# The Cyclicality of Wages and Match Quality*

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November 2018

**Abstract**

I estimate the cyclicality of real wages for job stayers, and hires from both employment and from unemployment, using an administrative matched employer-employee dataset from Germany. I find that the wages of new hires appear to be *less* procyclical than the wages of job stayers. I propose an explanation based on countercyclical selection on match quality: when aggregate productivity is low, worker-firm matches have to be unusually productive to warrant job creation. The presence of the match quality selection effect is supported by the relationship between the initial aggregate conditions and the subsequent risk of separation: jobs started when unemployment is high are at a decreased risk of ending with a separation to unemployment, which suggests that they are positively selected. Finally, I show that the match quality selection effect arises in a Diamond-Mortensen-Pissarides search and matching model with match-specific productivity and turnover costs.

*JEL classification:* E24, E32, J41, J63, J64

*Keywords:* Wages, Unemployment, Cyclicality, Search and Matching

*I am indebted to Andres Drenik and Jón Steinsson for guidance and support. I thank Andreas Mueller and Hassan Afrouzi, Jennifer LaO, Christian Moser, Emi Nakamura, Stephanie Schmitt-Grohé, and seminar participants at the Columbia University for comments and discussions.

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# 1 Introduction

Unemployment is volatile relative to aggregate shocks, as discussed in Shimer (2005) and Pissarides (2009). Changes in incentives for job creation are an important driver of unemployment, since it is driven more by fluctuations in job creation and job finding than by fluctuations in separations.[1] The incentives for job creation depend on the expected cost of labor, which is proxied by the wages of new hires. Consequently, the cyclical behavior of wages is crucial for understanding the cyclical behavior of unemployment.

I provide new evidence on the cyclical behavior of real wages. I argue that countercyclical selection with respect to the quality of the match between a worker and a firm is reflected in the estimates of real wage cyclicality: the selection effect makes the wages of new hires appear less procyclical than they are. This view is supported by findings from German administrative microdata. I show that the match quality selection effect arises naturally in a Diamond-Mortensen-Pissarides search and matching model with match-specific productivity and turnover costs.

To investigate the cyclicality of wages, I estimate the relationship between the real wages and the unemployment rate using a matched employer-employee administrative dataset from Germany. The dataset allows for differentiating between two types of hires,[2] from employment and unemployment,[3] and addressing the potential biases: due to worker heterogeneity, as discussed in Bils (1985) and Solon, Barsky and Parker (1994); due to occupational down- or upgrading; and due to the differences between cyclicality of employment at high- and low-paying firms.[4]

Contrary to expectations, the wages of new hires are *less* procyclical than the wages of job

[1]See Hall (2005) or Shimer (2012) for a discussion of the decomposition of unemployment fluctuations.

[2]The differentiation between hires from employment and unemployment has been neglected in the wage cyclicality literature until recently. Notable recent exceptions are Getler, Huckfeldt and Trigari (2016) who find that the wages of hires from employment are more procyclical and the wages of hires from unemployment are no more cyclical than those of job stayers, and Haefke, Sonntag, and van Rens (2013), who find that changes in the wages of hires from unemployment closely follow aggregate labor productivity.

[3]Throughout the paper, "unemployment" refers to both unemployment and non-employment.

[4]Recently, Moscarini and Postel-Vinay (2012), Kahn and McEntarfer (2014), and Haltiwanger, Hyatt and McEntarfer (2015) investigated the cyclical properties of employment and employment growth for different categories of firms. Their findings raise the possibility that lower-paying firms are responsible for a higher share of employment and hires during downturns, which would introduce procyclical bias into the estimates of wage cyclicality.

stayers. This effect is stronger for hires from employment than for hires from unemployment. This counterintuitive result requires an explanation.

I propose an explanation based on countercyclical changes in the quality of firm-worker matches. Aggregate productivity has a direct effect on wages, as well as an indirect effect due to selection on match quality that acts in the opposite direction to the direct effect. During downturns, worker-firm pairs have to be unusually productive to warrant job creation. The average match quality for new hires is higher than for job stayers. Low aggregate productivity has a direct, negative effect on wages, as well as an indirect positive effect on the wages of new hires. In contrast, the opposite happens during upturns, as even low-quality matches are productive enough to be created. High aggregate productivity has a direct, positive effect on wages, as well as an indirect negative effect on the wages of new hires.

The presence of the match quality selection effect is empirically validated. As observed in Bowlus (1995), matches of better quality, which I conceptualize as match-specific productivity, should last longer. I investigate the relationship between risk of separation to unemployment, a proxy for match quality, and the unemployment rate at the start of a job. The relationship is negative: higher unemployment at the start of a job is associated with a decreased risk of a job ending with a separation to unemployment. This association is stronger for hires from employment than for hires from unemployment. These results support my hypothesis that matches started during downturns are positively selected, especially when they are created by a job-to-job transition.

Finally, I build a stochastic Diamond-Mortensen-Pissarides type model. The key features of the model are match-specific productivity and a hiring cost[5] that is incurred only when a job is created.[6] To be consistent with the results on job duration, the model features endogenous separations.

When aggregate productivity is low, the matches of new hires have high match-specific

[5]Hiring costs were added to the search and matching models in Braun (2006), Nagypal (2007), Silva and Toledo (2009) and Yashiv (2006).

[6]The presence of a firing cost would have the same effect. When firing workers is costly, during downturns some surviving matches generate a negative surplus, while all new matches have to generate a positive surplus. As I discuss later, the German labor market is characterized by the presence of both hiring and firing costs that are higher than in the US.

productivity, because only such matches are productive enough to cover a hiring cost. The previously created matches with low match-specific productivity are destroyed. The existing matches with medium and high match-specific productivity are productive enough to survive, even though some of them are not productive enough to cover a hiring cost. The matches of job stayers are a mixture of matches created in previous periods which are productive enough to survive, and matches created in recent periods of low productivity. Consequently, the distribution of match-specific productivity of new hires stochastically dominates the distribution of match-specific productivity of job stayers.

When aggregate productivity is high, even matches with low match-specific productivity are productive enough to cover a hiring cost. The matches for job stayers are a mixture of matches that survived previous periods of low aggregate productivity and matches created in recent periods of high productivity. Consequently, the distribution of match-specific productivity of job stayers stochastically dominates the distribution of match-specific productivity of new hires.

In the model, the match quality selection effect is present for both job stayers and new hires. The selection effect is stronger for new hires than for job stayers, which dampens procyclicality of the wages of new hires relative to the wages of job stayers.

I calibrate the model using external sources to inform the value of a hiring cost and the distribution of match-specific productivity. I compare the cyclical properties of the model-generated wages and the observed wages. The model-generated wages have similar cyclical properties as the observed wages: the wages of new hires are *less* procyclical than the wages of job stayers.

## 2 Related Literature

The main empirical part of this paper belongs to the literature on the cyclical properties of real wages. In the next section, I discuss how the results of this paper relate to previous empirical findings on wage cyclicality.

Bowlus (1995) introduced the idea that the relationship between the conditions at the start of a job and the subsequent risk of separation carries information about the cyclical properties of the match quality for new hires. To the best of my knowledge, this paper is the first to conduct such an analysis controlling for firm heterogeneity and using a large matched sample of firm and workers. As I discuss later, my results suggest that controlling for firm heterogeneity plays a crucial role in the analysis of the relationship between the conditions at the start of a job and the subsequent risk of separation.

The key elements of the model I use are match-specific productivity and hiring costs. Both features appeared previously in the theoretical literature. There are papers that assessed the hiring costs for Germany and the US, using survey data.

## 2.1 Cyclicality of Wages

How do real wages react to business cycle conditions? At least since the Dunlop-Tarshis-Keynes exchange, this simple question has been the subject of a large body of research and is still not fully answered. In recent years, the interest in this issue was renewed after Shimer (2005) argued that the Diamond-Mortensen-Pissarides search and matching model had difficulty reconciling fluctuations in unemployment and fluctuations in productivity. As emphasized in Pissarides (2009), establishing how real wages behave over the business cycle is crucial for understanding cyclical fluctuations in unemployment. This paper belongs to a recent wave of papers that use microdata to investigate the cyclicality of wages.

Up to the early 1990s, the consensus, based on studies using aggregate data, was that real wages in the US were acyclical or, at best, weakly procyclical. These studies were suspected to suffer from various forms of composition bias. As Stockman (1983) surmised, the composition of the labor force changes over the business cycle: hours and employment of low-wage workers are more procyclical than hours and employment of all workers, which induces a countercyclical bias in an aggregate measure of wages. An opposite procyclical bias was identified in Chirinko (1980) as arising from high cyclical sensitivity of high-wage industries such as durables manufacturing and construction.

The use of individual level data shattered the previous consensus, starting with Bils (1985) and Solon, Barsky and Parker (1994). Wages were usually found to be procyclical.

Newer papers differentiate not only between job stayers and new hires but also hires from unemployment and employment. A recent example is Haefke, Sonntag and van Rens (2013), which uses CPS cross-sectional data and finds that the elasticity of wages with respect to labor productivity is higher for hires from unemployment than for job stayers, and even higher for hires from employment, although the standard errors are large. A different conclusion is reached in Gertler, Huckfeldt and Trigari (2016), which uses SIPP panel data and to finds that the wages of job stayers are slightly procyclical, the wages of hires from unemployment are acyclical and the wages of hires from employment are procyclical.

Studies of the US labor market suffer from data limitations. Suitable datasets are, at best, panels. They contain scanty information on employers and often unsatisfactory information on workers. Wages, earnings and hours are plagued by measurement error. The use of administrative datasets reduces measurement error issues and allows to control for various potential sources of composition bias. Recent examples are Carneiro, Guimaraes and Portugal (2012) and Martins, Solon and Thomas (2012), which use Portuguese Quadros de Pessoal, a matched employer-employee dataset. In the first paper, the cyclicality of wages is estimated with controls for worker, job and occupation fixed effects. The wages of new hires are found to be more procyclical than the wages of job stayers. The second paper concentrates on hiring wages for a set of entry jobs, which are found to be quite procyclical. Due to limitations of the dataset, these papers cannot differentiate between hires from employment and those from unemployment.

For Germany, Stueber (2017) used a similar source of data as my paper, the employment biographies generated by the German social security system, but considered the period 1977-2009 at a yearly frequency. The wages of new hires were found to be no more procyclical, when controlling for worker and employer-occupation fixed effects, than the wages of job stayers.

## 2.2 Match Quality

Is match quality higher or lower in jobs started in periods of high unemployment than those started in periods of low unemployment? Match quality, however defined, is not directly observable. A traditional proxy for job quality is job duration - a better match should last longer. Using job duration until transition to different employment or unemployment as a proxy for match quality is equivalent to investigating the instantaneous probability of separation conditional on previous survival (the hazard rate). The sign and strength of the relationship between the risk of separation and the unemployment rate at the start of a job carries information about the cyclical behavior of match quality.

Bowlus (1995), the first to use job duration as a proxy for match quality, found that a higher initial unemployment rate increased the subsequent risk of separation. This finding, suggestive of procyclical match quality, motivated Barlevy (2002) to formulate a theory of sullying recessions. Baydur and Mukoyama (2018) used the competing risks model, finding that a higher initial unemployment rate increased the risk of job-to-job transition but decreased the risk of separation into nonemployment.

These papers used panel data from the National Longitudinal Survey of Youth, which precluded controlling for firm heterogeneity. Kahn (2008) exploited a small matched dataset of Fortune 500 firms and their employees. Controlling for firm heterogeneity switched the sign of the relationship between the separation risk and the initial unemployment rate from positive to negative. I observe a similar phenomenon in my data, which suggests that the average match quality for new hires might be countercyclical in the US as well as in Germany, contrary to previous findings.

## 2.3 Match-Specific Productivity

The presence of match-specific productivity, or equivalently the idiosyncratic price of output, is common in the search and matching literature. The standard assumption is that new matches start with the same match-specific productivity, which later evolves, as in Mortensen and Pissarides (1994), Pissarides (2009), and Fuijta and Ramey (2012). Matches were allowed to start with randomly drawn productivity in Mortensen (1982) and Mortensen and Nagypal (2007b). However, the consequences of the presence of match-specific productivity for the cyclical properties of wages were not investigated.

A paper closely related to mine is Gertler, Huckfeldt and Trigari (2016). They build a model with match-specific productivity and endogenous on-the-job search. The model generates a procyclical selection effect for new hires from employment. An interesting implication is that jobs created by a job-to-job transition during downturns should be at an increased risk of ending with a subsequent job-to-job transition. The implication was not investigated in the paper.

The consequences of match quality selection for wages appear in a different context in Hagedorn and Manovskii (2013). They argue that when wages depend on current conditions and match-specific productivity, past selection over match quality makes wages appear to depend on past labor market conditions summarized by the lowest unemployment rate during a job spell. Their preferred proxies for match quality are derived from measures of labor market tightness during a job spell and an employment cycle. In future empirical work, I plan to use information on past and future labor market conditions to control for match quality in the estimation of cyclicality of wages, along the lines of Beaudry and DiNardo (1991) and Hagedorn and Manovskii (2013), but with a focus on the most adverse labor market conditions which a job survives.

## 2.4 Turnover Costs

Turnover (hiring or firing) costs were added to the search and matching model in Braun (2006), Nagypal (2007), Silva and Toledo (2009) and Yashiv (2006). Turnover costs improve the performance of the model by making firms' net profits more responsive to changes in productivity.

Muehlemann and Pfeifer (2016) use a German firm-level survey from the 2000s to assess the recruitment and adaptation costs generated by job creation. The average total hiring cost in Germany was equal to more than 2 months of wage payments, with two-thirds of this cost incurred when a worker was hired, and one-third generated by vacancy creation and screening of applications. I use the provided ratio of a hiring cost to wages in my model calibration. For the US, Dube et al. (2010) assess the average total hiring cost to be approximately 1.1 of the monthly wages in California, which suggests that the hiring cost should be twice as high in Germany as in the US.

A characteristic feature of the German labor market is that the firing costs are high. Unlike in the US, an employee with a permanent contract that is dismissed on operational grounds is entitled to severance pay equal to half of a monthly wage for each year of tenure, up to 12 monthly wages for most workers, and even more for older workers with long tenure.

# 3 Data

I use a German matched employer-employee dataset data provided by the Research Data Centre of the Federal Employment Agency at the Institute for Employment Research (IAB). The Linked Employer-Employee Data Longitudinal Model 1993-2010 (LIAB LM 9310) contains administrative data on all workers that were employed at any time between 1999 and 2009 in one of the establishments covered by the 2000-2008 panel of the IAB Establishment Panel. The sample of establishments is drawn from the population of all establishments with employees covered by social security and stratified with respect to industry, size and federal state. A detailed description is provided in Klosterhuber, Heining and Seth (2014).

For each worker, I have information on all employment spells covered by social security between 1993 and 2010: an establishment identifier; sex; education; working hours (full-time or part-time); employment status (indicators for special status such as traineeship, partial retirement and others); daily earnings; occupation, with 120 occupational categories; and other information. Job tenure can be precisely calculated.

The dataset lacks precise information on working hours, but I observe whether a worker works full-time or part-time. Workers are classified as full-time if their contracted hours are the usual working hours in the establishment. Consequently, when I restrict the sample to full-time workers, the firm fixed effects control for differences in working hours across establishments.

The observations with daily earnings above the legally mandated contribution assessment ceiling (Beitragsbemessungsgrenze) are topcoded. More than 10% of the observations are affected. Using the Tobit regression with the same set of control variables as for the censored sample is computationally infeasible. Instead, to establish that it is implausible that my results are affected by censoring, I use a robustness check the replaces worker and firm fixed effects with the CHK estimates from Card, Heining, Kline (2013). They estimated a Mincer-type wage model with additive fixed effects for workers and establishments for all West German workers covered by social security. The estimated worker fixed effects represent a component of a wage that a worker receives wherever he works, controlling for his observable characteristics. The estimated firm fixed effects represent a wage component common to all workers in a firm, controlling for their observable and unobservable characteristics. The

IAB provided a supplementary dataset with the CHK estimates.

The main sample is restricted to the spells of employment in West German establishments that are the 2000-2008 panel cases of the IAB Establishment Panel. I restrict the sample to men aged 20-60. This restriction is adopted for comparability with earlier studies.

# 4 Empirical Results

In this section, I discuss the specification and the results for the estimation of the cyclicality of wages, and for the estimation of the relationship between the risk of separation and the initial conditions.

## 4.1 Wages: Specification

The specification for estimating the cyclicality of wages is the same as in Gertler, Huckfeldt and Trigari (2016). Data are at a monthly frequency. Let $w_{it}$ denote the real wage paid in period $t$ to individual $i$. The wage equation is

$$log\, w_{it} = \pi u_t + \pi_E NH_E(i,t)u_t + \pi_U NH_U(i,t)u_t + \alpha_i + \beta_{j(i)} + \boldsymbol{\gamma}_x' \boldsymbol{x}_{it} + \epsilon_{it} \qquad (1)$$

where $u_t$ is the unemployment rate, $NH_E(i,t)$ and $NH_U(i,t)$ are indicator variables that take value one for new hires from employment and from unemployment, respectively. The controls are worker fixed effects $\alpha_i$, firm fixed effects $\beta_{j(i)}$, where $j(i)$ denotes $i$'s employer, and additional variables contained in vector $\boldsymbol{x}_{it}$: indicators for both types of new hires; a time trend (calendar-month dummies and a quadratic polynomial in time); an education-specific cubic polynomial in age; a cubic polynomial in tenure when applicable; and occupation fixed effects.

Hires from employment are identified as workers that started their current job no more than 14 days after the end of their previous employment and without registering as an unemployed or a jobseeker. Hires from unemployment are identified as workers that started their current job more than 14 days after the end of their previous employment or after registering as an unemployed or a jobseeker. The results are robust to changing the cutoff for differentiation between hires from employment and unemployment to 31 days and to 7 days.

In Table 1, I present the estimates of the wage cyclicality for few variants of specification (1). The results for the full specification are in column (7). Columns (1)-(6) show results for specifications without some of the control variables. The results from the Tobit regression that uses an uncensored sample, with the CHK effects replacing worker and firm fixed effects, are shown in column 5 of Table 2. Columns (1)-(4) show results for variants of specification (1) used for comparisons with the Tobit regression. The estimates for a sample that includes part-time workers are shown in column 6 of 2.

The coefficients of interest are $\pi$, the semielasticity of wages with respect to the unemployment rate $u_t$, and the incremental effects for hires from employment and from unemployment, $\pi_E$ and $\pi_U$. The cyclicality of wages is captured by $\pi$, $\pi + \pi_E$, and $\pi + \pi_U$ for job stayers, new hires from unemployment and employment, respectively.

## 4.2 Wages: Results

The results in the first four columns on Table 1 show the estimates of $\pi$, $\pi_E$, and $\pi_U$ for the specifications that sequentially add more controls for worker heterogeneity: observable workers' characteristics in column (2), worker fixed effects in column (3), and occupation fixed effects in column (4). The estimates of wage cyclicality decrease substantially when controls are added. An exception are occupation fixed effects, which addition leaves the estimates essentially unchanged.

These results are consistent with both job stayers and new hires having better observable and unobservable characteristics when unemployment is higher. Cyclical occupational up- or down-grading seems to have negligible effects.

The addition of firm fixed effects lowers the estimates in comparison with the specification without any controls, as the comparison of columns (5) and (1) reveals. By themselves, these results suggest countercyclical changes in the quality of firms that retain and hire workers, although the firm fixed effects are difficult to interpret on their own since they might pick up differences in workforce characteristics or differences in usuak working hours across firms.

The estimates from the specifications without and with firm fixed effects in addition to full worker controls, presented in columns (3)-(4) and (6)-(7), reveal that the addition of firm fixed effects is unimportant for the wage cyclicality of job stayers but lowers the cyclicality of wages for new hires, in particular hires from unemployment, which suggests countercyclical changes in the quality of hiring firms.

The results for the full version of specification (1), shown in column (7) of Table 1, indicate that the wages of job stayers are procyclical. However, the wages of new hires are less procyclical than than the wages of job stayers, since the incremental effects $\pi_E$ and $\pi_U$ are estimated to be positive. This effect is more pronounced for hires from employment than from unemployment. The addition of controls for occupations is again unimportant, as shown by the similarity of the results in columns (7) and (6) which are obtained for the specifications with and without occupation fixed effects, respectively.

The robustness check that estimates the wage cyclicality for the whole sample yields reassuring results, presented in Table 2. I compare the results of the Tobit estimation on the whole sample, column (5), to the analogous results in column (1) from the estimation that uses only the uncensored observations. Both specifications use the CHK estimates as controls for worker and firm heterogeneity. The estimated wage cyclicality is similar. In turn, the estimates in column (1) are similar to the estimates in column (2), with occupation fixed effects, and the estimates in columns (1) and (2) are similar to the fixed-effects results in columns (3) and (4).

I estimate the wage equation using a sample that includes part-time workers, adding fixed effects for working hours and employment status. The results in column 6 of Table 2 are, again, qualitatively similar to the main results.

### 4.3 Separation Risk: Specification

The risk of separation is captured by the hazard rate defined as the instantaneous probability that worker $i$ experiences an event, in this case a separation, conditional on the event not happening up to time $t$ and the information set summarized in vector $\boldsymbol{z}_{it}$:

$$h_{it} = lim_{\Delta t \to 0} \frac{P(t \leq T_{event} < t + \Delta t | T_{event} \geq t,\ \boldsymbol{z}_{it})}{\Delta t},$$

where $T_{event}$ is a random variable, which value is the time when the event happens.

I use the Cox (1972) model of the hazard rate. The hazard rate takes the functional form

$$h_{it} = \tilde{h}_{jt} exp(\boldsymbol{\beta}' \boldsymbol{z}_{it} + \epsilon_{it})$$

where $\beta$ is a vector of parameters common for all observations, and $\tilde{h}_{jt}$ is the baseline hazard rate, which might differ across subsets (strata) of observations, in this case firms $j = j(i)$. For comparisons with previous papers, I estimate two versions of the Cox model: unstratified, in which the baseline hazard rate is the same for all firms, $\tilde{h}_{jt} = \tilde{h}_t$; and stratified, in which the baseline hazard rate $\tilde{h}_{jt}$ is allowed to vary across firms.

The stratified Cox model is a modification of the Cox proportional hazards model that allows the baseline hazard to differ across strata. Stratification in the Cox model is a counterpart of adding fixed effects to linear models. The strata in my estimation are firms, which allows for differences in the baseline hazard across firms.

The information set for worker $i$ at time $t$, captured in a vector $\boldsymbol{z}_{it}$, includes the unemployment rate at the start of a job, $u_{ij}^{initial}$, the indicator for hires from unemployment, $H_{ij}^U$, the indicator interacted with the initial unemployment rate, a time trend, initial wage, current unemployment rate and its square and other controls for observable worker heterogeneity.

The main estimation equation is

$$h_{it} = \tilde{h}_{jt} exp(\alpha u_{ij}^{initial} + \alpha_U H_{ij}^U u_{ij}^{initial} + \boldsymbol{\gamma}_x' \boldsymbol{x}_{it} + \epsilon_{it}), \tag{2}$$

where $\boldsymbol{x}_{it}$ contains elements of $\boldsymbol{z}_{it}$ other than $u_{ij}^{initial}$ and $H_{ij}^U u_{ij}^{initial}$.

For comparisons with previous papers, I estimate the equation (2) in the stratified and unstratified version and pool together 2 types of hires, estimating

$$h_{it} = \tilde{h}_{jt} exp(\alpha u_{ij}^{initial} + \boldsymbol{\gamma}_x' \boldsymbol{x}_{it} + \epsilon_{it}), \tag{3}$$

also in the stratified and unstratified version.

The results of the estimation of (2) with and without stratification across firms are in Tables 3 and 4, respectively. The results of the estimation of (3) with and without stratification across firms are in Tables 5 and 6. Columns (1) present the results for separations pooled together, columns (2) for separations to employment, columns (3) for separations to unemployment.

In specification (2), the coefficients of interest are $\alpha$, which captures the relationship between the initial unemployment rate and the subsequent risk of separation for hires from employment, and the incremental effect $\alpha_U$ for hires from unemployment. For hires from unemployment, the relationship between the initial unemployment rate and the subsequent risk of separation is captured by $\alpha + \alpha_U$. In specification (3), the coefficient of interest is $\alpha$.

## 4.4 Separation Risk: Results

The main results from the stratified Cox model with the incremental effect for hires from unemployment, presented in Table 3, suggest that a higher initial unemployment rate decreases the subsequent risk of separation to unemployment but not the risk of a job-to-job transition. This cyclical property is present, but attenuated, for hires from unemployment. When both types of separations are considered together, as in some previous papers, the relationship between the initial unemployment rate and the subsequent risk of separation is negative.

The unstratified Cox model yields different results, presented in Table 4. A higher initial unemployment rate decreases the subsequent risk of separation to employment. When both types of separations are considered together, the relationship between the initial unemployment rate and risk of separation is positive for hires from employment, although not significant for both types of hires considered together, as shown in column (1) of Table 6.

Controlling for firm heterogeneity has similar effects as in Kahn (2008), which used a small matched dataset with on large US firms and their employees. This raises a possibility that the estimates of the relationship between the initial unemployment rate and the subsequent risk of separation that neglect firm heterogeneity are biased.

I conclude that firm-worker matches established in times of higher unemployment appear to be of better quality. In the next sections, I conceptualize match quality as match-specific productivity, randomly drawn when a worker and firm meet and fixed for the duration of employment.

# 5 Selection Effect: Stylized Example

I illustrate the match quality selection effect with a stylized example: aggregate productivity takes two values, low $y_1$ and high $y_2$; match-specific productivity has three values $z_1$, $z_2$, and $z_3$, such that $z_1 < z_2 < z_3$; and agents are myopic, discounting with factor 0. I leave vacancy creation decision unspecified, assuming only that vacancies are created in both aggregate states.

A worker in a match with match-specific productivity $z$ produces $zy$ when aggregate productivity is $y$, receiving a fraction $\tau$ of his output. His employer receives $(1-\tau)zy$. The worker quits if his wage $\tau zy$ is lower than the unemployment benefit $b$.[7] The probability that an exogenous separation occurs is $\delta$.

When an unemployed worker and a vacancy-posting firm meet, they draw value $z$ of match-specific productivity from a fixed distribution. The firm has to incur a sunk cost $h$, but only if a job is created. The firm wants to create a job if its per-period earnings would cover the hiring cost, $(1-\tau)zy \geq h$. The worker wants the job if his wage would be no less than the unemployment benefit, $\tau zy \geq b$.

Figure 1 summarizes the model under parameter values ensuring that the match quality selection effect is present. The parameters have to satisfy the inequalities

$$z_3 \geq \frac{h}{(1-\tau)y_1} > z_2 \geq \frac{b}{\tau y_1} > z_1 \geq \frac{h}{(1-\tau)y_2} \tag{4}$$

which is possible. When aggregate productivity is high, all possible matches produce enough output to be preferable to unemployment for workers and to justify job creation for firms. There are no endogenous separations. When aggregate productivity is low, the lowest-productivity matches are destroyed, because workers find unemployment preferable. The medium-productivity matches are preferable to unemployment for workers, but are not productive enough to cover the hiring cost, which means that the existing medium-productivity matches survive but there no new medium-productivity matches.

The wages and job durations generated in a model that satisfies the condition (4) have the same cyclical properties as found in data. The relationship between the initial unemploy-

[7] For clarity of exposition, I assume that a firm and a worker split the match output $zy$, not the surplus $zy - b$. The reasoning goes through when they split the surplus instead.

ment rate and the subsequent risk of separation is negative, because the only endogenous separations are those of workers that quit low-productivity matches when aggregate productivity is low. I proceed to show that the wages of new hires are less procyclical than the wages of job stayers. The cyclical properties of wages result from the properties of the distributions of match-specific productivity for new hires and job stayers. The distribution of match-specific productivity for new hires stochastically dominates the distribution of match-specific productivity for job stayers when aggregate productivity is low, but the reverse happens when aggregate productivity is high.

The distribution of match-specific productivity for new hires is the same as the underlying distribution of match-specific productivities when aggregate productivity is high. When aggregate productivity is low, all match-specific productivities of new hires are equal to $z_3$. Consequently, the mean wages of new hires are $\overline{w}_2^H = \tau y_2 \mathbb{E}z$ and $\overline{w}_1^H = \tau y_1 z_3$, in upturns and in downturns, respectively.

When aggregate productivity is high, job stayers belong to one of three groups: workers that were hired during the current upturn, with the same match-specific productivity distribution as the underlying distribution of match-specific productivities, which mean is $\mathbb{E}z$; workers that were hired during a previous upturn and remained employed during a downturn, with a match-specific productivity distribution that is a truncation of the underlying distribution of match-specific productivities without $z_1$, which mean is $\mathbb{E}z|z > z_1$; and workers that were hired during a previous downturn, who are employed exclusively in matches with productivity $z_3$. Let the fractions of the second and third group of workers in the total number of employed workers be $\pi$ and $\pi'$.

The distribution of match-specific productivity for job stayers during upturns is a mixture of three distributions. Two of these distributions stochastically dominate the match-specific productivity distribution for new hires, one of them is the same distribution. Consequently, the distribution of match-specific productivity for job stayers stochastically dominates the distribution of match-specific productivity for new hires.

The mean wage of job stayers is

$$\overline{w}_2^S(\pi, \pi') = (1 - \pi - \pi')\tau y_2 \mathbb{E}z + \pi \tau y_2 \mathbb{E}z|z > z_1 + \pi' \tau y_2 z_3$$

where $\pi$, $\pi' \in [0, 1]$, such that $\pi + \pi' \in [0, 1]$, depend on the rate of exegenous separations, and history of vacancy creation and of aggregate states. The mean wage of job stayers, $\overline{w}_2^S(\pi,\pi')$, is higher than the mean wage of new hires, $\overline{w}_2^H = \tau y_2 \mathbb{E}z$, as long as $\pi + \pi' < 1$.

When aggregate productivity is low, job stayers belong to one of two groups: workers that were hired during the current or a previous downturn, who are employed exclusively in matches with productivity $z_3$; or workers that were hired during a previous upturn and remain employed during a downturn, with a match-specific productivity distribution that is a truncation of the underlying distribution of match-specific productivities without $z_1$, which mean is $\mathbb{E}z|z > z_1$. Let the fraction of the second group of workers in the total number of employed workers be $\gamma$.

The distribution of match-specific productivity for job stayers during downturns is a mixture of 3 distributions. One of these distributions is stochastically dominated by the match-specific productivity distribution for new hires, the other two are the same distribution. Consequently, the distribution of match-specific productivity for new hires stochastically dominates the distribution of match-specific productivity for job stayers.

The mean wage of job stayers is

$$\overline{w}_1^S(\gamma) = (1-\gamma)\tau y_1 z_3 + \gamma \tau y_1 \mathbb{E}z|z > z_1$$

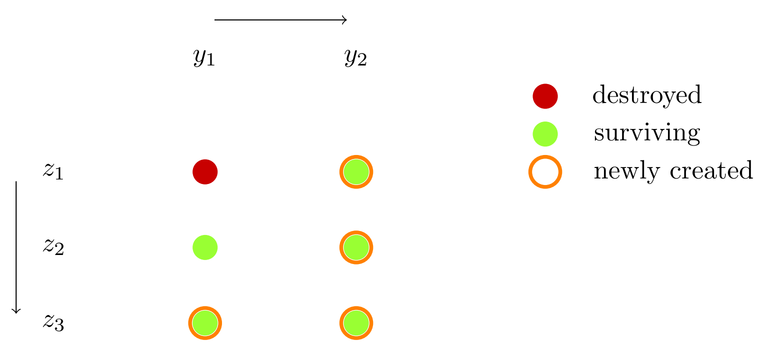
where $\gamma \in [0,1]$ depends on the rate of exegenous separations, and history of vacancy creation and of aggregate states. The mean wage of job stayers, $\overline{w}_1^S(\gamma)$, is lower than the mean wage of new hires, $\overline{w}_1^H = \tau y_1 z_3$, as long as $\gamma > 0$.

The inequalities $\overline{w}_1^S(\gamma) < \overline{w}_1^H$ and $\overline{w}_2^H < \overline{w}_2^S(\pi,\pi')$ imply that

$$\frac{\overline{w}_1^S(\gamma) - \overline{w}_2^S(\pi,\pi')}{\overline{w}_2^S(\pi,\pi')} < \frac{\overline{w}_1^H - \overline{w}_2^H}{\overline{w}_2^H}, \ \frac{\overline{w}_2^H - \overline{w}_1^H}{\overline{w}_1^H} < \frac{\overline{w}_2^S(\pi,\pi') - \overline{w}_1^S(\gamma)}{\overline{w}_1^S(\gamma)}. \tag{5}$$

When $\overline{w}_1^H < \overline{w}_2^h$, which is guaranteed by assuming that $y_1 z_3 < y_2 \mathbb{E}z$, inequalities (5) show that, in percentage terms, the mean wages of new hires are less responsive to aggregate productivity than the mean wages of job stayers. Consequently, regressing the logarithms of wages on aggregate productivity or unemployment, as done in the regression (1), leads to the conclusion that the wages of new hires are less procyclical than the wages of job stayers, even though all wages are equally and fully responsive to aggregate conditions.

Figure 1: Match Quality Selection Effect




# 6 Model

I build a variant of the Diamond-Mortensen-Pissarides search and matching model. The two crucial elements of the model are match-specific productivity and a hiring cost.

## 6.1 Model Outline

There is a continuum of workers with measure one and a continuum of firms. Each firm turns one unit of labor into $r(y, z)$ units of output, where $r$ is an increasing function of aggregate productivity $y$ and match-specific productivity $z$. I use the standard production function $r(y, z) = yz$. The unemployed workers receive a flow benefit $b$.

The workers and firms are risk-neutral. They maximize the expected sum of periodical incomes. The discount factor is $\beta$.

The aggregate productivity, $y$, is the same for all firms, with values in the set $Y = \{y_1, y_2, .., y_{N_Y}\}$, where $y_1 < y_2 < ... < y_{N_Y}$ and $N_Y \geq 2$. The aggregate productivity $y$ is updated to $\hat{y}$ at the beginning of the next period with probability $f_Y(y, \hat{y})$, where $f_Y : Y^2 \to [0, 1]$.

The match-specific productivity, $z$, with values in the set $Z = \{z_1, z_2, .., z_{N_Z}\}$, where $z_1 < z_2 < ... < z_{N_Z}$ and $N_Z \geq 2$, is fixed for each match after being drawn from a probability distribution with a cumulative distribution function $F_Z$ . The match-specific productivity is drawn when a worker and a firm meet, but before a worker is hired.

The notation for value functions is standard. The value of match to the firm, the value of match to the worker, the value of unemployment to the worker, and the match surplus are denoted as $J(y,z)$, $W(y,z)$, $U(y)$, and $S(y,z) = J(y,z) + W(y,z) - U(y)$.

The Nash bargaining divides the match surplus. The contract between a firm and its employee specifies the wage $w(y,z)$. The wage equalizes the worker's surplus $W(y,z)-U(y)$ with $\tau S(y,z)$, where $\tau \in [0,1]$ is the workers' bargaining power parameter.

There is a hiring cost $h \geq 0$ that has to paid in the first period of employment. This is a sunk cost that is incurred only if a job is created and that does not enter into the match surplus.

The firms create vacancies which meet workers through a frictional meeting process. The number of meetings is determined by a CRS matching function $M(u,v)$, which depends on the mass of created vacancies, $v$, and the mass of workers looking for jobs, $u$. The probabilities that the workers and vacancies meet is $M(u,v)/u$ for workers and $M(u,v)/v$ for the vacancies, which can be written as functions of labor market tightness $\theta = v/u$. An unemployed worker meets a vacancy with probability $p(\theta) = M(1,\theta)$, a vacancy meets a worker with probability $q(\theta) = M(\theta^{-1}, 1)$.

The zero profit condition determines vacancy creation. The firms' expected profit from vacancy creation depends on the probability of meeting a worker and the expected value of meeting a worker, denoted as $\tilde{J}(y)$. If the expected value exceeds the cost of vacancy creation, $c > 0$, vacancies are created until the expected profit is driven to zero. If the expected value is less than the cost of vacancy creation, no vacancies are created. Labor market tightness is determined in the equilibrium as

$$\theta(y) = \begin{cases} q^{-1}(c/\tilde{J}(y)), & \text{if } \tilde{J}(y) \geq c \\ 0. & \text{if } \tilde{J}(y) < c. \end{cases} \tag{6}$$

Matches are destroyed if the surplus $S(y,z)$ is negative and with the exogenous separation probability $\delta \in (0,1)$. For simplicity, I assume that the workers who lose a job cannot find a new one in the same period.

## 6.2 Value Functions

The match surplus $S$ is a sum of the firm's surplus, $J$, and the worker's surplus, $W - U$, where $W$ and $U$ are the value of employment and unemployment. The Nash bargaining leads to the condition

$$\frac{J(y,z)}{1-\tau} = S(y,z) = \frac{W(y,z) - U(y,z)}{\tau}.$$

The value accruing to an unemployed worker is

$$\begin{aligned} U(y) =& b + \beta \mathbb{E}\Big[\big(1 - p(\theta(\hat{y}))\big)U(\hat{y}) \\ & + p(\theta(\hat{y})) \int \mathbb{1}\{(1-\tau)S(\hat{y},z) < h\} dF_Z(z) U(\hat{y}) \\ & + p(\theta(\hat{y})) \int \mathbb{1}\{(1-\tau)S(\hat{y},z) \geq h\} W(\hat{y},z) dF_Z(z)\Big] \end{aligned}$$

which can be rewritten as

$$U(y) = b + \beta \mathbb{E}\Big[U(\hat{y}) + p(\theta(\hat{y})) \int \mathbb{1}\{(1-\tau)S(\hat{y},z) \geq h\} \tau S(\hat{y},z) dF_Z(z)\Big].$$

The value accruing to an employed worker is

$$\begin{aligned} W(y,z) =& w(y,z) + \beta \mathbb{E}\Big[\delta U(\hat{y}) \\ & + (1-\delta)\mathbb{1}\{S(\hat{y},z) < 0\} U(\hat{y}) \\ & + (1-\delta)\mathbb{1}\{S(\hat{y},z) \geq 0\} W(\hat{y},z)\Big] \end{aligned}$$

which can be rewritten as

$$W(y,z) = w(y,z) + \beta \mathbb{E}\Big[U(\hat{y}) + (1-\delta)\mathbb{1}\{S(\hat{y},z) \geq 0\} \tau S(\hat{y},z)\Big].$$

The value accruing to a firm employing a job stayer is

$$J(y,z) = r(y,z) - w(y,z) + \beta \mathbb{E}(1-\delta)\mathbb{1}\{S(\hat{y},z) \geq 0\} J(\hat{y},z)$$

which can be rewritten as

$$J(y,z) = r(y,z) - w(y,z) + \beta \mathbb{E}(1-\delta)\mathbb{1}\{S(\hat{y},z) \geq 0\}(1-\tau) S(\hat{y},z).$$

The surplus $S$ can be rewritten as

$$\begin{aligned} S(y,z) =& r(y,z) - b + \beta \mathbb{E}\Big[(1-\delta)\mathbb{1}\{S(\hat{y},z) \geq 0\} S(\hat{y},z) - \\ & p(\theta(\hat{y})) \int \mathbb{1}\{(1-\tau)S(\hat{y},\hat{z}) \geq h\} \tau S(\hat{y},\hat{z}) dF_Z(\hat{z})\Big]. \end{aligned} \tag{7}$$

The expected value of meeting a worker is

$$\tilde{J}(y) = \int \mathbb{1}\{(1-\tau)S(y,z) \geq h\}((1-\tau)S(y,z) - h)dF_Z(z). \tag{8}$$

## 6.3 Equilibrium

The equations (6)-(8) define a functional operator. An equilibrium is a surplus function $S$ satisfying the equation (7), where a market tigthness function $\theta$ is dictated by the equations (8) and (6). The equilibrium is a fixed point of a functional operator.

The equilibrium operator is not continuous, which is the the only obstacle that precludes proving the equilibrium existence with the use of the Brouwer's fixed-point theorem.[8] I consider a proxy of the model. In the proxy model, the equilibrium operator is continuous. I prove the equilibrium existence for the proxy model in Appendix C. If, in the equilibrium, the proxy model reduces to the original model, then the equilibrium of the proxy model is also an equilibrium of the original model.

I use the Brouwer's theorem, which does not guarantee the equilibrium uniqueness and is not constructive. However, I take the advantage of the properties of the equilibrium operator, which can be decomposed in a sum of its increasing and decreasing parts. I adopt a method that numerically narrows the space of potential equilibria, which I discuss in Appendix D.

## 6.4 Match Creation and Match Survival Thresholds

When aggregate productivity is $y$, a match with match-specific productivity $z$ is not endogenously destroyed if the condition $S(y,z) \geq 0$ is satisfied, and can be created if the condition $S(y,z) \geq h$ is satisfied. When $S(y,z)$ is increasing in the second argument, $z$, there are match-specific productivity thresholds for match survival and match creation,

$$z^s(y) = \min_{z \in Z}\{S(y,z) \geq 0\}$$

and

$$z^c(y) = \min_{z \in Z}\{S(y,z) \geq h\},$$

[8]The standard method of proving the equilibrium existence and uniqueness by proving that the equilibrium operator satisfies Blackwell's sufficient conditions, as in Mortensen and Nagypal (2007b), is not applicable, because terms of the type $\mathbb{1}\{x \geq 0\}x$ introduce non-convexity.

with the following properties: $z > z^s(y)$ implies that $S(y,z) \geq 0$ and a match with match-specific productivity $z$ is not endogenously destroyed; $z > z^c(y)$ implies that $S(y,z) \geq h$ and a match with match-specific productivity $z$ can be created; and $z^c(y) \leq z^s(y)$, the threshold for match creation is more demanding than for match survival. When $S(y,z)$ is also increasing in the first argument, $y$, the thresholds are non-increasing functions of aggregate productivity, $y$.

For the highest aggregate productivity, $y_{N_Y}$, it can be assumed without loss of generality that the thresholds for match survival and match creation coincide, $z^c(y_{N_Y}) = z^s(y_{N_Y})$, which together with $z^s(y) \leq z^c(y)$ implies that

$$z^s(y) \leq z^c(y) \leq z^c(y_{N_Y}) = z^s(y_{N_Y}) \tag{9}$$

for any aggregate productivity $y$.

## 6.5 Match-Specific Productivity for New Hires and Job Stayers

To illustrate the selection effect it is sufficient to consider two aggregate productivity states, low $y_1$ and high $y_2$. In this section, I show that the selection effect is present if there are some matches that can survive but cannot be created when aggregate productivity is low, $z^s(y_1) < z^c(y_1)$, which together with (9) implies that

$$z^s(y_1) < z^c(y_1) \leq z^c(y_2) = z^s(y_2). \tag{10}$$

There are four groups of workers whose match-specific productivity distributions I consider, new hires and job stayers when aggregate productivity is low and when aggregate productivity is high.

A match-specific productivity distribution for new hires, $H(z;y)$, is a truncation of the underlying match-specific productivity distribution, $F$, that restricts its domain to match-specific productivities that are above the match creation threshold

$$H(z;y) = \frac{F(z)}{1 - F(z^c(y))}.$$

For high aggregate productivity, $y_2$, the distributions $H(z;y)$ and $F(z)$ coincide.

The inequalities (10) guarantee that there are some matches that can survive but cannot be created when aggregate productivity is low. The match-specific productivity distribution for such matches is

$$P(z) = \frac{F(z)}{1 - F(z^s(y_1))}.$$

When aggregate productivity is low, job stayers belong to one of two groups: workers that were hired during the current or a previous episode of low productivity, whose match-specific productivity distribution is $H(z; y_1)$; or workers that were hired during an episode of high productivity and remain employed during a downturn, whose match-specific productivity distribution is $P(z)$. The match-specific productivity distribution for job stayers is

$$G(z; y_1, \gamma) = (1 - \gamma)H(z, y_1) + \gamma P(z)$$

where $\gamma \in [0, 1]$, the fraction of the second group of workers, decreases in the duration of low-productivity episode.

When aggregate productivity is high, job stayers belong to one of three groups: workers that were hired during the current episode of high productivity, whose match-specific productivity distribution is $H(z; y_2)$; workers that were hired during a previous episode of high productivity and remained employed during a previous episode of low productivity, whose match-specific productivity distribution is $P(z)$; or workers that were hired during a previous episode of low productivity, whose match-specific productivity distribution is $H(z; y_1)$. The match-specific productivity distribution for job stayers is

$$G(z, y_2, \pi, \pi') = (1 - \pi - \pi')H(z, y_2) + \pi P(z) + \pi' H(z, y_1)$$

where the fractions of the second and third group of workers are $\pi$ and $\pi'$, which decrease in the duration of high-productivity episode.

The inequalities (10) imply that the match-specific productivity distributions can be ordered the sense of first-order stochastic dominance. The ordering $H(z; y_2), P(z) \prec H(z; y_1)$ implies the ordering

$$H(z; y_2) \prec G(z, y_2, \pi, \pi'), G(z; y_1, \gamma) \prec H(z; y_1). \tag{11}$$

The first-order stochastic dominance ordering (11) implies inequalities between the means of the four distributions. The mean match-specific productivity for new hires when aggregate productivity is high is the lowest of the four means, the mean match-specific productivity for new hires when aggregate productivity is low is the of the four means, and the means for job stayers lie between these two extremes. When the means of $H(z;y_2)$, $G(z,y_2,\pi,\pi')$, $G(z;y_1,\gamma)$ and $H(z;y_1)$ are denoted as $\bar{z}_2^H$, $\bar{z}_2^S(\pi,\pi')$, $\bar{z}_1^S(\gamma)$ and $\bar{z}_1^H$, the inequalities between the means are

$$\bar{z}_2^H < \bar{z}_2^S(\pi,\pi'),\ \bar{z}_1^S(\gamma) < \bar{z}_1^H \tag{12}$$

The inequalities (12) lead to a conclusion that the mean match-specific productivity for new hires is countercyclical, rising when aggregate productivity is lower, and that its cyclical changes are smaller than the cyclical changes in the mean match-specific productivity for job stayers.

# 7 Calibration

The calibration is at a monthly frequency. The model has 11 parameters. For 4 parameters I use paremeters values that are common in the literature. I calibrate 7 remaining parameters using external information.

The key features of the model are a hiring cost and match-specific productivity. I use external sources to inform the value of the hiring cost and the match-specific productivity distribution.

The hiring cost $h$ is calibrated to be approximately 1.3 of the mean monthly wage, as calculated by Muehlemann and Pfeifer (2016) from a survey of German firms.

I follow the literature and assume that match-specific productivity has a lognormal distribution with a standard deviation $\sigma$. The data moment used to calibrate $\sigma$ is the standard deviation of the residual log wages, taken from Card, Heining, and Kline (2013), who estimated the Mincer equation for log wages using the whole universe of German labor market biographies.

The parameters $\beta$, $\eta$, $\tau$ and $\rho$ have values standard in the literature. The aggregate productivity is either low, $1-\sigma_y$, or high, $1+\sigma_y$. Tha parameter $\sigma_y$ targets the standard deviation 0.02 of log labor productivity, as in Shimer(2005).

The exogenous separation rate $\delta = 0.095$ is equal to the lower values of the monthly separation rate in the 2000s calculated in Nordmeier (2014) and consistent with previous calculations in Elsby et al. (2013).

The unemployment benefit $b$ is calibrated to target 0.4 of mean monthly labor income, as in Krause and Uhlig (2012) for the post-Hartz period.

I use the standard matching function $M(u, v) = \kappa u^{\eta} v^{(1-\eta)}$.[9]. The vacancy creation cost $c$ and the matching function efficiency parameter $\kappa$ are jointly calibrated to match the mean monthly job finding rate calculated in Nordmeier (2014) and Elsby et al. (2013), around $0.055 - 0.07$, and the mean monthly unemployment rate 0.09.

The model-generated wages have the key cyclical property matching my empirical findings: the estimated incremental effect $\hat{\pi}_U = 0.46$ is positive and significant. The calibration, its fit and the results on the cyclicality of wages are summarized in Appendix E.

The figure 2 shows the simulated response of mean wages to a positive shock to aggregate productivity. After the shock the mean wages of job stayers and new hires increase. The mean wages wages of job stayers are higher than for new hires, due to the presence of workers that were survived or were hired during previous episodes of low productivity, and gradually decrease to the mean wages of new hires as the share of workers hired during the current episode of high productivity.

Conversely, the figure 3 shows the simulated response of mean wages to a negative shock to aggregate productivity. After the shock the mean wages of job stayers and new hires decrease. The mean wages wages of job stayers are lower than for new hires, due to the presence of workers that were hired during previous episodes of high productivity, and gradually increase to the mean wages of new hires as the share of workers hired during the current episode of low productivity.

# 8 Conclusions

The relationship between the business cycle and real wages is one of the oldest topics in macroeconomics. I explored the previously neglected possibility that the cyclical changes in average match quality are reflected in the estimates of wage cyclicality. Using German administrative microdata, I found evidence of the presence of countercyclical selection on

[9]The function has to be truncated by the condition $M(u, v) \leq min\{u, v\}$, which is equivalent to a restriction $\theta \in [\kappa^{1/\eta}, \kappa^{1/(1-\eta)}]$

match quality for new hires. The estimates of both the wage cyclicality and the relationship between the initial conditions and the subsequent risk of separation support my hypothesis of the match quality selection effect.

I showed that the match quality selection effect arises in a standard Diamond-Mortensen-Pissarides search and matching model with two additional features: match-specific productivity and turnover costs. More generally, these two fairly realistic features could generate the same selection effect in models with different wage-setting mechanisms. An example would be a model with staggered multiperiod Nash bargaining in which workers' wages are negotiated for the first time when they are hired.[10] Without the selection effect, the wages of new hires would be more procyclical than the wages of job stayers, which are not fully flexible. With the selection effect induced by match-specific productivities and turnover costs, the observed procyclicality of the wages of new hires relative to job stayers would be attenuated. The estimation of the cyclicality of model-generated wages could lead to an incorrect conclusion that the wages of new hires were no more or not much more procyclical than the wages of job stayers.

My empirical results suggest that the match quality selection effect is stronger for hires from employment than from unemployment. In future work, I will incorporate on-the-job search to account for job-to-job transitions. In the present form, my model would not generate the stronger selection effect for hires from employment than for hires from unemployment. However, a conceptually easy modification should resolve this issue. For simplicity, I made match-specific productivity an inspection good, known to workers and firms immediately upon meeting. I could relax this assumption, making match-specific productivity partially an experience good. Then, worker-firm pairs receive a signal about match-specific productivity upon meeting. If they agree to form a match, the underlying productivity is revealed during first few months of its duration.[11] For hires from unemployment, the same force driving the selection effect in the baseline model appears in the generalized model. For hires from employment, the selection effect is enhanced: during downturns, the employed

[10] Unlike Gertler and Trigari (2009), where workers hired in-between wage renegotiations receive the ongoing wage.

[11] This is consistent with the observation that risk of separation is elevated during first few months on job, and goes down sharply afterwards.

workers are concerned about a risk of job loss in a new match, since unemployment spells are longer in expectation, and demand a higher signal about match quality to accept an offer of a job-job transition.

In future empirical work, I plan to use information on past and future labor market conditions[12] as controls for match quality in the estimation of cyclicality of wages. This methodology could be applied to data on wages from Germany as well from the US. My results on the risk of separation, taken together with previous results for the US, raise an interesting possibility that the countercyclical selection effect for new hires affects the estimates of wage cyclicality for the US labor market.

[12] Along the lines of Beaudry and DiNardo (1991) and Hagedorn and Manovskii (2013), but with the addition of information about the most adverse labor market conditions which a job survives.

# References


[1] Baley, Isaac, Ana Figueiredo, and Robert Ulbricht. 2018. "Mismatch Cycles." Unpublished.

[2] Barlevy, Gaudi. 2002. "The Sullying Effect of Recessions." *Review of Economic Studies* 69(1), 65-96.

[3] Baydur, Ismail and Toshihiko Mukoyama. 2018. "Employment Duration and Match Quality over the Business Cycle." Unpublished.

[4] Beaudry, Paul, and John DiNardo. 1991. "The Effect of Implicit Contracts on the Movement of Wages over the Business Cycle: Evidence from Micro Data." *Journal of Political Economy* 99(4): 665-88.

[5] Bils, Mark. 1985. "Real Wages over the Business Cycle: Evidence from Panel Data." *Journal of Political Economy* 93(4): 666-689.

[6] Bowlus, Audra. 1995. "Matching Workers and Jobs: Cyclical Fluctuations in Match Quality." *Journal of Labor Economics* 13(2): 335-350.

[7] Braun, Helge. 2006. "(Un)Employment Dynamics: The Case of Monetary Policy Shocks." Society for Economic Dynamics Meeting Papers 87.

[8] Card, David, Jorg Heining, and Patrick Kline. 2013. "Workplace Heterogeneity and the Rise of West German Wage Inequality." *Journal of Economics* 128(3): 967-1015.

[9] Carneiro, Anabela, Paulo Guimaraes, and Pedro Portugal. 2012. "Real Wages and the Business Cycle: Accounting for Worker, Firm, and Job Title Heterogeneity." *Economic Journal: Macroeconomics* 4(2): 133-152.

[10] Chirinko, Robert. 1980. "The Real Wage Rate over the Business Cycle." *Review of Economics and Statistics* 62(3): 459-461.

[11] Collatz, Lothar. 1966. "Functional analysis and numerical mathematics." New York: Academic Press.

[12] Den Haan, Wouter, Garey Ramey, and Joel Watson. 2000. "Job destruction and propagation of shocks." *American Economic Review* 90(3): 482-498.

[13] Dube, Arindrajit, Eric Freeman, and Michael Reich. 2010. "Employee Replacement Costs." Working Paper Series, Institute of Industrial Relations, UC Berkeley.

[14] Elsby, Michael, Bart Hobijn, and Ayşegül Şahin. 2013. "Unemployment Dynamics in the OECD." *Review of Economics and Statistics* 95(2): 530-548.

[15] Gertler, Mark, Christopher Huckfeldt, and Antonella Trigari. 2016. "Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires." National Bureau of Economic Research Working Paper 22341.

[16] Gertler, Mark, and Antonella Trigari. 2009. "Unemployment Fluctuations with Staggered Nash Wage Bargaining." *Journal of Political Economy* 117(1): 38-86.

[17] Hagedorn, Marcus, and Iourii Manovskii. 2013. "Job Selection and Wages over the Business Cycle." *American Economic Review* 103(2): 771-803.

[18] Hall, Robert. 2005. "Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years." *NBER Macroeconomics Annual* 20: 101-137.

[19] Haltiwanger, John, Henry Hyatt, and Erika McEntarfer. 2015. "Reallocation of Workers Across Employers by Firm Size and Firm Wage." National Bureau of Economic Research Working Papers 21235.

[20] Kahn Lisa. 2008. "Job Durations, Match Quality and the Business Cycle: What We Can Learn from Firm Fixed Effects." Unpublished.

[21] Kahn, Lisa, and Erika McEntarfer. 2014. "Employment Cyclicality and Firm Quality." National Bureau of Economic Research Working Paper 20698.

[22] Klosterhuber, Wolfram, Jorg Heining, and Stefan Seth. 2014. "Linked employer-employee-data from the IAB: LIAB longitudinal model 1993-2010 (LIAB LM 9310)." Datenreport. Documentation on Labour Market Data 201308_en, Institut fuer Arbeitsmarkt- und Berufsforschung (IAB), Nuernberg.

[23] Krause, Michael, and Harald Uhlig. 2012. "Transitions in the German labor market: Structure and crisis." *Journal of Monetary Economics* 59(1): 64-79.

[24] Krause, Michael, and Harald Uhlig. 2012. "Transitions in the German labor market: Structure and crisis." *Journal of Monetary Economics* 59(1): 64-79.

[25] Martins, Pedro, Gary Solon, and Jonathan Thomas. 2012. "Measuring What Employers Do about Entry Wages over the Business Cycle: A New Approach." *Economic Journal: Macroeconomics* 4(4): 36-55.

[26] Mortensen, Dale. 1982. "The Matching Process as a Noncooperative Bargaining Game." In *Economics of Information and Uncertainty*, edited by John J. McCall, 233-258. Chicago: University of Chicago Press.

[27] Mortensen, Dale, and Eva Nagypal. 2007a. "Labor-market Volatility in Matching Models with Endogenous Separations." *Scandinavian Journal of Economics* 109(4): 645-665.

[28] Mortensen, Dale, and Eva Nagypal. 2007b. "More on Unemployment and Vacancy Fluctuations." *Review of Economic Dynamics* 10(3): 327-347.

[29] Mortensen, Dale, and Christopher Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies* 61(3): 397-415.

[30] Moscarini, Giuseppe, and Fabien Postel-Vinay. 2012. "The Contribution of Large and Small Employers to Job Creation in Times of High and Low Unemployment." *American Economic Review* 102(6): 2509-2539.

[31] Muehlemann, Samuel, and Harald Pfeifer. 2016. "The Structure of Hiring Costs in Germany: Evidence from Firm-Level Data." *Industrial Relations: A Journal of Economy and Society* 55(2): 193-218.

[32] Mustre-del-Rio, Jose. 2017. "Job Duration Over the Business Cycle." Unpublished.

[33] Nagypal, Eva. 2007. "Labor-Market Fluctuations and On-the-Job Search." Unpublished.

[34] Nordmeier, Daniela. 2014. "Worker flows in Germany: Inspecting the time aggregation bias." *Labour Economics* 28: 70-83.

[35] Pissarides, Christopher. 2000. "Equilibrium Unemployment Theory." Cambridge, MA: MIT Press.

[36] Pissarides, Christopher. 2009. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica* 77(5): 1339-1369.

[37] Pissarides, Christopher, and Barbara Petrongolo. 2001. "Looking into the Black Box: A Survey of the Matching Function" *Journal of Economic Literature* 39(2): 390-431.

[38] Silva, Jose, and Manuel Toledo. 2009. "Labor Turnover Costs and the Behavior of Vacancies and Unemployment" *Macroeconomic Dynamics* 13(1): 76-96.

[39] Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95(1): 25-49.

[40] Shimer, Robert. 2012. "Reassessing the Ins and Outs of Unemployment." *Review of Economic Dynamics* 15(2): 127-148.

[41] Solon, Gary, Robert Barsky, and Jonathan Parker. 1994. "Measuring the Cyclicality of Real Wages: How Important Is Composition Bias?" *Quarterly Journal of Economics* 109(1): 1-26.

[42] Stockman, Alan. 1983. "Aggregation Bias and the Cyclical Behavior of Real Wages." Unpublished .

[43] Stueber, Heiko. 2017. "The Real Wage Cyclicality of Newly Hired and Incumbent Workers in Germany." *The Economic Journal* 127(600): 522-546.

[44] Yashiv, Eran. 2006. "Evaluating the performance of the search and matching model." *European Economic Review* 50(4): 909-936.

# A Appendix: Wage Cyclicality

Table 1: Wage Cyclicality Estimates

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| $\hat{\pi}$ | $0.634^{***}$ | $0.350^{***}$ | $-0.097^{**}$ | $-0.091^{**}$ | $0.392^{***}$ | $-0.097^{**}$ | $-0.091^{**}$ |
| | (0.058) | (0.057) | (0.042) | (0.041) | (0.052) | (0.042) | (0.042) |
| $\hat{\pi}_E$ | $1.543^{**}$ | $1.598^{**}$ | $0.769^{***}$ | $0.773^{***}$ | $1.553^{***}$ | $0.706^{***}$ | $0.718^{***}$ |
| | (0.671) | (0.737) | (0.223) | (0.238) | (0.379) | (0.161) | (0.178) |
| $\hat{\pi}_U$ | $3.167^{***}$ | $2.783^{**}$ | $0.410^{**}$ | $0.450^{**}$ | $1.086^{**}$ | $0.338^{**}$ | $0.384^{**}$ |
| | (1.149) | (1.086) | (0.201) | (0.201) | (0.500) | (0.169) | (0.172) |
| adj. R-sq | 0.014 | 0.227 | 0.866 | 0.866 | 0.396 | 0.867 | 0.867 |
| No of observations | 24751079 | 24751079 | 24736866 | 24663419 | 24751073 | 24736866 | 24663419 |
| No of firms | 3434 | 3434 | 3427 | 3421 | 3428 | 3427 | 3421 |
| No of workers | 443987 | 443987 | 429774 | 426886 | 443981 | 429774 | 426886 |
| Worker controls | No | Yes | Yes | Yes | No | Yes | Yes |
| Occupation FE | No | No | No | Yes | No | No | Yes |
| Worker FE | No | No | Yes | Yes | No | Yes | Yes |
| Firm FE | No | No | No | No | Yes | Yes | Yes |

*Notes*:

* p< .1, ** p<.05, *** p<.01; time-clustered standard errors in parentheses; uncensored observations for full-time non-trainee workers.

Table 2: Wage Cyclicality Estimates - Robustness

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| $\hat{\pi}$ | $-0.121^{***}$ | $-0.123^{***}$ | $-0.097^{**}$ | $-0.091^{**}$ | $-0.228^{***}$ | $-0.018$ |
| | (0.043) | (0.043) | (0.042) | (0.042) | (0.042) | (0.044) |
| $\hat{\pi}_E$ | $0.922^{***}$ | $0.911^{***}$ | $0.706^{***}$ | $0.718^{***}$ | $0.916^{***}$ | $0.718^{***}$ |
| | (0.250) | (0.242) | (0.161) | (0.178) | (0.215) | (0.160) |
| $\hat{\pi}_U$ | 0.389 | $0.544^{*}$ | $0.338^{**}$ | $0.384^{**}$ | 0.306 | 0.121 |
| | (0.327) | (0.305) | (0.169) | (0.172) | (0.291) | (0.121) |
| adj. R-sq | 0.765 | 0.774 | 0.867 | 0.867 | - | 0.935 |
| No of Obs. | 24751079 | 24677279 | 24736866 | 24663419 | 29221597 | 26703724 |
| No of Firms | 3434 | 3428 | 3427 | 3421 | 3439 | 3433 |
| No of Workers | 443987 | 440746 | 429774 | 426886 | 474070 | 479089 |
| Worker controls | Yes | Yes | Yes | Yes | Yes | Yes |
| Occupation FE | No | Yes | No | Yes | No | Yes |
| Worker FE | No | No | Yes | Yes | No | Yes |
| Worker CHK | Yes | Yes | No | No | Yes | No |
| Firm FE | No | No | Yes | Yes | No | Yes |
| Firm CHK | Yes | Yes | No | No | Yes | No |
| Only uncensored | Yes | Yes | Yes | Yes | No | Yes |
| Only full-Time | Yes | Yes | Yes | Yes | Yes | No |

*Notes*:

* p< .1, ** p<.05, *** p<.01; time-clustered standard errors in parentheses.

# B Appendix: Separation Risk

Table 3: Estimates for Job Duration, Stratification

| | All Separations | EE Separations | EU Separations |
|---|---|---|---|
| | (1) | (2) | (3) |
| $\hat{\alpha}$ | $-4.532^{***}$ | $-0.211$ | $-13.18^{***}$ |
| | (1.693) | (2.122) | (1.509) |
| $\hat{\alpha}_U$ | 0.412 | 2.366 | $5.037^{***}$ |
| | (1.031) | (1.556) | (1.093) |
| No of observations | 8465856 | 8465856 | 8465856 |
| No of firms | 4137 | 4137 | 4137 |
| No of workers | 269334 | 269334 | 269334 |

*Notes*:

* p< .1, ** p<.05, *** p<.01; time-clustered standard errors in parentheses; stratification by establishment.

Table 4: Estimates for Job Duration, No Stratification

| | All Separations | EE Separations | EU Separations |
|---|---|---|---|
| | (1) | (2) | (3) |
| $\hat{\alpha}$ | $4.170^{*}$ | $11.91^{***}$ | $-9.941^{***}$ |
| | (2.254) | (3.286) | (1.710) |
| $\hat{\alpha}_U$ | $-4.910^{**}$ | $-5.862^{*}$ | $4.389^{***}$ |
| | (2.111) | (3.032) | (1.658) |
| No of observations | 8465856 | 8465856 | 8465856 |
| No of firms | 4137 | 4137 | 4137 |
| No of workers | 269334 | 269334 | 269334 |

*Notes*:

* p< .1, ** p<.05, *** p<.01; time-clustered standard errors in parentheses; stratification by establishment.

Table 5: Estimates for Job Duration, All Hires, Stratification

| | All Separations | EE Separations | EU Separations |
|---|---|---|---|
| | (1) | (2) | (3) |
| $\hat{\alpha}$ | $-3.959^{***}$ | 0.731 | $-9.044^{***}$ |
| | (1.397) | (1.726) | (1.270) |
| No of observations | 8465856 | 8465856 | 8465856 |
| No of firms | 4137 | 4137 | 4137 |
| No of workers | 269334 | 269334 | 269334 |

*Notes*:

* p< .1, ** p<.05, *** p<.01; time-clustered standard errors in parentheses; stratification by establishment.

Table 6: Estimates for Job Duration, All Hires, No Stratification

| | All Separations | EE Separations | EU Separations |
|---|---|---|---|
| | (1) | (2) | (3) |
| $\hat{\alpha}$ | 1.467 | $9.872^{***}$ | $-7.623^{***}$ |
| | (1.528) | (2.435) | (1.424) |
| No of observations | 8465856 | 8465856 | 8465856 |
| No of firms | 4137 | 4137 | 4137 |
| No of workers | 269334 | 269334 | 269334 |

*Notes*:

* p< .1, ** p<.05, *** p<.01; time-clustered standard errors in parentheses.

# C Appendix: Equilibrium Existence

The equilibrium operator, denoted as $\mathcal{T}$, is defined by the equations (6)-(8). I use notation $P(y) = p(\theta(y))$ for the composite vacancy-meeting probability, and subscript $S$ for the dependence on $S$, writing $\mathcal{T}$ as

$$\begin{aligned}\mathcal{T}S(y,z) =& r(y,z) - b + \beta\mathbb{E}\Big[(1-\delta)\mathbb{1}\{S(\hat{y},z) \geq 0\}S(\hat{y},z) - \\ & P^S(\hat{y})\int \mathbb{1}\{(1-\tau)S(\hat{y},\hat{z}) \geq h\}\tau S(\hat{y},\hat{z})dF_Z(\hat{z})\Big],\end{aligned}$$

where

$$\theta^S(y) = \begin{cases} q^{-1}(c/\tilde{J}^S(y)), & \text{if } \tilde{J}(y) \geq c \\ 0. & \text{if } \tilde{J}(y) < c \end{cases}$$

and

$$\tilde{J}^S(y) = \int \mathbb{1}\{(1-\tau)S(y,z) \geq h\}((1-\tau)S(y,z) - h)dF_Z(z).$$

The operator $\mathcal{T}$ is not continuous. There are at most two sources of discontinuity: the components $\mathbb{1}\{(1-\tau)S(y,z) \geq h\}\tau S(y,z)$ and, potentially, the vacancy-meeting probability $P^S(y)$.[13]

[13] The components $\mathbb{1}\{S(y,z) \geq 0\}S(y,z)$ and $\mathbb{1}\{(1-\tau)S(y,z) \geq h\}((1-\tau)S(y,z) - h)$ are continuous with respect to $S$, similarly to a function $\mathbb{1}\{x \geq 0\}x$, which is continuous with respect to $x$.

In the proxy model, I replace the indicator function $\mathbb{1}\{(1-\tau)S(y,z) \geq h\}$ by a function defined as

$$a^S(y,z) = \begin{cases} 0, & \text{if } (1-\tau)S(y,z) - h \leq 0 \\ \frac{1}{d}\big((1-\tau)S(y,z) - h\big), & \text{if } 0 \leq (1-\tau)S(y,z) - h \leq d \\ 1, & \text{otherwise,} \end{cases}$$

where $d$ is a small positive number.

The function $a$ has an intuitive explanation in the context of job creation decisions: when a firm's share of surplus does not cover the hiring cost, a job is not created; when a firm's share of surplus is noticeably higher than the hiring cost, a job is created; when a firm's share of surplus is only slightly higher than the hiring cost, a job creation decision is randomized, with the creation probability increasing in the net profit from job creation.

The second potential source of discontinuity is the vacancy-meeting probability, $P^S(y)$. Under certain regularity conditions on $q$ and $p$, which are satisfied for a matching function $M(u,v) = \frac{uv}{(u^\eta + v^\eta)^{1/\eta}}$, the function $P^S(y)$ depends continuosly on $S$. However, for the calibration exercise I use the Cobb-Douglas matching function $M(u,v) = \kappa u^\eta v^{(1-\eta)}$, which makes $\theta^S(y)$ jump at $\tilde{J}^S(y) = c$. In this case, I have to replace the original vacancy-meeting probability, which is

$$P^S(y) = \kappa^{1/\eta}\Big(\frac{\tilde{J}^S(y)}{c}\Big)^{(1-\eta)/\eta}\mathbb{1}\{\tilde{J}^S(y) \geq c\}$$

by

$$\tilde{P}^S(y) = \kappa^{1/\eta}\Big(\frac{\tilde{J}^S(y)}{c}\Big)^{(1-\eta)/\eta}\alpha^S(y) \tag{13}$$

where

$$\alpha^S(y) = \begin{cases} 0, & \text{if } c \geq \tilde{J}(y) \\ \frac{1+e}{-e}\frac{c}{\tilde{J}(y)} + \frac{1+e}{e}, & \text{if } c + ce \geq \tilde{J}(y) \geq c \\ 1, & \text{if } \tilde{J}(y) \geq c + ce, \end{cases}$$

where $e$ is a small positive number. The replacement function $\tilde{P}^S(y)$ is equal to $P^S(y)$ when

$\tilde{J}^S(y) \notin (c, c + ce)$ and depends continuously on $S$.[14] This modification corresponds to a situation where some workers decide against looking for a job when economic conditions are so bad that the net expected firm's profit from vacancy creation conditional on meeting a worker is close to zero, and where the proportion of such workers approaches zero continuously when the net expected firm's profit from vacancy creation conditional on meeting a worker approaches zero.

I define the proxy equilibrium operator as

$$\begin{aligned}\tilde{\mathcal{T}}S(y,z) =& r(y,z) - b + \beta\mathbb{E}\Big[(1-\delta)\mathbb{1}\{S(\hat{y},z) \geq 0\}S(\hat{y},z) - \\ & \tilde{P}^S(\hat{y}) \int a^S(\hat{y},\hat{z})\tau S(\hat{y},\hat{z})dF_Z(\hat{z})\Big],\end{aligned}$$

where $\tilde{P}^S = P^S$ if functions $p$, $q$ satisfy certain regularity conditions, and where $\tilde{P}^S$ is defined by equation (13) for the Cobb-Douglas matching function.

An equilibrium of the (proxy) model is a fixed point of an operator, $\tilde{\mathcal{T}}$, that maps a functional space, $\mathcal{S}$, into itself. To prove the existence of an equilibrium using the Brouwer's fixed point theorem, I show that the space $\mathcal{S}$ contains its own image under $\tilde{\mathcal{T}}$, that the space is convex and compact, and that the operator $\tilde{\mathcal{T}}$ is continuous.

I define the space of potential surplus functions, $\mathcal{S}$, by the condition $S \in \mathcal{S}$ iff $S : Y \times Z \to [\underline{S}, \overline{S}]$. The space is endowed with the maximum norm. The bounds

$$\overline{S} = \big(r(y_{N_Y}, z_{N_Z}) - b\big)/\big(1 - (1-\delta)\beta\big)$$

and

$$\underline{S} = r(y_1, z_1) - b - \beta\tau\overline{S}.$$

are such that the space $\mathcal{S}$ contains its own image under $\tilde{\mathcal{T}}$. It is easily checked that if $\underline{S} \leq S(y,z) \leq \overline{S}$ for all $y, z$, then $\underline{S} \leq \tilde{\mathcal{T}}S(y,z) \leq \overline{S}$ for all $y, z$ follows .

Compactness of $\mathcal{S}$ follows from the Bolzano-Weierstrass theorem applied to $[\underline{S}, \overline{S}]^{|Y \times Z|}$, since set $Y \times Z$ is finite. Convexity of $\mathcal{S}$ is obvious.

It remains to prove that the operator $\tilde{\mathcal{T}}$ is continuous.

**Lemma C.1.** *The operator $\tilde{\mathcal{T}}$ is continuous, if (1) $\tilde{P}^S = P^S$, $p$ is a differentiable function with a derivative which is bounded and bounded away from zero, and $q$ an invertible and*

[14] Truncating the matching function $M(u,v) = \kappa u^{\eta} v^{(1-\eta)} \leq min\{u,v\}$ leads to restriction $P^S(y),\ \tilde{P}^S(y) \leq 1$.

*differentiable function with a derivative that is bounded away from zero on* $[0, A]$*, for all* $A < \infty$*, with* $q(0) = 1$*. The operator* $\tilde{\mathcal{T}}$ *is continuous, if (2)* $\tilde{P}^S$ *is defined by the equation (13).*

*Proof.* It is sufficient to show that there exists constant $D$ such that

$$|\mathcal{T}S_2(y,z) - \mathcal{T}S_1(y,z)| \le D||S_1 - S_2||$$

for all $S_1, S_2 \in \mathcal{S}$ and all $y \in Y$, $z \in Z$.

Examination of the definition of $\tilde{\mathcal{T}}$ reveals that it suffices to prove that

$$|\tilde{P}^{S_2}(y) - \tilde{P}^{S_2}(y)| \le C||S_2 - S_1||.$$

for some constant $C$.

In the case (1), when functions $p$, $q$ satisfy some regularity conditions, it is sufficient to prove the existence of constants $A$, $B$ such that

$$|\theta^{S_2}(y) - \theta^{S_1}(y)| \le A||S_1 - S_2||,$$

$$|p(\theta^{S_2}(y)) - p(\theta^{S_1}(y))| \le B|\theta^{S_2}(y) - \theta^{S_1}(y)|,$$

because then $C = AB$ satisfies the required condition. Since $p$ is differentiable with a derivative that is bounded and bounded away from zero, constant $B = max_{x\in[0,\infty)}|p\prime(x)|$ can be used.

The last step is to show that $A = max_{x\in[1,\overline{S}]}|\frac{1}{q\prime(q^{-1}(x))}|/c$ satisfies the required condition.

There are four cases to consider: $\tilde{J}^{S_2}(y), \tilde{J}^{S_1}(y) \ge c$, $\tilde{J}^{S_2}(y) \ge c > \tilde{J}^{S_1}(y)$, $\tilde{J}^{S_1}(y) \ge c > \tilde{J}^{S_2}(y)$ and $c > \tilde{J}^{S_2}(y), \tilde{J}^{S_1}(y)$. It holds that $\tilde{J}^S$ is bounded from above by $\overline{S}$ and that

$|\tilde{J}^{S_2}(y)) - \tilde{J}^{S_1}(y)| \leq ||S_1 - S_2||$. In the first case, it holds that

$$
\begin{aligned}
&|\theta^{S_2}(y) - \theta^{S_1}(y)| = |q^{-1}(c/\tilde{J}^{S_2}(y)) - q^{-1}(c/\tilde{J}^{S_1}(y))| = |\int_{c/\tilde{J}^{S_1}(y)}^{c/\tilde{J}^{S_2}(y)} q^{-1\prime}(x)dx| \\
&\leq max_{x\in[c/\tilde{J}^{S_2}(y), c/\tilde{J}^{S_1}(y)]}|q^{-1\prime}(x)| * |\frac{c}{\tilde{J}^{S_2}(y)} - \frac{c}{\tilde{J}^{S_1}(y)}| \\
&\leq max_{x\in[1,\overline{S}]}|q^{-1\prime}(x)| * |\frac{c}{\tilde{J}^{S_2}(y)} - \frac{c}{\tilde{J}^{S_1}(y)}| \\
&\leq max_{x\in[1,\overline{S}]}|\frac{1}{q\prime(q^{-1}(x)}| * |\frac{c}{\tilde{J}^{S_2}(y)} - \frac{c}{\tilde{J}^{S_1}(y)}| \\
&\leq max_{x\in[1,\overline{S}]}|\frac{1}{q\prime(q^{-1}(x)}| * |(\tilde{J}^{S_1}(y) - \tilde{J}^{S_2}(y))/c| \\
&\leq A||S_1 - S_2||.
\end{aligned}
$$

In the second case, we have that $\theta^{S_1}(y) = 0$, and similar steps as above applied with substitution of 0 for $\theta^{S_1}(y)$ yield

$$
\begin{aligned}
&|\theta^{S_2}(y) - \theta^{S_1}(y)| = |\theta^{S_2}(y) - 0| = |q^{-1}(c/\tilde{J}^{S_2}(y)) - q^{-1}(1)| \\
&\leq max_{x\in[1,\overline{S}]}|\frac{1}{q\prime(q^{-1}(x)}| * |c - \tilde{J}^{S_2}(y))/c| \\
&=\leq max_{x\in[1,\overline{S}]}|\frac{1}{q\prime(q^{-1}(x)}| * (\tilde{J}^{S_2}(y)) - c)/c \\
&=\leq max_{x\in[1,\overline{S}]}|\frac{1}{q\prime(q^{-1}(x)}| * (\tilde{J}^{S_2}(y)) - \tilde{J}^{S_1}(y)))/c \\
&\leq A||S_1 - S_2||.
\end{aligned}
$$

The third case is analogous. Finally, in the fourth case, we have that $|\theta^{S_2}(y) - \theta^{S_1}(y)| = 0$.

In the case (2), when $\tilde{P}^S$ is defined by the equation (13), there are five cases to consider: $\tilde{J}^{S_2}(y), \tilde{J}^{S_1}(y) \geq c + ce$, $\tilde{J}^{S_2}(y) \geq c + ce \geq \tilde{J}^{S_1}(y) \geq c$, $c + ce \geq \tilde{J}^{S_2}(y), \tilde{J}^{S_1}(y) \geq c$, $c + ce \geq \tilde{J}^{S_2}(y) \geq c \geq \tilde{J}^{S_1}(y)$ and $c > \tilde{J}^{S_2}(y), \tilde{J}^{S_1}(y)$. In each of these cases, it is easy to find $C_i$, $i \in \{1, 2, 3, 4, 5\}$, such that $|\tilde{P}^{S_2}(y) - \tilde{P}^{S_2}(y)| \leq C_i||S_2 - S_1||$. The largest of $C_i$ is the desired constant $C$.

$\square$

# D Appendix: Monotone Iteration

To narrow down the set of possible equilibria, I use a method known in numerical functional analysis, discussed in Collatz (1966). Consider a functional operator $\mathcal{T} : \mathcal{S} \to \mathcal{S}$, where $\mathcal{S}$ is a space of real-valued functions from $X$ to a compact set, which contains bounds $\underline{S}$, $\overline{S} \in \mathcal{S}$ such that $\forall_{x\in X}\ \underline{S}(x) \leq S(x) \leq \overline{S}(x)$.

Suppose that $\mathcal{T}$ can be decomposed into an increasing (monotone) operator $\mathcal{T}^1$ and a decreasing (antitone) operator $\mathcal{T}^2$: there are $\mathcal{T}^1, \mathcal{T}^2 : \mathcal{S} \to \mathcal{S}$ such that $\forall_{x\in X}\ \mathcal{T}(x) = \mathcal{T}^1(x) + \mathcal{T}^2(x)$ and such that if $\forall_{x\in X}\ S_1(x) \leq S_2(x)$ for $S_1, S_2 \in \mathcal{T}$, then $\forall_{x\in X}\ \mathcal{T}^1 S_1(x) \leq \mathcal{T}^1 S_1(x)$ and $\mathcal{T}^1 S_1(x) \geq \mathcal{T}^1 S_1(x)$.

We can define two sequences of functions, $\underline{S}_n$ and $\overline{S}_n$, where the nitial elements are $\underline{S}_0 = \underline{S}$ and $\overline{S}_0 = \overline{S}$. The subsequent elements are defined as

$$\underline{S}_{n+1} = \mathcal{T}^1 \underline{S}_n + \mathcal{T}^2 \overline{S}_n$$

and

$$\overline{S}_{n+1} = \mathcal{T}^1 \overline{S}_n + \mathcal{T}^2 \underline{S}_n.$$

**Lemma D.1.** *The inequalities*

$$\underline{S}_0(x) \leq \underline{S}_1(x) \leq ... \leq \underline{S}_n(x) \leq \overline{S}_n(x) \leq ... \leq \overline{S}_1(x) \leq \overline{S}_0(x)$$

*hold for all $n \in \mathbb{N}$ and $x \in X$.*

*Proof.* By induction. The inequality $\underline{S}_0(x) \leq \overline{S}_0(x)$ holds by assumption. From $\underline{S}_n(x) \leq \overline{S}_n(x)$ it follows that

$$\underline{S}_{n+1}(x) = \mathcal{T}^1 \underline{S}_n(x) + \mathcal{T}^2 \overline{S}_n(x) \leq \mathcal{T}^1 \overline{S}_n(x) + \mathcal{T}^2 \underline{S}_n(x) = \overline{S}_{n+1}(x)$$

from the monotone properties of $\mathcal{T}^1, \mathcal{T}^2$. $\square$

**Lemma D.2.** *For any fixed point $S^*$ of the operator $\mathcal{T}$ and any $n \in \mathbb{N}$, the inequalities*

$$\underline{S}_n(x) \leq S^*(x) \leq \overline{S}_n(x)$$

*hold for all $x \in X$.*

*Proof.* By induction. The inequality $\underline{S}(x) = \underline{S}_0(x) \leq S^*(x) \leq \overline{S}_0(x) = \overline{S}(x)$ holds by assumption. From $\underline{S}_n(x) \leq S^*(x) \leq \overline{S}_n(x)$, it follows that

$$\underline{S}_{n+1}(x) = \mathcal{T}^1 \underline{S}_n(x) + \mathcal{T}^2 \overline{S}_n(x) \leq S^*(x) \leq \mathcal{T}^1 \overline{S}_n(x) + \mathcal{T}^2 \underline{S}_n(x) = \overline{S}_{n+1}(x)$$

from the monotone properties of $\mathcal{T}^1, \mathcal{T}^2$.

$\square$

From the first lemma, it follows that, for all $x \in X$, an ascending and bounded from above sequence $\overline{S}_n(x)$ and a descending and bounded from below sequence $\underline{S}_n(x)$ have limits, $\underline{\underline{S}}(x)$ and $\overline{\overline{S}}(x)$, since functions from the space $\mathcal{S}$ have values in a compact set. Consequently, it is possible to numerically narrow down the set of fixed points of $\mathcal{T}$, by constructing $\underline{S}_n$ and $\overline{S}_n$ and finding their limits, which is done by iteration.

Both the original operator $\mathcal{T}$ and the proxy operator $\tilde{\mathcal{T}}$ are decomposable into monotone and antitone parts. For the operator $\mathcal{T}$, these parts are $\mathcal{T}^1$ and $\mathcal{T}^2$ such that

$$(\mathcal{T}^1 S)(y,z) = r(y,z) - b + \beta \mathbb{E}\Big[(1-\delta)\mathbb{1}\{S(\hat{y},z) \geq 0\}S(\hat{y},z)\Big]$$

and

$$(\mathcal{T}^2 S)(y,z) = -\beta \mathbb{E}\Big[P^S(\hat{y}) \int \mathbb{1}\{(1-\tau)S(\hat{y},\hat{z}) \geq h\}\tau S(\hat{y},\hat{z}) dF_Z(\hat{z})\Big].$$

For the operator $\tilde{\mathcal{T}}$, these parts are $\tilde{\mathcal{T}}^1$ and $\tilde{\mathcal{T}}^2$ such that

$$(\tilde{\mathcal{T}}^1 S)(y,z) = r(y,z) - b + \beta \mathbb{E}\Big[(1-\delta)\mathbb{1}\{S(\hat{y},z) \geq 0\}S(\hat{y},z)\Big]$$

and

$$(\tilde{\mathcal{T}}^2 S)(y,z) = -\beta \mathbb{E}\Big[\tilde{P}^S(\hat{y}) \int a^S(\hat{y},\hat{z})\tau S(\hat{y},\hat{z}) dF_Z(\hat{z})\Big].$$

# E Appendix: Calibration

Table 7: Model Parameters

| Value | Description | Target/Source |
|---|---|---|
| $h = 1.9$ | hiring cost | Muehlemann and Pfeifer (2016) |
| $\sigma = 0.3$ | sd of log match productivity | Card, Heining, and Kline (2013) |
| $\beta = 0.9966$ | discount factor | standard, annual interest rate 4.17% |
| $\eta = 0.5$ | matching function elasticity | standard, Pissarides and Petrongolo (2001) |
| $\tau = 0.5$ | workers' bargaining power | standard, Hosios' condition, $\tau = \eta$ |
| $\sigma_y = 0.02$ | aggregate productivity sd | standard, Shimer (2005) |
| $\rho = 1/24$ | transition probabilities | standard, 2-year long recessions |
| $\delta = 0.0095$ | exogenous separation rate | separations, Elsby et al. (2013), Nordmeier (2014) |
| $b = 0.42$ | unemployment benefit | Krause and Uhlig (2012) |
| $c = 0.42$ | vacancy creation cost | unemployment, job finding, Elsby et al. (2013), Nordmeier (2014) |
| $\kappa = 0.35$ | matching efficiency | unemployment, job finding, Elsby et al. (2013), Nordmeier (2014) |

Table 8: Model Fit

| Outcome | Target | Description |
|---|---|---|
| 1.29 | 1.3 | hiring cost relative to $\overline{w}$ |
| 0.11 | 0.14 | sd of (residual) log wages |
| 0.37 | 0.4 | unemployment benefit relative to $\overline{w}$ |
| 0.12 | 0.09 | unemployment rate |
| 0.01 | 0.01 | separation rate |
| 0.07 | 0.055-0.07 | job finding rate |

*Notes*:

Results from simulations of 2400 monthly observations on 10000 workers with 51 possible match-specific productivities, $\overline{w}$ denotes mean labor income.

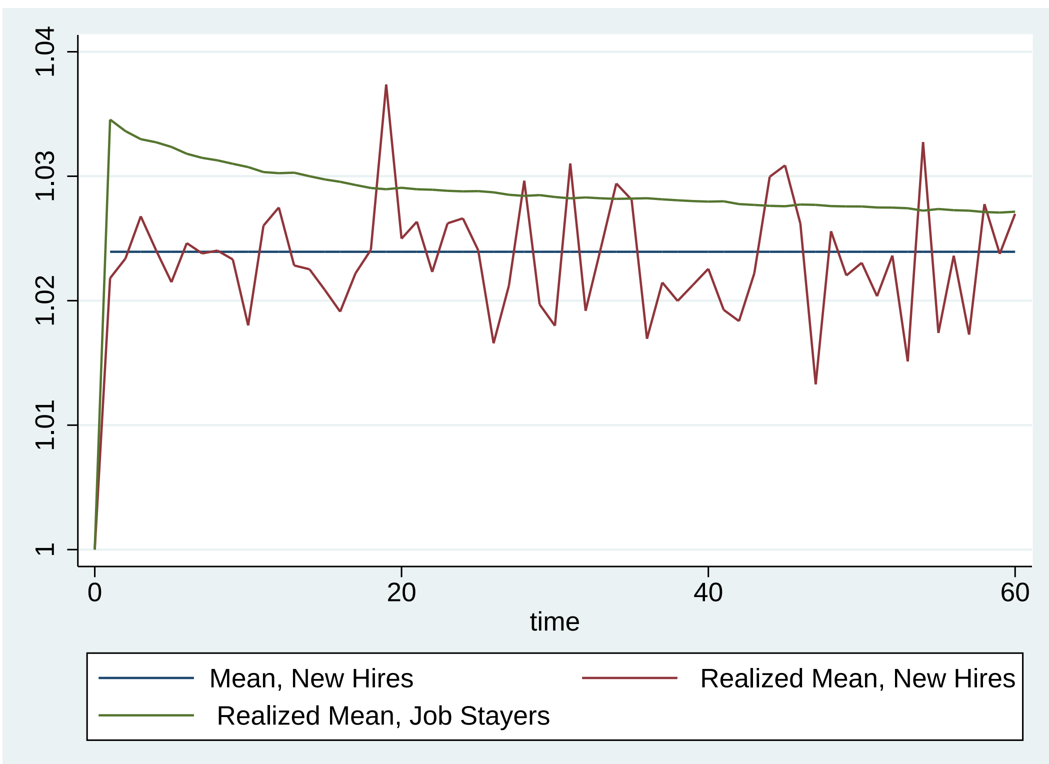
Table 9: Wage Cyclicality Estimates for Model-Generated Wages

| | |
|---|---|
| $\hat{\pi}$ | $-1.85$ |
| $\hat{\pi}_U$ | 0.46 |

*Notes*:

Average values from 5 simulations of 2400 monthly observations on 20000 workers with 51 possible match-specific productivities.

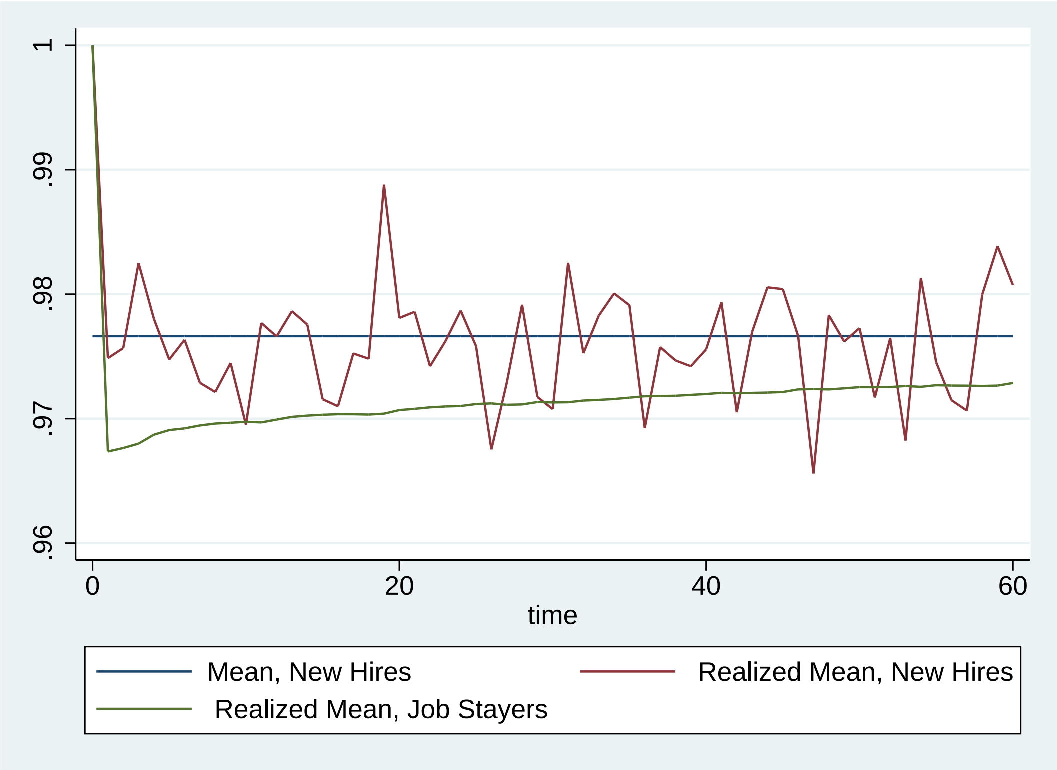
Figure 2: Response of Wages to a Positive Aggregate Shock




*Notes*:

The series are normalized by the mean wage for new hires in the low productivity state. The series start with the economy in the low productivity state and depict a simulated response of the mean wage for job stayers and the mean wages for new hires, realized and expected, to a positive change in aggregate productivity.

Figure 3: Response of Wages to a Negative Aggregate Shock




*Notes*:

The series are normalized by the mean wage for new hires in the high productivity state. The series start with the economy in the high productivity state and depict a simulated response of the mean wage for job stayers and the mean wages for new hires, realized and expected, to a negative change in aggregate productivity.