^2 \ 0 \ \sigma_3^2 \ 0 \ \sigma_4^2 \ 0)$$

This state space representation, in combination with the Kalman Filter, can be used to estimate  $\alpha_t$  for  $t = 1, \dots, T$ . The first entry of each  $\hat{\alpha}_t$  for  $t = 1, \dots, T$ , gives an estimate for the FCI  $C_t$ , for  $t = 1, \dots, T$ .

## 5 Model Estimation

This section begins with a brief explanation of the algorithms that are used to estimate the model given in the previous section. I then describe in detail how I apply the algorithms to estimate the parameters of the model.

### 5.1 Introduction to Estimation Algorithms

#### The Kalman Filter

The Kalman Filter is a recursive algorithm that provides an estimate of the unobserved state in a state space model at time  $t$  based on all available information at  $t$ . Dynamic time series models that have unobserved variables

can be represented in state space form.

At time  $t$ , the Kalman Filter forms an optimal predictor of the next observation using a prediction equation. Once a new observation becomes available, it is then incorporated into the estimate of the state vector using updating equations. The filtering equations are applied recursively as each new observation becomes available.

Let  $\hat{\alpha}_t$  denote the estimate of  $\alpha_t$  using all information from  $t = 1 \dots t$  including the current observation  $Y_t$ . Let  $\hat{\alpha}_{t|t-1}$  be the estimate of  $\alpha_t$  given all information up to time  $t - 1$ . The prediction equations are:

$$\hat{\alpha}_{t|t-1} = T_t \hat{\alpha}_{t-1} + c_t \quad (5.1)$$

$$P_{t|t-1} = T_t P_{t-1} T_t' + R_t Q_t R_t', \quad t = 1, \dots, T. \quad (5.2)$$

with prediction error  $v_t = Y_t - Z_t \hat{\alpha}_{t|t-1} - d_t$ . The updating equations are

$$\hat{\alpha}_t = \hat{\alpha}_{t|t-1} + P_{t|t-1} Z_t' F_t^{-1} (Y_t - Z_t \hat{\alpha}_{t|t-1} - d_t) \quad (5.3)$$

$$P_t = P_{t|t-1} - P_{t|t-1} Z_t' F_t^{-1} Z_t P_{t|t-1}, \quad t = 1, \dots, T \quad (5.4)$$

where  $F_t = Z_t P_{t|t-1} Z_t' + H_t$ . Taking Harvey's suggestion,  $\hat{\alpha}_0$  is set to a vector of zeroes and  $P_0$  as a matrix with very large numbers on the diagonal, indicating the high variance of the initial estimate for  $\hat{\alpha}_0$ , which is based off of no information.

The Kalman Filter is used to form an estimate of the financial conditions index, given the other parameters of the model. Once the model is represented in state space form, as presented in section 4, the Kalman Filter gives an estimate of  $\alpha_t = \begin{bmatrix} C_t & C_{t-1} & e_{1t} & e_{1,t-1} & e_{2t} & e_{2,t-1} & e_{3t} & e_{3,t-1} & e_{4t} & e_{4,t-1} \end{bmatrix}'$  for  $t = 1, \dots, T$ . The first entry of  $\hat{\alpha}_t$  is an estimate of the financial conditions index.

## The Metropolis-Hastings Algorithm

This section contains a brief overview of the Metropolis-Hastings Algorithm, which is used to estimate the stochastic volatility series of the model. In this section, the notation and description of the algorithm is based on the overview of the Metropolis-Hastings Algorithm given in Chib & Greenberg (1995).

Monte Carlo Markov Chain methods are used to simulate complex multivariate distributions. For example, they can be used to generate the first and second moments of multiple parameters that arise from a complex distribution. A very basic and preliminary description of the algorithm is as follows: The objective is to generate samples from a complex target density  $\pi(x)$ . Suppose that there is some density that can generate candidates for the elements of the distribution that depends on the current state of the process. The candidate-generating density is  $q(x, y)$  - when the process is at the point  $x$ , the density generates a value  $y$ . There are various ways to choose  $q(x, y)$ . It is also necessary to have  $\alpha(x, y)$ , which is the probability of moving from  $x$  to  $y$ , since the choice of  $q(x, y)$  can result in a move from  $x$  to  $y$  too often or from  $y$  to  $x$  too rarely. If a move is not made, the process returns  $x$  again from the target distribution. The first few thousand draws are rejected so that the starting value has no effect on the distribution obtained.

The algorithm is as follows:

- Repeat for  $j = 1, 2, \dots, N$
- Generate  $y$  from  $q(x^{(j)}, \cdot)$  and  $u$  from  $U(0, 1)$ .
- If  $u \leq \alpha(x^{(j)}, y)$  then set  $x^{(j+1)} = y$ .
- Else set  $x^{(j+1)} = x^{(j)}$ .
- Return the values  $\{x^{(1)}, x^{(2)}, \dots, x^{(N)}\}$ .

The mean and variance of the values returned, after discarding an initial subset of values, approximate the target distribution. The actual procedure used to sample the stochastic volatility series is a Metropolis-Hastings algorithm and is detailed in Lopes & Polson (2010).

## Gibbs Sampling

Gibbs Sampling can actually be considered a special case of the Metropolis-Hastings algorithm. It is important enough to deserve its own section, since it serves as a framework for estimating the entire model.

A basic overview of Gibbs Sampling is as follows: suppose the goal is to estimate  $k$  variables  $z_t$  for  $t = 1, 2, \dots, k$ , given the complete set of conditional densities  $f(z_t|z_{j \neq t}), t = 1, 2, \dots, k$ . Then, given some arbitrary starting values  $(z_2^0, \dots, z_k^0)$ , execute the following steps.

- Draw  $z_1^1$  from  $f(z_1|z_2^0, \dots, z_k^0)$
- Draw  $z_2^1$  from  $f(z_2|z_1^1, z_3^0, \dots, z_k^0)$
- ...
- Draw  $z_k^1$  from  $f(z_k|z_1^1, z_2^1, \dots, z_{k-1}^1)$

These steps are repeated  $J$  times, with enough of the initial iterations discarded to remove the effect of the starting values  $(z_2^0, \dots, z_k^0)$ . It has been shown that the joint and marginal distributions of  $(z_1^j, z_2^j, \dots, z_k^j)$  converge to the actual joint and marginal distributions as  $J$  approaches infinity. This framework allows for the complex model given in Section 4 to be estimated iteratively (see the following section).

## 5.2 The Estimation Procedure

For this section, the following notation adapted from Kim & Nelson (1999) is used:

$$\tilde{C}_T = \begin{bmatrix} C_1 & C_2 & \dots & C_T \end{bmatrix}'$$

$$\begin{aligned}
\tilde{h}_T &= [h_1 \quad h_2 \quad \dots \quad h_T]' \\
\tilde{\phi} &= [\phi_1 \quad \phi_2]' \\
\tilde{\psi} &= [\psi_1 \quad \psi_2 \quad \psi_3 \quad \psi_4]', \tilde{\psi}_i = [\psi_{i1} \quad \psi_{i2}] \\
\tilde{\sigma}^2 &= [\sigma_1^2 \quad \sigma_2^2 \quad \sigma_3^2 \quad \sigma_4^2]' \\
\tilde{\lambda} &= [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4]'
\end{aligned}$$

The following steps are repeated 10,000 times, with the first 2,000 draws discarded. The mean and the variance of the draws of the parameters, including the volatility  $\tilde{h}_T$  and the index  $\tilde{C}_T$ , give the mean and variance of my estimates for the parameters of the model described in Section 4.

- Step 1: Conditional on the data and all parameters of the model, generate  $\tilde{C}_T$ , the financial conditions index.
- Step 2: Conditional on  $\tilde{C}_T$  and  $\tilde{h}_T$ , generate  $\tilde{\phi}$ , the coefficients of the AR(2) model that describes the index.
- Step 3: Conditional on  $\tilde{C}_T$ , and data for the  $i$ -th series, generate  $\tilde{\psi}_i$ ,  $\lambda_i$ , and  $\sigma_i^2$  for  $i = 1, 2, 3, 4$ .  $\lambda_i$  relates the index to the  $i$ -th series, and the  $\tilde{\psi}_i$  and  $\sigma_i^2$  model the persistence and variance of the errors  $e_{it}$  from Equation 4.1.
- Step 4: Sample  $\tilde{h}_T$ , the volatility of the series, via random walk Metropolis-Hastings
- Step 5: Conditional on  $\tilde{h}_T$  generate  $\mu$ ,  $\beta$ ,  $\tau^2$ , the parameters that determine the characteristics of the stochastic volatility of the index.

Steps 1-3 are adapted from Chapter 8 of Kim & Nelson (1999) and Steps 4-5 are adapted from Lopes & Polson (2010). The following sections give an

overview of how the methods developed by Kim & Nelson (1999) and Lopes & Polson (2010) were used to draw the parameters in each step of the Gibbs Sampler.

**Step 1: Generate  $\tilde{C}_t$ , conditional on the parameters of the model and the data  $\tilde{y}_T$**

The state space form of a model is not unique. For this section, a different form is used than the one presented in Section 4. The state space model in Section 4, which is useful for backing out the values of the index given the other parameters in the model, was presented by way of example, since it is more simple to derive than the representation given in this section. In the Gibbs Sampling framework, values of the index from  $t = 1, \dots, T$  must be drawn a distribution. The procedure to draw values of the index  $\tilde{C}_T$  requires the covariance matrix of the transition equation,  $Q$ , to have a  $JXJ$  block that is positive definite, with the rest of the matrix equal to zero. The state space model in Section 4 does not have a matrix  $Q$  with these characteristics, while the following state space model does. The measurement equation is:

$$\begin{bmatrix} y_{1t}^* \\ y_{2t}^* \\ y_{3t}^* \\ y_{4t}^* \end{bmatrix} = \begin{bmatrix} \lambda_1 & -\lambda_1\psi_{11} & -\lambda_1\psi_{11} \\ \lambda_2 & -\lambda_2\psi_{21} & -\lambda_2\psi_{22} \\ \lambda_3 & -\lambda_3\psi_{31} & -\lambda_3\psi_{32} \\ \lambda_4 & -\lambda_4\psi_{41} & -\lambda_4\psi_{42} \end{bmatrix} \begin{bmatrix} C_t \\ C_{t-1} \\ C_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{4t} \end{bmatrix},$$

$$E(e_t e_t') = H = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}$$

and the transition equation is:

$$\begin{bmatrix} C_t \\ C_{t-1} \\ C_{t-2} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_{t-1} \\ C_{t-2} \\ C_{t-3} \end{bmatrix} + \begin{bmatrix} e^{h_t/2} w_t \\ 0 \\ 0 \end{bmatrix},$$

$$Q_t = \begin{bmatrix} e^{h_t} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

where  $y_{it}^* = y_{it} - \psi_{i1}y_{i,t-1} - \psi_{i2}y_{i,t-2}$ ,  $i = 1, 2, 3, 4$ .

The following procedure generates  $\tilde{C}_T$ .  $Q$  is singular, so let  $Q^*$  be the 1x1 block of  $Q$  that is positive-definite. Denote the first row of  $\alpha_{t+1}$  as  $\alpha_{t+1}^*$ . Generate  $\alpha_t$ ,  $t = T, T-1, \dots, 1$  from the following distributions:

$$\alpha_T | y_T \sim N(\alpha_{T|T}, P_{T|T}), \quad (5.5)$$

$$\alpha_t | y_t, \alpha_{t+1}^* \sim N(\alpha_{t|t, \alpha_{t+1}^*}, P_{t|t, \alpha_{t+1}^*}), t = T-1, T-2, \dots, 1 \quad (5.6)$$

. The mean and variances of the previous distributions are obtained in the following way:

- Run the Kalman filter algorithm to calculate  $\alpha_{t|t} = E(\alpha_t | y_t)$  and  $P_{t|t} = Cov(\alpha_t | y_t)$  for  $t = 1, 2, \dots, T$  and save them. The last iteration of the Kalman filter gives  $\alpha_{T|T}$  and  $P_{T|T}$ , which are used to generate  $\alpha_T$  using Equation 5.5.
- For  $t = T-1, T-2, \dots, 1$ , given  $\alpha_{t|t}$  and  $P_{t|t}$ , let  $Z^*$  be the first  $J$  rows of  $Z$  and  $v_{t+1}^*$  be the first  $J$  rows of  $v_{t+1}$ . In the financial conditions index model,  $J=1$ , so  $Q^*$  is a scalar. The notation of the following equations, however, generalizes to cases where  $J > 1$  and  $Q^*$  is not a scalar. The mean and variance of the distribution in Equation 5.6 are

obtained from the following equations:

$$\alpha_{t|t, \alpha_{t+1}^*} = E(\alpha_t | y_t, \alpha_{t+1}^*) = \alpha_{t|t} + P_{t|t} Z^{*'} (Z^{*'} P_{t|t} Z^{*'} + Q^*)^{-1} (\alpha_{t+1}^* - Z^* \alpha_{t|t})$$

$$P_{t|t, \alpha_{t+1}^*} = Cov(\alpha_t | y_t, \alpha_{t+1}^*) = P_{t|t} - P_{t|t} Z^{*'} (Z^{*'} P_{t|t} Z^{*'} + Q^*)^{-1} Z^* P_{t|t}$$

The first entry of generated  $\alpha_t$  for  $t = 1, \dots, T$  gives  $\tilde{C}_T$  for the current iteration of the Gibbs Sampler.

### Step 2: Generating $\tilde{\phi}$ , conditional on $\tilde{C}_T$

Consider Equation 4.2, rewritten as  $C_t = X\tilde{\phi} + q_t$ , where  $X = [C_{t-1}, C_{t-2}]$  and  $q = e^{h_t/2} w_t$ . Before the start of the Gibbs Sampling procedure in the previous subsection, a multivariate normal prior for  $\tilde{\phi}$  is defined, given by  $\tilde{\phi} \sim N(\alpha, A)$ . When little is known about  $\tilde{\phi}$ , the priors are defined somewhat arbitrarily, for example as  $\alpha = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$  and  $A = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$ . As more information about the mean and covariance matrix of  $\tilde{\phi}$  is gained from results of initial executions of the estimation program, the priors are adjusted. Given the series  $C_t$  drawn in in the previous step of the Gibbs Sampler, calculate a posterior for  $\phi$ : This posterior distribution is  $\tilde{\phi} | C_t \sim N(\bar{\alpha}, \bar{A})$ , where

$$\bar{\alpha} = (A^{-1} + X'X)^{-1}(A^{-1}\alpha + X'C_t),$$

$$\bar{A} = (A^{-1} + X'X)^{-1}$$

. Drawing from a multivariate normal distribution with mean  $\bar{\alpha}$  and variance  $\bar{A}$  gives the estimate of  $\tilde{\phi}$  for the current iteration of the Gibbs Sampler. The draw is only retained if the roots of  $\phi(L) = 0$  lie outside the unit circle, since  $\tilde{C}_T$  is stationary.

**Step 3: Generating  $\tilde{\psi}_i, \tilde{\lambda}_i, \tilde{\sigma}_i^2$ , conditional on  $\tilde{y}_{iT}$  and  $\tilde{C}_T$**

Conditional on  $\tilde{C}_T$ , equations 4.1 and 4.3 are four independent regression models, with autocorrelated disturbances. The prior distributions of the parameters are

$$\begin{aligned}\lambda_i | \tilde{\psi}_i, \sigma_i^2 &\sim N(a_i, A_i), \\ \tilde{\psi}_i | \lambda_i, \sigma_i^2 &\sim N(b_i, B_i) \\ \tilde{\sigma}_i^2 | \lambda_i, \tilde{\psi}_i &\sim IG(v_i/2, f_i/2).\end{aligned}$$

The parameters of the posterior distributions are calculated in the following way.

For  $\lambda_i$ , consider the equation  $\tilde{y}_{iT}^* = \lambda \tilde{C}_T^* + \epsilon_i$ , where  $y_{it}^* = y_{it} - \psi_{i1}y_{i,t-1} - \psi_{i2}y_{i,t-2}$  and  $C_t^* = C_t - \psi_{i1}c_{t-1} - \psi_{i2}c_{t-2}$ .  $\lambda_i$  is generated from  $N(\bar{a}_i, \bar{A}_i)$ , where

$$\begin{aligned}\bar{a}_i &= (A_i^{-1} + \sigma_i^{-2} \tilde{C}_T^{*'} \tilde{C}_T^*)^{-1} (A_i^{-1} a_i + \sigma_i^{-2} \tilde{C}_T^{*'} \tilde{y}_{iT}^*), \\ \bar{A}_i &= (A_i^{-1} + \sigma_i^{-2} \tilde{C}_T^{*'} \tilde{C}_T^*)^{-1}.\end{aligned}$$

To generate  $\psi_i$ , focus on Equation 4.3:

$$e_{it} = \psi_{i1}e_{i,t-1} + \psi_{i2}e_{i,t-2} + \epsilon_{it},$$

where  $e_{it} = y_{it} - \lambda_i C_t$ . Consider this equation in matrix form:

$$\tilde{e}_{iT} = E_i \tilde{\psi}_i + \epsilon_i,$$

where  $\epsilon_i \sim N(0, \sigma_i^2 I_{T-2})$  for  $i = 1, 2, 3, 4$ . Given the prior distribution for  $\tilde{\psi}_i$ , the parameters are generated as follows.  $\tilde{\psi}_i | \lambda_i, \sigma_i^2, \tilde{C}_T, \tilde{y}_{it} \sim N(\bar{b}, \bar{B})$ , where

$$\begin{aligned}\bar{b} &= (B_i^{-1} + \sigma_i^{-2} E_i' E_i)^{-1} (B_i^{-1} b_i + \sigma_i^{-2} E_i' \tilde{e}_{iT})' \\ \bar{B} &= (B_i^{-1} + \sigma_i^{-2} E_i' E_i)^{-1}\end{aligned}$$

Similarly to when  $\tilde{\phi}$  was drawn, only retain the draw of  $\tilde{\psi}$  if the roots of  $\psi(L) = 0$  lie outside the unit circle.

To generate  $\sigma_i^2$ , focus on the equation used to draw  $\tilde{\psi}_i$ ,  $\tilde{e}_{iT} = E_i\tilde{\psi}_i + \epsilon_i$ . Combine the likelihood with the prior distribution, and draw  $\sigma_i^2$  from

$$\sigma_i^2 | \tilde{\psi}_i, \tilde{\lambda}_i, \tilde{C}_T, \tilde{y}_{iT} \sim IG\left(\frac{v_i + (T-2)}{2}, \frac{f_i + (\tilde{e}_{iT} - E_i\tilde{\psi}_i)'(\tilde{e}_{iT} - E_i\tilde{\psi}_i)}{2}\right).$$

#### Step 4 and 5: Generating $\tilde{h}_t, \mu, \beta, \tau$

This section is adapted from a lecture on stochastic volatility models based on the work of Lopes & Polson (2010). Prior information about the parameters needed for Gibbs Sampling and the Metropolis-Hastings algorithm are as follows, where  $\theta = [\mu, \beta]'$ :

$$h_0 \sim N(a, A)$$

$$\theta | \tau^2 \sim N(m, \tau^2 M)$$

$$\tau^2 \sim IG(v/2, vs^2/2)$$

$$h_t | h_{t-1}, h_{t+1}, \theta, \tau^2 \sim N(u_t, V^2)$$

$$h_T | h_{T-1}, \theta, \tau^2 \sim N(u_T, \tau^2)$$

where

$$u_t = \left(\frac{1-\beta}{1+\beta^2}\right)\mu + \left(\frac{\beta}{1+\beta^2}\right)(h_{t-1} + h_{t+1})$$

$$V^2 = \tau^2(1+\beta^2)^{-1}$$

$$u_T = \mu + \beta h_{T-1}$$

#### Sampling $(\theta, \tau^2 | \tilde{h}_T, h_0)$

Calculate the posterior distribution by combining the likelihood and the prior. The posterior distribution is given by the normal-inverse gamma distribution

$$(\theta, \tau^2) \sim NIG(\bar{m}, \bar{M}, \bar{v}, \bar{s}^2).$$

The parameters of the NIG distribution are given by the following, where  $X = (1_T, h_{0:(T-1)})$ .

$$\begin{aligned}\bar{v} &= v + T \\ \bar{M} &= (M^{-1} + X'X)^{-1} \\ \bar{m} &= (M^{-1} + X'X)^{-1}(M^{-1}m + X'\tilde{h}_T) \\ \bar{s} &= \frac{vs^2 + (\tilde{h} - X\bar{m})'(\tilde{h} - X\bar{m}) + (\bar{m} - m)'M^{-1}(\bar{m} - m)}{\bar{v}}\end{aligned}$$

**Sampling**  $(h_0|\theta, \tau^2, h_1)$

Combine  $h_0 \sim N(a, A)$  and  $h_1|h_0 \sim N(\mu + \beta h_0, \tau^2)$ , in order to draw  $h_0$  from  $h_0|h_1 \sim N(\bar{a}, \bar{A})$ , where

$$\begin{aligned}\bar{A} &= (A^{-1} + \beta^2\tau^{-2})^{-1} \\ \bar{a} &= (A^{-1} + \beta^2\tau^{-2})^{-1}(A^{-1}a + \beta\tau^{-2}(h_1 - \mu))\end{aligned}$$

**Sampling**  $\tilde{h}_T$  using Metropolis-Hastings

The procedure is summarized as follows: For each  $t = 1, \dots, T$ , generate  $h_t$  by a random move from the previous draw of  $h_t$ . Choose whether to accept or reject the new  $h_t$  by calculating the ratio of the value of the likelihood functions of the equations that include  $h_t$  (equations 4.2 and 4.4) for the new  $h_t$  and the old  $h_t$ , using the other parameters generated in previous steps of the Gibbs Sampler. Then, this ratio is compared to a random number. If the ratio is greater than the random number, the draw for  $h_t$  in that iteration of the Gibbs Sampler is set to the new  $h_t$ . Otherwise, it is set to the old  $h_t$ . This generally results in a move when the candidate improves the likelihood, but also allows for a move sometimes even if the new candidate does not improve the likelihood. This allows for the whole distribution of a parameter to be sampled, even if the distribution is complex. Formally, the procedure is as follows. Let  $V_t^2 = V^2$  for  $t = 1, \dots, T - 1$  and  $V_T^2 = \tau^2$ , and  $v_h^2$  be

the tuning variance of the algorithm. Let  $\hat{w}_t = C_t - \phi_1 C_{t-1} - \phi_2 C_{t-2}$ , for  $t = 1, \dots, T$ , the residual of equation 4.2. Repeat the following steps for  $t = 1, \dots, T$ , where  $j$  indicates we are in the  $(j+1)$ -ith iteration of the Gibbs Sampler:

- 1. Current state:  $h_t^{(j)}$
- 2. Sample  $h_t^*$  from  $N(h_t^{(j)}, v_h^2)$
- 3. Compute the acceptance probability

$$\alpha = \min\left\{1, \frac{f_N(h_t^*; u_t, V_t^2) f_N(\hat{w}_t; 0, e^{h_t^*})}{f_N(h_t^{(j)}; u_t, V_t^2) f_N(\hat{w}_t; 0, e^{h_t^{(j)}})}\right\}$$

- 4. New state,  $h_t^{(j+1)}$ , is  $h_t^*$  with probability  $\alpha$ ,  $h_t^{(j)}$  with probability  $1 - \alpha$ .

## 6 Results

In this section I present the results that analyze the relationships between the estimated U.S. and Canadian financial conditions indices. As the U.S. economy dwarfs the Canadian economy, in this section, causality statements are generally made in the context of the U.S. affecting Canada.

### 6.1 Estimated Results for the Indices

Figure 5 shows the standard deviation-normalized plots of the U.S. and Canadian FCIs that I estimate from the data using the model in Section 4. I have assumed that financial conditions in the U.S. and Canada are mean zero and stationary, without a time trend of increasing or decreasing financial conditions from 1984 to 2014. So, the indices are mean zero, and generally fluctuate closely around the mean.

There are notable periods where the indices drop significantly. The most significant is the multiple standard deviation drop that occurs during the

Great Recession. In addition, it is clear that in all three U.S. recessions from 1984 to 2014, both the U.S. and Canadian indices drop during the recession, and increase during the recovery after the recession, though not all of these drops correspond to a recession signal as defined in the next section. Periods considered to be times of stability and growth in financial markets, such as 2003-2006 and the mid-1980s, do correspond to periods of above average financial conditions for both countries. The indices are noisy, however, and fluctuate considerably. Comparing the indices to the data series plotted in Figure 1 and 2, the indices do appear to have captured the common factor in the four input series. Spikes that occur only in one market, such as the 1987 stock market crash, or the jump in commodity prices in the early 1990s, do not show up as significant events in the index. Generally, the indices have a pattern that is less volatile and more smooth than the stock, interest rate, commodity, and currency series.

The first two columns in Table 6 and Table 7 give the parameter estimates for the U.S. and Canada full FCI models.  $\phi_1$  is significant for both of the FCIs but  $\phi_2$  is not, which means that lags of more than one month of each FCI do not predict the current value of the FCI. The  $\lambda_i$ s for the stock, commodity and currency series are significant for both FCIs, suggesting that all three series are making up a significant part of the index. The  $\lambda_4$  for the interest rate series, however, is not significant, indicating that the interest rate series has little influence on the financial conditions index that I have created. The general lack of significance of the  $\psi_{ij}$ s shows that errors in the prediction of the series based on the financial conditions index have little autocorrelation.

The  $\beta$  term in the stochastic volatility part of the model is close to zero, which indicates that volatility is not persistent in the financial index, contrary to previous work which has identified high persistence of volatility in financial markets. The  $\tau^2$  term is significant, which does signal the existence of stochastic volatility in the series. However, it is much smaller than the  $\tau^2$

terms of the input series, excluding the currency basket, meaning the index has less volatility than the input series. The intercept of the AR(1) regression of  $h_t$ ,  $\mu$ , does not provide information about the characteristics of the volatility of the indices.

The volatility estimates,  $e_t^h$ , for the U.S. and Canada FCIs is given in Figure 6. The pattern of the volatility for the two indices is similar. However, the plots are dominated by the volatility that occurred during the Great Recession, so little inference can be made in terms of the signaling power of the volatility estimation. Using a standard deviation threshold is infeasible since the volatility observations, excluding those near the Great Recession, are all within one standard deviation of the mean. Visually, however, some of the spikes in volatility estimates do make sense. Post 2009, spikes in the U.S. and Canada correspond to periods of heightened uncertainty related to the Eurozone crisis. There is a spike in the U.S. index, but not the Canadian index, for the 1987 stock market crash. 2001, which corresponds to the bursting of the tech bubble and the 9/11 attacks, also corresponds to periods of higher volatility in the U.S. and Canada. In general, however, the series fluctuate closely around the mean except for during the Great Recession.

To show the robustness of the Canadian index compared to American index, I have also presented the parameter estimates for Index 3, the index without the exchange rate for the U.S. and Canada, and Index 2, the index with data only up to 2008 in Table 6 and 7. Excluding the exchange rate measure, the U.S. parameter on the oil price becomes insignificant but the Canadian parameter on the commodity basket that was significant in Index 1 remains significant. Canada appears to have a much more defined link between the various components of the financial markets than the U.S. does. With data only up to 2008, the stock market dominates the U.S. index, whereas the parameter estimates for the Canadian index do not change markedly for Index 2 compared to Index 1. In Index 2 for the U.S., none of the  $\lambda$  parameters are significant, while the stock, commodity, and exchange

rate  $\lambda$ s remain significant for Canada.

## 6.2 Signaling

Table 8 and 9 show the signaling power of the financial conditions indices for the U.S. and Canada that I have created. The various thresholds that I use to indicate that there is a disruption in financial markets at various confidence levels are multiples of each index's standard deviation. I use a two-tailed significance threshold, but in the following paragraph I focus my analysis on periods of financial disruption, so I interpret only the negative signals related to these periods and ignore positive signals. At a 90% significance level,  $1.65*SD$ , the U.S. index signals the collapse of the dot-com bubble, the Great Recession, and various months that correspond to peaks in the Eurozone crisis. The Dec. 1984 signal is most likely due to variance that occurs in the initial months that the index is estimated on. Nov. 1992 appears to be a false signal. The U.S. index does not signal the 1990-1991 recession. At a 95% confidence level,  $1.96*SD$ , the U.S. index signals just the dot-com collapse and the Great Recession, while at a 99% confidence level,  $2.58*SD$ , it only signals the Great Recession.

I also compare the signaling capacity of Index 2 to the index that was estimated on the full dataset, Index 1. A plot of the original index and the limited index from 1985 to 2007 for the U.S. is given in Figure 12. Defining a signal as 1.645 standard deviation drop in the index, Index 2 correctly signals the 1987 stock market crash, the early 1990s recession in the U.S., and the 2001 recession. Index 1, however, is dwarfed by the Great Recession. The spikes for 1987 and the early 1990s recession appear to be too small to register as a crisis. The 2001 recession does appear, and there are other apparently false signals. For the full index, the drop in financial conditions that occurred during the Great Recession dominates every other financial disruption in the last 30 years. The domination of the Great Recession in the U.S. index and the insignificance of many of the parameters of Index 2, estimated on data

without the Great Recession, could indicate that my model has overfit the 2008 outlier.

At a 90% significance, The Canadian index signals the Russian financial crisis, the Asian financial crisis in 1997-1998, the dot-com collapse, disruptions possibly related to the 9/11 terrorist attacks, and a deepening of the Eurozone crisis in Sept. 2011. The Canadian index does not signal the early 1990s recession that occurred in Canada. At a 95% confidence level only the 2001 and Great Recession disruptions appear and at a 99% confidence level, as for the U.S. index, only the Great Recession is signaled.

Table 10 and 11 show the signals that the differences in the indices give, which could suggest periods when financial conditions in the U.S. and Canada, which are normally very correlated, diverged. A 90% confidence level appears to give too many false signals, so I focus on a 95% confidence level. For an indication that the U.S. is below Canada, the 1988, 1993, 2000, and 2005 signals appear to be false. The Jan. 2001 and 2010 signals are most likely significant, however. In early 2001, Canada, as a resource-based economy, may have been less affected by the tech bubble collapse than the U.S. The signals in 2009 and 2010 correspond to a period of time where Canada was recovering from the Great Recession much faster than the U.S. was.

For a signal that Canada is below the U.S., the collapse of CCB and Northland bank in Sept. 1985 shows up at a 95% confidence level. In addition, the difference in the indices also signals some months during the Asian financial crisis and parts of the financial crisis as periods where the Canadian financial conditions were significantly worse than American financial conditions. However, there are many signals that are difficult to explain, suggesting that the indices capacity to accurately signal differences in Canadian and U.S. financial markets is limited.

### 6.3 Correlation of the U.S. and Canadian FCIs

The U.S. and Canadian indices are clearly closely related, and the correlation coefficient of the two FCIs is 0.76. The Canadian index is expected to be closely correlated to the U.S. index, since Canada is a small open economy that does not have power over world prices, interest rates or exchange rates, and relies heavily on exports, mainly to the United States, to drive the economy.

Figure 10 shows the 12-month trailing correlation of the U.S. and Canadian index, which gives, for each month, the correlation of the indices over the previous 12 months. Generally, the trailing correlation of the two indices ranges between 0.7 and 0.9, but there are significant periods where this correlation breaks down. The early to mid 1990s were a time of a deep recession in Canada, classified by the C.D. Howe Institute as having equal severity to the 2008 recession, occurring at the same time as a mild recession in the United States. In addition, the 1994-1995 Mexican crisis may have contributed to the decrease in financial conditions in Canada that did not correspond to as sustained a decrease in financial conditions in the U.S. In the early 2000s the trailing correlation also breaks down, corresponding to the 9/11 terrorist attacks and the tech bubble collapse. 9/11 certainly affected Canadian markets, but, according to my analysis, not to the same extent as it did the U.S. markets. Furthermore, the tech bubble collapse most likely also caused more widespread disruption in the U.S. compared to Canada, which is a resource based and export driven economy.

The hypotheses I have come up with in this section are corroborated by Figure 7, which shows Canadian financial conditions remarkably lower than the U.S. conditions in the mid 1990s and generally higher in the early 2000s. In general, trailing correlation of the indexes breaks down for a short period after the onset of a crisis in either country. For the U.S. crises, correlation is generally high through the beginning of the crisis, and only breaks down at the end of the recession, which is in line with previous literature on contagion.

The phenomenon of correlation breaking down after recessions, however, has not been well documented.

Cross-correlation functions are used to suggest the number of lags to be included in a VAR model. I estimate the cross-correlation of the U.S. and Canadian index to motivate the choice of the VAR model in Section 6.3. In motivating the choice of the VAR, the cross-correlation plot also provide a measure of the number of lags of the U.S. index that predict the Canadian index, and vice-versa.

When testing the cross-correlation of the Canadian index using lags of the U.S. index, the cross-correlation is affected by the structure of the Canadian index and common trends that the U.S. and Canadian series have over time. So, in order to interpret the results of a cross-correlation, the series are pre-whitened. To prewhiten the U.S. and Canadian indices, I estimate an AR(1) model for each:

$$\begin{aligned} C_t^{(CAN)} &= \phi_1 C_{t-1}^{(CAN)} + e_{1t} \\ C_t^{(US)} &= \phi_2 C_{t-1}^{(US)} + e_{2t} \end{aligned}$$

Then, I calculate

$$\hat{e}_{1t} = C_t^{(CAN)} - \hat{\phi}_1 C_{t-1}^{(CAN)}$$

and

$$\hat{e}_{2t} = C_t^{(US)} - \hat{\phi}_2 C_{t-1}^{(US)}$$

for  $t = 1, \dots, T$  and calculate the cross correlation of the residuals. The plot of the cross-correlation of the pre-whitened FCIs is in Figure 8. The cross-correlation plot shows that the correlation between the current value of the Canadian index and values of the U.S. index dies out after a 1 month lag in the U.S. index. The current value of the U.S. index is correlated only with the current value of the Canadian index.

The cross-correlation result suggests that the U.S. index leads the Canadian index by one month, since the correlation of the Canadian index with a 1 month lag in the U.S. index is significant. This is an important finding, show-

ing that the Canadian FCI, in following the U.S. index, may be significantly affected by changes in the U.S. FCI. However, it also suggests that the U.S. index provides little information about the Canadian index more than one month in advance, and the Canadian index contains almost no information about future values of the U.S. index at all. So, current changes in U.S. and Canadian financial markets are not closely related to future changes in each other's financial markets. In general, trends in the FCIs are not persistent, and are very difficult to predict.

For the estimated volatility of the indices, the correlation coefficient between the two volatility series, at 0.73, is similar to the correlation of the actual level of the index. This suggests that not only the level of financial conditions, but also the uncertainty that accompanies fluctuations in the level of the FCIs is closely related in the U.S. and Canada. Figure 9 gives the cross-correlation plot of the volatility. In order to calculate the cross-correlation, the volatility series is also pre-whitened, in the same process as above, with  $C_t^{(CAN)}$  replaced with  $h_t^{(CAN)}$  and  $C_t^{(US)}$  replaced with  $h_t^{(US)}$ . The plot shows little correlation between the current levels of Canadian volatility and lags of the U.S. volatility beyond the current and previous level of U.S. volatility. For lags of more than one month, U.S. volatility cannot predict future values of Canadian volatility, and Canadian volatility cannot predict future values of U.S. volatility. Previous studies, as well as my estimates in Section 2, show the persistence of volatility as very high in financial markets. Given these results, I expected to find cross-correlation at longer lags than appear in Figure 9. The cross-correlation plots, and the parameter estimates for the volatility do not support previous evidence that financial series have high persistence of volatility. This suggests that high persistence of volatility is a characteristic of individual financial markets, but that the common component of multiple financial markets does not have this persistence. It is possible that this finding is due to the noise of the financial data that makes estimation difficult, and the domination of the volatility series by the Great

Recession. The enormous spike in volatility that occurs in 2008 prevents the fluctuations from 2008-2013 from having clear, significant trends, which could explain why persistence in the indices are so low.

## 6.4 The Effect of Shocks in the U.S. Market on the Canadian Market

The descriptions of vector autoregression (VAR) models and variance decomposition in this section and some of the notation are adapted from Enders (1995). A VAR model is a multiequation time series model that captures linear interdependencies among multiple time series. Using the Akaike Information Criterion (AIC) lag length selection, as well as the cross-correlation test described in Section 6.3, I found that a VAR(1) model best described the relationship between the U.S. and Canada FCIs. A VAR model lets  $C_t^{(CAN)}$  be affected by current and past realizations of  $C_t^{(US)}$  and past realizations of itself, and allows  $C_t^{(US)}$  to be affected by current and past realizations of  $C_t^{(CAN)}$  and past realizations of itself. The VAR standard form is derived from the bivariate system

$$\begin{aligned} C_t^{CAN} &= -b_{12}C_t^{US} + \gamma_{11}C_{t-1}^{CAN} + \gamma_{12}C_{t-1}^{US} + \epsilon_{yt} \\ C_t^{US} &= -b_{21}C_t^{CAN} + \gamma_{22}C_{t-1}^{US} + \gamma_{21}C_{t-1}^{CAN} + \epsilon_{zt} \end{aligned}$$

where  $\epsilon_{yt}$  and  $\epsilon_{zt}$  are uncorrelated, white-noise disturbances with standard deviations of  $\sigma_y$  and  $\sigma_z$ . In the two variable case, the standard form VAR(1), derived from the previous system, is

$$\begin{aligned} C_t^{CAN} &= a_{11}C_{t-1}^{CAN} + a_{12}C_{t-1}^{US} + e_{1t} \\ C_t^{US} &= a_{21}C_{t-1}^{CAN} + a_{22}C_{t-1}^{US} + e_{2t}. \end{aligned}$$

$e_{it}$  for  $i = 1, 2$  are stationary, uncorrelated processes, with zero mean and constant variance.  $e_{1t}$  and  $e_{2t}$  are correlated, however, and the variance-covariance matrix of the shocks is

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_2^2 \end{bmatrix}$$

I estimated the parameters of the following VAR(1) model and used the model to make inferences on how a U.S. shock affects the Canadian financial market over time. The estimated VAR model, with Canada ordered first, is as follows (standard errors of each of the estimates are underneath the estimates):

$$C_t^{CAN} = 0.443C_{t-1}^{CAN} + 0.538C_{t-1}^{US}$$

(0.0552)      (0.085)

$$C_t^{US} = -0.0911C_{t-1}^{CAN} + 0.914C_{t-1}^{US}$$

(0.0317)      (0.0486)

There are 352 observations. The  $R^2$  for the regression with the Canadian index as the dependent variable is 0.56 and the  $R^2$  for the prediction of the U.S. index as the dependent variable is 0.66. All parameters are significant at a 99% significance level, meaning the Canadian index is predicted by one lag of itself and the U.S. index, and the U.S. index is predicted by one lag of itself and the Canadian index. However, the relative size of the coefficients differ. The Canadian FCI is dependent on the U.S. and Canadian FCI in approximately equal parts. For the U.S. FCI, however, the coefficient on the lag of the U.S. FCI is ten times larger than the coefficient on the Canadian FCI. The covariance matrix of the residuals is

$$\hat{\Sigma} = \begin{bmatrix} 0.252 & 0.0872 \\ 0.0872 & 0.08330 \end{bmatrix}.$$

The variance of the Canadian residuals is higher than the variance of the U.S. residuals, and the correlation between the residuals is 0.60.

## VAR and the Impulse Response

A VAR can be written as a vector moving average (VMA).

$$\begin{bmatrix} C_t^{CAN} \\ C_t^{US} \end{bmatrix} = \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix} \begin{bmatrix} \epsilon_{yt-i} \\ \epsilon_{zt-i} \end{bmatrix},$$

where

$$\phi_i = \left[ 1/(1 - b_{12}b_{21}) \right] \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^i \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix}$$

The four elements of  $\phi_{jk}(0)$  are the impact multipliers. For example,  $\phi_{12}(0)$  is the instantaneous impact of a U.S. shock, a one unit change in  $\epsilon_{zt}$ , on the Canadian index,  $C_t^{CAN}$ . The accumulated effects of impulses in  $\epsilon_{zt}$  or  $\epsilon_{yt}$  can be obtained by summing the coefficients of the impulse response functions. So, the long-run effect of a change in a U.S. shock,  $\epsilon_{zt}$ , on the Canadian index,  $C_t^{CAN}$ , is  $\sum_{i=1}^{\infty} \phi_{12}(i)$ . Figure 11 shows the effect of a U.S. shock on the Canadian financial conditions index, which is a plot of the impulse response function  $\phi_{12}(i)$  against  $i$ . After a positive shock in the U.S. index, within two months, there is a sharp increase in Canadian financial conditions. This impulse response dies out after 24 months. It is important to note that there are additional restrictions necessary on the two-variable VAR system in order to identify the impulse responses, since the VAR model is overidentified. The constraint used is that I assume that a shock in the Canadian index,  $\epsilon_{yt}$ , has no direct effect on the U.S. index,  $C_t^{US}$ , although there is an indirect effect in that lagged values of the Canadian index still affect the U.S. index. In the case of the index VAR, this assumption involves imposing an ordering on the indices, so that the U.S. index leads the Canadian index. Since the size of the U.S. economy dwarfs the Canadian economy, and changes in the Canadian index are often a direct result of

events in the U.S., this seems to be a very reasonable assumption for the index VAR.

### Variance Decomposition

Given the current value of  $C_t^{CAN}$  and  $C_t^{US}$ , and the estimated coefficients of the VAR model, the VAR model can be used to forecast future values of the indices. Using the VMA representation of the model, the n-step ahead forecast of the indices, letting  $c_t = \begin{bmatrix} C_t^{CAN} \\ C_t^{US} \end{bmatrix}$ , is

$$c_{t+n} = \sum_{i=0}^{\infty} \phi_i \epsilon_{t+n-i}.$$

The n-period forecast error is

$$c_{t+n} - \hat{c}_{t+n} = \sum_{i=0}^{n-1} \phi_i \epsilon_{t+n-i}.$$

From this equation, the n-step ahead forecast error variance of the Canadian index is derived:

$$\sigma_y(n)^2 = \sigma_y^2[\phi_{11}(0)^2 + \phi_{11}(1)^2 + \dots + \phi_{11}(n-1)^2] + \sigma_z^2[\phi_{12}(0)^2 + \phi_{12}(1)^2 + \dots + \phi_{12}(n-1)^2]$$

This forecast error variance of the Canadian index can be decomposed into proportions due to Canadian shocks and U.S. shocks. These proportions, respectively, are

$$\frac{\sigma_y^2[\phi_{11}(0)^2 + \phi_{11}(1)^2 + \dots + \phi_{11}(n-1)^2]}{\sigma_y(n)^2}$$

and

$$\frac{\sigma_z^2[\phi_{12}(0)^2 + \phi_{12}(1)^2 + \dots + \phi_{12}(n-1)^2]}{\sigma_y(n)^2}$$

The forecast error variance decomposition explains the proportion of the movements in one index due to its own shocks versus shocks to the other index. This allows inference to be made on the exogeneity of the indices - if  $\epsilon_{yt}$  shocks explain little of the forecast error variance of  $C_t^{US}$ , then that is a good indication that  $C_t^{US}$  is exogenous. The ordering constraint that applied for the impulse response functions also applies for the variance decomposition. So, to, obtain variance decomposition results for the Canadian index, I estimated a VAR model with the Canadian index ordered first, and for the results for the U.S. index, I estimated a VAR model with the U.S. index ordered first. Using variance decomposition, I found that a U.S. shock explains 5% of the short-run variance of the forecast error in the Canadian index, and 20% of the long-run variance of the forecast error in the Canadian index when forecasting two years ahead. The Canadian index is significantly affected by shocks in the U.S. index. In comparison, a Canadian shock explains 0% of the short-run variance and 5% of the long-run variance of the forecast error in the U.S. index when forecasting two years ahead. Financial conditions in the U.S. have a far more significant effect on conditions in Canada, compared to the effect of conditions in Canada on the U.S., as expected.

### **Granger Causality Test**

In the two equation VAR(1) model,  $C_t^{CAN}$  does not Granger-cause  $C_t^{US}$  if and only if  $a_{21} = 0$ .  $C_t^{US}$  does not Granger-cause  $C_t^{CAN}$  if and only if  $a_{12} = 0$ . This is a weaker condition than exogeneity, which would require all current and past values of  $C_t^{CAN}$  to have no significant effect on  $C_t^{US}$ .

The Granger Causality test is conducted using an F-test. A Granger causality test rejects the hypothesis that the U.S. does not Granger-cause the Canadian index at a 99% significance level. This is an added confirmation that fluctuations in financial conditions in the U.S. significantly affect financial conditions in Canada. However, the Granger causality test also rejects the hypothesis that Canada does not Granger-cause the U.S. index at

a 99% significance level.  $a_{21}$  is significant, and, surprisingly, it is negative, which would indicate a negative Canadian shock would result in a positive U.S. response in the next period. It is unlikely that changes in Canadian financial conditions regularly affect U.S. financial conditions, especially in a period where there has not been any serious Canadian financial collapse that would have resulted in major changes to the U.S. economy and financial markets. The result of the Granger causality test can be explained by an identification issue with my VAR model. Perhaps there is a third series, which could be described as a series of world shocks, that affects both the U.S. and Canada, and has been left out of the VAR. This conclusion is supported by the correlation coefficient of  $e_{1t}$  and  $e_{2t}$ , which is high, and could indicate the presence of some third series affecting both the U.S. and Canadian indices.

## 7 Conclusion

The dynamic factor model estimation with stochastic volatility was successful. I found that the indices I created had some signaling power, but were dominated by the 2008-2009 disruption to the financial markets. I found that the collapse of CCB and Northland Bank, the Asian Financial Crisis, and some months during the Great Recession were periods where Canada had significantly worse financial conditions than the U.S. My indices also confirm that Canada did recover faster than the U.S. after the Great Recession and was not as affected by the tech bubble collapse. However, relying on the differences in the FCIs did result in false signals that did not appear to be related to any major structural differences in the U.S. and Canada. My correlation results were also interesting, showing a lack of long-term cross-correlation between the indices as well as their volatility. This does corroborate the literature that shows that financial markets are very difficult to predict. In the analysis of the volatility of the indices, it was surprising

that the volatility of the series did not show more persistence. My results suggest that while individual financial time series do show highly persistent volatility, an estimate of a common component of the financial market does not.

By perturbing the data series used to estimate the indices, I find that the Canadian financial market, which is a part of Canada's small, open economy, has much more of a robust link between commodity markets, the stock market, the exchange rate, and interest rates than the U.S. market.

In addition, the creation of the FCIs allows for various hypotheses to be posed and tested regarding the relationship between U.S. and Canada financial conditions. As expected, I found that U.S. shocks had a significant effect on the Canadian financial market, whereas Canadian shocks had little significance in the U.S. market. U.S. shocks explain over 20% of Canadian forecast error variance, while Canadian shocks explain less than 5% of U.S. forecast error variance. I did find, however, that the Canadian index does Granger-cause the U.S. index, which was unexpected and could indicate that the VAR model I estimated is missing some third series that affects both the U.S. and Canada.

There are many aspects of my approach that can be improved. The noise of financial data made estimation of the financial conditions indexes and inference related to the resulting series difficult. Volatility estimation especially was not robust and results for the U.S. index varied greatly with small changes in the hyperparameters or input data. It would be useful to create other financial conditions indices, using approaches other than estimation using a dynamic factor model, to see how robust the results I came up with were. Furthermore, the FCIs that I created contain a very limited amount of information about the financial markets in the U.S. and Canada. Creating FCIs with many more input series and repeating my analyses would be useful. There are many other measures that could be included and are useful for gauging financial conditions, such as implied volatility in the options

market, interest rate and funding cost measures, short and long-term bond prices, futures prices and derivatives prices.

Though the indices that I create are noisy and do not include as many series as would be optimal, I do show that a dynamic factor models, which have generally been used to create indices of real economic indicators, can be used to estimate indices based on financial data. Furthermore, it is feasible to add stochastic volatility with a persistence parameter to the dynamic factor model and estimate the index with stochastic volatility, which is not something that has been documented in the literature. The financial conditions indices that are created in this way can be used to test hypotheses about the relationships between financial markets in different countries, and are able to identify differences between the effect of shocks in the small, open Canadian market on the U.S. and shocks in the large, dominant U.S. market on Canada.

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Table 1: Correlation of Input Series for the FCIs

	TSX	BCPI	CAD /USD	TED (Can.)	SP500	WTI	USD	TED (U.S.)
TSX	1.00	0.231	0.303	-0.034	0.778	0.133	-0.173	-0.123
BCPI	0.231	1.00	0.424	0.041	0.084	0.710	-0.166	0.063
CAD/USD	0.303	0.424	1.00	0.027	0.241	0.312	-0.361	0.045
TED (Can.)	-0.034	0.041	0.027	1.00	-0.043	0.034	0.036	0.460
SP500	0.778	0.084	0.241	-0.043	1.00	-0.015	-0.148	-0.137
WTI	0.133	0.710	0.312	0.034	-0.015	1.00	-0.119	0.067
USD	-0.173	-0.165	-0.361	0.036	-0.148	-0.119	1.0000	0.018
TED (U.S.)	-0.123	0.063	0.045	0.460	-0.137	0.067	0.018	1.00

Table 2: Recessions in the U.S. and Canada since 1984

<b>Canada</b>	<b>U.S.</b>
March 1990 - April 1992	July 1990 - March 1991
October 2008 - May 2009	March 2001 - November 2001 December 2007 - June 2009

Table 3: Events Causing Financial Stress in the U.S. and Canada since 1984

<b>Event</b>	<b>Canada Only</b>
CCB and Northland Bank failures (1985, Canada)	Yes
October 1987 Stock Market Crash	No
Asian Crisis (1997-1998)	No
Mexican crisis (1994-1995)	No
Russian debt default (1998)	No
LTCM collapse (1998)	No
High-tech bubble collapse (2000)	No
Subprime Mortgage Crisis (2007-2009)	No
Eurozone Crisis (2010-2012)	No

Table 4: U.S. Individual Volatility Parameter Estimates

Parameter	S&P 500	WTI	Currency Basket	TED Spread
$\mu$	1.4051 (0.5113)	1.2798 (0.3588)	2.97 (0.4031)	0.0722 (0.0589)
$\beta$	0.4686 (0.1910)	0.5156 (0.1361)	-0.03 (0.1239)	0.9238 (0.0602)
$\tau^2$	0.3524 (0.1298)	0.3219 (0.1183)	0.0888 (0.1566)	0.4648 (0.1404)

Table 5: Canada Individual Volatility Parameter Estimates

Parameter	S&P TSX	Commodity Basket	USD/CAD	Canada TED Spread
$\mu$	2.0234 (0.4852)	0.3642 (0.1588)	1.2694 (0.5382)	0.1019 (0.0884)
$\beta$	0.2426 (0.1747)	0.8458 (0.0913)	0.5242 (0.19)	0.9258 (0.0715)
$\tau^2$	0.3747 (0.1296)	0.3061 (0.132)	0.3061 (0.0954)	0.4697 (0.1787)

Table 6: Parameter Estimates of the Financial Conditions Indices

Parameter	FCI (U.S.)	FCI (Can.)	FCI-ex (U.S.)	FCI-ex (Can.)	FCI-pre08 (U.S.)	FCI-pre08 (Can.)
$\phi_1$	0.5893 (0.0818)	0.5582 (0.0799)	0.5996 (-0.0831)	0.6206 (0.0814)	0.5777 (0.0904)	0.5777 (0.0919)
$\phi_2$	-0.01519 (0.07461)	-0.024 (0.0709)	-0.0048 (0.076)	-0.0306 (0.0732)	-0.01 (0.0854)	-0.021 (0.085)
$\lambda_1$	1	1	1	1	1	1
$\lambda_2$	1.213 (0.5118)	2.8811 (0.3852)	0.5947 (0.5632)	2.27 (0.5515)	-0.033 (0.594)	1.717 (0.4889)
$\lambda_3$	-1.4352 (0.5010)	2.6005 (0.3565)	- -	- -	-0.0603 (0.6341)	1.3282 (0.526)
$\lambda_4$	-0.13972 (0.2386)	0.0091 (0.2621)	-0.269 (0.312)	-0.061 (0.318)	-0.1061 (0.254)	0.1152 (0.3272)
$\sigma_1^2$	19.285 (1.502)	18.67 (1.461)	19.146 (1.515)	19.098 (1.5)	17.68 (1.62)	18.77 (1.699)
$\sigma_2^2$	17.31 (1.586)	12.74 (1.694)	18.048 (1.45)	15.031 2.215	16.81 (1.482)	16.8 (1.944)
$\sigma_3^2$	17.82 (1.848)	13.406 (1.593)	- -	- -	17.79 (1.619)	16.963 (1.842)
$\sigma_4^2$	18.321 (1.3901)	20.23 (1.535)	18.141 (1.404)	20.22 (1.55)	15.05 (1.32)	19.13 (1.67)
$\psi_{11}$	0.0328 (0.0562)	0.0183 (0.0556)	0.0211 (0.0577)	0.034 (0.0557)	-0.0224 (0.0665)	0.0296 (0.0645)
$\psi_{12}$	0.2618 (0.0613)	0.0938 (0.0724)	0.2899 (0.0564)	0.1053 (0.0802)	0.2719 (0.0638)	0.0884 (0.0727)
$\psi_{21}$	-0.0168 (0.0651)	0.1324 (0.0714)	-0.222 (0.0533)	-0.155 (0.053)	0.0693 (0.063)	0.2021 (0.0717)
$\psi_{22}$	-0.2201 (0.05167)	-0.1548 (0.5355)	-0.0352 (0.0546)	-0.024 (0.056)	-0.367 (0.0592)	-0.1552 (0.0625)
$\psi_{31}$	-0.0328 (0.0562)	-0.017 (0.0547)	- -	- -	-0.047 (0.0645)	-0.0521 (0.0626)
$\psi_{32}$	-0.04408 (0.0559)	-0.0022 (0.0655)	- -	- -	-0.1212 (0.062)	-0.1226 (0.0692)
$\psi_{41}$	0.0098 (0.0598)	-0.0642 (0.644)	-0.035 (0.0546)	0.018 (0.0651)	0.031 (0.062)	-0.0816 (0.0662)
$\psi_{42}$	-0.1832 (0.0518)	0.0733 (0.0533)	-0.1829 (0.0522)	0.0737 (0.0537)	-0.1412 (0.0587)	0.1127 (0.0613)

Table 7: Volatility Parameter Estimates of the Financial Conditions Indices

Parameter	FCI (U.S.)	FCI (Can.)	FCI-ex (U.S.)	FCI-ex (Can.)	FCI-pre08 (U.S.)	FCI-pre08 (Can.)
$\mu$	-0.5189 (0.0887)	-0.4684 (0.141)	-0.431 (0.105)	-0.414 (0.128)	-0.589 (0.123)	-0.4944 (0.0951)
$\beta$	-0.0931 (0.1364)	0.0421 (0.2132)	0.0241 (0.1649)	0.1013 (0.171)	-0.026 (0.154)	0.0382 (0.1634)
$\tau^2$	0.0232 (0.0097)	0.0387 (0.0137)	0.02257 (0.01086)	0.025 (0.0136)	0.0214 (0.0085)	0.02563 (0.0122)

Table 8: Disruption Signalling of U.S. Index

90% Confidence	95% Confidence	99% Confidence
Dec. 1984 Nov. 1992 March 2001 - April 2001 Aug. 2008 - Feb. 2009 June 2010 Sept. 2011 June 2012	April 2001 Aug. 2008 - Jan. 2009	Sept. 2008-Dec. 2008

Table 9: Disruption Signalling of Canadian Index

90% Confidence	95% Confidence	99% Confidence
Sept. 1998 March 2001 Oct. 2001 Sept. 2008 - Jan. 2009 Sept. 2011	March 2001 Oct. 2001 Sept. 2008 - Jan. 2009	Sept. 2008 - Jan. 2009

Table 10: Divergence Signal, U.S. Below Canada

90% Confidence	95% Confidence	99% Confidence
Dec. 1984	Dec. 1984	July 1988
Feb. 1988	June-July 1988	Oct. 2005
June-July 1988	Dec. 1993	
April 1991	Feb. 2000	
Dec. 1993	Jan. 2001	
Feb. 2000	April 2005	
Jan. 2001	Sept.-Oct. 2005	
March 2003	June 2008	
April 2005	Feb. 2009	
July - Oct. 2005	Jan. - Feb. 2010	
June - July 2008		
Feb. 2009		
Jan. - Feb. 2010		

Table 11: Divergence Signal, Canada Below U.S.

90% Confidence	95% Confidence	99 % Confidence
April 1985	Oct.-Dec. 1985	-
August 1985	May 1994	
Sept-Dec. 1985	April 1995	
May 1994	Sept.-Nov. 1998	
March-April 1995	Nov. 2008	
Sept. -Nov. 1998	Aug. 2009	
Feb. 2006		
Nov. 2008		
Aug. 2009		

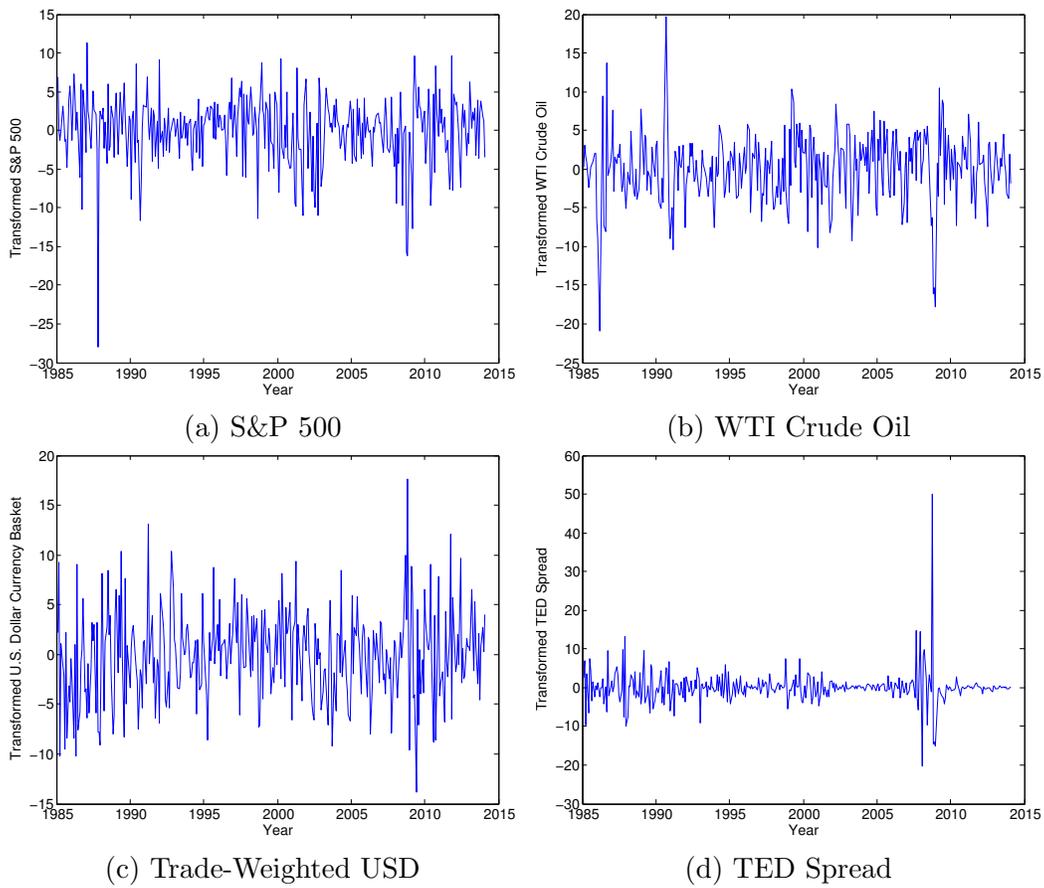


Figure 1: Components of the U.S. Index

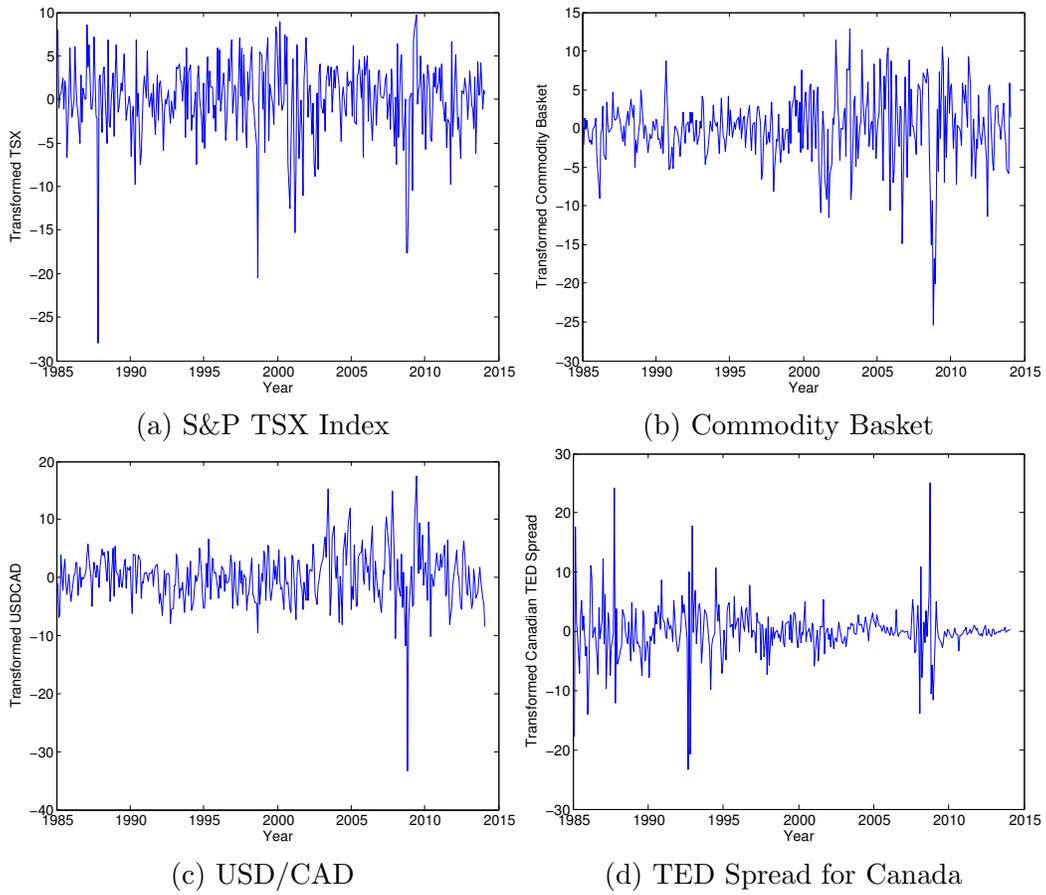
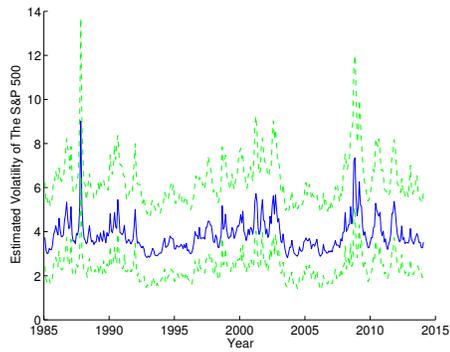
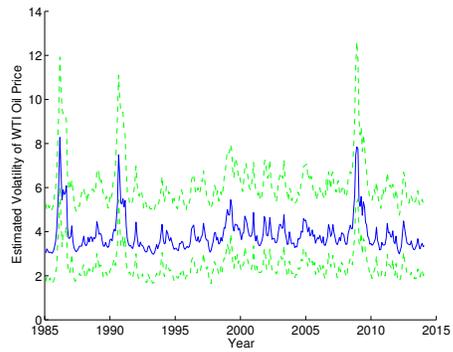


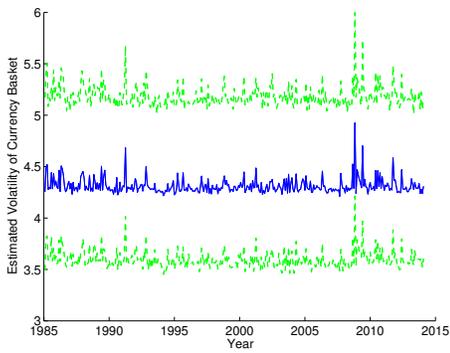
Figure 2: Components of the Canadian Index



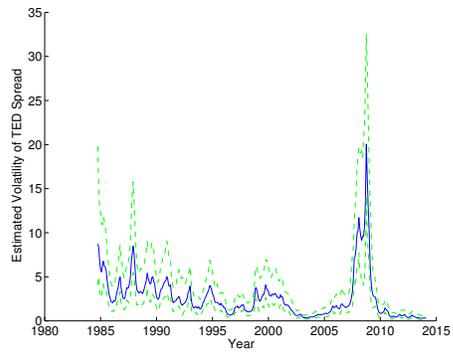
(a) S&P 500



(b) WTI Crude Oil

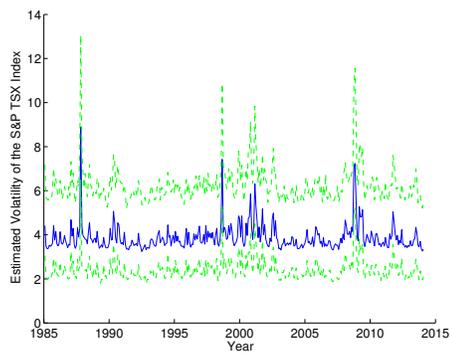


(c) Trade-Weighted USD

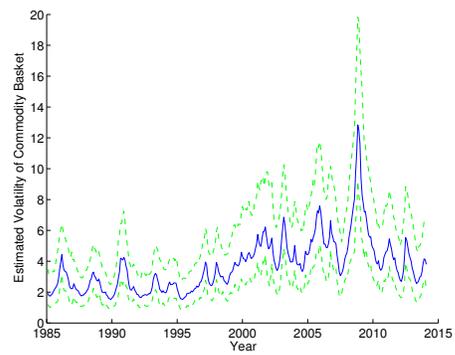


(d) TED Spread

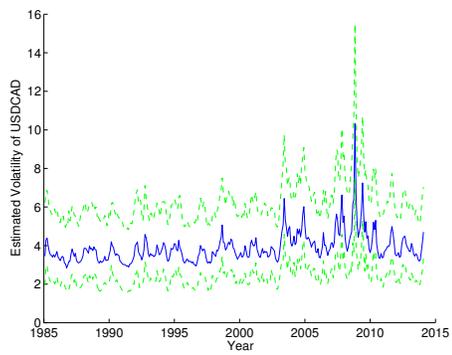
Figure 3: Volatility of Inputs to the U.S. Index



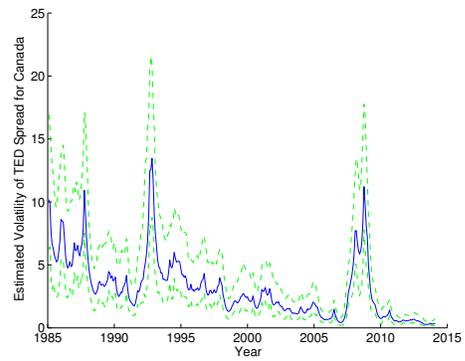
(a) S&P TSX Index



(b) Commodity Basket



(c) USD/CAD



(d) TED Spread for Canada

Figure 4: Volatility of Inputs to the Canadian Index

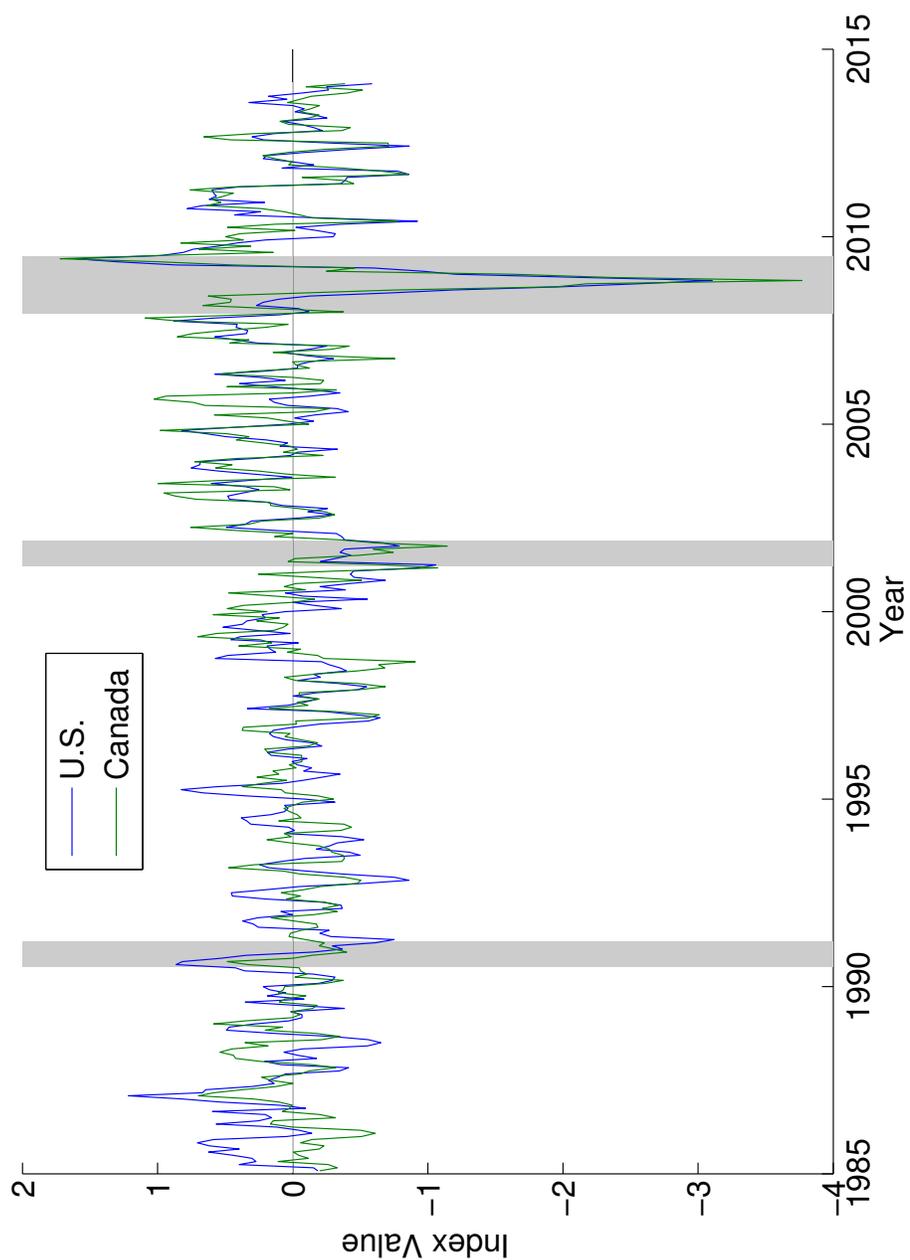


Figure 5: U.S. and Canadian FCIs

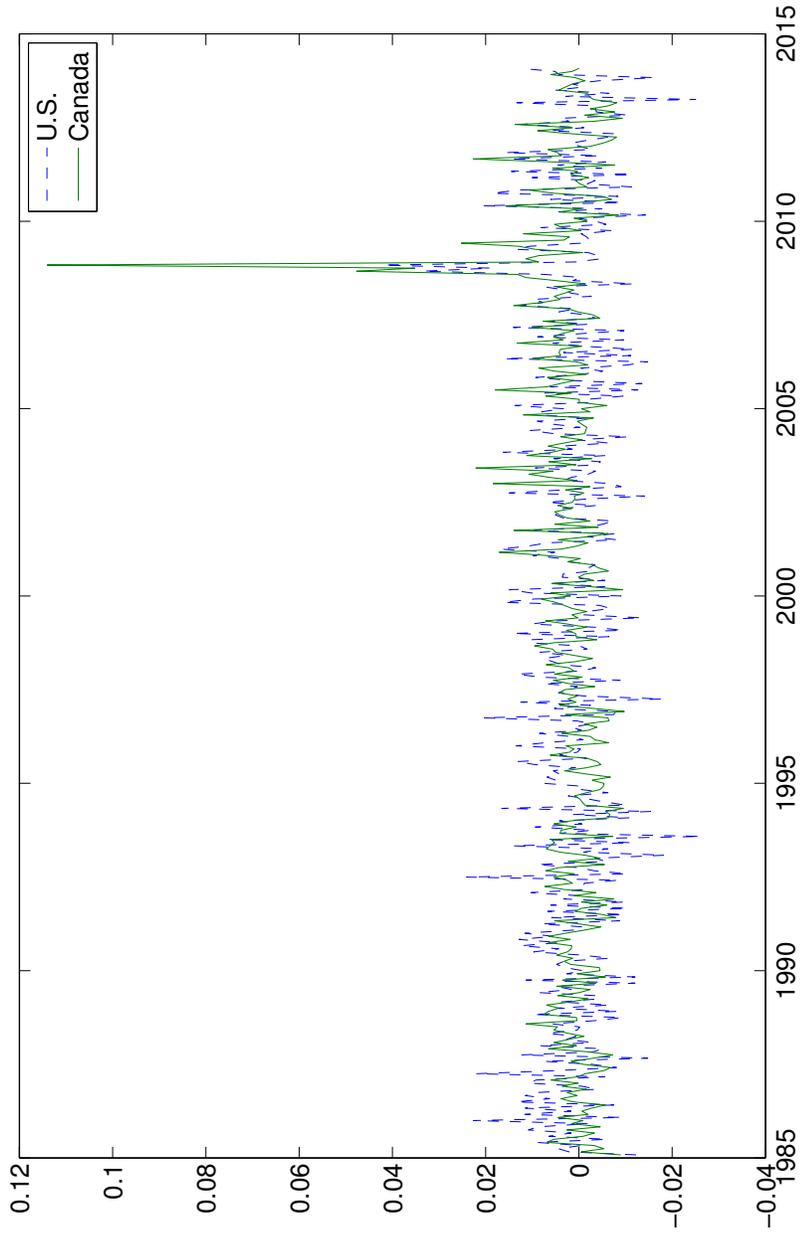


Figure 6: Estimated Volatility

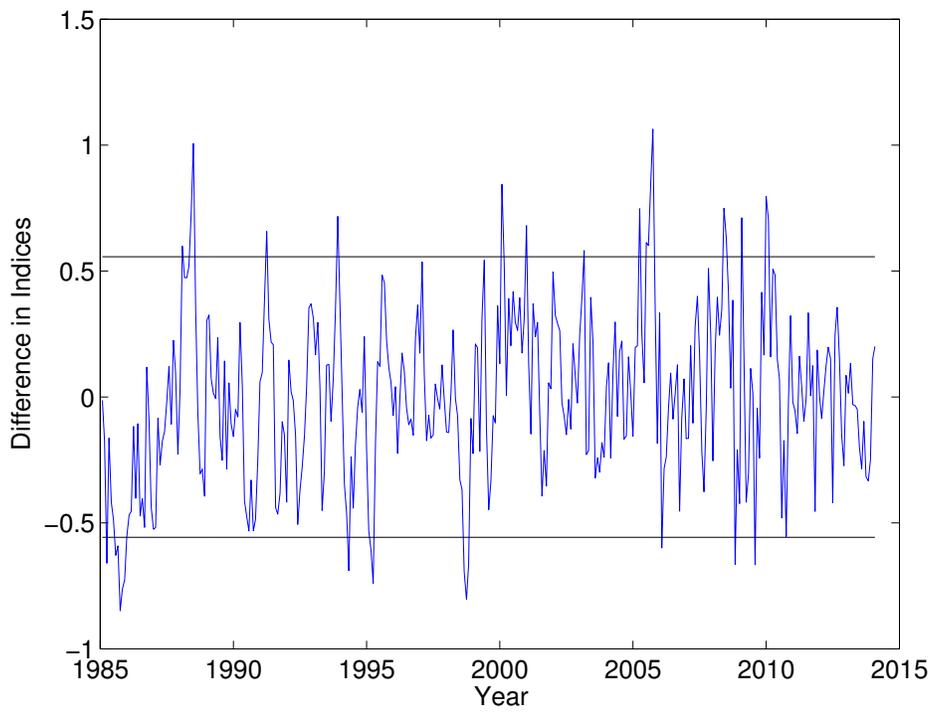


Figure 7: Difference in the Canadian and U.S. Indexes

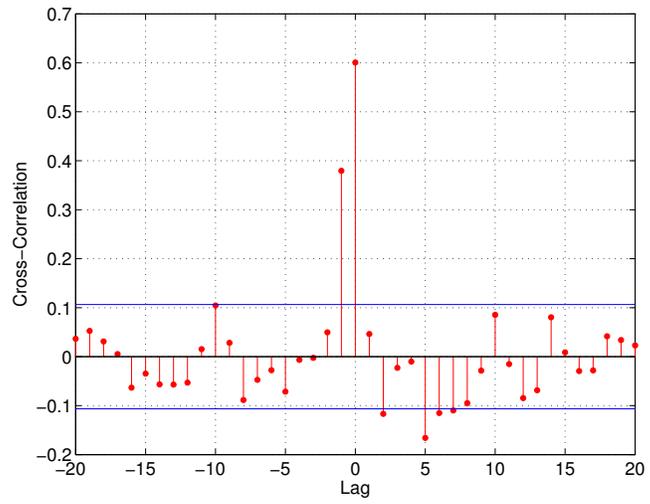


Figure 8: Cross-Correlation of the Canadian FCI with the U.S. FCI

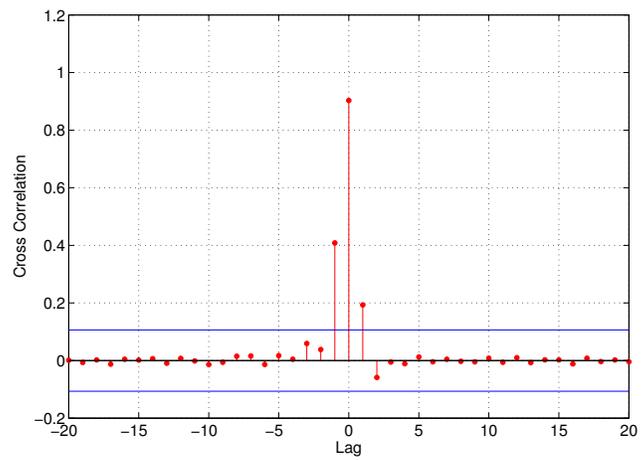


Figure 9: Cross-Correlation of the Volatility of Canadian FCI with the Volatility of the U.S. FCI

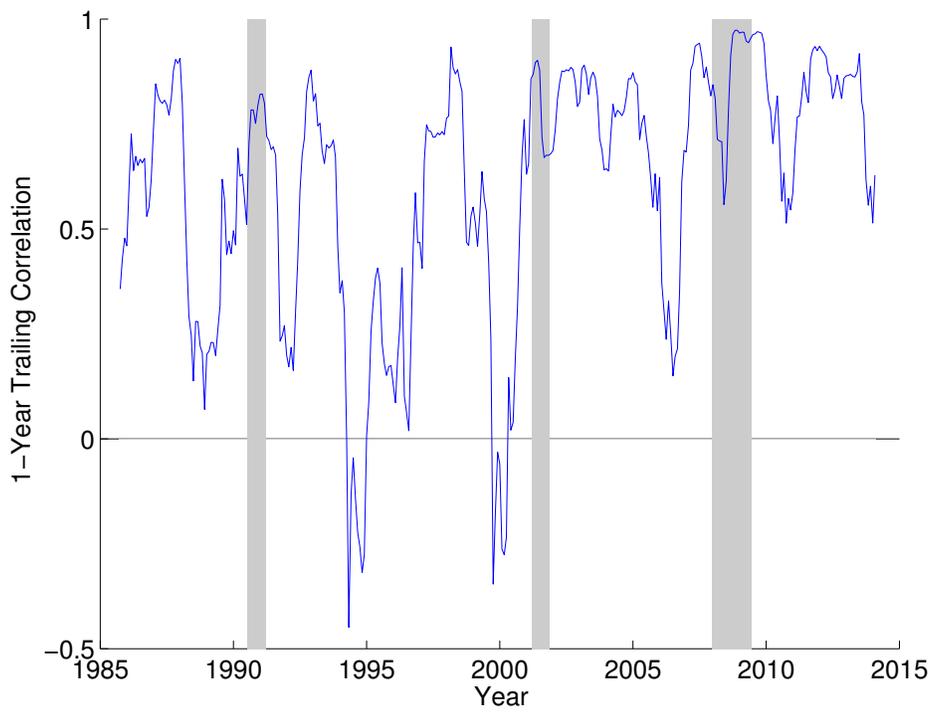


Figure 10: 1-Year Trailing Correlation of the Canadian FCI and the U.S. FCI

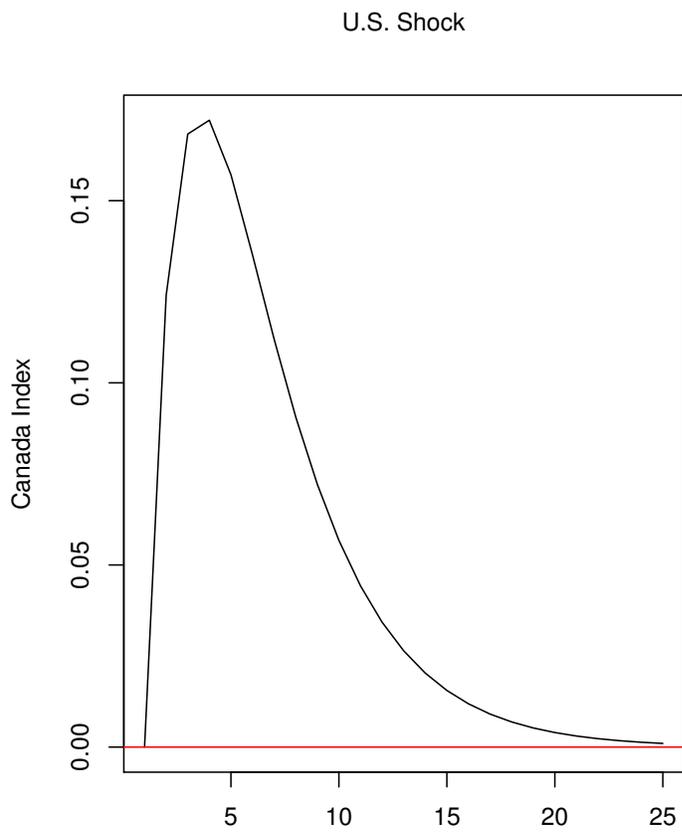


Figure 11: Impulse Response of Canadian Index to a U.S. Shock

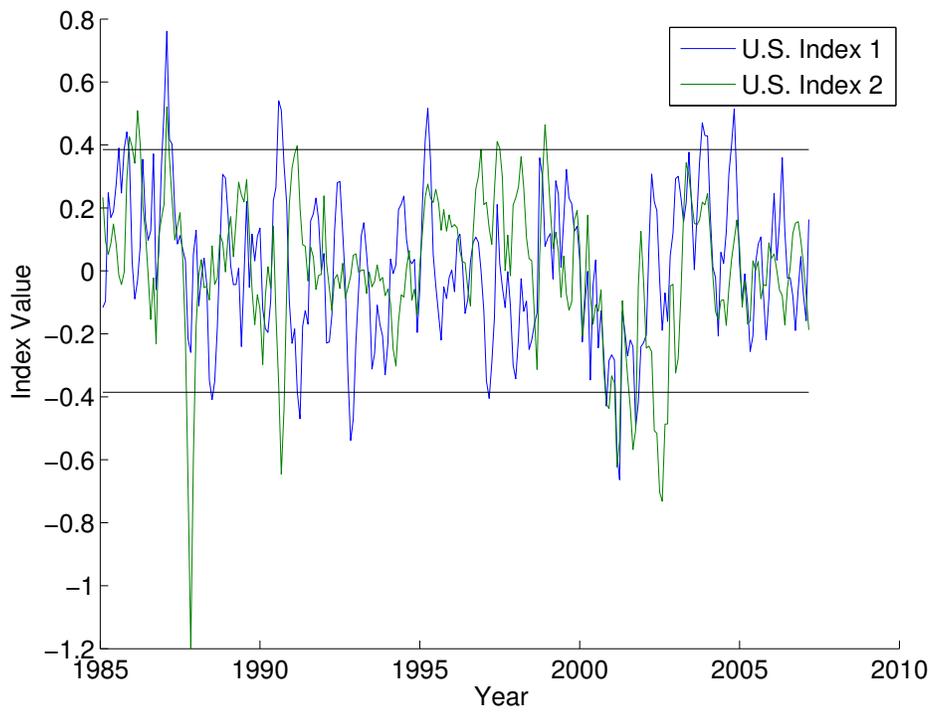


Figure 12: Index using Data Including Great Recession and without Great Recession