Cumulative Voting
Increases Minority Turnout and Representation.
An experiment.*

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Abstract

Under majoritarian election systems, securing participation and representation of minorities remains an open problem, made salient in the US by its history of voter suppression. One remedy recommended by the courts is Cumulative Voting (CV): each voter has as many votes as open positions and can cumulate votes on as few candidates as desired. Theory predicts that CV encourages the minority to overcome obstacles to voting: although each voter is treated equally, CV increases minority's turnout relative to the majority, and the minority's share of seats won. A lab experiment based on a costly voting design strongly supports both predictions.

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"And so the world watches America—the only great power in history made up of people from every corner of the planet, comprising every race and faith and cultural practice—to see if our experiment in democracy can work. [...] The jury is still out." (Obama, 2020, p. xvi).

1 Introduction

The fragility of American democracy, rooted historically in slavery, manifests itself in the persistent efforts to disenfranchise racial and linguistic minorities, Black Americans first and foremost. The watershed passing of the Voting Rights Act (VRA) in 1965 and the judiciary decisions that followed aimed at remedying the inequality. But under America's majoritarian electoral system, granting representation to minority groups is difficult. The creation of majority-minority districts is problematic on many fronts, and racial gerrymandering was deemed unconstitutional by the Supreme Court in *Shaw v. Reno* (1993), a position reaffirmed by the Court in subsequent decisions. In 2013, in *Shelby County v. Holder*, the Court voided the VRA formula used to identify jurisdictions subject to federal oversight of their electoral rules, effectively blocking the reach of the VRA. How to secure political participation by and representation of minorities remains an unsolved problem.[1]

The number of elected Black representatives declined sharply at the start of the 20c., a result both of aggressive tactics of disenfranchisement and of the shift to multi-member districts–districts where voters elect multiple members. Multi-member districts limit the patronage of party machines and contain the costs and arbitrariness of gerrymandering, a particularly salient advantage to our contemporary eyes. If voters are asked to cast a single vote for each open position, however, a majority, no matter how slender, can control every seat. As noted already by Charles Dodgson (aka Lewis Carroll) in 1884, such mechanical majority control can be reduced by adopting alternative voting systems that vary the number of votes that voters can cast for each candidate. We study one such system: Cumulative Voting (CV).

Under CV, each voter has as many votes as there are open seats, and the candidates with more votes win, as under simple plurality. However, each voter is allowed to distribute the votes freely among any number of candidates. In an election for four open positions, for example, each voter has four votes, but can cast all on a single candidate, or give one vote each to four candidates, or distribute the votes as desired over any number of candidates. CV treats every voter equally; yet, a cohesive minority can ensure itself some victories by cumulating its vote. CV was used for more than 100 years, from 1870 to 1980, to elect representatives to the Illinois State House and is the rule now in tens of local jurisdictions, often as the remedy imposed by the courts in response to voting rights lawsuits. Outside local politics, it is used to elect corporate boards in approximately 10% of S&P 500 companies.[2]

Empirically, CV correlates with an increase in the number of elected minority representatives (Brockington et al. 1998, Bowler et al., 2003), as well as in the public goods provided to minority communities (Pildes and Donoghue, 1995). In addition, its use appears to increase minority participation in the political system. Matching individual jurisdictions using CV to "twin" others using standard multi-member districts elections, Bowler et al. (2001) report a consistent 5% increase in minority turnout. In evaluating these results, however, the problem is that the adoption of CV is not random: it typically follows voting rights litigation, presumably indicating heightened sensitivity to minority representation and stronger involvement.

[1]For a brief panoramic summary of voter suppression in the US and the role of the VRA, see Grofman et al. 1992.

[2]See Bowler et al. 2003 for a short history of CV. Other useful sources are Bowler et al. 1999, Engstrom 2010, Pildes and Donoghue 1995. For a strong defense of CV, see Guinier 1994. Updated information on the current use of CV is reported in fairvote.org.

The historical evidence then must be accompanied by experimental testing. It is such testing that we conduct in this paper.

Higher minority turnout under CV is usually attributed to more optimistic prospects of affecting the electoral outcome. In the lab, the hypothesis maps directly into an experiment where payoffs depend on one's own group achieving electoral success but voting is individually costly. Do participants, and participants on the minority side in particular, overcome those costs more often when votes can be cumulated?

The costly voting model we implement is the classic tool for studying instrumental voting (Riker and Ordeshook 1968, Palfrey and Rosenthal 1983, Ledyard 1984). The model does not explain the level of turnout observed in large elections, but has good predictive power for the small elections we run in the lab (Levine and Palfrey, 2007), and, most importantly, is widely agreed to capture the comparative statics properties of different elections: turnout is predicted to increase when elections are closer, when they are more salient, when voting costs are lower, when the electorate is smaller. It is this type of comparative effect that interests us here: does minority turnout increase when the voting rule changes to CV?

CV induces a complicated strategic problem: voters must decide not only whether to vote or not, but also how to allocate their votes among the candidates. In jurisdictions that have adopted it, the use of CV relies on the coordinating role of the leadership of each party. Such coordination requires two related steps. First, the leadership must decide how many candidates to nominate; second, voters need direction on how to distribute their votes. The literature is rife with colorful examples of leaders' fast learning after mistakenly nominating too many or too few candidates, and of their detailed instructions to their constituency (Bowler et al., 1999, Sawyer and MacRae, 1962, Pildes and Donoghue, 1995). Once the correct number of candidates is chosen, for any level of turnout, electoral success is maximized by voters spreading all their votes equally. A simplified "equal-and-even" variant of CV, where voters must divide their votes equally among the candidates they choose has been adopted in Peoria, IL. Even in the absence of such a constraint, Sawyer and MacRae (1962) and Goldburg (1994) document that equal spreading of votes over all candidates nominated by one's party was the norm during the 110 years of CV in the Illinois General Assembly, the longest historical experience with CV, and probably one where behavior converged to incorporate its lessons.

In line with the evidence, our experiment attributes to the leaders the task of solving the voters' coordination problem. The only decision participants make is whether to turn out to vote or to abstain, given the number of candidates in the party list and the requirement that votes be equally spread. The number of candidates is set optimally, given the number of open seats, and the size of each party.[3] We run different treatments, varying not only the voting rule but also the number of open seats and the relative size of the minority. For all our parametrizations, theory suggests that CV should increase the minority's turnout relative to the majority's, as well as the fraction of seats won by minority candidates. Both predictions are satisfied in every case. The experiment confirms CV's potential to increase both the minority's turnout and its electoral success.

We are aware of two laboratory experiments on CV, both focusing on strategic voting. Gerber et al. (1998) analyze an election with two open seats where the majority group is divided over its preferred candidate, while the minority is united. Cooper and Zillante (2012) study a spatial model with no parties and multiple open seats, where several candidates and voters are located on an ordered ideological space and voters choose strategically for whom to vote and how many votes to cast. Both papers ignore the leaders' coordinating role–an aspect that seems to us central to actual implementations of CV–and neither has voting

[3]Setting optimally the number of candidates implies taking into account its impact on voters' turnout decisions, in line with models of leaders' enforced social norms in voting (Levine and Mattozzi, 2020).

costs nor studies the question of turnout. Closer to us are experiments that focus directly on turnout, and especially on relative turnout for groups of different sizes (Levine and Palfrey, 2007). CV is an example of "semi-proportional" voting rules, rules that can approach proportional representation without the creation a proportional electoral system.[4] Thus particularly relevant is the experiment in Herrera et al. (2014), comparing turnout under majoritarian and proportional elections, and qualifying the commonly held belief that turnout is higher in the latter case.

The American electoral system has been fashioned by its British roots and by the founders' resolution to hamper the emergence of factions. Both have resulted in opposition to proportional representation. Electoral rules like CV have the potential to afford representation to groups that often do share common goals but are confined to permanent minority status by majoritarian voting. And they can do so without the rigidity and arbitrariness of majority-minority single districts, or of quota systems. They deserve more study.

2 The Model

An electorate of N potential voters selects $K > 1$ representatives for a commission. Each position is identical to the others, and all positions are simultaneously decided in the election. The N voters are divided into two parties: M, the majority party with M members, and m, the minority party with $m < M$ members, where $M + m = N$. Each party p proposes a list of K candidates and all candidates on a party list are perceived as identical by the voters, whose goal is simply to maximize the number of positions won by their party.

A voter whose party wins k positions has utility $u(k)$, increasing in k. We denote by V the value of controlling all positions and assume that $u(k)$ is linear in k, or $u(k) = (k/K)V$. The assumption of linearity simplifies the lab implementation and the theoretical analysis but can also be defended on substantive grounds: while controlling a majority of seats is important, empirically any "place at the table" has value, and the more places the higher the value. The linearity assumption captures an exercise of committee power that is proportional to the number of seats a party has won.

Each voter has K votes, and the K candidates with most votes are elected. If there are ties, all tied candidates are treated identically: after the highest voted candidates are elected, the remaining $k \leq K$ open positions are filled by selecting k winners randomly among the tied candidates. We study two electoral systems, *multi-seat plurality* (MP), and *cumulative voting* (CV). Under MP, each voter can cast at most one vote for each candidate. Under CV, the voter can distribute the votes over as many candidates and in any manner the voter desires, as long as the overall budget of K votes is satisfied. Before voting takes place, party leaders communicate public instructions on how to vote. We denote by v_p the recommended vote distribution by the leader of party p.[5] Under MP, a voter's limit of one vote per candidate makes the instructions trivial; under CV, the leaders' instructions solve the voters' coordination problem.

In equilibrium, voters follow the instructions of their party leader, and the leaders' instructions maximize the number of seats won by party candidates. Voters and leaders share a common goal, and following the party's instructions is a best response for a voter when other party voters do as well. We can think of the voting game as a game between the two party leaders who directly control the distribution of MK votes, for party M, or mK votes, for party m, over the K candidates on each party's list. Call Z (z) the number of seats

[4] Other such systems are Limited Voting and Single Transferable Vote.

[5] That is, we call v_p^{ik} the recommended number of votes on candidate k_p to be cast by voter i who belongs to party p, where $v_p^{ik} \geq 0$, and $\sum_{k_p} v_p^{ik} = K$. We call v_p^i the vector of recommended votes to be cast by voter i, and v_p the profile of votes to be cast by all voters in party p over all party candidates. Thus v_{m} is a matrix of size $[m \times K]$, and v_{M} a matrix of size $[M \times K]$.

won by party M (m). An equilibrium is a pair $\{v^*_{\mathsf{M}}(m, M, K), v^*_{\mathsf{m}}(m, M, K)\}$ such that $Z(v^*_{\mathsf{M}}, v^*_{\mathsf{m}}) \geq Z(v_{\mathsf{M}}, v^*_{\mathsf{m}})$ for all $v_{\mathsf{M}} \neq v^*_{\mathsf{M}}$ and $z(v^*_{\mathsf{M}}, v^*_{\mathsf{m}}) \geq z(v^*_{\mathsf{M}}, v_{\mathsf{m}})$ for all $v_{\mathsf{m}} \neq v^*_{\mathsf{m}}$. We focus on equilibria in weakly undominated strategies where party leaders always instruct voters to cast votes on party candidates only.

Before introducing voting costs, it is instructive to describe the properties of the two voting systems when voting is costless.

2.1 A short detour to costless voting

With no reason for any voter to abstain or to refrain from casting all votes, under MP the party leader recommends voting for all candidates on the party list. In the absence of voting costs, party M wins all seats: each M candidate receives M votes, and each m candidate receives $m < M$ votes.

CV, on the other hand, grants the minority the possibility of some representation. Suppose for example that all m voters in the minority party concentrate all their votes on a single candidate, who thus receives mK votes. The minority candidate is certain to win a seat if there is at least one majority candidate with fewer than mK votes: the minority candidate competes with the weakest of the majority candidates. In turn, if the majority attempts to win all seats, the weakest majority candidate will have most votes when the MK total majority votes are distributed equally among all K majority candidates, and each receives $MK/K = M$ votes. Hence the minority can guarantee itself a seat if $mK > M$, or $m > M/K$. This ratio, known as the *threshold of exclusion*, is a fraction of M: for example, a minority that is half the size of the majority can guarantee itself a seat if the number of open seats is three or more.

How many seats can the minority win? Sawyer and MacRae (1962) provide an elegant early analysis for a specific example; more recently, practitioners of CV have developed a formula that calculates boundaries on the number of seats that can be won, given K, M, and m.[6] The formula ignores the strategic interaction between the two parties, but the answer it yields can be grounded in a full strategic analysis. As we prove in the online appendix:

Proposition 1. *In the absence of voting costs, for all m, $M > m$, and K, in all equilibria of the voting game:*

$$\begin{aligned}
&z = 0 && \text{if } m < M/K \\
&z \in \left(\frac{Km - M}{M + m}, \frac{Km + m}{M + m}\right) && \text{if } m > M/K \text{ and } \frac{Km - M}{M + m} \notin \mathbb{Z}_+ \\
&z = \begin{cases} \frac{Km-M}{M+m} & \text{with prob } m/(M+m) \\ \frac{Km+m}{M+m} & \text{with prob } M/(M+m) \end{cases} && \text{if } m \geq M/K \text{ and } \frac{Km - M}{M + m} \in \mathbb{Z}_+
\end{aligned}$$

Because $(Km + m)/(M + m) - (Km - M)/(M + m) = 1$, if $\frac{Km-M}{M+m}$ is not an integer, there exists a unique integer value of z in the relevant interval; if $\frac{Km-M}{M+m}$ is an integer, then $\frac{Km+m}{M+m}$ is one as well.

The result follows because the linearity of $u(k)$ renders the leaders' game a constant-sum two-player game. All equilibria of such a game must then yield the same payoffs the players can guarantee themselves, i.e., the maximin payoffs. Extending the reasoning applied above yields the proposition.

In the absence of voting costs, CV makes it possible for the minority to win some seats, in contrast to the monopoly of power granted to the majority under MP. But in realistic applications a crucial question is

[6] CV-calculators can be found online. See, for example, sbbizlaw.com, or Wikipedia https://en.wikipedia.org/wiki/Cumulative_voting.

turnout. We extend our model to include voting costs.

2.2 Voting costs

Suppose now that each voter i faces a cost of voting c_i, drawn randomly and independently across voters from a common distribution $F(c)$ with support $[\underline{c}, \overline{c}]$. Realized costs are private information, but the distribution $F(c)$ is common knowledge and does not depend on party affiliation. The cost c_i represents the cost of going to the polls and is independent of the number of votes cast. A voter whose party wins k positions has utility $U_i(k)$, given by:

$$U_i(k) = \begin{cases} u(k) - c_i & \text{if voter } i \text{ voted} \\ u(k) & \text{if voter } i \text{ abstained} \end{cases}$$

The focus is on turnout: what realizations of the voting cost will induce a voter to participate in the election, as opposed to abstaining?

The game now has three stages. First, the leaders publicly issue voting recommendations; then, after observing privately the realization of her voting cost and given her expectations of turnout and voting by all, each voter decides whether or not to turnout; finally, voters at the polls choose how to cast their votes. Note that leaders' voting instructions influence the turnout decisions, an effect the leaders internalize. We focus on a semi-symmetric equilibrium such that: within each party, all voters follow the same turnout strategy, and the leader's instructions are symmetric for all voters; at the polls, it is weakly undominated for all voters to obey their party's voting instructions. The equilibrium of the game is a pair of cost thresholds $\{c_{\mathsf{M}}, c_{\mathsf{m}}\}$ and a pair of voting instructions $\{v^*_{\mathsf{M}}, v^*_{\mathsf{m}}\}$ such that in each party p, with $p \in \{\mathsf{M}, \mathsf{m}\}$ and $p' \neq p$, given v^*_p, $v^*_{p'}$, and $c_{p'}$, all $i \in p$ with $c_i < c_p$ strictly prefer to vote, and all $i \in p$ with $c_i > c_p$ strictly prefer to abstain, and, given c_p, $c_{p'}$, and $v^*_{p'}$, v^*_p maximizes the expected number of seats won by p.[7]

We study the two voting systems in turn.

2.2.1 Multi-winner plurality (MP)

Under MP, conditional on being at the polls, the voter's weakly dominant strategy is to vote for all K party candidates, and such are the leaders' instructions. Each candidate receives one vote from all party voters who turnout. Although the standard approach to costly voting considers single winner elections, typically over two alternatives, it can be easily generalized for our purposes.[8]

Call S_p the number of voters who turn out for party p. Each M candidate receives S_{M} votes, and each m candidate receives S_{m} votes. Thus only three outcomes are possible: either $S_{\mathsf{M}} > S_{\mathsf{m}}$, and all K positions are won by M candidates; or $S_{\mathsf{M}} < S_{\mathsf{m}}$, and all K positions are won by m candidates; or $S_{\mathsf{M}} = S_{\mathsf{m}}$, and all K positions are tied, with K majority and K minority candidates all having the same number of votes. Under a tie, the K winners are chosen randomly among all tied candidates. We denote by Eu^{MP}_T the expected utility gain from winning seats under MP in case of a tie. Then:

$$Eu^{MP}_T = \sum_{k=0}^{K} \frac{\binom{K}{k}\binom{K}{K-k}}{\binom{2K}{K}} u(k) = V/2$$

[7]With voting costs, the strategic choices of the voters cannot be ignored and the game cannot be reduced to a game between the two party leaders. The full description of the game and of the players' strategies is in the appendix.

[8]Arzumanian and Polborn (2017) study costly voting with multiple candidates but a single winner. Our model is closer to the traditional two-candidate, one-winner set-up, with each party list being the parallel to the party candidate.

where the second equality follows from $u(k) = (k/K)V$.

As in single-winner elections, a voter from party p facing opposite party p' must weigh her cost of voting against the expected utility gain from influencing the outcome. Denoting by S_{p-i} the number of voters who turn out in party p ignoring i, voter i can influence the outcome either by breaking ties (when $S_{p-i} = S_{p'}$; an event with probability denoted by π_p^T) or by making ties (when $S_{p-i} = S_{p'} - 1$, with probability π_p^{T-1}). Thus the thresholds $\{c_{\mathsf{M}}, c_{\mathsf{m}}\}$ solve the system of equations:

$$c_{\mathsf{m}} = [u(K) - Eu_T^{MP}]\pi_{\mathsf{m}}^T + [Eu_T^{MP} - u(0)]\pi_{\mathsf{m}}^{T-1} = (V/2)(\pi_{\mathsf{m}}^T + \pi_{\mathsf{m}}^{T-1})$$
$$c_{\mathsf{M}} = [u(K) - Eu_T^{MP}]\pi_{\mathsf{M}}^T + [Eu_T^{MP} - u(0)]\pi_{\mathsf{M}}^{T-1} = (V/2)(\pi_{\mathsf{M}}^T + \pi_{\mathsf{M}}^{T-1})$$

or:

$$c_{\mathsf{m}} = (V/2)\pi_{\mathsf{m}} \tag{1}$$
$$c_{\mathsf{M}} = (V/2)\pi_{\mathsf{M}} \tag{2}$$

where $\pi_p = \pi_p^T + \pi_p^{T-1}$ is the pivotal probability for a voter of party p. Note that the linearity of the utility function implies that the equilibrium equations (1) and (2) do not depend on K. Because voters, once they turn out, vote for the full party list, the problem is then formally identical to the classic costly voting problem with a single winner and two alternatives. The problem is well-known, and we leave the expressions for the pivot probabilities to the online appendix.

Given equilibrium $\{c_{\mathsf{m}}, c_{\mathsf{M}}\}$, we can derive the probabilities of winning different number of positions. In line with the winner-take-all spirit of MP, seats will be shared between the two parties only in the case of ties, that is, only when equal numbers of voters turn out to vote from the two parties. Beyond this observation, the derivation of the probabilities of winning different numbers of seats is straightforward, and again we leave the expressions to the online appendix.

2.2.2 Cumulative voting (CV)

Under CV, the leaders' instructions need to specify the candidates on whom votes should be cumulated, and how a voter who has turned out should split her votes among such candidates. In our symmetric model, Proposition 1 and its proof show that when turnout is known, party leaders can ask voters to split their votes equally over a subset of candidates, with no loss. With voting costs and uncertain turnout, however, the analysis is much more complex. We rely instead on the "equal-and-even" variant of CV and impose equal spreading of votes in our model and in the experiment.

We call the candidates identified by the leaders *viable*. With equal spreading of votes, the problem simplifies to the choice of the number of viable candidates by the leaders, and the turnout decision by individual voters. We denote by g for party m and by G for party M the optimal number of party candidates, chosen by the leaders, and by c_{m} and c_{M} the equilibrium cost thresholds that determine turnout in each party, where $g = g\,(G, c_{\mathsf{m}}, c_{\mathsf{M}})$, $G = G\,(g, c_{\mathsf{m}}, c_{\mathsf{M}})$, $c_{\mathsf{m}} = c_{\mathsf{m}}\,(g, G, c_{\mathsf{M}})$, $c_{\mathsf{M}} = c_{\mathsf{M}}\,(g, G, c_{\mathsf{m}})$.[9] Note that for any positive turnout, if $g < K$, party M is guaranteed $\min[G, K - g]$ seats, and similarly, if $G < K$, party m is guaranteed $\min[g, K - G]$ seats. The positions effectively contested are $g + G - K$.

For given g and G, equilibrium cost thresholds trade off, as is standard, costs of voting and expected

[9]Note that once at the polls, the objectives of the voters and of the leaders coincide. Hence a voter will not deviate from the party's instructions. We describe the full game more precisely in the appendix.

utility gains from influencing the election. As before, a voter can break an existing tie, or can cause a tie, but if the party's viable candidates are fewer than the number of seats, the voter can now move the outcome from a loss to a win of all contested positions. Consider the problem for $i \in \mathsf{M}$. Call $S_{\mathsf{M-i}}$ the number of M voters turning out to vote, excluding i. By voting, i breaks a tie if $(K/G)S_{\mathsf{M-i}} = (K/g)S_{\mathsf{m}}$, or $S_{\mathsf{M-i}} = S_{\mathsf{m}}(G/g)$; i causes a tie if $(K/G)(S_{\mathsf{M-i}}+1) = (K/g)S_{\mathsf{m}}$, or $S_{\mathsf{M-i}} = S_{\mathsf{m}}(G/g) - 1$. In addition, voter i can shift M from losing to winning all contested positions if both $(K/G)S_{\mathsf{M-i}} < (K/g)S_{\mathsf{m}}$ and $(K/G)(S_{\mathsf{M-i}}+1) > (K/g)S_{\mathsf{m}}$, or $S_{\mathsf{M-i}} \in (S_{\mathsf{m}}(G/g)-1, S_{\mathsf{m}}(G/g))$.

We denote by π_p^T, as before, the probability that the votes of a member of party p break the tie on all contested positions, by π_p^{T-1} the probability that they induce a tie, and by π_p^W the probability that they moves party p from losing to winning all contested positions. In equilibrium, if $c_{\mathsf{M}} \in (0,1)$ and $G+g > K$,[10] it must solve:

$$c_{\mathsf{M}} = [u(G) - Eu^{CV}_{T,\mathsf{M}}(G,g)]\pi^T_{\mathsf{M}} + [Eu^{CV}_{T,\mathsf{M}}(G,g) - u(K-g)]\pi^{T-1}_{\mathsf{M}} + [u(G) - u(K-g)]\pi^W_{\mathsf{M}}$$

where:

$$Eu^{CV}_{T,\mathsf{M}}(G,g) = \sum_{x=0}^{G} u(x)\binom{G}{x}\binom{g}{K-x}/\binom{G+g}{K} = \frac{G}{g+G}V.$$

Or:

$$c_{\mathsf{M}} = \frac{V(g+G-K)}{K}\left[\frac{G}{g+G}\pi^T_{\mathsf{M}} + \frac{g}{g+G}\pi^{T-1}_{\mathsf{M}} + \pi^W_{\mathsf{M}}\right] \tag{3}$$

The problem is analogous for minority voters. The equilibrium condition for an interior threshold c_{m} yields:

$$c_{\mathsf{m}} = [u(g) - Eu^{CV}_{T,\mathsf{m}}(G,g)]\pi^T_{\mathsf{m}} + [Eu^{CV}_{T,\mathsf{m}}(G,g) - u(K-G)]\pi^{T-1}_{\mathsf{m}} + [u(g) - u(K-G)]\pi^W_{\mathsf{m}}$$

where:

$$Eu^{CV}_{T,\mathsf{m}}(G,g) = \sum_{x=0}^{g} u(x)\binom{g}{x}\binom{G}{K-x}/\binom{G+g}{K} = \frac{g}{g+G}V.$$

Or:

$$c_{\mathsf{m}} = \frac{V(g+G-K)}{K}\left[\frac{g}{g+G}\pi^T_{\mathsf{m}} + \frac{G}{g+G}\pi^{T-1}_{\mathsf{m}} + \pi^W_{\mathsf{m}}\right] \tag{4}$$

The pivot probabilities and the probabilities of winning different numbers of position in case of ties can be derived as under MP, but taking into account that the number of candidates, in each party, now may differ from the number of seats. We leave them to the appendix.

2.2.3 Equilibria under MP and CV for the experimental parametrizations

We have constructed equilibria for sets of parameters that include those we use in the experiment. Figure 1 shows the equilibrium turnout rates in the two parties, and the expected fraction of seats won by the minority under the two voting systems for different majority margins, in the upper and lower panels, different values of M, on the horizontal axis, and $K = 2$ or 4. (Recall that K does not affect these outcomes under MP).[11]

The figure highlights two main regularities. First, CV consistently increases the relative turnout of the

[10] If $G + g \leq K$, there are no contested positions and $c_{\mathsf{M}} > 0$ cannot be supported in equilibrium.

[11] Recall that with F uniform, c_p corresponds to the expected turnout rate in party p (we found a unique equilibrium in all cases). Note the surprising lack of a consistent underdog effect ($c_{\mathsf{m}} > c_{\mathsf{M}}$) in the MP model, a finding we discuss in the appendix. Raising K to 6 does not change the qualitative results.

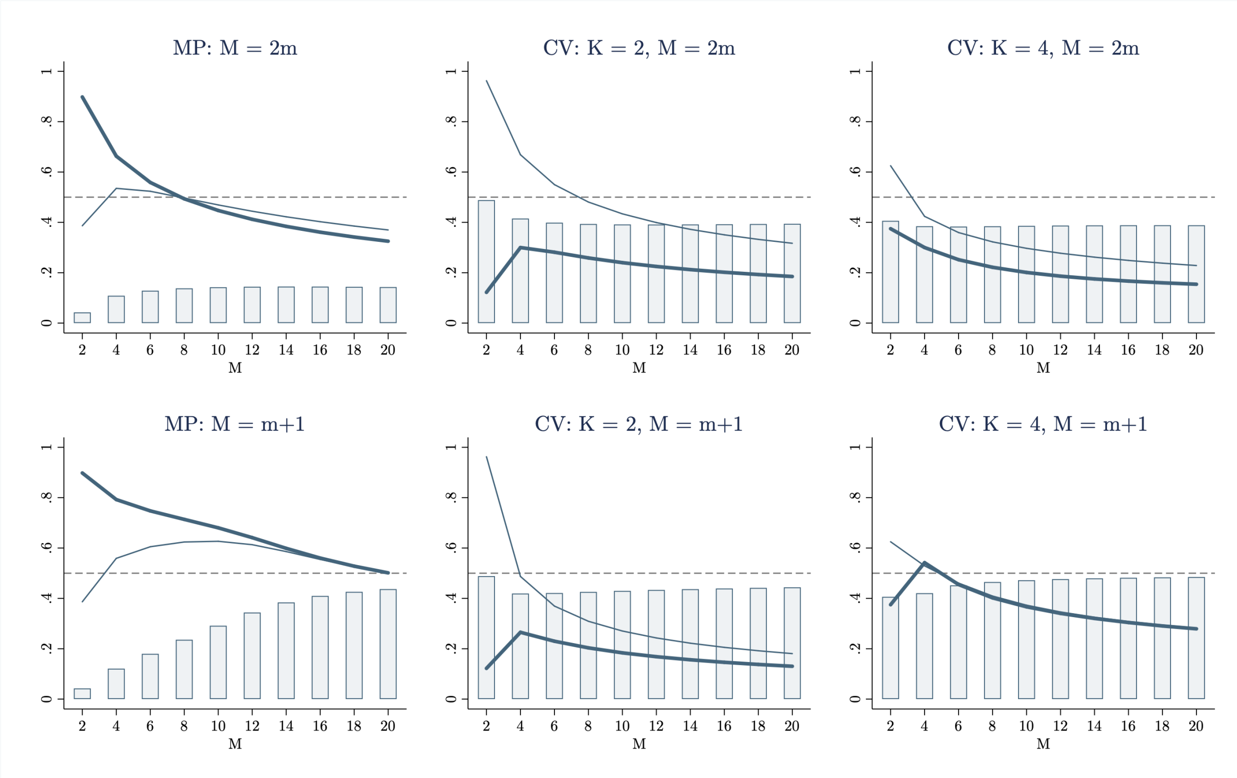

Figure 1: *Expected turnout rates and share of minority seats, MP and CV.* The thick lines correspond to c_{M}, the thin lines to c_{m}; the bars correspond to the expected share of minority seats. F is uniform over $[0,1]$; $V = 4$.

minority: whether the majority margin is large or small, whether the number of open seats is just enough for CV to differentiate itself from MP ($K = 2$, in the central panels), or is higher ($K = 4$ in the panels on the right), the ratio $c_{\mathsf{m}}/c_{\mathsf{M}}$–the ratio of the expected turnout rate of the minority to the expected turnout rate of the majority–is higher under CV than under MP. This remains true whether the electorate is small or large, unless the difference in size of the two parties becomes negligible (for $M = m+1$ and large M), and turnout equalizes for the two parties under both MP and CV. Second, the expected fraction of seats won by the minority is consistently higher under CV. The effect is most striking when the minority is relatively small ($M = 2m$), and its expected share of seats never rises above 14% under MP, less than half its share of the electorate, as opposed to being consistently close to 40% under MP.

The difference in outcomes between MP and CV is driven by the number of viable candidates. In all cases, the minority party sets $g < K$ under CV; when $G = K$, the minority's cumulation of votes results in a higher probability of affecting the outcome, incentivizing turnout; when $G < K$, the difference in turnout probabilities is reduced, but the share of minority victories is boosted by the seats left uncontested by the majority. In the figure, when $K = 2$, $G = K = 2$, but $g = 1$ for all M and m; when $K = 4$, $G = 3$ always; $g = 2$ if $m = M/2$; $g = 3$ otherwise. Thus in the lower right panel, $g = G = 3$, and the two parties' turnout probabilities converge, but the uncontested seat keeps the share of expected minority victories higher than under MP.

We can compare the results to minority victories in the absence of voting costs, and thus of turnout effects. Under MP, as we know, the minority never wins any seat, as opposed to the less than proportional but positive share predicted with voting costs. Under CV and costless voting, the expected share of minority victories is $1/2$ if $m = M - 1 > 1$, and either $1/3$ (if $K = 2$) or $1/4$ (if $K = 4$) if $m = M/2$. Accounting for turnout thus softens the impact of relative party size, reducing expected minority victories for $m = M - 1$, but increasing them for $m = M/2$. Under CV, the minority always achieves substantive representation but is expected to maintain its minority status in the allocation of seats.

3 The Experiment

The experiment reproduces exactly the theoretical model. Our main focus is the comparison between turnout, for both parties, and the fraction of minority victories under the two voting rules, MP and CV. To evaluate the robustness of the results and to test the power of the theoretical framework we implemented four different parametrizations: while we kept $M = 4$ throughout the experiment, we varied m between 2 and 3; for each m, we set $K = 2$ and $K = 4$. Thus we ran a $2 \times 2 \times 2$ design, with 8 treatments in total. In all treatments, voting costs were drawn independently across participants from a uniform distribution with support $[0, 100]$, and V, the value of controlling all positions, was set at 400.

In line with the theory, the number of candidates fielded by each party under CV was set exogenously, at the theoretically optimal value for each parametrization. The design thus mimics the numerical simulations, with $M = 4$. We reproduce it in Table 1, together with the theoretical predictions.

M, m	K	Rule	G, g	c_{m}	c_{M}	(c_{m} / c_{M})	Exp. Min. Seats	Exp. Min. Share
4, 2	2	MP	2, 2	0.54	0.66	0.8	0.22	0.11
4, 2	2	CV	2, 1	0.67	0.30	2.2	0.84	0.42
4, 2	4	MP	4, 4	0.53	0.66	0.8	0.44	0.11
4, 2	4	CV	3, 2	0.42	0.30	1.4	1.52	0.38
4, 3	2	MP	2, 2	0.56	0.79	0.7	0.24	0.12
4, 3	2	CV	2, 1	0.49	0.27	1.9	0.84	0.42
4, 3	4	MP	4, 4	0.56	0.79	0.7	0.48	0.12
4, 3	4	CV	3, 3	0.53	0.54	1.0	1.68	0.42

Table 1: *Experimental Design and Predictions.* $F(c)$ is Uniform$[0, 100]$; $V = 400$.

The table dictates the hypotheses to test. The first set concerns the propensities to turnout, $\{c_{\mathsf{m}}, c_{\mathsf{M}}\}$.[12] First, under all parametrizations, the relative turnout of minority to majority voters is higher under CV than under MP (H1). Second, under MP, the turnout rate is higher for the majority; under CV, it is higher for the minority (H2). The second set of hypotheses concerns expected minority victories. First, in all parametrizations the expected share of seats won by the minority is higher under CV than under MP (H3). Second, the share varies with the voting rule but, for each rule, is very close to constant across parametrizations (H4).

We conducted the experiment between August and October 2020, with participants recruited using the Columbia Experimental Laboratory for the Social Sciences (CELSS)' Orsee website[13]. Most subjects were undergraduate students at Columbia University or Barnard College. All sessions were online due to the

[12] Recall that with F Uniform, c_p is the expected fraction of members of p who choose to vote.
[13] Greiner (2015).

COVID-19 pandemic: participants received instructions and communicated with experimenters using the Zoom videoconferencing software, and accessed the experiment interface on their personal computer's web browser. The experiment is programmed in z-Tree (Fischbacher, 2007) and was run virtually using z-Tree Unleashed (Duch et al., 2020). Each experimental session lasted about 90 minutes with average earnings of \$23. With the exception of a more visual style for the instructions, the experiment developed very similarly to in-person experiments in the lab.[14]

The experimental implementation followed classic costly voting experiments (Levine and Palfrey, 2007). Participants acted as eligible voters; at each round, each drew an independent voting cost and decided whether or not to vote. If the decision was to vote, the participant's K votes were divided equally among the candidates fielded by the voter's party, a number that equaled K under MP and g or G under CV. At the end of the round, an outcome screen reported the party affiliations of the K winning candidates and the number of members of each party who had voted. During each session, the minority size m was kept fixed, and participants played 15 consecutive rounds each of four treatments, CV and MP for each of $K = 2$ and $K = 4$, in an order that changed across sessions.[15] Each participant's final earnings corresponded to the sum of their earnings from one randomly drawn round from each treatment (in addition to the \$5 show-up fee).

In each round, two groups of m and M minority and majority members respectively were formed randomly. Party affiliations were kept constant within each treatment to facilitate learning but were assigned randomly across treatments. For given m, either 2 or 3, we ran two experimental sessions for each of four orders of treatments. Thus eight sessions were conducted with $m = 2$ (12 subjects per session), and eight with $m = 3$ (14 subjects per session), for a total of 208 experimental subjects.

4 Experimental Results

Figure 2 reports the frequency of turnout for minority and majority voters in the different treatments, in the upper panels, and the relative frequency of the two parties in the lower panel.[16]

For all experimental values of K and m, minority turnout is higher under CV than under MP. The effect is particularly strong for $m = 2$, but remains consistently positive, if more muted, with $m = 3$ as well. The difference remains positive even when theory predicts instead a decline in turnout. Interestingly, with the single exception of $m = 3$ and $K = 2$, the predicted decline in turnout for the majority under CV is not observed in the lab: majority turnout remains mostly constant.

The lower panel shows the final effect on relative turnout frequencies: in all cases, the minority's relative turnout rate increases under CV. The theory also predicts that the minority's turnout rate should be lower than the majority's in all MP treatments, and weakly higher in all CV treatments. The prediction is satisfied with one exception: with $m = 3$ and $K = 4$, the minority's turnout frequency increases with CV but remains below the majority's; the theory singles out this case as the one for which the effect is weakest.

In Tables 2 and 3, we regress subjects' turnout decisions on each subject's voting cost realization and on a series of dummy variables, reflecting K, m, the voting system, and all cross effects among the three variables, controlling for the round number, and the order of treatments in the session. The excluded case is

[14] The online appendix contains a reproduction of the instructions.

[15] The four treatments were ordered so that only parameter was changed at each step. Thus we ran four different orders: $\{CVK2, MPK2, MPK4, CVK4\}$, $\{MPK2, CVK2, CVK4, MPK4\}$, $\{MPK4, CVK4, CVK2, MPK2\}$, and $\{CVK4, MPK4, MPK2, CVK2\}$.

[16] In all figures below, reported theoretical predictions are calculated on the basis of the realized experimental cost draws.

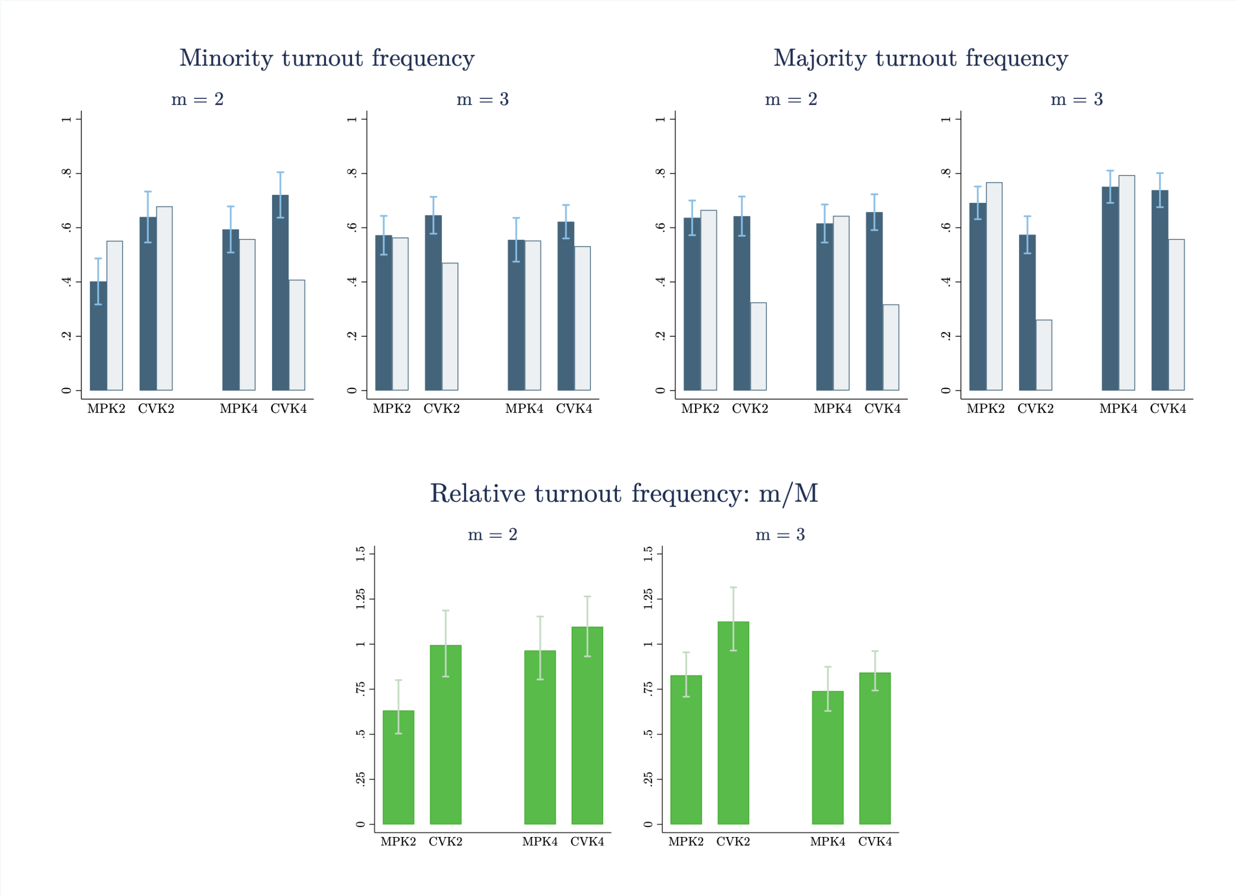

Figure 2: *Turnout frequencies.* Upper panels: the 95% CI's reflect standard errors clustered at the individual level. Lower panel: the 95% CI's are calculated from 10,000 Monte Carlo simulations that allow correlation in turnout decisions at the individual level. The grey bars in the upper panel correspond to the theory.

$m = 2$, $K = 2$, MP, with treatment order 1.[17] Standard errors, clustered at the individual level to account for multiple observations from each individual, are reported in parentheses under each parameter, above p-values, reported in brackets. We report results from both a linear probability and a probit model. The results are fully consistent across the two models.

Table 2 studies data from the minority. Under MP, the increase in the minority's relative size from $m = 2$ to $m = 3$ increases turnout, and so does the increase in the number of seats from $K = 2$ to $K = 4$, although the two effects are muted when they occur together. The most interesting result for us is the strongly positive effect of CV, especially but not only when m is small and K is small. The order of treatments has no detectable effect, while experience (or fatigue) at later experimental rounds causes a significant but quantitatively small decline in turnout.

[17] Recall that the orders of treatment are: $\{CVK2, MPK2, MPK4, CVK4\}$ (order 1), $\{MPK2, CVK2, CVK4, MPK4\}$ (order 2), $\{MPK4, CVK4, CVK2, MPK2\}$ (order 3), and $\{CVK4, MPK4, MPK2, CVK2\}$ (order 4).

Table 2: Frequency of Turnout - Minority

	Linear Probability Model	Probit Model
K = 4	0.184	0.601
	(0.052)	(0.172)
	[0.000]	[0.000]
m = 3	0.171	0.559
	(0.052)	(0.176)
	[0.001]	[0.002]
CV	0.236	0.770
	(0.061)	(0.205)
	[0.000]	[0.000]
(K = 4) $\times (m = 3)$	-0.199	-0.641
	(0.069)	(0.228)
	[0.004]	[0.005]
(K = 4) $\times CV$	-0.087	-0.249
	(0.082)	(0.281)
	[0.291]	[0.375]
(m = 3) $\times CV$	-0.149	-0.457
	(0.078)	(0.261)
	[0.058]	[0.080]
(K = 4) $\times (m = 3) \times CV$	0.065	0.159
	(0.104)	(0.353)
	[0.537]	[0.652]
Treatment order=2	-0.024	-0.095
	(0.044)	(0.147)
	[0.579]	[0.520]
Treatment order=3	0.044	0.144
	(0.043)	(0.147)
	[0.304]	[0.328]
Treatment order=4	0.026	0.083
	(0.046)	(0.154)
	[0.574]	[0.587]
Round	-0.007	-0.024
	(0.002)	(0.006)
	[0.000]	[0.000]
Voting Cost	-0.008	-0.026
	(0.000)	(0.002)
	[0.000]	[0.000]
Constant	0.862	1.142
	(0.049)	(0.167)
	[0.000]	[0.000]
Observations	4800	4800
R^2	0.276	

Voter turnout is measured as a binary 0-1 subject decision. Standard errors are clustered at the individual subject level.

Table 3 shows the results for the majority. Under MP, the regression finds a small and only marginally significant increase in turnout with $m = 3$. Under CV, there is a decline in turnout in the $m = 3$, $K = 2$ case, but the most noticeable result is the lack of the predicted decline in the other parametrizations. Overall, majority turnout remains relatively constant, regardless of the voting system or the values of K and m. For the majority too we find no effect of treatment order and a small significant decline in turnout at later rounds.

Table 3: Frequency of Turnout - Majority

	Linear Probability Model	Probit Model
K = 4	-0.002	-0.004
	(0.033)	(0.110)
	[0.949]	[0.972]
m = 3	0.083	0.303
	(0.044)	(0.153)
	[0.064]	[0.048]
CV	0.008	0.030
	(0.035)	(0.120)
	[0.810]	[0.805]
(K = 4) $\times(m = 3)$	0.048	0.171
	(0.048)	(0.172)
	[0.318]	[0.320]
(K = 4) $\times CV$	0.020	0.070
	(0.045)	(0.155)
	[0.657]	[0.651]
(m = 3) $\times CV$	-0.137	-0.470
	(0.052)	(0.176)
	[0.009]	[0.008]
(K = 4) $\times(m = 3) \times CV$	0.082	0.284
	(0.066)	(0.232)
	[0.218]	[0.220]
Treatment order=2	0.033	0.128
	(0.045)	(0.158)
	[0.456]	[0.418]
Treatment order=3	0.025	0.090
	(0.045)	(0.158)
	[0.581]	[0.567]
Treatment order=4	0.011	0.046
	(0.049)	(0.170)
	[0.824]	[0.787]
Round	-0.005	-0.019
	(0.001)	(0.004)
	[0.000]	[0.000]
Voting Cost	-0.008	-0.026
	(0.000)	(0.001)
	[0.000]	[0.000]
Constant	1.039	1.793
	(0.045)	(0.171)
	[0.000]	[0.000]
Observations	7680	7680
R^2	0.246	

Voter turnout is measured as a binary 0-1 subject decision. Standard errors are clustered at the individual subject level.

Summarizing then, CV increases the minority's turnout, both in absolute terms and relative to the majority, supporting both hypotheses H1 and H2. The most noticeable deviation from the theory is the majority's persistently high turnout under CV.[18]

Figure 2 invites a natural question: CV affects positively the minority's propensity to vote, but does the aggregate effect mirror a widespread change in behavior, or the changed attitude of a few outliers? Figure 3 addresses the question by reporting the CDF's of the individual cutpoints estimated from our data.[19]

[18]As a result, total turnout, predicted to fall under CV, remains in fact constant in the lab. We discuss the data on total turnout in the appendix.

[19]See the online appendix for details on the estimation.

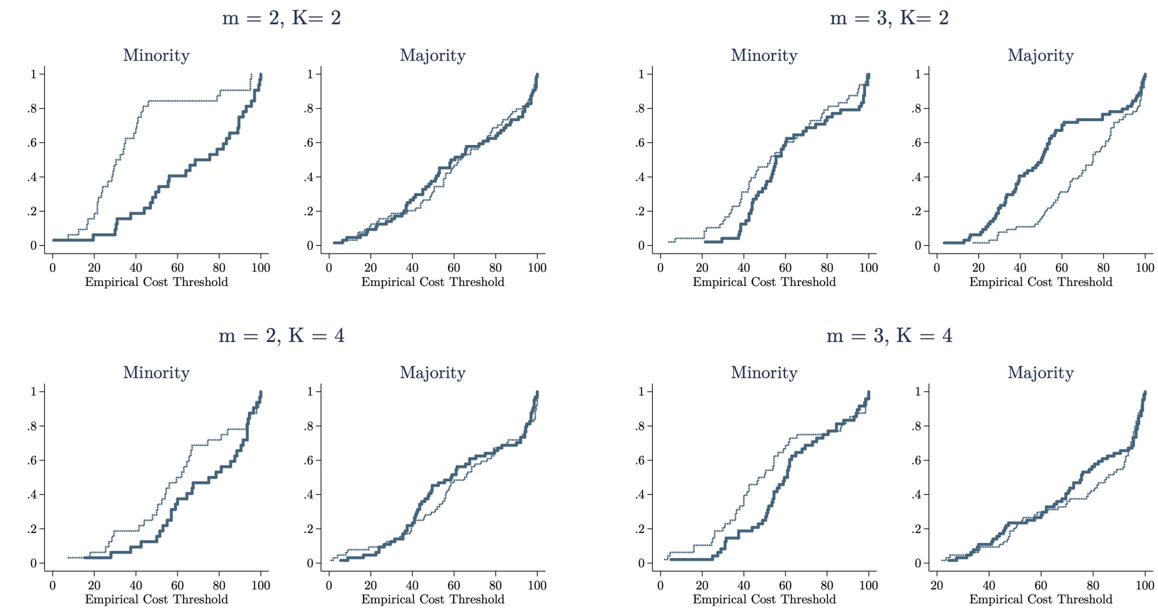

Figure 3: *CDF's of Cost Cutpoints.* The thin lines correspond to MP; the solid lines to CV.

The minority's higher propensity to vote under CV operates throughout the distribution of individual cutpoints. Especially with $m = 2$, when the minority is half the majority, the move to CV causes an unambiguous shift rightward of the whole distribution: the minority cutpoints distribution under CV FOSD's the distribution under MP. The effect is particularly strong when $K = 2$, and under CV the minority cumulates all votes on a single candidate ($g = 1$): CV makes the prospect of winning one seat realistic and encourages participation. When $m = 3$ and the difference in size between the two parties is smaller, the minority still shifts homogeneously towards higher turnout but the move is less pronounced.

As for the majority, with $m = 2$, majority members barely modify their propensity to vote. With $m = 3$, the conclusion is similar when $K = 4$; when $K = 2$, however, under CV the majority is certain of one victory and doubts to be able to control the second seat. The result, as predicted, is a consistent decline in voting throughout the majority's cutpoint distribution: the distribution under MP FOSD's the distribution under CV. As remarked earlier, it is the only case in which the predicted decline in majority's turnout is seen in the data.

Figure 3 also shows the heterogeneity in behavior we see in the lab. Under both voting rules and for all parametrizations, the theoretical CDF's have a single step at the equilibrium cutpoint. In line with previous results from similar experiments[20], this is not what we see in the data.

Did CV help the minority secure more seats? Figure 4 shows that the answer is positive.

[20]For example, Levine and Palfrey (2007).

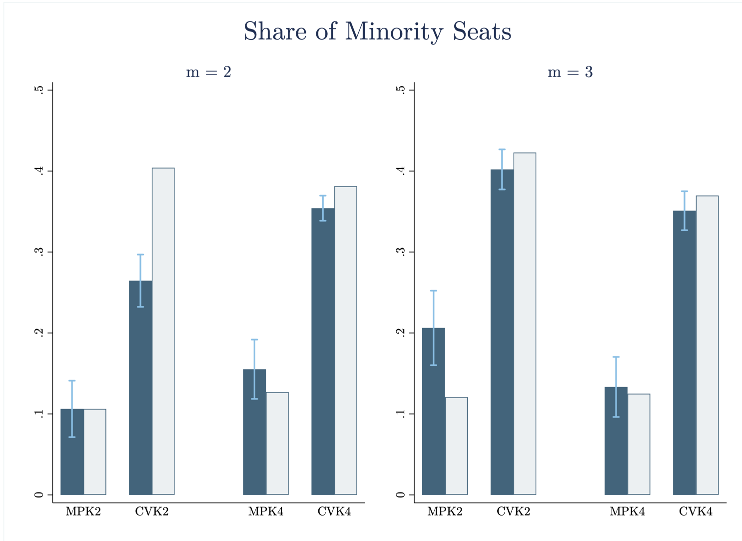

Figure 4: *Share of seats won by the minority.* Blue columns correspond to the data, grey to the theory. The 95% CI's are calculated from standard errors clustered at the level of the voting group.

The message from Figure 4 is confirmed by Table 4, which reports the results of an OLS regression of the share of minority victories on the same regressors used earlier–dummies for the treatment parameters and CV, and their cross effects, controlling for treatment order and experimental round. As usual, the excluded case corresponds to MP, $m = 2$, $K = 2$, and treatment order 1. The unit of observation is the $m+M$-subjects voting group, and standard errors are clustered at the voting group level. For every parametrization, CV increases the fraction of minority victories, and does so very significantly, both quantitatively and statistically. As expected, so does the increase in the minority size ($m = 3$). Here again there is no effect of treatment order, and neither is there a change from earlier to later rounds, reflecting the previous finding of a parallel and equally small decline in turnout for both parties as the sessions proceeded.

We conclude that the data strongly support hypothesis H3.[21] As for hypothesis H4, supported too is the prediction that the differences in minority victories should be larger across voting rules, for given parametrization, than across parametrizations, for given voting rule. The point prediction of constant share of minority victories across parametrizations, for given voting rule, fares slightly less well, with the largest exception the fewer than expected successes of the minority under CV when $K = 2$ and $m = 2$, reflecting the unexpected high turnout of the majority in this treatment. Even in this case, however, the share of minority seats is more than double what it is under MP.

[21] Note that the share of minority victories remains under 50% in all cases.

Table 4: Share of Minority Victories

	% Minority Seats
K = 4	0.049
	(0.026)
	[0.065]
m = 3	0.100
	(0.029)
	[0.001]
CV	0.158
	(0.025)
	[0.000]
(K = 4) $\times (m = 3)$	-0.122
	(0.040)
	[0.002]
(K = 4) $\times CV$	0.041
	(0.033)
	[0.215]
(m = 3) $\times CV$	0.037
	(0.037)
	[0.308]
(K = 4) $\times (m = 3) \times CV$	-0.019
	(0.048)
	[0.695]
Treatment order=2	-0.012
	(0.016)
	[0.441]
Treatment order=3	0.011
	(0.016)
	[0.505]
Treatment order=4	0.021
	(0.017)
	[0.221]
Round	0.001
	(0.001)
	[0.392]
Constant	0.092
	(0.023)
	[0.000]
Observations	1920
R^2	0.148

Standard errors are clustered at the group level.

5 Conclusions

As we write, a new administration has entered the White House and made the strengthening of voting rights for minorities one of its priorities.[22] Together with the abolition of unequal obstacles to exercising the right to vote, thought should be given to the voting rules themselves, a reflection that will become very salient with the drawing of new legislative district maps after the 2020 census. Local jurisdictions have built

[22] "We need to restore and expand the Voting Rights Act [...] and continue to fight back against laws that many states are engaged in to suppress the right to vote, while expanding access to the ballot box for all eligible voters." (President Joseph Biden, speech at the White House, January 26, 2021)

experience with multi-member districts and voting rules that grant the minority the possibility of meaningful representation. Such rules deserve more publicity and more study.

This paper presents the results of a controlled lab experiment testing Cumulative Voting (CV), a voting rule for electing representatives to multi-seat bodies that allows a voter to cumulate votes on a subset of candidates, at the cost of not casting votes on the others. As predicted by the theory, we have found that CV encourages turnout by the minority party, relative to plurality, and increases the share of seats that the minority wins.

By design, the experiment sidestepped problems of coordination among the voters, because the historical experience teaches, plausibly, that ensuring such coordination is the responsibility of party leaders. But even in such a case, richer scenarios can be easily constructed, where not all candidates are equivalent, or not all are equally valued by different voters. Future experiments should consider such environments.

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A Appendix A

A.1 CV when voting is costless

Proposition 1. *In the absence of voting costs, for all m, $M > m$, and K, in all equilibria of the voting game:*

$$
\begin{aligned}
&z = 0 && \text{if } m < M/K \\
&z \in \left(\frac{Km-M}{M+m}, \frac{Km+m}{M+m}\right) && \text{if } m > M/K \text{ and } \frac{Km-M}{M+m} \notin \mathbb{Z}_+ \\
&z = \begin{cases} \frac{Km-M}{M+m} & \text{with prob } m/(M+m) \\ \frac{Km+m}{M+m} & \text{with prob } M/(M+m) \end{cases} && \text{if } m \geq M/K \text{ and } \frac{Km-M}{M+m} \in \mathbb{Z}_+
\end{aligned}
$$

Because $(Km+m)/(M+m) - (Km-M)/(M+m) = 1$, if $\frac{Km-M}{M+m}$ is not an integer, there exists a unique integer value of z in the relevant interval; if $\frac{Km-M}{M+m}$ is an integer, then $\frac{Km+m}{M+m}$ is one as well.

Proof. As described in the text, the result follows because the linearity of $u(k)$ renders the parties game a constant-sum two-player game. The proposition exploits the properties of such games: in all equilibria, payoffs must equal the payoffs from the players' maximin strategies. Suppose first that $m < M/K$. Then the M party can guarantee itself all K seats by dividing its votes equally over K candidates, and the m party cannot win any seat. Suppose then $m > M/K$. For any v_{M}, party m maximizes the probability of winning z seats by dividing its votes equally over z candidates, and guarantees itself z seats if $mK/z > MK/(K-z+1)M$, or $z < (Km+m)/(M+m)$. At the same time, party M maximizes the probability of winning $(K-z)$ seats by dividing its votes equally over $K-z$ candidates, and guarantees itself $K-z$ seats if $MK/(K-z) > mK/(z+1)$, or $z > (Km-M)/(M+m)$. We require z to be an integer. Since $(Km+m)/(M+m) - (Km-M)/(M+m) = 1$, if $(Km-M)/(M+m)$ is not an integer, then there exists a unique integer value of $z \in \left(\frac{Km-M}{M+m}, \frac{Km+m}{M+m}\right)$ and such strategies are an equilibrium: neither party has a profitable deviation. Hence an equilibrium exists. By the properties of constant-sum two-player games, it then follows that all equilibria must deliver such an outcome: many equilibria are possible,with votes divided unequally among the candidates and more than z or Z candidates from each party receiving positive votes, but in all equilibria the m party wins z seats, and the M party $Z = K - z$ seats.[23]

Finally, suppose $m \geq M/K$ and $(Km-M)/(M+m)$ is an integer. Then the m party can guarantee itself $(Km-M)/(M+m) \equiv \underline{z}$ seats, but can do better by spreading votes equally over $(Km-M)/(M+m)+1 = (Km+m)/(M+m) \equiv \overline{z}$ candidates. Similarly, the M party can guarantee itself $K-\overline{z}$ seats, but can do better by spreading votes equally over $K-\overline{z}+1 = K-\underline{z}$ candidates. In equilibrium then, party m (M) spreads its votes equally over $\overline{z}$ ($K-\underline{z}$) candidates; a total of $K+1$ candidates receive votes, and all are tied with $[K/(K+1)](M+m)$ votes each. The tie-break rule selects K winners randomly from the $K+1$

[23] For example, suppose $K = 4$, $M = 4$, and $m = 3$, which is one of our experimental parametrizations. In the absence of voting costs, in all equilibria $z = 2$. One possible set of equilibrium strategies is $v^i_{\mathsf{m}} = \{2, 2, 0, 0\}$ for all $i \in \mathsf{m}$, and $v^i_{\mathsf{M}} = \{2, 2, 0, 0\}$ for all $i \in \mathsf{M}$. Thus two candidates in each party list receive votes and are elected: the two winning candidates in party M receive 8 votes each, and the two winning candidates in party m receive 6 votes each. But other equilibria are possible. For example, holding voting unchanged in the minority party, three candidates from party M may receive $\{7, 7, 2\}$ votes each. However there is no equilibrium that allows either party to win more than two seats.

candidates. It then follows that:

$$
\begin{aligned}
prob(z=\overline{z}) &= \frac{\binom{K-\underline{z}}{K-\overline{z}}}{\binom{K+1}{K}} = 1-\frac{\overline{z}}{K+1} = \frac{M}{m+M} \\
prob(z=\underline{z}) &= 1-prob(z=\overline{z}) = \frac{m}{m+M}
\end{aligned}
$$

□

The proposition replicates the formulas reported online. Its contribution is the remark that if the game is constant-sum, such outcomes are not only the result of the two players' maximin strategies, but must hold in all equilibria.

A.2 Costly voting

A.2.1 Multi-winner plurality (MP)

We report here the binomial formulas for the pivot probabilities. Under MP, such formulas are well-known (see for example Levine and Palfrey, 2007).

$$
\begin{aligned}
\pi_{\mathsf{m}}^{T-1} &= \sum_{x=0}^{m-1}\binom{m-1}{x}\binom{M}{x+1}F(c_{\mathsf{m}})^x[1-F(c_{\mathsf{m}})]^{m-1-x}F(c_{\mathsf{M}})^{x+1}[1-F(c_{\mathsf{M}})]^{M-(x+1)} \\
\pi_{\mathsf{m}}^{T} &= \sum_{x=0}^{m-1}\binom{m-1}{x}\binom{M}{x}F(c_{\mathsf{m}})^x[1-F(c_{\mathsf{m}})]^{m-1-x}F(c_{\mathsf{M}})^{x}[1-F(c_{\mathsf{M}})]^{M-x}
\end{aligned}
$$

and:

$$
\begin{aligned}
\pi_{\mathsf{M}}^{T-1} &= \sum_{x=1}^{m}\binom{m}{x}\binom{M-1}{x-1}F(c_{\mathsf{m}})^x[1-F(c_{\mathsf{m}})]^{m-x}F(c_{\mathsf{M}})^{x-1}[1-F(c_{\mathsf{M}})]^{M-1-(x-1)} \\
\pi_{\mathsf{M}}^{T} &= \sum_{x=0}^{m}\binom{m}{x}\binom{M-1}{x}F(c_{\mathsf{m}})^x[1-F(c_{\mathsf{m}})]^{m-x}F(c_{\mathsf{M}})^{x}[1-F(c_{\mathsf{M}})]^{M-1-x}
\end{aligned}
$$

The frequency of minority victories is sensitive to the relative turnout rates of the two parties, captured by the ratio of the two thresholds, $c_{\mathsf{m}}/c_{\mathsf{M}}$. Although the study of costly voting models has identified an "underdog effect"–the tendency for the minority's turnout rate to be higher than the majority's, or $c_{\mathsf{m}} > c_{\mathsf{M}}$– the existence of such an effect is sensitive to the exact specification of the model. It has been proven in a number of scenarios: when the voting cost is fixed and equal for all (Taylor and Yildirim, 2010a); when voters' direction of preferences is randomly drawn (Ledyard, 1984; Taylor and Yildirim, 2010b); when the size of the electorate is uncertain (Herrera et al., 2014; Krishna and Morgan, 2015).

In the specification used above, relative turnout under MP depends on V, the value of winning all seats. Because the model is widely used but this observation is missing from the literature, we make it explicit in the following remark.

Remark. *For any finite M, m, and F there exists a finite $\widehat{V}(M,m)$ such that if $V=\widehat{V}$, then $c_{\mathsf{m}} = c_{\mathsf{M}}$.*

Proof. Call $\widehat{c}$ the median of $F(c)$. Straightforward manipulations of the pivot probabilities show that if

$c_{\mathsf{m}} = c_{\mathsf{M}} = \widehat{c}$, and thus $F(c_{\mathsf{m}}) = 1 - F(c_{\mathsf{M}}) = 1/2$, then $(\pi_{\mathsf{m}}^{T} + \pi_{\mathsf{m}}^{T-1}) = (\pi_{\mathsf{M}}^{T} + \pi_{\mathsf{M}}^{T-1}) = (1/2)^{M+m-1}\binom{M+m}{m}$. Hence for any M and m, $c_{\mathsf{m}} = c_{\mathsf{M}} = \widehat{c}$ is an equilibrium as long as $\widehat{c} = (V/2)(1/2)^{M+m-1}\binom{M+m}{m}$, or $V = \widehat{c}\left(2^{(M+m)}/\binom{M+m}{m}\right) = \widehat{V}$. $\square$

Note that our focus is not on the two parties' relative turnout per se, but on the impact on such relative turnout of the voting rule.

The derivation of the probabilities of winning different numbers of seats is straightforward. Consider the problem from the perspective of a minority voter. Begin with the probability of losing all positions, $\Pr(W_{\mathsf{m}} = 0)$. Such probability equals the probability that either all minority candidates receive strictly fewer votes than the majority candidates, or that all candidates are tied but minority candidates lose all tie-breaks. Or, $\Pr(W_{\mathsf{m}} = 0) = \Pr(S_{\mathsf{m}} < S_{\mathsf{M}}) + \Pr[(S_{\mathsf{m}} = S_{\mathsf{M}}) \cap (\mathsf{m} \text{ loses all tie-breaks})]$. That is:

$$
\begin{aligned}
&\Pr(W_{\mathsf{m}} = 0) = \\
&= \sum_{S_{\mathsf{M}}=1}^{M} \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}}[1 - F(c_{\mathsf{M}})]^{M-S_{\mathsf{M}}} \sum_{S_{\mathsf{m}}=0}^{S_{\mathsf{M}}-1} \binom{m}{S_{\mathsf{m}}} F(c_{\mathsf{m}})^{S_{\mathsf{m}}}[1 - F(c_{\mathsf{m}})]^{m-S_{\mathsf{m}}} + \\
&+ \sum_{S_{\mathsf{M}}=0}^{M} \binom{M}{S_{\mathsf{M}}}\binom{m}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}}[1 - F(c_{\mathsf{M}})]^{M-S_{\mathsf{M}}} \times \\
&\times F(c_{\mathsf{m}})^{S_{\mathsf{M}}}[1 - F(c_{\mathsf{m}})]^{m-S_{\mathsf{M}}} \left(1/\binom{2K}{K}\right)
\end{aligned}
$$

Similarly, the probability that m wins all positions, $\Pr(W_{\mathsf{m}} = K)$ equals the probability that either all minority candidates receive strictly more votes than the majority candidates, or that all candidates are tied but minority candidates win all tie-breaks.Or, $\Pr(W_{\mathsf{m}} = K) = \Pr(S_{\mathsf{m}} > S_{\mathsf{M}}) + \Pr[(S_{\mathsf{m}} = S_{\mathsf{M}}) \cap (\mathsf{m} \text{ wins all tie-breaks})]$. That is:

$$
\begin{aligned}
&\Pr(W_{\mathsf{m}} = K) = \\
&= \sum_{S_{\mathsf{M}}=0}^{m-1} \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}}[1 - F(c_{\mathsf{M}})]^{M-S_{\mathsf{M}}} \sum_{S_{\mathsf{m}}=S_{\mathsf{M}}+1}^{m} \binom{m}{S_{\mathsf{m}}} F(c_{\mathsf{m}})^{S_{\mathsf{m}}}[1 - F(c_{\mathsf{m}})]^{m-S_{\mathsf{m}}} + \\
&+ \sum_{S_{\mathsf{M}}=0}^{M} \binom{M}{S_{\mathsf{M}}}\binom{m}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}}[1 - F(c_{\mathsf{M}})]^{M-S_{\mathsf{M}}} \times \\
&\times F(c_{\mathsf{m}})^{S_{\mathsf{M}}}[1 - F(c_{\mathsf{m}})]^{m-S_{\mathsf{M}}} \left(1/\binom{2K}{K}\right)
\end{aligned}
$$

The probabilities of other numbers of minority victories can be derived in the same fashion. The probability of electing w minority candidates, with $w \in (0, K)$ equals the probability that all candidates are tied and m wins w tie-breaks. Thus:

$$\Pr(W_{\mathsf{m}} = w) =$$
$$= \sum_{S_{\mathsf{M}}=0}^{M} \binom{m}{S_{\mathsf{M}}} F(c_{\mathsf{m}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{m}})]^{m-S_{\mathsf{M}}} \times$$
$$\times \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M-S_{\mathsf{M}}} \binom{K}{w} \binom{K}{K-w} / \binom{2K}{K}$$

For given M, m, K, $F(c)$, and $\{u(k)\}$, the equilibrium yields expected turnout rates for voters of the two parties, the probabilities of winning $0, 1, .., K$ positions for each party, and ex ante expected utility for an M and an m voter.[24]

A.2.2 Cumulative Voting (CV)

Equilibrium The presence of voting costs makes voters independent strategic players. The game now has $m + M + 2 = N + 2$ players, the voters of each party and the two party leaders. It has three stages. In the first stage, the two party leaders issue public voting recommendations v_p, where v_p^{ik} is the number of votes party voter i is instructed to cast on party candidate k_p, conditional on i having turned out to vote. In the second stage, each voter i privately observes the realization of her idiosyncratic voting cost c_i, and all voters decide contemporaneously whether or not to turn out. In the third stage, voters at the poll choose how to cast their votes. We denote by x_p the realized vote distribution in party p, of the same dimension of v_p.

The distribution F from which voting costs are independently drawn is common knowledge, but the cost realizations are private information. The randomness of the costs creates uncertainty, and all players make decisions so as to maximize their expected payoffs, forming expectations about voters' realized turnout. Party leaders maximize $Eu_p(k)$, or equivalently the expected number of seats won by the party; voters maximize $EU_{i,p}(k, c_i)$, their net utility taking into account not only the expected number of seats won but also their private voting cost.

Note that leaders' instructions are conditional on a voter being at the polls. When at the polls, the voter's objective coincides with the objective of the party leader, and equilibrium instructions must be such that, when a voter expects others to follow the instructions, it is optimal for her to follow them as well. In the text and in the experiment we impose equal spreading of votes. Thus we simplify the leaders' problem to the choice of the number of viable candidates, r_v in m and R_v in M, or $v_{\mathsf{m}}^{ik} = K/r_v$, $v_{\mathsf{M}}^{ik} = K/R_v$, and, once at the polls, give voters no choice on how to distribute their votes: $x_{\mathsf{m}}^{ik} = K/r_v$, $x_{\mathsf{M}}^{ik} = K/R_v$. As is well-known, the voters' optimal turnout strategy is a cutpoint strategy: there is a maximum cost $\bar{c}_{i,p}$ such that voter i in party p prefers to turn out if $c_i < \bar{c}_{i,p}$ and prefers to stay home if $c_i > \bar{c}_{i,p}$.

We restrict attention to semi-symmetric equilibria in weakly undominated strategies. That is, we focus on equilibria with the following features: all voters in the same party follow the same strategy; party leaders instruct all voters in the party to split their votes in the same fashion, and to cast votes exclusively for own party candidates; at the polls voters follow leaders instructions as long as doing so is weakly undominated.

We denote by g (G) the equilibrium number of viable candidates in party m (M). An equilibrium then is a set of strategies $\{G, g, c_{\mathsf{M}}, c_{\mathsf{m}}\}$ such that (i) all $i \in \mathsf{M}$ with $c_i < c_{\mathsf{M}}(c_{\mathsf{m}}, G, g)$ strictly prefer to vote, and all $i \in \mathsf{M}$ with $c_i > c_{\mathsf{M}}(c_{\mathsf{m}}, G, g)$ strictly prefer to abstain, and all $i \in \mathsf{m}$ with $c_i < c_{\mathsf{m}}(c_{\mathsf{M}}, G, g)$ strictly

[24] Note in particular that if $u(K) - Eu_T^{MP} = Eu_T^{MP} - u(0)$, or $Eu_T^{MP} = [u(K) - u(0)]/2$, the equilibrium thresholds $\{c_{\mathsf{m}}, c_{\mathsf{M}}\}$ are identical to the thresholds that solve the corresponding costly voting problem with a single winner.

prefer to vote, and all $i \in \mathsf{m}$ with $c_i > c_{\mathsf{m}}(c_{\mathsf{M}}, G, g)$ strictly prefer to abstain; and (ii) $EZ(G, g, c_{\mathsf{M}}, c_{\mathsf{m}}) \geq EZ(R_v, g, c_{\mathsf{M}}, c_{\mathsf{m}})$ for all $R_v \neq G$, and $Ez(g, G, c_{\mathsf{M}}, c_{\mathsf{m}}) \geq Ez(r_v, G, c_{\mathsf{M}}, c_{\mathsf{m}})$ for all $r_v \neq g$.

Pivot probabilities and probabilities of winning different numbers of seats Consider first the perspective of a majority voter. The pivot probabilities correspond to the probabilities of the three events described in the text–breaking a tie (if $(K/G)S_{\mathsf{M-i}} = (K/g)S_{\mathsf{m}}$), making a tie (if $(K/G)(S_{\mathsf{M-i}} + 1) = (K/g)S_{\mathsf{m}}$), or moving the outcome from a loss to a win on all contested positions (if $S_{\mathsf{M-i}} \in (S_{\mathsf{m}}(G/g) - 1, S_{\mathsf{m}}(G/g)))$. Note that since $S_{\mathsf{M}-i}$ and S_{m} are non-negative integers, the first event is only possible if either G/g is an integer, or $S_{\mathsf{M}-i} = S_{\mathsf{m}} = 0$; the second event is only possible if G/g is an integer, and the third event is only possible if G/g is not integer.

The equations corresponding to the pivot probabilities are logically straightforward:

$$\widetilde{\pi}_{\mathsf{M}}^{T} = I_Q[(G/g)S_{\mathsf{m}}] \sum_{S_{\mathsf{m}}=0}^{m} \left\{ \binom{m}{S_{\mathsf{m}}} F(c_{\mathsf{m}})^{S_{\mathsf{m}}} [1 - F(c_{\mathsf{m}})]^{m-S_{\mathsf{m}}} \right.$$
$$\left. \binom{M-1}{(G/g)S_{\mathsf{m}}} F(c_{\mathsf{M}})^{(G/g)S_{\mathsf{m}}} [1 - F(c_{\mathsf{M}})]^{M-1-(G/g)S_{\mathsf{m}}} \right\}$$

$$\widetilde{\pi}_{\mathsf{M}}^{T-1} = I_Q[(G/g)S_{\mathsf{m}}] \sum_{S_{\mathsf{m}}=1}^{m} \left\{ \binom{m}{S_{\mathsf{m}}} F(c_{\mathsf{m}})^{S_{\mathsf{m}}} [1 - F(c_{\mathsf{m}})]^{m-S_{\mathsf{m}}} \right.$$
$$\left. \binom{M-1}{(G/g)S_{\mathsf{m}} - 1} F(c_{\mathsf{M}})^{(G/g)S_{\mathsf{m}}-1} [1 - F(c_{\mathsf{M}})]^{M-1-[(G/g)S_{\mathsf{m}}-1]} \right\}$$

and

$$\widetilde{\pi}_{\mathsf{M}}^{W} = (1 - I_Q[(G/g)S_{\mathsf{m}}]) \sum_{S_{\mathsf{m}}=0}^{m} \left\{ \binom{m}{S_{\mathsf{m}}} F(c_{\mathsf{m}})^{S_{\mathsf{m}}} [1 - F(c_{\mathsf{m}})]^{m-S_{\mathsf{m}}} \right.$$
$$\left. \binom{M-1}{\lfloor (G/g)S_{\mathsf{m}} \rfloor} F(c_{\mathsf{M}})^{\lfloor (G/g)S_{\mathsf{m}} \rfloor} [1 - F(c_{\mathsf{M}})]^{M-1-\lfloor (G/g)S_{\mathsf{m}} \rfloor} \right\}$$

where $I_Q[(G/g)S_{\mathsf{m}}] = 1$ if $(G/g)S_{\mathsf{m}}$ is an integer, and 0 otherwise, and $\lfloor x \rfloor$ is the floor function, denoting the greatest integer smaller or equal to x.[25]

The problem is analogous for a minority voter. The relevant equations are:

$$\widetilde{\pi}_{\mathsf{m}}^{T} = I_Q[(g/G)S_{\mathsf{M}}] \left\{ \sum_{S_{\mathsf{M}}=0}^{M} \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M-S_{\mathsf{M}}} \right.$$
$$\left. \binom{m-1}{(g/G)S_{\mathsf{M}}} F(c_{\mathsf{m}})^{(g/G)S_{\mathsf{M}}} [1 - F(c_{\mathsf{m}})]^{m-1-(g/G)S_{\mathsf{M}}} \right\}$$

$$\widetilde{\pi}_{\mathsf{m}}^{T-1} = I_Q[(g/G)S_{\mathsf{M}}] \sum_{S_{\mathsf{M}}=1}^{M} \left\{ \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M-S_{\mathsf{M}}} \right.$$
$$\left. \binom{m-1}{(g/G)S_{\mathsf{M}} - 1} F(c_{\mathsf{m}})^{(g/G)S_{\mathsf{M}}-1} [1 - F(c_{\mathsf{m}})]^{m-1-[(g/G)S_{\mathsf{M}}-1]} \right\}$$

[25]We are also using the convention $\binom{n}{y} = 0$ if $y > n$.

$$\widetilde{\pi}^{W}_{\mathsf{m}} = (1 - I_Q[(g/G)S_{\mathsf{M}}]) \sum_{S_{\mathsf{M}}=0}^{M} \left\{ \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M - S_{\mathsf{M}}} \right.$$
$$\left. \binom{m-1}{\lfloor (g/G)S_{\mathsf{M}} \rfloor} F(c_{\mathsf{m}})^{\lfloor (g/G)S_{\mathsf{M}} \rfloor} [1 - F(c_{\mathsf{m}})]^{m-1-\lfloor (g/G)S_{\mathsf{M}} \rfloor} \right\}$$

The probabilities of the minority winning different numbers of position can be derived as under MP, but taking into account that the number of candidates, in each party, now may differ from the number of seats. The probability of the minority losing all seats must be 0 if $G < K$; if instead $G \geq K$, then as before it equals the probability that either all minority candidates receive strictly lower votes than the majority candidates, or that all candidates are tied but minority candidates lose all tie-breaks. That is:

$$\Pr(W_{\mathsf{m}} = 0 | G \geq K) =$$
$$= \sum_{S_{\mathsf{M}}=1}^{M} \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M - S_{\mathsf{M}}} \sum_{S_{\mathsf{m}}=0}^{X(S_{\mathsf{M}})} \binom{m}{S_{\mathsf{m}}} F(c_{\mathsf{m}})^{S_{\mathsf{m}}} [1 - F(c_{\mathsf{m}})]^{m - S_{\mathsf{m}}} +$$
$$+ \sum_{S_{\mathsf{M}}=0}^{M} \binom{M}{S_{\mathsf{M}}} \binom{m}{(g/G)S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M - S_{\mathsf{M}}} \times$$
$$\times F(c_{\mathsf{m}})^{(g/G)S_{\mathsf{M}}} [1 - F(c_{\mathsf{m}})]^{m-(g/G)S_{\mathsf{M}}} I_Q[(g/G)S_{\mathsf{M}}] \binom{G}{K} / \binom{G+g}{K}$$

where:

$$X(S_{\mathsf{M}}) = \begin{cases} (g/G)S_{\mathsf{M}} - 1 & \text{if } (g/G)S_{\mathsf{M}} \text{ is an integer} \\ \lfloor (g/G)S_{\mathsf{M}} \rfloor & \text{otherwise} \end{cases}$$

The probability of electing w minority candidates, with $w \in (0, g)$ is 0 if $K - G > w$; it equals the probability that all candidates are tied and m wins w tie-breaks if $K - G < w$, and equals the probability either that all are tied and m loses all tie-breaks or that all m candidates receive fewer votes if $K - G = w$. Thus:

$$\Pr(W_{\mathsf{m}} = w | K - G \leq w) =$$
$$= \sum_{S_{\mathsf{M}}=0}^{M} \binom{m}{(g/G)S_{\mathsf{M}}} F(c_{\mathsf{m}})^{(g/G)S_{\mathsf{M}}} [1 - F(c_{\mathsf{m}})]^{m-(g/G)S_{\mathsf{M}}} \times$$
$$\times \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M - S_{\mathsf{M}}} I_Q[(g/G)S_{\mathsf{M}}] \binom{g}{w} \binom{G}{K-w} / \binom{G+g}{K} +$$
$$+ I_{K-G=w} \sum_{S_{\mathsf{M}}=1}^{M} \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M - S_{\mathsf{M}}} \times$$
$$\left(\sum_{S_{\mathsf{m}}=0}^{X(S_{\mathsf{M}})} \binom{m}{S_{\mathsf{m}}} F(c_{\mathsf{m}})^{S_{\mathsf{m}}} [1 - F(c_{\mathsf{m}})]^{m - S_{\mathsf{m}}} \right)$$

where $I_Q[(g/G)S_{\mathsf{M}}\ ]$ and $X(S_{\mathsf{M}}\ )$ are defined as above, and $I_{K-G=w}$ is an indicator function taking value 1 if $K - G = w$ and 0 otherwise.

Finally, the probability of electing g minority candidates equals 1 if $K - G \geq g$, it equals the probability that either all minority candidates receive more votes or that all candidates are tied and the g minority candidates win all tie-breaks. That is:

$$\Pr(W_{\mathsf{m}} = g | K - G < g) =$$
$$= \sum_{S_{\mathsf{M}}=0}^{M} \binom{m}{(g/G)S_{\mathsf{M}}} F(c_{\mathsf{m}})^{(g/G)S_{\mathsf{M}}} [1 - F(c_{\mathsf{m}})]^{m-(g/G)S_{\mathsf{M}}} \times$$
$$\times \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M-S_{\mathsf{M}}} I_Q[(g/G)S_{\mathsf{M}}] \binom{G}{K-g} / \binom{G+g}{K} +$$
$$\sum_{S_{\mathsf{m}}=1}^{m} \binom{m}{S_{\mathsf{m}}} F(c_{\mathsf{m}})^{S_{\mathsf{m}}} [1 - F(c_{\mathsf{m}})]^{m-S_{\mathsf{m}}} \times$$
$$\left(\sum_{S_{\mathsf{M}}=0}^{Y(S_{\mathsf{m}})} \binom{M}{S_{\mathsf{M}}} F(c_{\mathsf{M}})^{S_{\mathsf{M}}} [1 - F(c_{\mathsf{M}})]^{M-S_{\mathsf{M}}} \right)$$

where:

$$Y(S_{\mathsf{m}}) = \begin{cases} (G/g)S_{\mathsf{m}} - 1 & \text{if } (G/g)S_{\mathsf{m}} \text{ is an integer} \\ \lfloor (G/g)S_{\mathsf{m}} \rfloor & \text{otherwise} \end{cases}$$

A.3 Experimental Results

A.3.1 Total turnout regression

Although our interest is primarily in the effect of the voting rule in increasing the minority's turnout rate, relative to the majority, a natural policy concern is the impact of CV on turnout as a whole. The results described so far, separately for minority and majority, can be aggregated to yield the answer, but it is convenient to report directly our findings on total turnout. Figure A1 reports barcharts of the experimental subjects' turnout frequency under the different parametrizations and the two voting rules, with no distinction based on party affiliation.

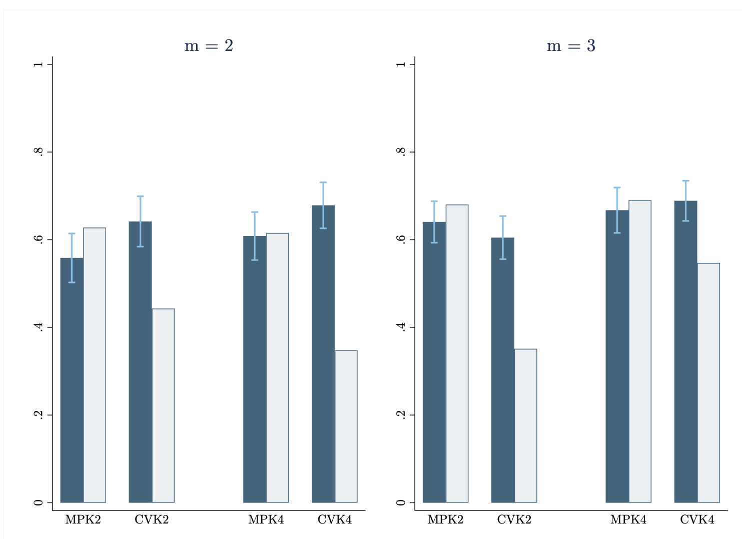

Figure A1: *Turnout rate (all voters).* The dark columns are the data, the light grey columns the theoretical predictions. 95% CI's are calculated from standard errors clustered at the individual level.

The main message is that turnout is little affected by the voting rule and the change in parametrization. The regression results reported in Table A1 lend precision to the broad message. The table is formatted as Tables 2 and 3, with standard errors (in parentheses) and p-values (in brackets) reported under each parameter; as in the previous tables, the excluded case is $m = 2$, $K = 2$, MP, with treatment order 1. Given MP, the increase in m from 2 to 3 increases the electorate's propensity to turnout, and given $m = 2$, switching from MP to CV again increases turnout. The positive effect of CV, however, is reduced under $m = 3$. The final lesson remains that the net effects are small.

Table A1: Frequency of Turnout - All Voters

	Linear Probability Model	Probit Model
K = 4	0.061	0.196
	(0.028)	(0.090)
	[0.031]	[0.029]
m = 3	0.099	0.329
	(0.037)	(0.120)
	[0.007]	[0.006]
CV	0.084	0.273
	(0.028)	(0.093)
	[0.003]	[0.003]
(K = 4) $\times (m = 3)$	-0.041	-0.132
	(0.040)	(0.133)
	[0.305]	[0.319]
(K = 4) $\times CV$	-0.016	-0.040
	(0.037)	(0.123)
	[0.657]	[0.743]
(m = 3) $\times CV$	-0.121	-0.385
	(0.037)	(0.124)
	[0.001]	[0.002]
(K = 4) $\times (m = 3) \times CV$	0.065	0.213
	(0.050)	(0.168)
	[0.190]	[0.205]
Treatment order=2	0.011	0.038
	(0.038)	(0.127)
	[0.767]	[0.767]
Treatment order=3	0.032	0.106
	(0.039)	(0.131)
	[0.406]	[0.418]
Treatment order=4	0.017	0.060
	(0.041)	(0.136)
	[0.682]	[0.661]
Round	-0.006	-0.020
	(0.001)	(0.003)
	[0.000]	[0.000]
Voting Cost	-0.008	-0.025
	(0.000)	(0.001)
	[0.000]	[0.000]
Constant	0.982	1.540
	(0.037)	(0.132)
	[0.000]	[0.000]
Observations	12480	12480
R^2	0.242	

Voter turnout is measured as a binary 0-1 subject decision. Standard errors are clustered at the individual subject level.

A.3.2 Individual behavior: monotonicity violations and cutpoints.

We briefly report here results on individual behavior. In cost of voting experiments, and more broadly in experiments where the equilibrium is in monotone cutpoint strategies, violations of monotonicity are informative not only about the accuracy of the theoretical predictions but also about the participants' understanding of the rules of the game. In our experiment, this is particularly important because a common objection to CV is that its strategic complexity is a difficult obstacle for voters. Although experimental participants limit themselves to the decision to turnout or not, the fact that under CV turnout implies casting multiple votes, and in one case fractional votes, for each candidate could indeed be confusing.

Figures A2 and A3 report histograms of the frequency of monotonicity violations by participants. For each participant, violations are counted as the minimum number of decisions that need to be modified for that participant's turnout decision to be fully monotonic in the voting cost realization: if i chooses to turnout for a cost realization $c_i = \widehat{c}$ then i should turn out for all $c_i < \widehat{c}$, and if i chooses to abstain for a cost realization $c_i = c'$ then i should abstain for all $c_i > c'$. The first bin in each histogram corresponds to no violations; the remaining bins to increasing deciles.The figures have multiple panels, for subjects in the minority (on the left) or in the majority (on the right), for different values of K, and for the two voting rules.

The figures establish two main results. First, monotonicity violations are not common: in all treatments and in each party, it is always the case that more than half of participants have at most a single violation (out of 15 rounds of play). The statement in fact undersells the evidence: over the full data set, 75 percent of participants have at most one violation; the corresponding shares within each party are more than three quarters (78 percent) in the majority, and more than two thirds (69 percent) in the minority. Second, and most important here, there is no systematic difference between the frequency of violations under MP and under CV: for the majority, the fraction with no more than one violation is 80 percent under CV and 77 percent under MP; for the minority, 72 percent under CV and 67 percent under MP. At least in the simplified structure of our experiment, the hypothesis that CV is more confusing for voters is not supported by any evidence of more random behavior.

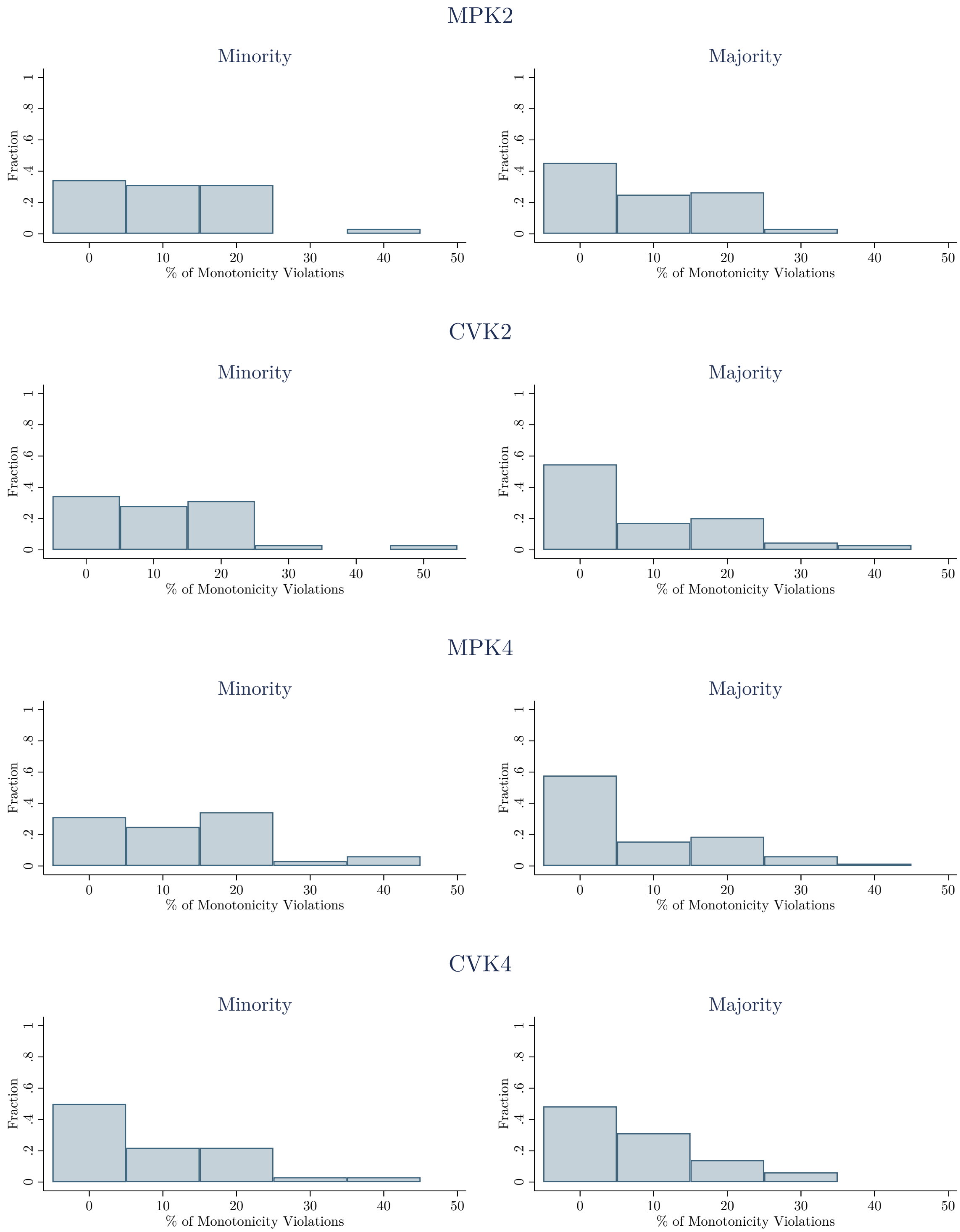

Figure A2: Monotonicity Violations ($m = 2$)

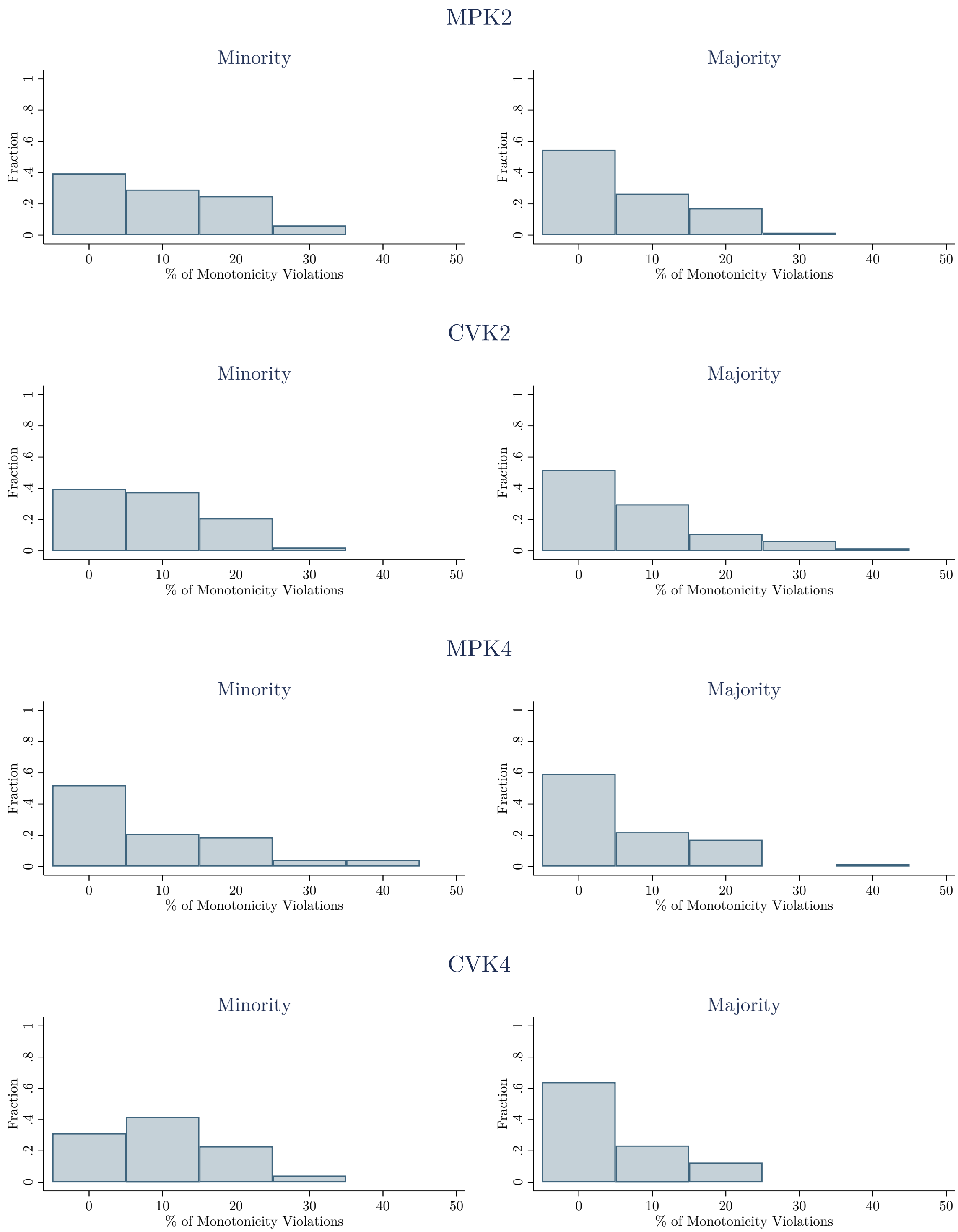

Figure A3: Monotonicity Violations ($m = 3$)

Monotone behavior can be consistent with very different strategies, a point made very clear by Figures A4 and A5. As in other experiments where equilibrium strategies are monotonic (Casella et al., 2006, Levine and Palfrey, 2007), we can use the minimization of monotonicity violations as guide to estimating individual cost cutpoints. Figures A4 and A5 report the results of such estimation: for each subject, the figures report the cutpoint that minimizes the frequency of violations.[26] The size of each circle is proportional to the number of subjects with the same cutpoint. The blue diamonds correspond to the average of the individual cutpoints, and the gray diamond to the theoretical prediction.

Figure A4: Cost Cutpoints ($m = 2$)

[26]In the case of multiplicity, the figures report the mean cutpoint. In a few cases (15 subjects out of 208), different ranges of cutpoints are consistent with minimizing multiplicity violations, and we have chosen the range that is closest to equilibrium. In all cases, we have verified that alternative choices do not change the qualitative results.

Figure A5: Cost Cutpoints ($m = 3$)

The first observation, already noted when discussing Figure 3 in the text, is the high heterogeneity in behavior. While the theoretical semi-symmetric equilibrium predicts a single cutpoint for each party, the data show large variation in behavior, in both parties and for both voting rules. Not surprisingly, average cutpoints are closer to the theory. Second, again in line with the discussion of turnout in the text, the figures show that for both $m = 2$ and $m = 3$, majority subjects, on average, are less sensitive to changes in treatments than theory suggests: the range spanned by the blue diamonds is narrower than the range spanned by the gray diamonds. In particular, relative to the theory, majority subjects on average under-vote under MP and over-vote under CV, a behavior that translates into the approximate constancy of the majority turnout rate across the two voting rules.

References for Appendix A

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Duch, M. L., Grossmann, M. R. P. and Lauer, T., 2020, "z-Tree unleashed: A novel client-integrating architecture for conducting z-Tree experiments over the Internet", *Journal of Behavioral and Experimental Finance,* 28.

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Ledyard, J., 1984, “The pure theory of large two-candidate elections”, *Public Choice*, 44, 7-41.

Levine, D., & Palfrey, T., 2007, “The Paradox of Voter Participation? A Laboratory Study”, *American Political Science Review*, 101, 143-158.

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B Appendix B

Experimental Instructions

We report in Appendix B the instructions for a representative experimental session, with $m = 2$ and 12 subjects and treatment order 1 ($\{CVK2, MPK2, MPK4, CVK4\}$). As discussed in the text, the sessions were held via the Zoom videoconferencing software and run virtually using z-Tree Unleashed (Duch et al., 2020). As a result, instructions were conveyed in more visual style than usual.

In this experiment, you are an eligible voter in an election called to fill several seats.

You will be assigned to one of two parties, either the **Purple** party or the **Orange** party.

If your party wins positions, you win points. However, voting is costly, so if you vote, you lose points.

Your decision in each round will be whether to **vote** or **not vote**.

The experiment will have 4 *Parts*, each with 15 *rounds.*

At the end of each Part, 1 round will be randomly drawn, and the points you earned in those rounds will be converted into monetary earnings.

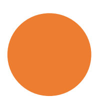
The conversion rate from points to dollars is: 100 points = $1.50 dollar.

Your final earnings will also include the show-up fee of $5.

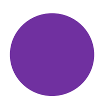
Part 1

Part 1 consists of 15 rounds.
Each round is a new election and will develop as follows:

At the beginning of Part 1, you will be randomly assigned to a party: either **Orange** or **Purple**.

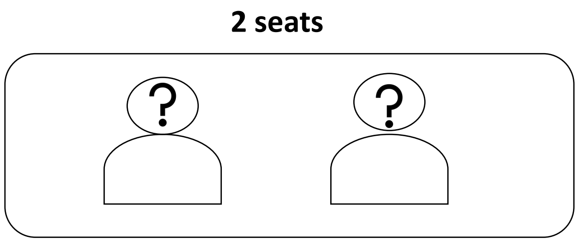


or



You will be part of a group of **6 voters**: **2 Purple** voters and **4 Orange** voters.

Your group will vote to elect a commission with 2 seats.

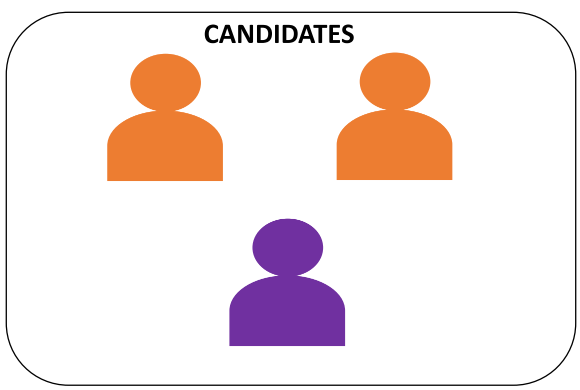


The 2 candidates with most votes win. If there is a tie, it is broken randomly by the computer.

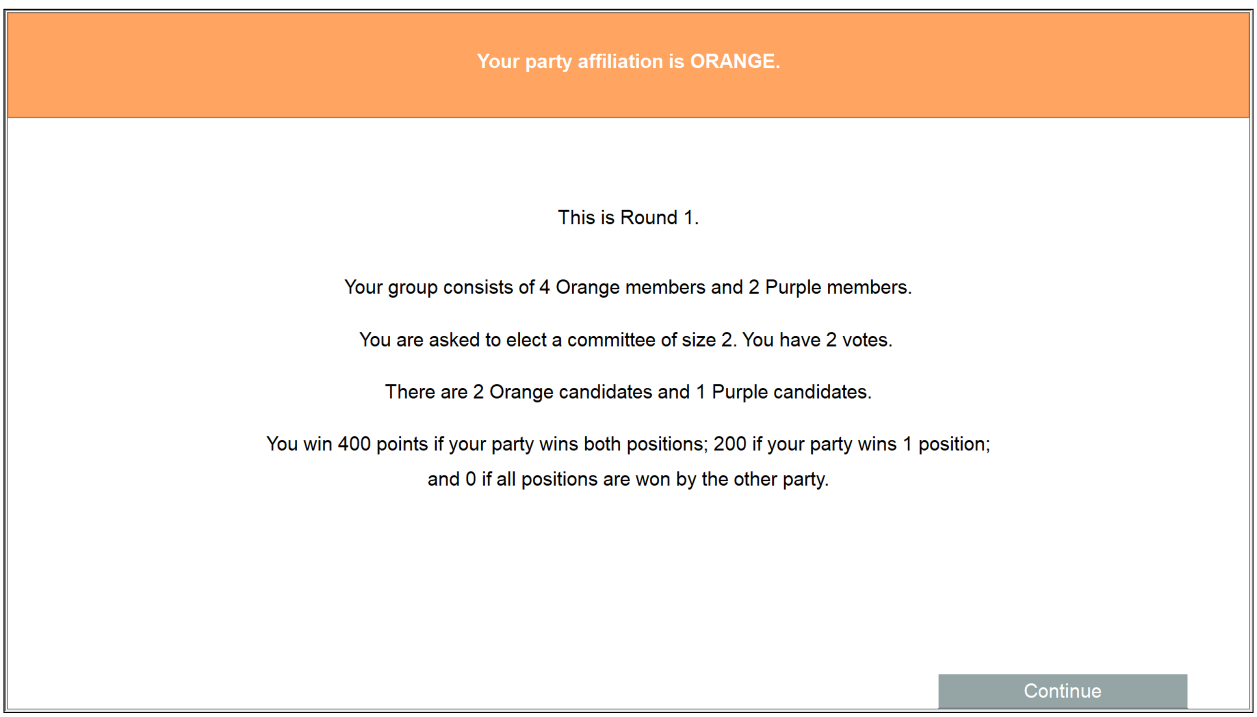
You win **400 points** if your party wins **both** positions, **200 points** if it wins **1** position, and **0** if it wins **none**.

To fill these seats, the Orange party has nominated **2 candidates**.

The **Purple** party has nominated **1 single candidate.**



The computer screen will remind you of this information, and will look as follows.



Whether you are **Orange** or **Purple**, your decision is whether to **vote** or **not vote**.

You are given an **endowment of 100 points**. But if you vote you suffer a voting cost.

The cost is random and can be any **integer between 0 and 100 with equal probability.**

Your cost is **independent** of everyone else's cost. Thus, your cost gives you no information about their costs.

You will be shown your cost before you choose to **vote** or **not vote**.

Your party affiliation is ORANGE.

This is Round 1.

Your voting cost for this Round only is

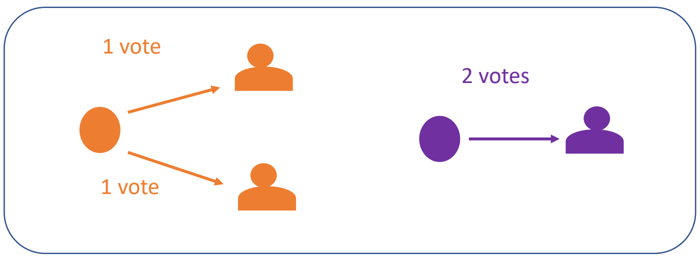
22

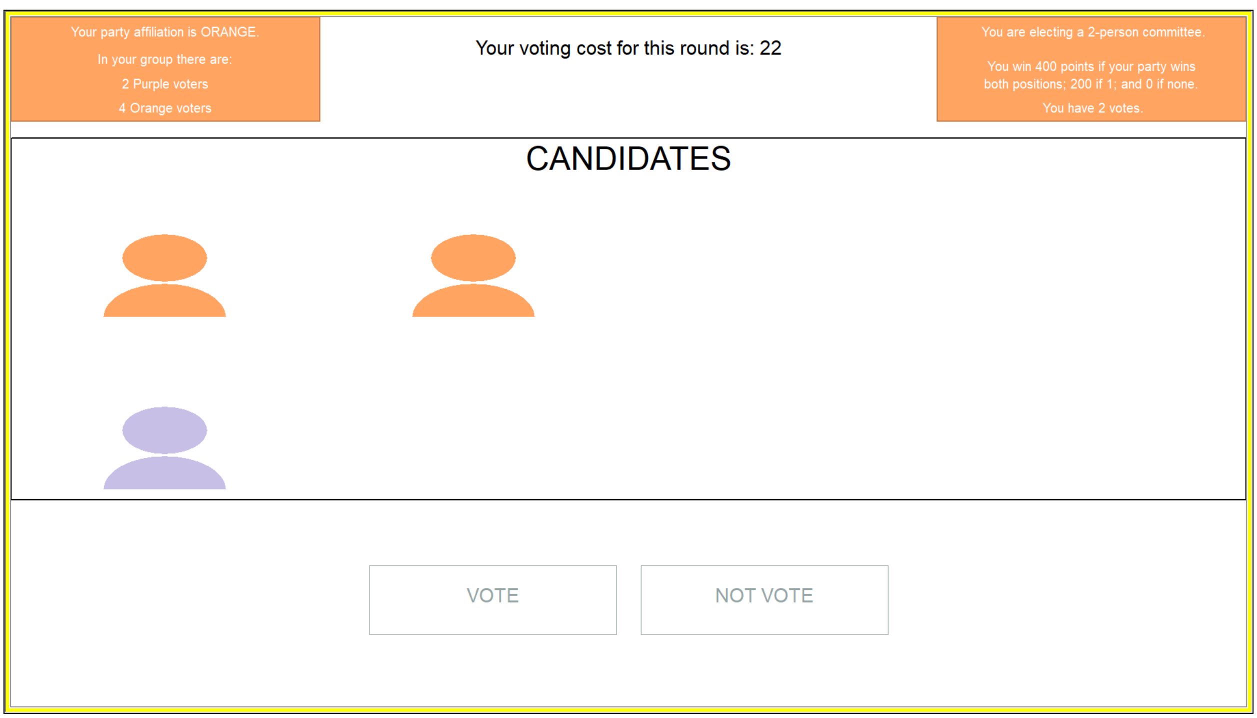
Because 2 positions are open, **each voter has 2 votes**.

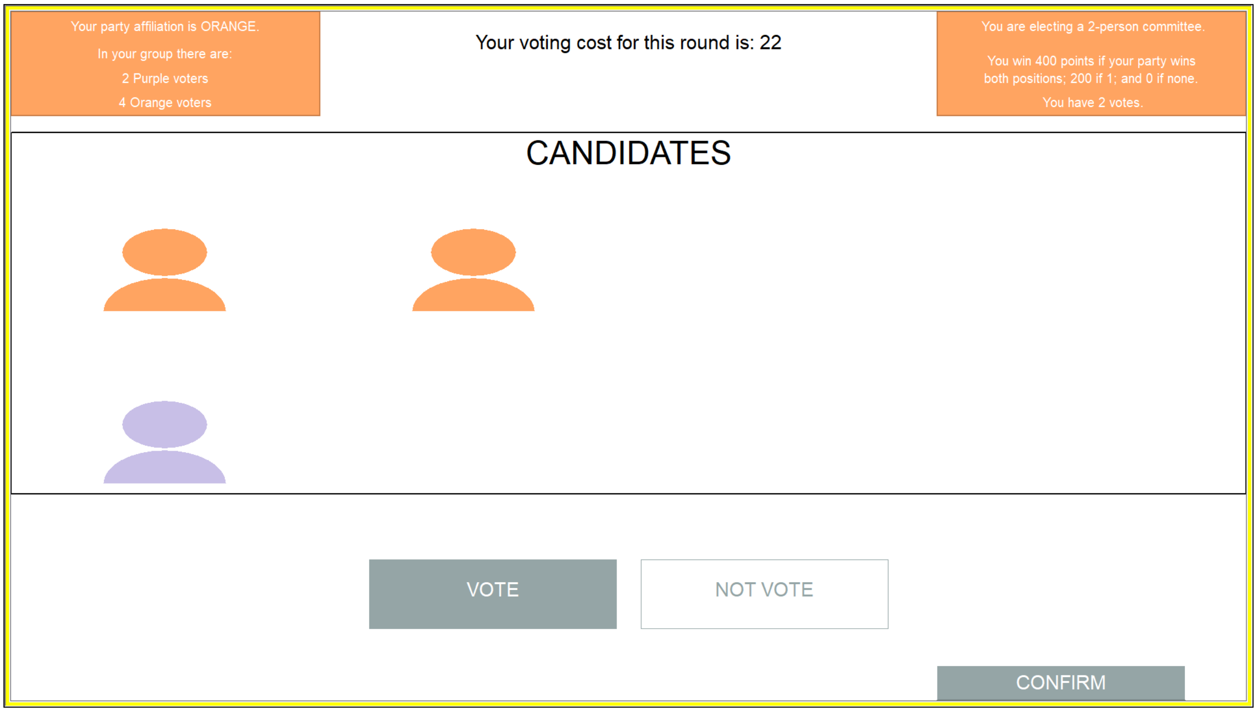
When you vote, all your votes are shared equally by all candidates of your party.

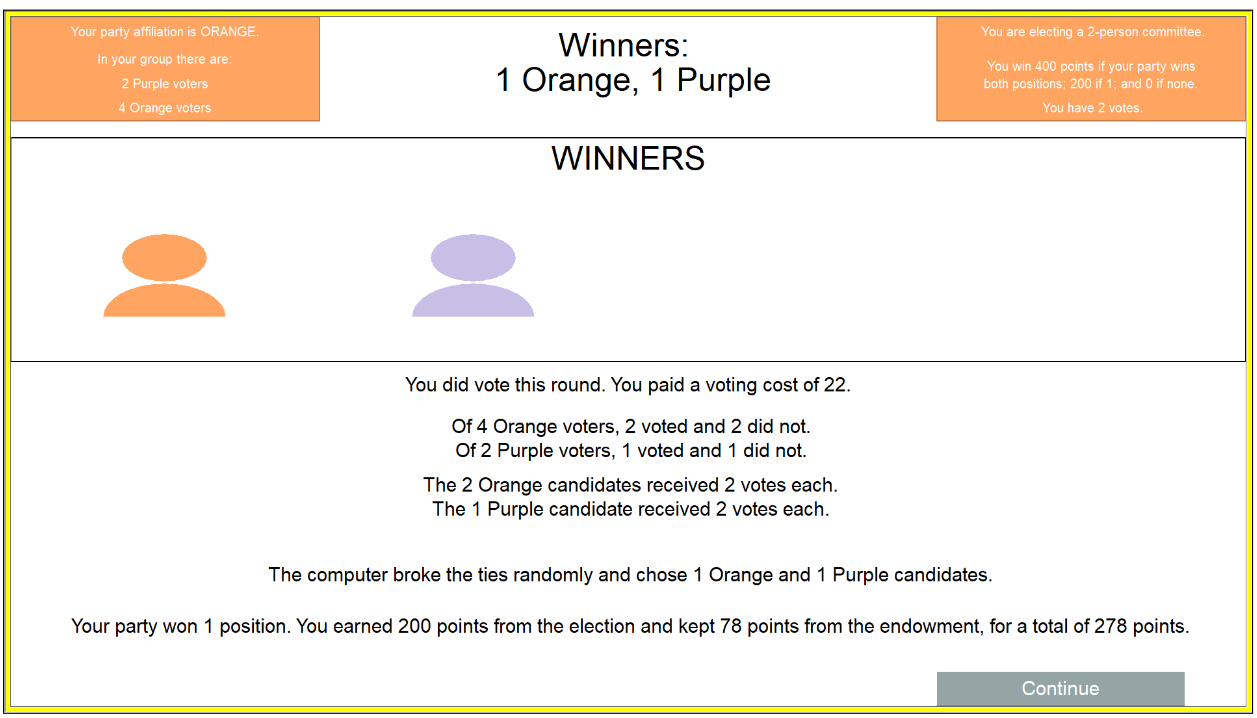
If your party is **Orange** and you vote, you cast **1 vote** for each of the 2 **Orange** candidates.

If your party is **Purple** and you vote, you cast **both of your votes** for the single **Purple** candidate.






Your party affiliation is ORANGE.
In your group there are:
2 Purple voters
4 Orange voters
Your voting cost for this round is: 22
You are electing a 2-person committee.
You win 400 points if your party wins both positions; 200 if 1; and 0 if none.
You have 2 votes.
CANDIDATES
VOTE
NOT VOTE
CONFIRM

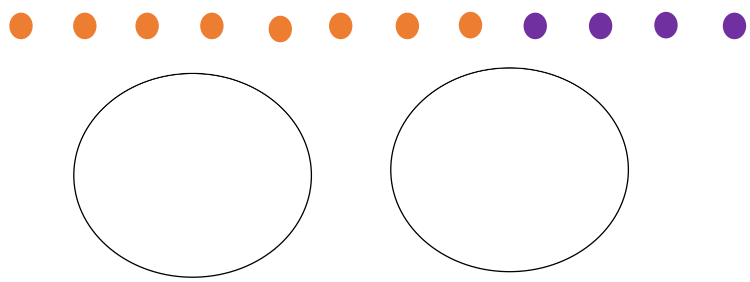

Your party affiliation is ORANGE.
In your group there are:
2 Purple voters
4 Orange voters
Winners:
1 Orange, 1 Purple
You are electing a 2-person committee.
You win 400 points if your party wins both positions; 200 if 1; and 0 if none.
You have 2 votes.
WINNERS
You did vote this round. You paid a voting cost of 22.
Of 4 Orange voters, 2 voted and 2 did not.
Of 2 Purple voters, 1 voted and 1 did not.
The 2 Orange candidates received 2 votes each.
The 1 Purple candidate received 2 votes each.
The computer broke the ties randomly and chose 1 Orange and 1 Purple candidates.
Your party won 1 position. You earned 200 points from the election and kept 78 points from the endowment, for a total of 278 points.
Continue

After the practice rounds, Part 1 will consist of 15 rounds.

Each round is a new election:

1. You will maintain the same party affiliation (**Orange** or **Purple**) for all of Part 1.

2. At each round, you will be rematched randomly into a group of 6 voters: **4 Orange** and **2 Purple**. Because groups are rematched, you will **not** be with the same five other subjects.



3. The **groups are independent**. What happens in one group does not affect the others.

4. The election will proceed as before (2 open seats, 2 votes per voter, **2 Orange candidates**, **1 Purple candidate**)

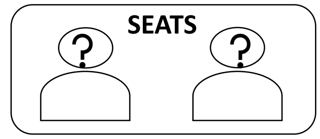
5. All random **voting costs will be drawn again**. Each voter will receive a new endowment of 100 points.

6. You and everyone else will decide again whether to vote or not.

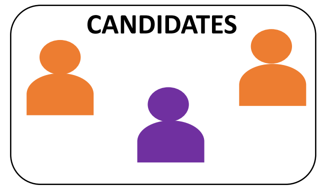
SUMMARY

Your group votes to fill **2 open seats** and **each voter has 2 votes**.

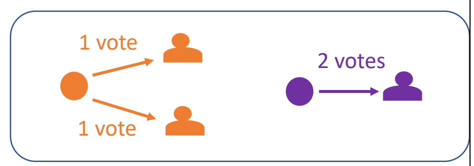
You win 400 points if your party wins both seats, 200 if 1, 0 if none.



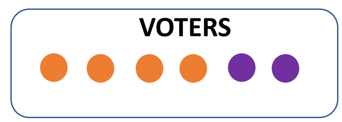
There are **2 Orange** candidates and **1 Purple** candidate.



Because each voter has 2 votes, an **Orange voter** who votes casts **1 vote** for each Orange candidate; a **Purple voter** who votes casts **2 votes** for the single Purple candidate.



The group consists of **4 Orange voters** and **2 Purple voters.**



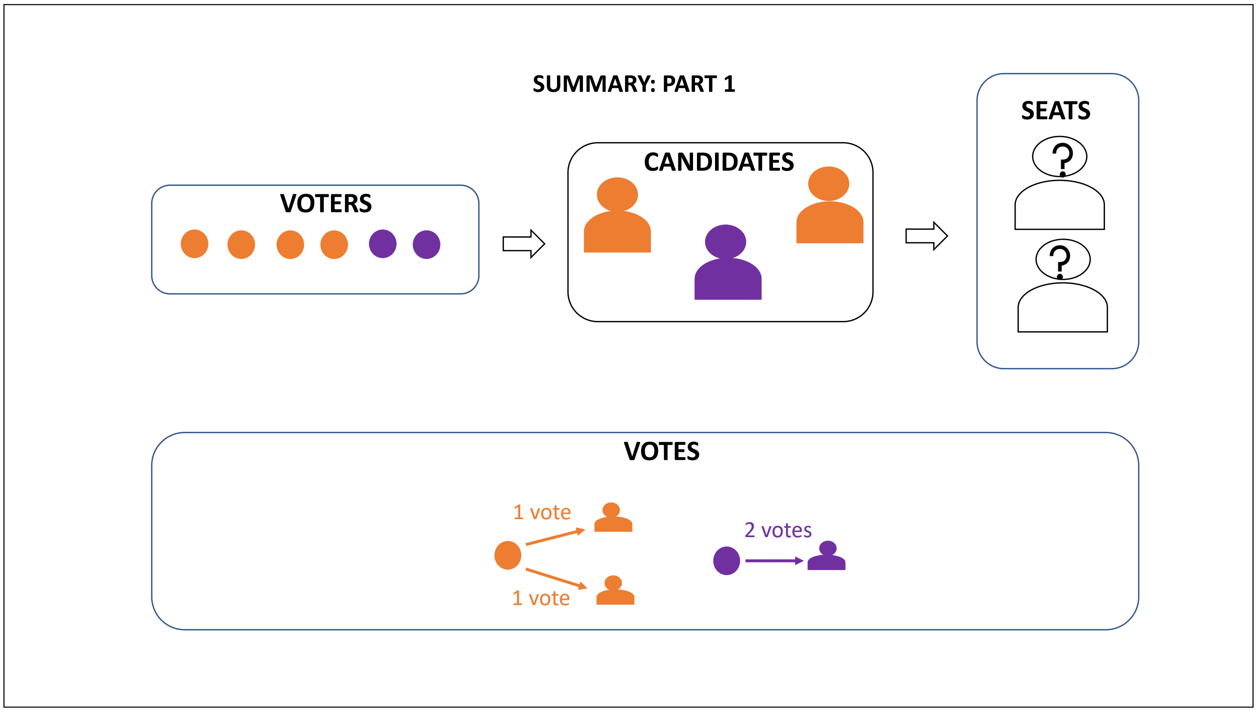
Voting is costly.

If you vote, you pay your voting cost whether or not your party wins any position.

If your party wins any position, you will receive points whether you voted or not.

Your party is more likely to win positions if you vote.

Your decision will be whether to vote or not.


SUMMARY: PART 1
VOTERS
CANDIDATES
SEATS
VOTES
1 vote
1 vote
2 votes

Part 2

Part 2 is very similar to Part 1.

The difference is in the number of candidates:

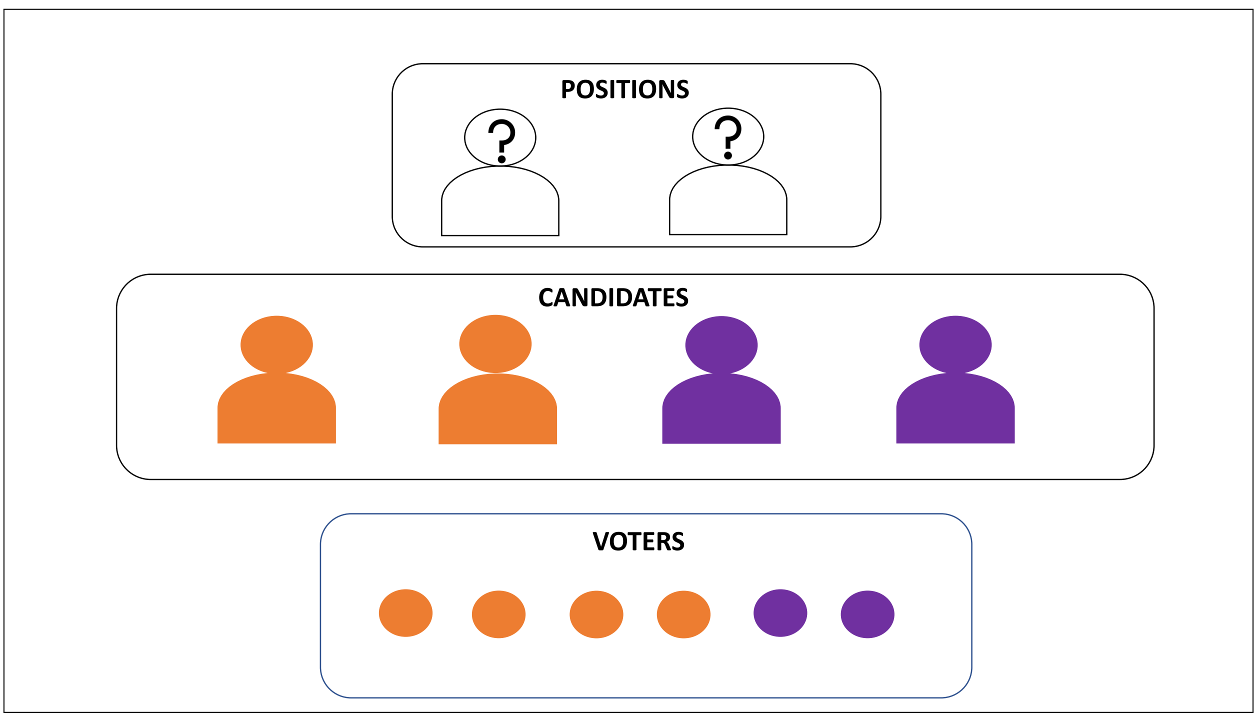
In part 2, both parties have nominated **2 candidates** each.

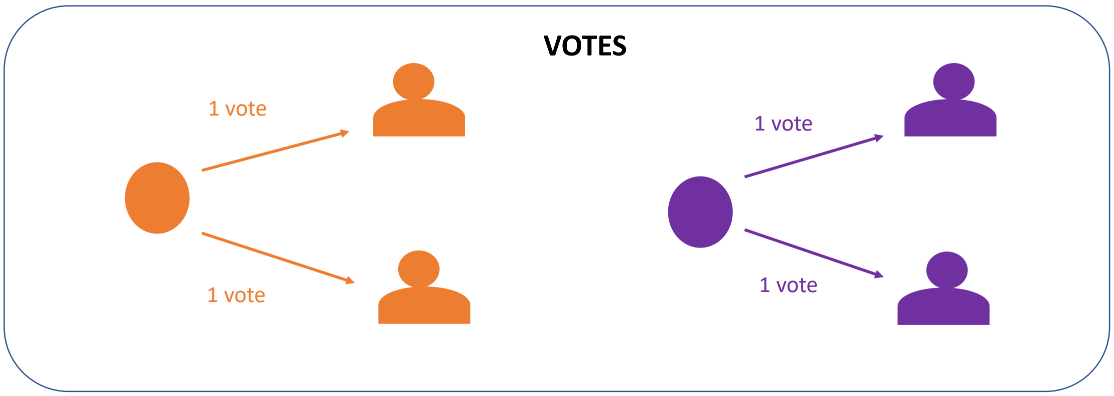
If you vote, **you cast 1 vote for each of your party's candidates**.

The two winning candidates will be the two candidates with most votes, as before.

You will be randomly reassigned a party affiliation, either **Purple** or **Orange**, which may differ from Part 1 and that you will keep for the whole of Part 2.

Everything else remains identical.





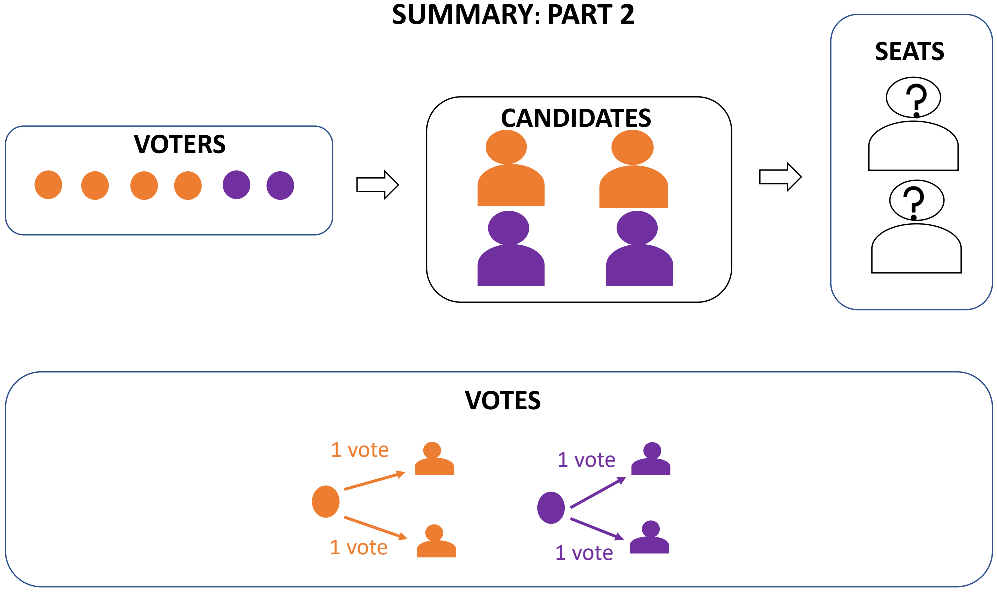
Part 2 will last 15 rounds.

Remember that:

- groups are reshuffled at each round.

- voting costs are redrawn at each round.

SUMMARY: PART 2



Part 3

Part 3 is very similar to Part 2.

There are three differences:

1. In Part 3, your group will elect a **4-person commission:** the election is for 4 open seats.

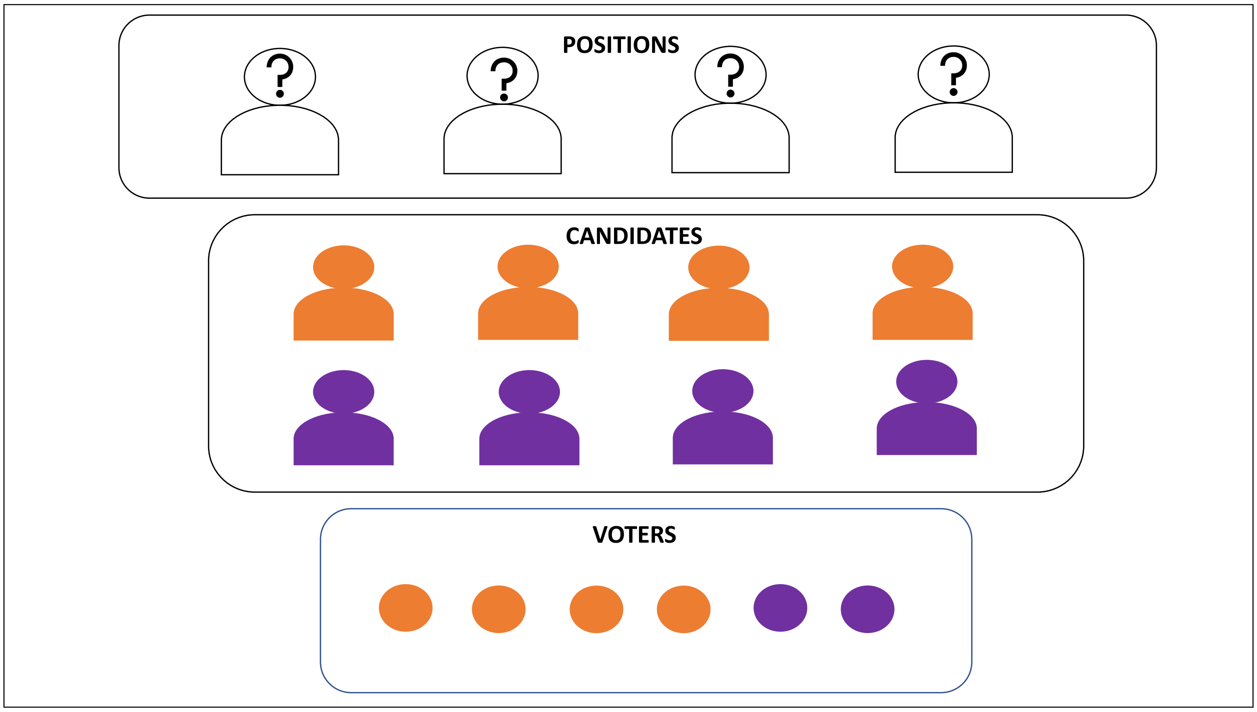
2. Each voter has **4 votes**.

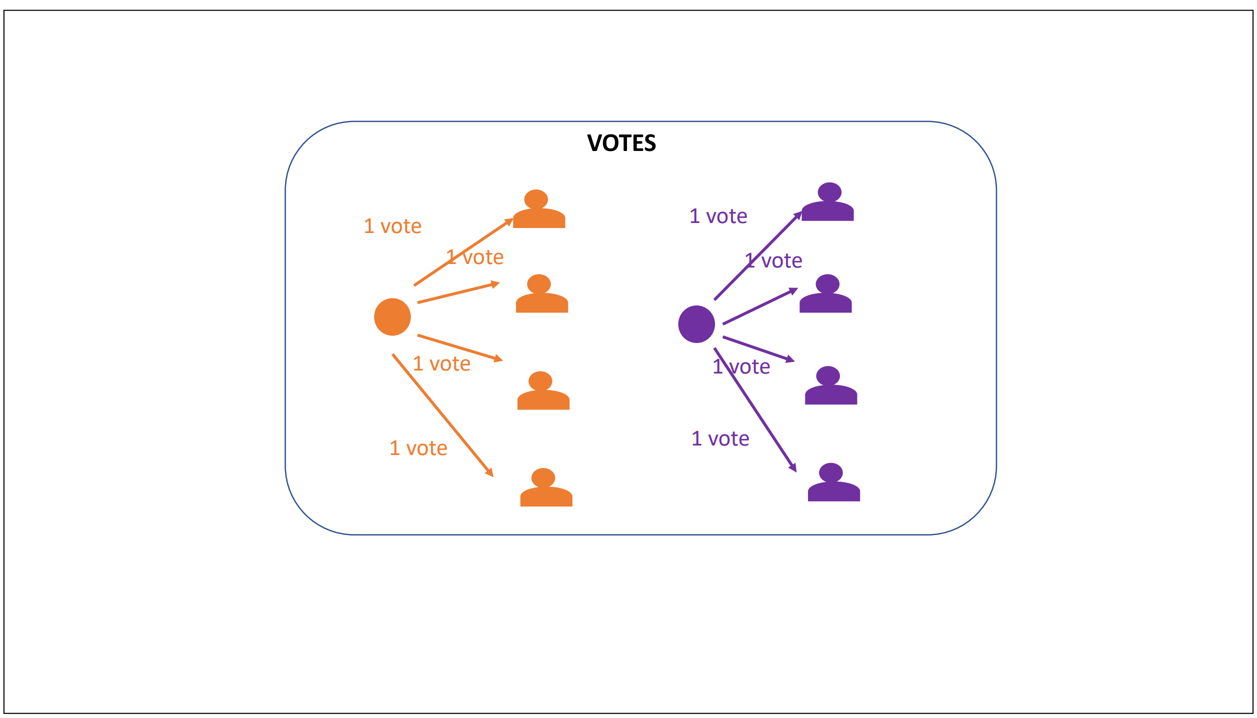
3. Each party has nominated **4 candidates.**

If you vote, you cast **1 vote for each of your party's candidates**.

The 4 winning candidates will be the 4 candidates with most votes.

In addition to the 100 points round endowment, you win 400 points if your party wins 4 positions, 300 if it wins 3, 200 if 2, 100 if 1, and 0 if it wins none.


POSITIONS
CANDIDATES
VOTERS

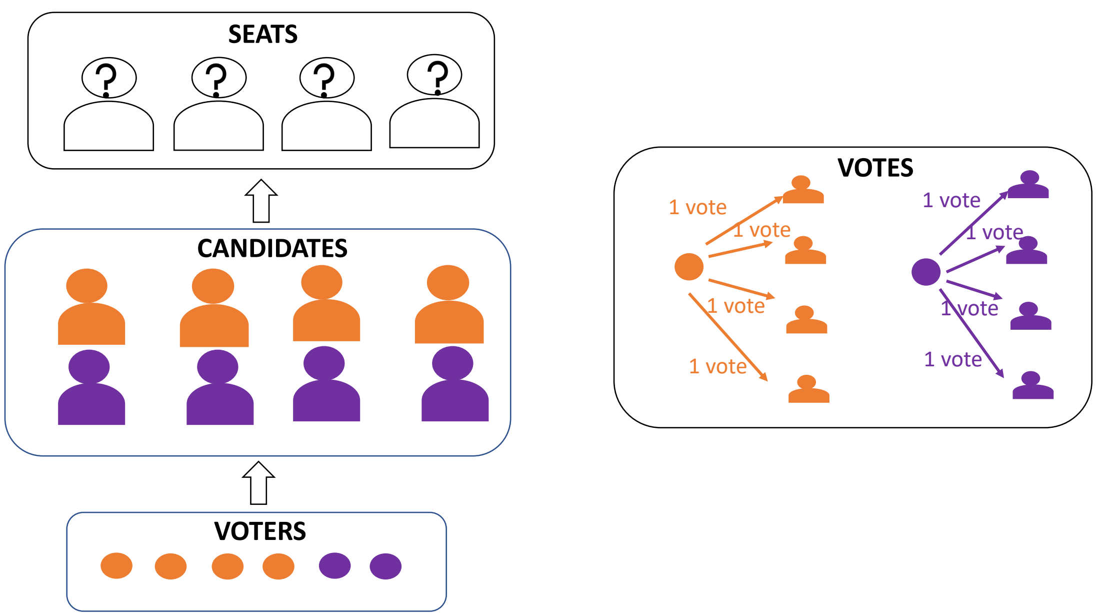

VOTES
1 vote
1 vote
1 vote
1 vote
1 vote
1 vote
1 vote
1 vote

You will be randomly reassigned a party affiliation, either **Purple** or **Orange**, which may differ from Part 2 and that you will keep for the whole of Part 3.

Everything else remains identical.

Part 3 will last 15 rounds.

SUMMARY: PART 3



Part 4

Part 4 is very similar to Part 3.

The difference is in the number of candidates:

The **Orange** party has nominated **3 candidates**.

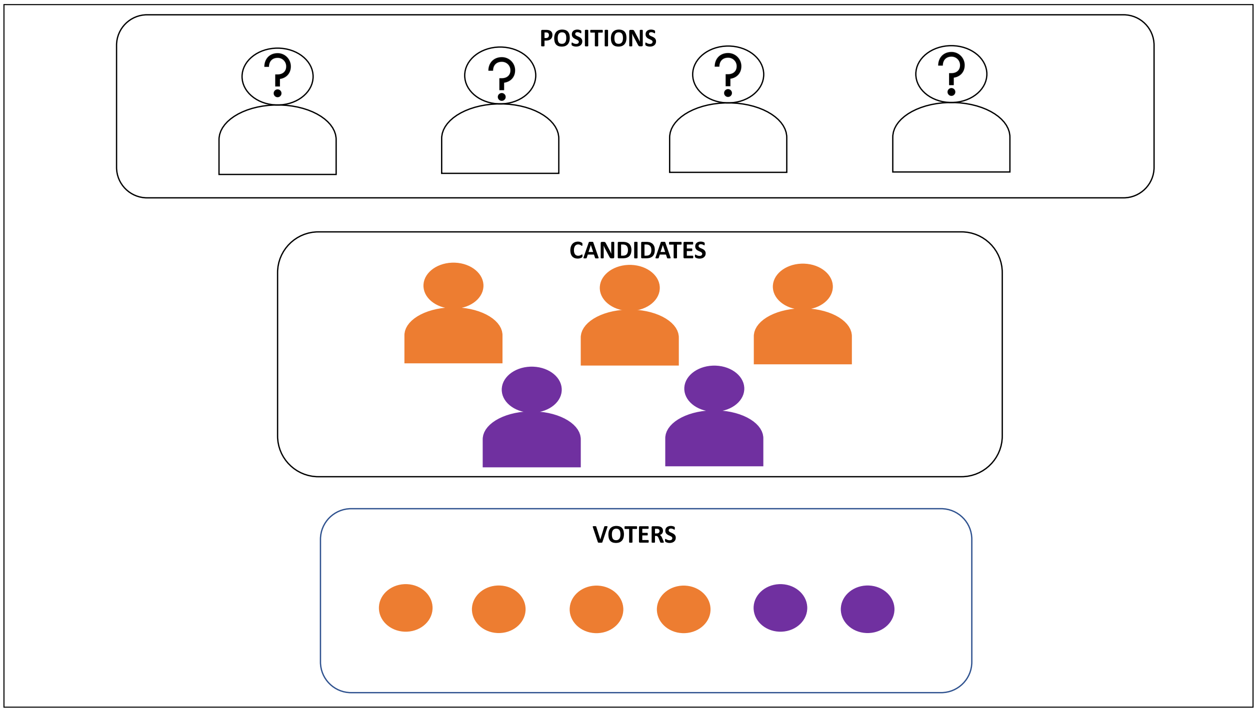
The **Purple** party has nominated **2 candidates.**

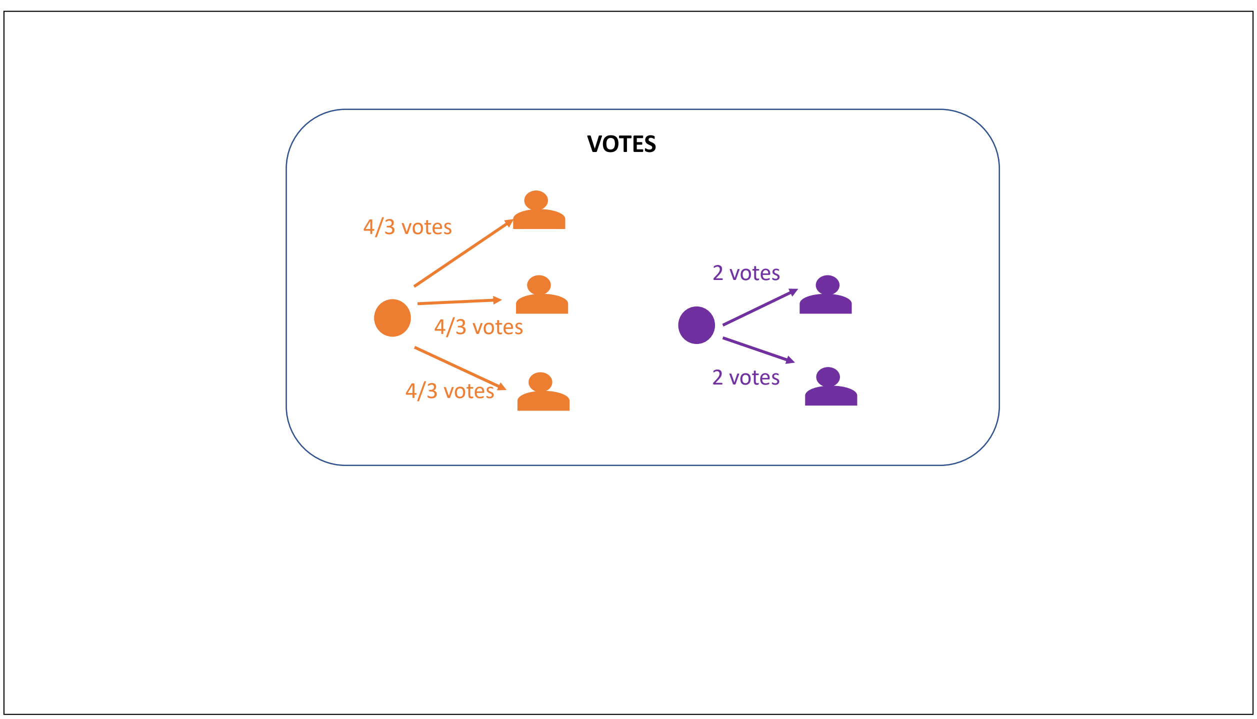
Because every voter has 4 votes:

If you vote and are **Orange**, you cast **4/3 votes** for each **Orange candidate**.

If you vote and are **Purple**, you cast **2 votes** for each **Purple candidate**.

The 4 winning candidates will be the 4 candidates with most votes, as before.


POSITIONS
?
?
?
?
CANDIDATES
VOTERS


VOTES
4/3 votes
4/3 votes
4/3 votes
2 votes
2 votes

You will be randomly reassigned a party affiliation, either **Purple** or **Orange**, which may differ from Part 3 and that you will keep for the whole of Part 4.

Everything else remains identical.

Part 4 will last 15 rounds.