Consumer Cities: The Role of Housing Variety*

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Abstract

Housing costs are key in understanding real income differences across space and time. Standard measures of housing costs do not account for availability differences, where some housing varieties are available in certain cities or time periods but not others. When households have idiosyncratic preferences over housing units, the set of available housing varieties in a city matters. This paper develops theoretically-founded housing price indices to measure housing costs that account for availability differences. To allow for flexible substitution patterns, I propose a method to jointly estimate the nests that varieties belong to and the elasticity of substitution across varieties within each nest. I find that households in larger cities benefit from having access to varieties not available in smaller cities. Utility-consistent housing prices reduce the elasticity of housing prices with respect to population by a half. Since housing is a third of household expenditure, this implies that we have systematically underestimated real income and overestimated residual amenities in larger cities. In contrast to previous estimates, I find that real income is increasing in city size after accounting for availability differences.

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I Introduction

Housing is a household's single largest expenditure item, accounting for a third of U.S. household expenditure. Therefore, measuring the cost of housing is key in understanding real income differences across space and time. Housing is not homogeneous and standard approaches have incorporated observable heterogeneity in price comparisons. Standard hedonic approaches are widely used in the construction of local housing price indices (e.g., Albouy 2016, Moretti 2013, Eeckhout et al. 2014, Baum-Snow and Han 2021) and underlie the construction of regional price parities by the Bureau of Economic Analysis.[1] Across time, repeat-sales indices are used to measure inflation by comparing transaction prices of the same housing unit (Case and Shiller 1989).

There is a key dimension that is unaccounted for by standard approaches to measuring housing costs: availability differences. Availability differences arise when some housing varieties are available in certain cities or time periods but not others.[2] When households have idiosyncratic preferences over housing units, the choice set of available housing varieties in a city matters.[3] Increases in the available set of varieties means that households benefit from a better match to their ideal housing unit.

This paper develops utility-consistent housing price indices to measure housing costs that account for availability differences. Since households substitute across housing varieties when faced with different prices and choice sets, it is important to allow for flexible substitution patterns when estimating price indices. I use a nested demand structure that partitions housing varieties into nests, which generates flexible substitution patterns as varieties within a nest are more similar to each other than compared to varieties in other nests.[4]

I propose a new data-driven method that jointly estimates both the elasticities of substitution and the nesting structure for a Nested Constant Elasticity of Substitution (CES) demand system. A data-driven nesting approach is important in two ways. First, it is not obvious how to classify housing varieties into nests. Second, there is no guarantee that a researcher-specified nesting structure corresponds to the true utility specification. Since my method relies on the prices and quantities of individual varieties, it also works in less data-rich settings where variety character-

[1] The hedonic approach estimates a regression of housing prices on physical characteristics and city fixed effects. The city fixed effects are then extracted and used as city-level housing price indices. For details underlying the hedonic adjustment to housing prices by the Bureau of Economic Analysis, see Aten 2005, Martin, Aten, and Figueroa 2011, and Rassier et al. 2021.

[2] I define a housing variety by the physical characteristics of the residential unit. Specifically, I consider a variety as defined by the full interaction of the decade the housing structure was originally built (or underwent substantial renovation), number of rooms, number of bedrooms, number of bathrooms, number of floors, structure type, lot size quintile, and distance categories from the Metropolitan Statistical Area (MSA) central business district (CBD).

[3] As evidence of these idiosyncratic preferences, households engage in costly search effort when purchasing a home. According to Zillow survey data, a typical buyer spends 4.4 months searching, with 18% of households spending more than 6 months (Zillow 2018). Han and Strange (2015) describe how the "multidimensional heterogeneity of buyer tastes" is an important reason why real estate agents are so prevalent and command significant fees. Finally, most households would not purchase a home without visiting. For example, survey data from Redfin shows that 80% of home buyers would have to visit before submitting an offer ("Buying a Home Sight Unseen," New York Times, July 2018).

[4] This nested demand structure is widely used in empirical industrial organization, international trade, and macroeconomics due to its tractability (e.g., Goldberg 1995, Broda and Weinstein 2006, 2010, and Handbury 2022).

istics are unobserved.

I find that households in larger cities benefit from having access to housing varieties unavailable in smaller cities. Utility-consistent housing price indices that account for housing variety availability differences reduce the elasticity of housing costs with respect to city population by 50% relative to a standard hedonic price index. These differences are quantitatively important. A standard hedonic approach results in predicted housing costs that are 2.2 times higher in New York (population 19.3 million) than Merced, CA (population 266,000). Utility-consistent price indices that account for variety differences imply housing costs that are only 1.5 times higher.

As households benefit from increased housing variety in larger cities, this means that we have underestimated real income in larger cities. The elasticity of real income with respect to population increases from -0.02 based on a standard hedonic approach to 0.02 based on utility-consistent housing price indices. Based on a standard hedonic approach, predicted real income is 7.2% lower in New York compared to Merced. In contrast, utility-consistent housing price indices imply the opposite: real income is 7.5% higher in New York compared to Merced.

In a spatial equilibrium where households are indifferent between cities, amenities are a compensating differential for real income differences.[5] As a result, accounting for the benefits of increased housing variety in larger cities revises downward estimates of amenities in larger cities. I find that recovered amenities based on a standard hedonic approach imply a counter-intuitive positive valuation for commute time. In contrast, after accounting for the increased housing variety in larger cities, commute time is negatively valued by households.

To micro-found the price indices, I develop a model of housing demand with differentiated housing varieties that accounts for the discrete nature of housing decisions. In the first stage, households decide on a city to live in and how to allocate their income between housing and other consumption. In the second stage, households choose a single housing variety and decide on the square feet of their unit. I show that the expected utility from housing services in the household's location choice decision can be expressed as a function of the nested CES housing price index. This is the result of a key timing assumption: although households know the price and quality of each housing variety in each city when they make their location choice, households draw idiosyncratic preferences over housing varieties after they choose a city to live in.[6]

A nested CES demand structure is useful due to the tractable and parsimonious exact price indices. As the CES price indices are separable in a component that measures variety differences and a component that measures relative prices, this allows me to easily decompose why the nested

[5]A literature has highlighted sorting on (unobserved) worker productivity to larger cities as a key driver of higher nominal incomes in large cities (Combes et al. 2010, De La Roca and Puga 2016, Davis and Dingel 2020). With sorting, higher real incomes in larger cities do not necessarily imply lower amenities as the Rosen-Roback spatial equilibrium condition only applies to homogeneous households.

[6]These idiosyncratic draws may reflect differentiation across units within a variety or idiosyncratic match draws that are ex-ante unknown to the household. For instance, Han and Strange (2015) state that "[b]uyers who are considering moving do not know which houses will suit their tastes until they search." I show in Appendix B that I can generate the same location choice probabilities and nested CES housing price index with a simultaneous choice over locations, housing nests, and housing varieties. Instead of the timing assumption, I make an analogous assumption that a household's preference draws for a housing variety are uncorrelated across locations.

CES approach generates different price indices than the standard hedonic approach. In contrast to a single nest structure, a nested CES relaxes the assumption of Independence of Irrelevant Alternatives for varieties in different nests. By allowing the elasticities of substitution to differ across nests, variety differences across cities will have differential impacts depending on the nests that the varieties belong to.

To construct the price indices, two key parameters are the nesting structure and the elasticities of substitution. To jointly estimate both parameters, I develop a new application of the panel Group Fixed Effects (GFE) estimator from Bonhomme and Manresa (2015). The method minimizes a least-squares criterion with respect to the demand parameters and all possible partitions of varieties.[7] The key idea behind the method is that varieties within a nest share the same nest-level elasticity of substitution and the same nest-level price index. The estimation algorithm iterates between a step that solves for the nesting structure with a k-means clustering algorithm and a step that solves for the elasticities of substitution with a standard fixed-effects regression. I show high accuracy of both the nest assignment and elasticity estimation in Monte Carlo simulations.

Utilizing a rich set of fixed effects based on the panel structure, I make three identifying assumptions: 1) fixed effects absorb the endogenous quantity response from homeowners, 2) demand and supply shocks are uncorrelated 3) nest-market-level fixed effects are systematically different across nests. The first two assumptions address omitted variable bias when I regress prices on quantities to estimate the inverse demand elasticity.[8] The third is an additional assumption from Bonhomme and Manresa (2015) that is needed to identify both the nest structure and demand parameters. In the case of nested CES demand, I show that the nest-market-level fixed effects have a structural interpretation corresponding to nest-level price indices and quality shocks.

To estimate the housing price indices, I use transaction-level prices, square feet, and housing characteristics from Zillow's ZTRAX database. My final sample includes 19 million transactions in 98 Metropolitan Statistical Areas (MSA) between 2005 and 2019. Since I use the transaction prices of housing units, the price indices I construct are measures of the prices available to a home buyer rather than a measure of the value of the overall housing stock. These prices are consistent with the location choice model: marginal households that decide to move to an MSA choose among the available housing varieties for sale.

I find important heterogeneity in housing characteristics and elasticities of substitution across the estimated nests. I estimate that the elasticities of substitution range from 6.5 to 9.9 across six nests in contrast to a single nest estimate of 8.3. As the estimation only uses prices and quantities of individual varieties, I can verify my estimated nest structure by analyzing the density of housing characteristics in each nest. I am able to sharply characterize three nests that account for 90% of

[7] The application of the GFE estimator to nested CES demand estimation is new: Bonhomme and Manresa prove statistical properties of the estimator and apply GFE to investigate the link between democracy and growth. Almagro and Manresa (2021 Slides) have concurrent work on applying the GFE estimator to a nested logit demand system.

[8] The demand elasticity in CES is also known as the elasticity of substitution. With the inclusion of fixed effects for each variety in each MSA, the demand parameters are estimated from relative movements in quantities and prices for each variety over time in each MSA. To absorb correlated trends in demand and supply shocks over time, I include linear time trends for each variety in each MSA.

the expenditure: 1) The McMansion nest that contains large single-family detached units built in the 1990s and 2000s, 2) The Suburban nest that contains smaller single-family detached units built in the post-war era, 3) The Urban nest that contains units close to the city center.[9]

Using the estimated nest structure and elasticities of substitution, I construct utility-consistent spatial price indices that measure how housing prices and the set of available varieties in each MSA compare to prices and the set of available varieties in a comparison MSA. There are two components of the CES price index comparison: the first measures variety differences (the variety adjustment) and the second measures the relative prices and shares of common varieties that are present in both comparison points (the common price index). The variety adjustment captures the welfare impact of variety differences across space and is a function of the relative expenditure shares on common varieties in the two comparison points, mediated by the elasticity of substitution. For varieties that are more substitutable, variety differences should matter less for welfare.

I find that the increased housing variety in larger cities is quantitatively important. Compared to a standard hedonic index which has a 0.19 elasticity with respect to population, the overall nested CES price index has a 0.09 elasticity with respect to population. This difference is primarily driven by the variety adjustment term of the nested CES price index, which has a negative 0.07 elasticity with respect to population. Hence, standard hedonic approaches overestimate housing prices in larger cities since they do not account for differences in the available choice set.

I show that the increased availability of housing varieties in larger cities is robust to two alternative specifications. First, I find a similar variety effect across city sizes when I follow Handbury and Weinstein (2014) and consider the comparison of each MSA to a representative national household that has access to all varieties.[10] Second, the population elasticity of the variety adjustment is robust to including average household income as a control.

The nested CES comparison generates larger variety adjustments across cities than the single nest CES comparison, demonstrating the importance of flexibly estimating substitution patterns. This difference arises due to a mechanism similar to the one highlighted in Ossa (2015), who analyzes gains from trade with heterogeneous sectors. When there are larger variety differences in nests with a lower elasticity of substitution, this will generate a larger variety adjustment compared to the single nest case.

Consistent with the cross-sectional results, I find that faster growing MSAs between 2005 and 2019 experienced relative increases in variety availability. This relative increase in variety offsets 20% of the relative increase in the common price component in faster growing MSAs. Comparing the MSA at the 90th percentile of population growth to the MSA at the 10th percentile, I find that the 15% relative increase in the common price index is partially offset by a 2.8% decline in

[9] Moreover, I find that the McMansion nest has the lowest price per square foot and the highest elasticity of substitution. The remaining three nests contain other urban units: on average they are more expensive, close to the city center, and medium-sized.

[10] The national household comparison asks how prices in each MSA compare to the quantity-weighted prices available to a national household and how the set of available varieties in each MSA compares to what is available nationally. For the national comparison, since the national household has access to all varieties, the common varieties are the set of varieties available in each city.

the variety adjustment. These results indicate the importance of accounting for housing variety differences not only across space, but also across time.

Since housing expenditures account for around a third of a household's budget, housing variety differences impact the measurement of real income across space. The benefits of increased housing variety in larger cities means that previous studies have underestimated real income in larger cities (Albouy 2011, 2016 and Diamond and Moretti 2021). Utility-consistent housing prices reverse the result from Albouy (2011, 2016) that real income falls with city size. Under my preferred specification, I find that once I account for housing availability differences, real income is increasing in city size.[11]

Differences in real income further impact the measurement of amenities in a spatial equilibrium. When households are indifferent between cities, amenities compensate households for real income differences. Because standard hedonic approaches underestimate real income in larger cities, they overestimate amenities in larger cities. As a result, recovered amenities using utility-consistent housing prices imply different valuations for characteristics that systematically covary with population size. After accounting for housing variety differences, I find that commuting is a negative amenity rather than a positive amenity, and the cost of air pollution doubles.

In the last part of the paper, I show that the increased variety in larger cities benefits high-income households the most. I investigate heterogeneity in housing costs by merging transaction-level mortgage applicant income data. I find that the increased variety in larger cities benefits households in all income quartiles. However, higher income households benefit more from the increased variety than compared to lower income households. As a result, the elasticity of utility-consistent housing prices with respect to population is 0.1 for the lowest income quartile compared to 0.06 for the highest income quartile.[12]

Finally, I investigate how real income varies by city size for households of different skill types. I map income-quartile-specific housing prices and housing expenditure shares to three skill types: incomplete high school (low skill), high school graduates (medium skill), and college graduates (high skill). In contrast to Diamond and Moretti (2021), I find that real income in larger cities based on the standard hedonic approach is underestimated for all three skill types: I estimate that the population elasticity of real income increases by 0.03 after accounting for housing variety differences.[13]

Related Literature. Housing costs are a key input in recent quantitative spatial models (e.g.,

[11]Card, Rothstein and Yi (2021) find that standard estimates of the effect of city size on nominal wages are overestimated due to sorting. To address unobserved worker heterogeneity, Card et al. use longitudinal data from the LEHD and include worker fixed effects. In contrast, I follow Albouy (2011, 2016) and Diamond and Moretti (2021) by controlling for observable worker demographics. I estimate an elasticity of 0.05 of household income with respect to city size in contrast to their elasticity of 0.03. Using their estimates will not affect the degree to which we have underestimated real income in larger cities.

[12]These results are related to a recent literature that has investigated the link between amenities and income, including Diamond (2016) and Almagro and Dominguez-Iino (2022).

[13]For low skill households, the population elasticity of real income increases from -0.05 to -0.02, while for medium skill households the population elasticity increases from -0.01 to 0.02. For high skill households, the population elasticity of real income increases from 0.02 to 0.05. Although high skill households benefit the most from increased variety availability in large cities, this benefit is offset by lower expenditure shares on housing relative to low skill households.

Redding and Rossi-Hansberg 2017 for a review, Ahflefdt et al. 2015, Diamond 2016). Spatial models typically feature both endogenous agglomeration and congestion forces that increases in population size (Allen and Arkolakis 2014, Allen and Donaldson 2022). As a result, housing costs that account for variety availability will affect calibration of these quantitative models, both for how we think about endogenous amenities and congestion costs across cities, and also the calibration of amenity residuals.

The paper proposes a joint estimation approach that is relevant to the trade and macroeconomics literature that relies on nested CES demand systems. Building upon Sato (1976) and Vartia (1976), Feenstra (1994) shows how to extend the exact price indices to account for variety differences. In recent work, Redding and Weinstein (2020) show how to account for quality differences. Handbury and Weinstein (2014) use a nested CES structure and find increased grocery availability in larger cities.

My model and estimation build upon the literature on housing demand. Bayer et al. (2007) and Calder-Wang (2020) apply discrete-choice models to estimate housing demand within a metropolitan area. I utilize a similar micro-foundation and evaluate cross-metropolitan differences in housing costs. The nesting structure is related to the housing search literature that has highlighted the importance of market segments, defined by housing characteristics and neighborhood, that households search over (Piazzesi, Schneider, and Stroebel, 2010).[14] The nests I estimate can be interpreted as market segments; as a result, the price indices capture both the substitution across varieties within a segment and the substitution across segments.

Finally, this paper contributes to the literature on hedonic housing estimation (e.g., see Sheppard (1999) for a review and Diewart et al. (2008)). In recent work, Epple, Quintero, and Sieg (2020) consider housing differentiated by a latent quality variable with non-homothetic preferences for quality. In contrast to their approach, my housing demand model is able to capture variety differences: what happens when some qualities are unavailable in one location versus another? Although most of my analyses assume homothetic preferences, in the last part of the paper I analyze how housing costs vary across city size by income quartile.

II Stylized Facts

To motivate the theory and structural model, I document two stylized facts about housing availability differences across cities. First, I find that large MSAs have substantially more housing varieties than small MSAs. Second, I find that there are systematic differences in the types of housing available in large versus small MSAs.

A key methodological question is how to define a housing variety. I define a housing variety by the full interaction of observable housing characteristics (e.g., number of rooms, bedrooms,

[14]Using real estate search data, Piazzesi et al. divide the San Francisco Bay Area into 500+ segments by zip code, price, and bathrooms. Their definition of a segment sits between my definition of a variety and nest, although this analogy is imperfect. The geography of their segments is much more granular while I have a much richer set of other housing characteristics that matter for household preferences.

structure type, decade built), and distance categories from the Metropolitan Statistical Area (MSA) central business district (CBD).[15] This granular definition allows me to measure the exact combination of housing characteristics that a household considers in their purchase decision. Since the definition of a variety includes the distance from the city center, housing prices can change systematically based on the distance from the city center, which is a key empirical and theoretical result in the urban literature.

II.1 Number of Varieties Across Cities

I use housing transaction data from the Zillow ZTRAX database to estimate how the number of unique varieties varies by city size. The ZTRAX database combines housing characteristics data from county assessor offices and housing transaction data from country recorder offices. I find that large cities have more unique varieties of housing transacted compared to small cities: there is an elasticity of 0.42 between the number of unique varieties and population size.[16] Figure 1 documents this positive relationship in 2015 for the 98 MSAs in my final sample (see Section V.1 for further details on the ZTRAX data). As a robustness check, Figure A5 shows similar elasticities for owner-occupied units and rental units in the American Community Survey.[17]

What can generate this positive relationship between MSA size and number of housing varieties? In Appendix A, I develop a spatial equilibrium model where a fixed cost of constructing new housing varieties leads to a larger number of equilibrium varieties in larger population areas. With CES demand over symmetric varieties and free entry, a larger population size increases the number of varieties since there is a fixed scale for each variety (due to a constant markup over marginal costs à la Krugman 1980). In a spatial equilibrium, a sufficient condition for a locally stable equilibrium is a sufficiently high elasticity of substitution across varieties and a high input share of land in the production of housing floor area.

II.2 Variety Differences Across Cities

To analyze the contribution of housing characteristics to the increased variety in larger cities, I omit each characteristic from the definition of a variety. I then re-compute the number of varieties under the new definition and re-estimate the elasticity of the number of varieties with respect to population. Figure 2 plots the change in the elasticity relative to the baseline estimate in Figure 1 as each characteristic is omitted one at a time from the definition of a variety.

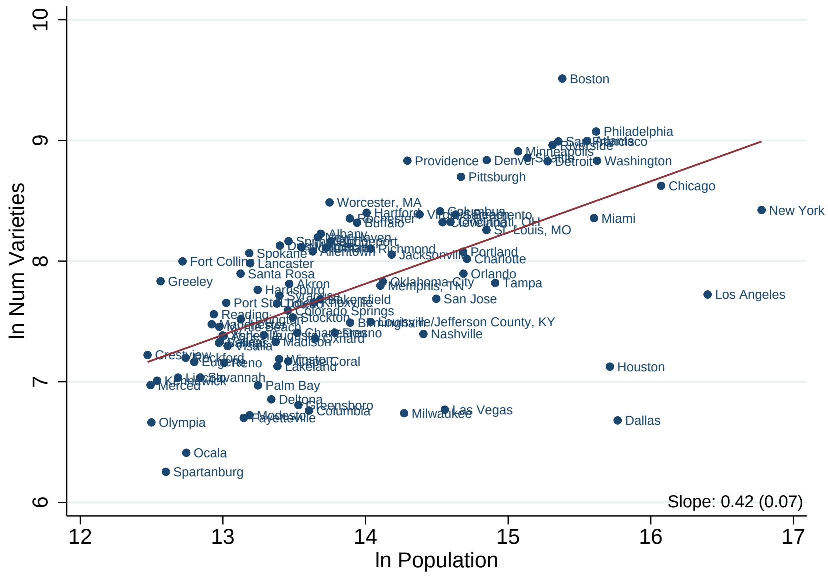
There is significant heterogeneity across characteristics in their contribution to the increased variety in larger cities. The largest decrease is caused by omitting the decade the unit was built,

[15] The full list of observable characteristics include the decade the housing structure was originally built (or underwent substantial renovation), number of rooms, number of bedrooms, number of bathrooms, number of floors, structure type, and lot size quintile.

[16] I run a regression of log number of unique varieties transacted on log population size with year fixed effects.

[17] Although the ACS has a wider sample of MSAs, there are several benefits of using the Zillow ZTRAX data. First, the ZTRAX dataset contains market prices rather than self-reported valuations as in the ACS. Second, there is substantially more detailed housing characteristics in the ZTRAX data, allowing me to define a more granular variety. Third, ZTRAX contains square feet and detailed location data, which the ACS data does not have.

Figure 1: Number of Varieties vs Population



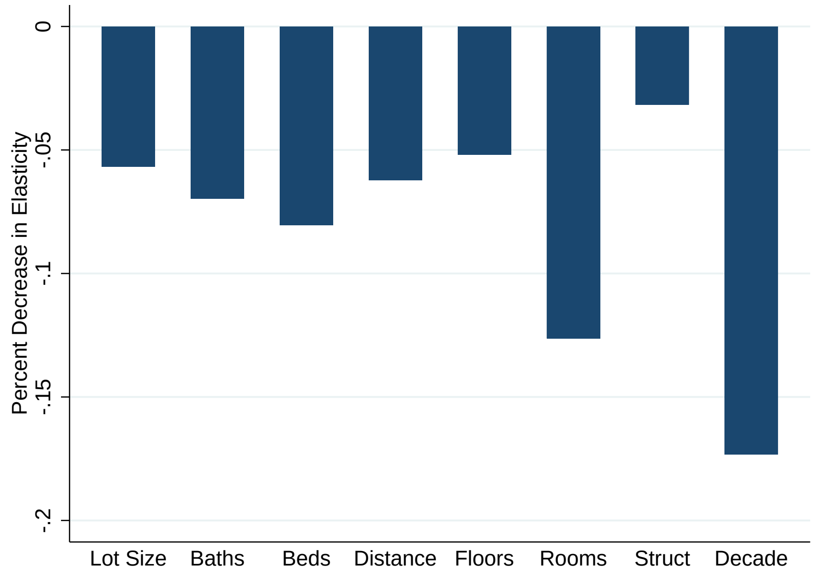
Notes: Figure plots the log number of unique housing varieties in each MSA versus log MSA population for 98 MSAs in ZTRAX data in 2015 (see Data section for further details).

which reduces the elasticity of the number of varieties with respect to population by 17%. The second largest decrease is caused by omitting the number of rooms, which leads to a 13% decrease. Since omitting distance from city center only leads a 6% decrease, this is suggestive that the increased number of varieties is not mechanically driven by the fact that more populous MSAs are also physically larger.

Next, I compare differences in the density of housing characteristics between small and large cities. I consider the set of unique varieties transacted each year from 2005 to 2019 and compare the share of varieties with each characteristic between the smallest ten cities and the largest ten cities in my sample. Figure 3 shows that relative to smaller cities, there is a higher density of older units and units with more rooms in larger cities. Transactions in smaller cities are located close to the city center, while larger cities have a peak density between 10 to 30 miles from the city center. For structure type, while both larger and smaller cities predominantly have transactions that are single-family detached (reflecting the fact that I use housing sale data), transactions in larger cities involve a more diverse set of structure types, including single-family attached and multi-unit apartments. Figure A7 in Appendix F presents the full set of comparisons across all characteristics.

A key parameter that governs how the differing choice sets affect household welfare in different cities is the elasticity of substitution across housing varieties. If housing varieties are perfect substitutes, then housing variety availability differences do not matter. However, when housing varieties are imperfectly substitutable, availability differences will matter for welfare.

Figure 2: Number of Varieties vs Population: Sensitivity to Characteristics



Notes: Figure plots the percentage change in the estimated elasticity of the number of varieties against population as each characteristic is dropped from the definition of a variety. A larger magnitude implies that the characteristic is a more important source of variety differences in larger cities.

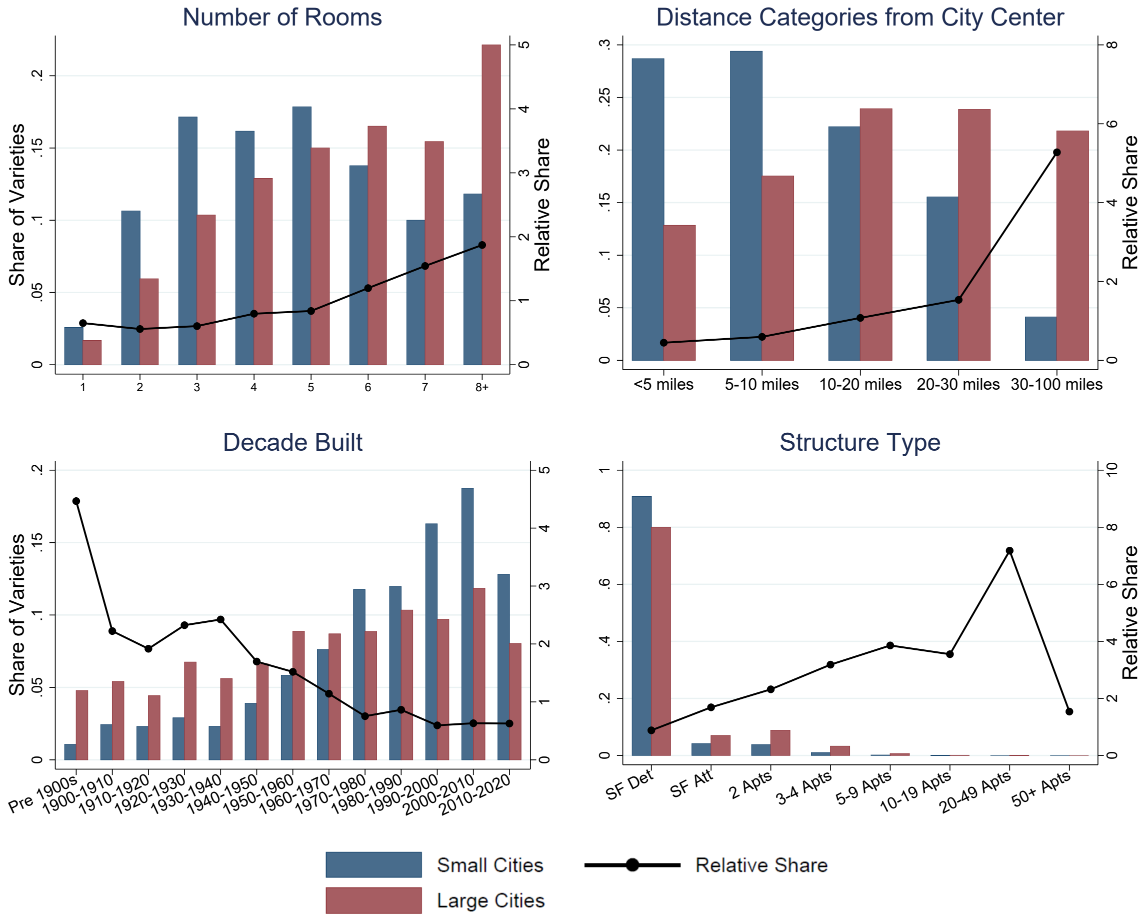
III Demand Model

To quantify the importance of housing variety availability differences, I develop utility-consistent price indices motivated by a model of a household's location decision and housing variety choice. The model accounts for the fact that households do not consume a composite of housing services over all varieties, but choose a single housing variety.

Since households substitute across housing varieties when faced with different prices and choice sets, it is important to allow for flexible substitution patterns in the demand system. I use a nested demand structure that partitions housing varieties into nests, where varieties within a nest are more similar to each other than compared to varieties in other nests. As a result, my nested CES price index summarizes housing prices and the available choice set of housing varieties, while generating flexible substitution patterns by allowing varieties to be more substitutable within nests.

The model builds upon earlier work investigating the link between CES and logit, including Anderson, de Palma, and Thisse (1987, 1992) and Verboven (1995). However, there is a subtle difference between my model and the discrete-continuous models in earlier work. The previous literature has shown that the aggregate demand across all households with discrete-continuous preferences is equivalent to a representative agent with CES preferences. Instead of integrating over all households in a location, I instead consider the household's expectation over their idiosyncratic housing variety shocks. This assumption then leads to the CES price index appearing

Figure 3: Housing Characteristic Differences Between Small and Large Cities



Notes: Figure plots the share of transactions with different housing characteristics in the bar plots. The blue bars correspond to the smallest 10 MSAs while the red bars correspond to the largest 10 MSAs in my sample (left y-axis). The black line is the relative share in the large MSAs versus small MSAs (right y-axis).

in the first stage of a household's location choice problem.

The model has a sequential two-stage structure. Within a period t (omitted for simplicity), a household j first chooses locations and then chooses a housing variety. Locations are denoted by $i = 1, ..., N$ and housing varieties are indexed by $v \in \Omega_i$, where Ω_i denotes the set of housing varieties available in location i.

In the first stage, a household j chooses a location i to reside in and decides how to allocate their location-specific wages between housing and other consumption expenditures.[18] In this location choice, households evaluate a location-specific wage, amenities, idiosyncratic location draws, and utility from other housing and other consumption. The utility from housing will be determined by the household's optimal behavior in the second stage.[19]

[18] Assumptions behind multi-stage budgeting are described in Blackorby et al. (1978) and Deaton and Muellbauer (1980).

[19] I assume that households are forward-looking (in terms of the two stages), so households will anticipate their

In the second stage, the household chooses a housing variety v and the square feet of housing services to consume in a nested discrete-continuous setup. Households pick a single variety to consume, and then choose the square feet given their housing expenditure. Conditional on location choices and housing expenditure, the nested discrete-continuous housing variety choice means that total square feet demand for each variety across households in a city is equal to demand by a representative household with nested CES preferences. This intensive-margin choice of square feet allows a tractable first stage compared to a pure discrete-choice setup.[20]

I make a key timing assumption: the idiosyncratic housing variety preferences are not revealed until the second stage, so that the household will choose a location and housing expenditures in expectation of the idiosyncratic housing variety preference draws. As a result of the timing assumption, the utility from expected housing services in the first stage will be a function of a nested CES housing price index. In contrast to a standard hedonic approach, the nested CES price index accounts for variety differences.

The timing assumption can be relaxed by considering a simultaneous choice over both locations and housing varieties. I show in Appendix B that I can generate the same location choice probabilities as in the main model. Furthermore, the nested CES housing price index summarizes the expected utility over the housing decision within a location. An important assumption is that each household's preference draws for a variety are uncorrelated across locations. Hence, the timing assumption restricts the sorting of households on housing characteristics.[21]

III.1 Second Stage

To solve the household's two-stage sequential problem, I work backwards from the second stage. Household j has already chosen location i and housing expenditure E_{ij}^H in the first stage. In the second stage, the household faces a discrete-choice problem over housing varieties indexed by v and a quantity decision of how much square feet to consume of that variety v, q_{ivj}.

$$\max_{v \in \Omega_i,\ q_{ivj} \geq 0} u_{ivj} = \ln q_{ivj} + \ln \varphi_{iv} + \varepsilon_{ivj} \tag{1}$$

$$\text{s.t.} \quad p_{iv} q_{ivj} \leq E_{ij}^H$$

where φ_{iv} represents the quality of housing variety v in location i that is common across all households and ε_{ivj} represents an idiosyncratic household quality draw for housing variety v in location

optimal choices in the subsequent stage.

[20] Dubé et al. (2022) discuss issues with expenditure aggregation in pure discrete-choice setups.

[21] A technical difference is that the simultaneous location and housing variety choice does not nest a Rosen-Roback equilibrium, because the nesting structure implies that the elasticity of substitution across locations has to be less elastic than the elasticity of substitution across housing nests. Hence, the elasticity of substitution across locations cannot tend toward infinity as in a Rosen-Roback equilibrium.

i.[22] u_{ivj} can be interpreted as a quality-adjusted log square feet of housing services.[23]

Households will choose to spend all of their housing expenditure on housing variety v, so that $q_{ivj} = \frac{E^H_{ij}}{p_{iv}}$. Heterogeneity across households in their housing expenditure will then lead to differences in square feet purchased. Substituting this quantity choice into (1), I can re-write the optimization problem as

$$\max_{v \in \Omega_i} u_{ivj} = \ln E^H_{ij} - \ln p_{iv} + \ln \varphi_{iv} + \varepsilon_{ivj} \tag{2}$$

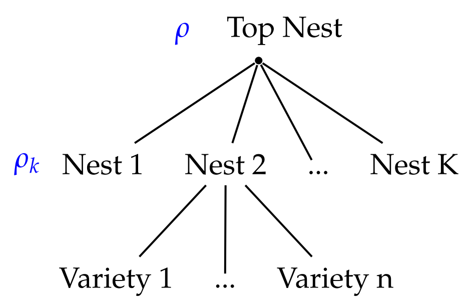
where ε_{ij} is distributed i.i.d. across households from a distribution F_i. I assume that F_i takes on a nested logit form where housing varieties are partitioned into K nests

$$F_i(\vec{\varepsilon}) = \exp\left[- \sum_{k \in \Omega^K_i} \left(\sum_{v \in \Omega_{ik}} e^{-\epsilon_{ivj}/\rho_k} \right)^{\rho_k/\rho} \right]$$

Ω^K_i denotes the available nests at location i (where the superscript capital K denotes a set of nests) and Ω_{ik} denotes the varieties available in nest k at location i. Finally, to be consistent with utility maximization, $0 < \rho_k < \rho, \forall k$.

Following the literature, it is standard to display a nested logit system as a tree diagram (note that preference draws for housing varieties are not drawn sequentially, but simultaneously):

Figure 4: Nested Logit Demand System



Notes: Figure plots the nesting structure for a logit demand system with K nests. Nests form a partition over housing varieties v. Although presented sequentially, the choice occurs simultaneously over housing nest k and variety v.

From McFadden (1981), the probability of choosing variety v is equal to

$$Pr(v^*_j = v) = \varphi^{\sigma_k - 1}_{iv} \left(\frac{p_{iv}}{\mathbb{P}^H_{ik}} \right)^{1-\sigma_k} \left(\frac{\mathbb{P}^H_{ik}}{\mathbb{P}^H_i} \right)^{1-\sigma} \tag{3}$$

[22]Notice that φ_{iv} allows for a nest-level quality shock that is common to all varieties within a nest k. Let $\varphi_{ik(v)}$ denote the component that is common to all varieties within a nest k. Without loss of generality, $\varphi_{iv} \equiv \varphi_{ik(v)} \tilde{\varphi}_{iv}$. It is convenient to normalize the quality shocks within a nest k, so that $\prod_{v \in k} \tilde{\varphi}_{iv} = 1$ so that $\left(\prod_{v \in k} \varphi_{iv}\right)^{\frac{1}{N_k}} = \varphi_{ik(v)}$.

[23]Anderson et al. (1989) show that a model where products are described by a bundle of characteristics and where individuals have ideal levels of each characteristic (a Lancaster approach) is equivalent to a logit discrete-choice model under certain conditions.

where v_j^* indicates household j's optimal housing variety choice and I have defined $\rho \equiv \frac{1}{\sigma-1}$ and $\rho_k \equiv \frac{1}{\sigma_k-1}$

$$\mathbb{P}_i^H = \left(\sum_{k\in\Omega_i^K}\left(\mathbb{P}_{ik}^H\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \quad \mathbb{P}_{ik}^H = \left(\sum_{v\in\Omega_{ik}}\varphi_{iv}^{\sigma_k-1}p_{iv}^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}} \tag{4}$$

Notice that the choice probabilities and price indices are homothetic as they do not depend on the housing expenditure of household j.

III.2 First Stage

In the first stage, household j chooses a location i to live in and how to allocate their wages w_{ij} between housing and non-housing consumption

$$\max_{i\in 1,\dots,n,\ E_{ij}^H\geq 0,\ E_{ij}^C\geq 0} \mathcal{U}_{ij} = \mathbb{B}_i\left(U_i^T(E_{ij}^T)\right)^{\mu_i}\left(U_i^H(E_{ij}^H)\right)^{1-\mu_i} z_{ij} \tag{5}$$

$$s.t. \quad E_{ij}^H + E_{ij}^T \leq w_{ij}$$

where $U_i^T(E_{ij}^T)$ is the utility of non-housing consumption as a function of other consumption expenditure, $U_i^H(E_{ij}^H)$ is the utility of housing consumption as a function of housing expenditure, $\mathbb{B}_i$ is the amenity of location i, and z_{ij} are idiosyncratic preference draws from a Fréchet distribution with shape parameter $\nu > 1$ over locations.[24]

For other consumption, there exists a price index $\mathbb{P}_i^T$ in location i so that

$$U_i^T(E_{ij}^T) \equiv \frac{E_{ij}^T}{\mathbb{P}_i^T}$$

At the first stage of the demand model, I assume that households know the housing prices, quality, and the set of available varieties in each location. However, the idiosyncratic housing draws ε are not yet realized. As a result, $U_i^H(E_{ij}^H)$ is the expected quality-adjusted square feet of housing services.

Following the properties of GEV, the expected value of the optimal housing choice in location i is

$$\mathbb{E}_\varepsilon\left(\max_{v\in\Omega_i} u_{ivj}(E_{ij}^H)\right) = \ln E_{ij}^H + \frac{1}{\sigma-1}\ln\left(\sum_{k\in\Omega_i^K}\left(\sum_{v\in\Omega_{ik}}\left(\frac{\varphi_{iv}}{p_{iv}}\right)^{\sigma_k-1}\right)^{\frac{\sigma-1}{\sigma_k-1}}\right) \tag{6}$$

To preserve the multiplicative form, I assume that $U_i^H(E_{ij}^H)$ is equal to the exponential of the expected value of the optimal housing choice in the second stage:

$$U_i^H(E_{ij}^H) = \exp\left[\mathbb{E}_\varepsilon\left(\max_{v\in\Omega_i} u_{ivj}(E_{ij}^H)\right)\right]$$

[24] I assume that there is a single price/rent ratio that applies to all locations that converts housing prices to per-period housing rents.

The exponential form can be relaxed by using a multiplicative Fréchet set-up. In Appendix C, I build upon Lind and Romando (2021) and define a multiplicative utility form with idiosyncratic preferences drawn from a multivariate max-stable Fréchet distribution. I show that the multiplicative Fréchet set-up generates the same choice probabilities and CES price indices as the nested logit.

Taking the exponential of (6), the expected utility from housing services is equal to

$$U_i^H(E_i^H) = \exp\left[\mathbb{E}\left(\max_{v\in\Omega_i} u_{ivj}(E_{ij}^H)\right)\right] = \frac{E_{ij}^H}{\mathbb{P}_i}$$

where the housing price index is given in (4).

I can now re-write the first stage optimization problem in (5) as

$$\max_{i\in 1,\ldots,n,\ E_{ij}^H\geq 0,\ E_{ij}^T\geq 0} \mathcal{U}_{ij} = \mathbb{B}_i\left(\frac{E_{ij}^T}{\mathbb{P}_i^T}\right)^{\mu}\left(\frac{E_{ij}^H}{\mathbb{P}_i}\right)^{1-\mu_i} z_{ij} \tag{7}$$

$$\text{s.t.} \quad E_{ij}^H + E_{ij}^T \leq w_{ij}$$

The presence of the nested CES price index in the indirect utility for household j is not because households have CES preferences and consume a composite bundle of housing varieties. Rather, I have provided a micro-foundation for why households will behave as if they faced a nested CES price index.

For any location i, the household's optimal housing and other consumption expenditures can be solved for in terms of wages

$$E_{ij}^H = (1-\mu_i)w_{ij} \qquad E_{ij}^T = \mu_i w_{ij}$$

Substituting this into the optimization equation now means that households choose a location i, given the price index for other consumption $\mathbb{P}_i^T$ and the housing price index $\mathbb{P}_i^H$

$$\max_i \mathcal{V}_{ij} = \mathbb{B}_i \underbrace{\left(\frac{w_{ij}}{\left(\mathbb{P}_i^T\right)^{\mu_i}\left(\mathbb{P}_i^H\right)^{1-\mu_i}}\right)}_{\text{Real Income}} z_{ij} \tag{8}$$

Real income is measured as wages over a Cobb-Douglas price index over other consumption prices and housing prices. This choice of Cobb-Douglas can be interpreted as a first-order approximation of a general utility function as in Albouy (2011). Compared to the standard approach in the urban literature, rather than a hedonic housing price index, $\mathbb{P}_i^H$ is a nested CES price index that accounts for substitution and availability of housing variety.

Using the fact that z_{ij} are idiosyncratic preference draws from a Fréchet distribution, the probability of choosing location i for a household j is given by

$$Pr(i_j^* = i) = \frac{\left(\mathbb{B}_i \frac{w_{ij}}{\left(\mathbb{P}_i^T\right)^{\mu}\left(\mathbb{P}_i^H\right)^{1-\mu}}\right)^{\nu}}{\sum_{\ell}\left(\mathbb{B}_{\ell} \frac{w_{\ell j}}{\left(\mathbb{P}_{\ell}^T\right)^{\mu}\left(\mathbb{P}_{\ell}^H\right)^{1-\mu}}\right)^{\nu}} \tag{9}$$

As $\nu \to \infty$, utilities are equalized across locations so that the model nests a Rosen-Roback equilibrium.

III.3 Market-Level demand

Let q_{iv} denote the total demand of square feet of variety v in location i. Integrating over all households in a location i, the total square feet demand of each variety is

$$q_{iv}(\mathbf{p}_{it}, \boldsymbol{\varphi}_{it}; \sigma) = \varphi_{iv}^{\sigma_k - 1} \frac{p_{iv}^{1-\sigma_k}}{(\mathbb{P}_{ik}^H)^{1-\sigma_k}} \frac{(\mathbb{P}_{ik}^H)^{1-\sigma}}{(\mathbb{P}_i^H)^{1-\sigma}} E_i^H \tag{10}$$

where the emphasis indicates the full vector of prices and quality shocks, and E_i^H is the total housing expenditure of households that decide to reside in location i

$$E_i^H = \int_{j:\ i_j^* = i} (1-\mu) w_{ij} dj$$

where i_j^* indicates household j's optimal location choice.

This market-level demand is equivalent to demand generated by a representative agent with nested CES preferences (see Anderson et al. (1987) and Verboven (1995)). This illustrates the close connection between the expected value of the maximum preference draw and CES price indices alluded to previously.

IV Estimation Strategy

This section describes my estimation strategy for the parameters of the nested CES housing price indices: the nesting structure, the lower-level elasticities of substitution across varieties within each nest, and top-level elasticity of substitution across nests. Standard approaches to estimating a nested demand system use a pre-specified nesting structure and sequentially estimate the demand parameters from the lowest-level to the top-level (McFadden 1980, Goldberg 1995, Hottman, Redding, and Weinstein, 2016).[25] I follow a similar sequential approach and consider the joint problem of estimating both the nesting structure and the demand parameters.

As a first step, I develop an estimation approach for the lower-level nest parameters assuming that I know the true nesting structure. I derive a structural estimating equation from the market-

[25] Given a nesting structure, researchers have also proposed jointly estimated all demand parameters using maximum likelihood.

level demand of each variety in each market, where a market is defined as an MSA and year. Since prices and quantities are determined in equilibrium, I make two identifying assumptions to address omitted variable bias. First, I assume that the quantity supplied response from homeowners is absorbed by a rich set of fixed effects. Second, I assume that conditional on the fixed effects, shocks to quantity supplied are uncorrelated with quality shocks. A key fixed effect is the nest-level fixed effect in each market, which absorbs the structural nest-level price indices.

These two identifying assumptions are reasonable since 85% of transactions are for second-hand homes. Decisions by homeowners to sell their homes (e.g., new job offer in different area, kids leaving for college) drive changes in quantity of square feet supplied that are uncorrelated with quality shocks. Based on these two identifying assumptions, I can regress price per square feet on the total quantity of each variety to identify the inverse elasticities of substitution.

Next, I propose a panel method to jointly estimate both the nesting structure and the lower-level elasticity of substitution within each nest. The panel method builds upon the estimating equation developed in the first part when the nests are known; it minimizes a least squares criterion with respect to the demand parameters and all possible partitions of varieties into nests. A data-driven nesting approach is important since 1) it is not obvious how to assign housing varieties into nests and 2) a pre-specified nesting structure is not guaranteed to correspond to the true utility specification.

My proposed method to estimate the nesting structure is a new application of the Group Fixed Effects (GFE) estimator from Bonhomme and Manresa (2015). The key idea behind the method is that varieties within a nest share the same elasticity of substitution and the same nest-level price index across markets. To identify the nests, I make a third identifying assumption that the nest-level fixed effects in each market are on average different across nests.

There are two key advantages of the joint estimation method. First, the proposed method uses systematic variation in prices and quantities in a structurally consistent way. In contrast, clustering housing varieties based on observable housing characteristics will form groups of housing varieties that are similar to each other. However, there is no guarantee that these groups are the nests that are relevant for household substitution patterns. Second, since only data on quantities and prices are needed, this estimation approach can be utilized in other settings where there is less data available on variety characteristics.

Given the estimates of the nesting structure and the lower-level elasticity of substitution, I then proceed to estimate the top-level elasticity of substitution across nests using two approaches. The first approach assumes that conditional on the fixed effects, the quantity variation at the nest level is uncorrelated with nest-level quality shocks. The second approach builds upon Hottman, Redding, and Weinstein (2016) and utilizes the structure of CES nests.

To evaluate the accuracy of the joint estimation procedure, I provide results from Monte Carlo simulations. In each simulation, I randomly assign varieties to nests and randomly draw lower-level and top-level parameters. Generating quality and quantity shocks, I can solve for the equilibrium prices and expenditure shares in each period. I then apply my proposed method and demonstrate that the nest-level demand elasticities are estimated precisely.

IV.1 Estimating Lower-Level Elasticities with Known Nesting Structure

As a first step, I describe my estimation approach for the lower-level demand elasticities given the true nesting structure. Given the mapping from housing variety v to nest k, denoted by $k(v)$, I can take logs of the market-level nested demand equation (10) and write prices in terms of quantities, price indices, and quality shock

$$\ln p_{ivt} = -\frac{1}{\sigma_{k(v)}} \ln q_{ivt} + \frac{\sigma_{k(v)} - \sigma}{\sigma_{k(v)}} \ln \mathbb{P}^H_{ik(v)t} + \frac{\sigma - 1}{\sigma_{k(v)}} \ln \mathbb{P}^H_{it} + \frac{1}{\sigma_{k(v)}} \ln E^H_{it} + \frac{\sigma_{k(v)} - 1}{\sigma_{k(v)}} \ln \varphi_{ivt} \quad (11)$$

where i is MSA, v denotes a housing variety, and t is time.

Consider the following estimating equation for the inverse-demand elasticities

$$\ln p_{ivt} = -\frac{1}{\sigma_{k(v)}} \ln q_{ivt} + \alpha_{ik(v)t} + \alpha_{iv} + \alpha_{iv} t + \epsilon_{ivt} \quad (12)$$

where $\alpha_{ik(v)t}$ are MSA X nest X year fixed effects, α_{iv} are MSA X housing variety fixed effects, and $\alpha_{iv}t$ are linear time trends for each housing variety in each MSA. The estimation strategy is to identify the inverse demand elasticity of each nest with changes over time in the supply of square feet for each housing variety, and relative changes across varieties within each nest in each MSA.

It is helpful to map the estimating equation to the structural market-level demand equation. The nest-level fixed effects in each market ($\alpha_{ik(v)t}$) absorb the price indices and aggregate expenditure $\frac{\sigma_{k(v)} - \sigma}{\sigma_{k(v)}} \ln \mathbb{P}^H_{ik(v)t} + \frac{\sigma - 1}{\sigma_{k(v)}} \ln \mathbb{P}^H_{it} + \frac{1}{\sigma_{k(v)}} \ln E^H_{it}$. The nest-level fixed effects in each market further absorb quantity and quality shocks that are common across varieties within each nest in each market. The variety-level fixed effects and variety-level time trend ($\alpha_{iv}, \alpha_{iv}t$) absorb market-level persistent components of quantity and quality shocks φ_{ivt}. Thus, the regression residual corresponds to

$$\epsilon_{ivt} \equiv \frac{\sigma_{k(v)} - 1}{\sigma_{k(v)}} \ln \varphi_{ivt}$$

where φ_{ivt} are the variety quality shocks. Appendix Section D.5 derives all three fixed effects in terms of fundamentals.

IV.2 Identifying Assumptions

Identification of the inverse demand elasticity requires that conditional on the fixed effects, quantity of square feet of variety v in market it is uncorrelated with quality shocks[26]

$$E[(\ln q_{ivt})\,(\ln \varphi_{ivt})\,|\alpha_{ik(v)t}, \alpha_{iv}, \alpha_{iv}t] = 0 \quad (13)$$

There are two sources of omitted variable bias in the demand estimation that the estimation strategy addresses. First, simultaneity bias can arise since demand and supply are determined

[26] $E[\ln \varphi_{ivt}|\alpha_{ik(v)t}, \alpha_{iv}, \alpha_{iv}t] = 0$ by the normalization discussed in footnote 22.

jointly in equilibrium.[27] Second, shocks to the quantity supplied of square feet may be correlated with quality shocks.[28]

The two sources of omitted variable bias can be seen by writing down a structural supply equation

$$\ln q_{ivt}^{S} = \ln f_{it}(\mathbf{p}_{it}) + \ln \omega_{ivt}$$

where the first term $\ln f_{it}(\mathbf{p}_{it})$ denotes the endogenous supply response by homeowners that is a function of prices, while the second term ω_{ivt} denotes the quantity supply shock. In equilibrium, quantity supplied is equal to quantity demand for each variety in each market. Hence, I can rewrite the identifying assumption in (13) as

$$E[(\ln f_{it}(\mathbf{p}_{it}))\,(\ln \varphi_{ivt})\,|\alpha_{ik(v)t}, \alpha_{iv}, \alpha_{iv}t] + E[(\ln \omega_{ivt})\,(\ln \varphi_{ivt})\,|\alpha_{ik(v)t}, \alpha_{iv}, \alpha_{iv}t] = 0$$

The first term on the LHS corresponds to covariance between the supply response by homeowners and the quality shock, while the second term on the LHS corresponds to the covariance between supply shocks and quality shocks.

In the supply equation, I focus on homeowners rather than new construction. Second-hand houses are the primary component of housing supply: 85% of housing transactions are for second-hand houses in my sample period (NAHB). Household decisions to move (and hence the decision to sell), are often unrelated to housing prices, generating exogenous variation in supply. Based on the March CPS supplement from 2005-2019, the majority of moves between 2005-2019 are for reasons unrelated to current housing prices, with 43% of moves due to family or employment reasons, 5% due to other non-housing reasons, and 19% wanting a better neighborhood, or new or better housing.[29]

Assumption 1: The fixed affects absorb the endogenous supply response by homeowners, so that

$$E[(\ln f_{it}(\mathbf{p}_{it}))\,(\ln \varphi_{ivt})\,|\alpha_{ik(v)t}, \alpha_{iv}, \alpha_{iv}t] = 0$$

I assume that the panel fixed effects address simultaneity bias by absorbing the effect of expected variety prices on quantity supplied. In Appendix D, I justify this assumption by deriving a market equilibrium when homeowners are not able to perfectly anticipate the price of their variety at the beginning of the period.[30] I assume that homeowners have rational expectations: homeowners understand the endogenous supply response and the fact that prices will clear the quantity supplied of floor space in the market equilibrium. I show that the quantity supplied is increasing in

[27] Since long-run new housing supply elasticities at the MSA level exist in the literature (Saiz 2010), one might use those elasticities to identify the demand curve. However, since around 80% to 90% of housing transactions are for second-hand homes, long-run new housing supply elasticities in the literature are not necessarily informative about the short-run supply elasticity of second-hand homes.

[28] It is standard in the trade literature to assume uncorrelated demand and supply shocks conditional on differencing over time and against a comparison product. For example, see Leamer (1981), Feenstra (1994), and Broda and Weinstein (2006).

[29] The remaining categories are 3% for cheaper housing and 7% for other housing reasons.

[30] See Anenberg (2014) and Qian, Mateen, and Zhang (2021) for evidence of imperfect expectations of the housing price.

two sets of parameters. The first set is composed of expected price indices and expected aggregate housing expenditure. The second set is composed of the expected quality shock and the expected supply shock.

Assumption 1 requires that the panel fixed effects absorb both sets of parameters. It is reasonable that the next-level fixed effects in each market $\alpha_{ik(v)t}$ absorb the parameters in the first term: the expected price indices and expected aggregate housing expenditures. This assumption is mild, allowing for the case where homeowners can perfectly foresee the aggregate price indices and expenditures or for the case where homeowners have adaptive expectations and form predictions based on previous aggregate prices and expenditures. For the second set of parameters, this will be absorbed by the three sets of fixed effects when homeowners generate an expectation for their variety's quality and supply shock based on three MSA-specific components: the nest average in the time period, the variety's average over time, and the variety's average growth.

It is worth discussing how my estimates will be affected if the form of expectations is incorrect. Suppose that homeowners perfectly observe variety prices rather than forming an expectation and supply shocks are uncorrelated with quality shocks. When the estimated coefficient is negative (as I find empirically), it will be an upward-biased estimate of the true inverse demand elasticity. In other words, the demand elasticity is overestimated.[31] Since the impact of variety differences in the housing price index is decreasing in the demand elasticity, the estimated welfare impact of variety differences across space will be a lower bound of the true welfare impact.

Assumption 2: After controlling for the fixed effects, supply and quality shocks are uncorrelated

$$E[(\ln \omega_{ivt})\,(\ln \varphi_{ivt})\,|\alpha_{ik(v)t}, \alpha_{iv}, \alpha_{iv}t] = 0$$

This assumption is reasonable given the fact that 85% of the market are second-hand homes. Since $\ln \omega_{ivt}$ are supply shocks that are unrelated to expected price of a variety, decisions of households to move (e.g., a new job offer, school district for children) are uncorrelated with the quality shock of their current housing variety.

One potential concern are moves to a better neighborhood or better home. If these moves are due to negative quality shocks of the current housing variety rather than due to the quality shock of a new home, this would induce a negative correlation between quality and supply shocks. However, with the inclusion of the three sets of fixed effects, several types of key quality shocks are absorbed. These include quality shocks for all varieties in the nest in a market, persistent quality shocks for a variety in an MSA, and persistent growth in the quality shock for a variety in an MSA. For there to be substantial bias, the quality shock of a variety has to be idiosyncratic compared to other varieties in the nest and has to deviate from the persistent trend and mean across time.

Given Assumptions 1 and 2 and the true nesting structure, the identifying assumption in (13) is satisfied and the inverse elasticities of substitution are identified. Notice that the elasticity of substitution across nests (σ) does not have to be identified at this stage of the estimation process.

[31] This bias is formalized and discussed in Leamer (1981).

Since I include nest-level fixed effects in each market, I compare variation in prices and quantities between varieties in the same nest. Thus, substitution across nests is absorbed by the nest-level fixed effects.

IV.3 Unknown Mapping Between Housing Varieties and Nests

I propose a method that solves the joint problem of estimating nest parameters and nest membership for K nests. The method is a new application of the panel Group Fixed-effects Estimator (GFE) from Bonhomme and Manresa (2015). Building upon the panel estimation equation (12) based on the structural market-level demand for a variety, I consider the joint problem of estimating both the lower-level elasticities of substitution and the nesting structure. Let $k(v)$ denote the nest membership of each variety v, so that $k(v) \in \{1, ..., K\}$.[32]

$$(\widehat{\sigma}_k, \widehat{\alpha}, \widehat{k}) = \underset{(\sigma_k,\alpha,k)\in\Sigma^K\times\mathcal{A}^{KNT}\times\Gamma_K}{\text{argmin}} \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{v\in\Omega_{it}} \underbrace{\left(\ln p_{ivt} + \frac{1}{\sigma_{k(v)}}\ln q_{ivt} - \alpha_{ik(v)t} - \alpha_{iv} - \alpha_{iv}t\right)^2}_{\epsilon_{ivt}} \tag{14}$$

The key idea behind this estimation approach is that varieties in a nest share the same elasticity of substitution and the same nest-level price indices and quality shocks. These common parameters correspond unobserved nest-level heterogeneity that can be estimated. As a preliminary step to reduce the computational burden and simplify the identifying assumptions, let $\ln \tilde{p}_{ivt}$ and $\ln \tilde{q}_{ivt}$ denote the residualized price and quantity with respect to their variety X MSA average (removing α_{iv}) and variety and MSA specific linear time trends (removing $\alpha_{iv}t$).

To solve the problem, I utilize an iterative procedure that solves the optimization problem for subsets of the parameters.[33] With a known nest membership, $k(v)$, then the fixed effects regression discussed in the Section IV.1 can be used to solve the least-squares criterion to estimate σ_k. Choosing $(\widehat{\sigma}_k, \widehat{\alpha})$ to minimize the least-squares criterion corresponds exactly to a standard fixed effects regression in (12).

Consider the case when the elasticities of substitution are known but the nest assignment is not known. With a known σ_k, $\ln \tilde{p}_{ivt} + \frac{1}{\sigma_{k(v)}} \ln \tilde{q}_{ivt}$ can be computed. Then, minimizing the least-squares criterion with respect to all possible partitions of varieties into nests and nest-level fixed effects, $(\widehat{\alpha}, \widehat{k})$, is exactly the objective of the k-means clustering algorithm. Although this is a challenging problem, the k-means algorithm is tractable and widely used in machine learning and engineering.[34]

[32]To see how the data structure maps to Bonhomme and Manresa, the groups are nests, the "time" variable is MSA x year, and the "individuals" are housing varieties. As a result, nest x MSA x year fixed effects, which absorb the nest-level price index for each market, are the group x time fixed effects.

[33]See Wright (2015) for a discussion of coordinate descent algorithms in solving problems with smooth and convex constraints.

[34]k-means has been used recently in the literature to cluster firms and households based on observable characteristics (e.g., Epple et al. 2020, Setzler and Tintelnot (2021), Almagro and Domínguez-Iino 2022). However, this contrasts with my proposed approach which solves a structural estimating equation with respect to both parameters and nest assignment. Rather than clustering on characteristics of a variety, my approach groups varieties based on systematic

To further understand how the estimation works, it is useful to define the centroid of the nest. The nest centroid in each market is the average of the data points $\left(\ln \tilde{p}_{ivt} + \frac{1}{\sigma_{k(v)}} \ln \tilde{q}_{ivt}\right)$ over all varieties assigned to the nest, or equivalently, the nest-level fixed effect in each MSA and year

$$\alpha_{ikt} \equiv \frac{1}{N_k} \sum_{k^*(v)=k} \left(\ln \tilde{p}_{ivt} + \frac{1}{\sigma_{k(v)}} \ln \tilde{q}_{ivt} \right)$$

where N_k is the number of varieties assigned to nest k, or $N_k = \sum_v \mathbb{1}_{k^*(v)=k}$. The k-means algorithm iteratively assigns each variety to the nest with the nearest centroid. At each iteration of the algorithm, the nest centroid is re-defined with the updated nest assignment. Thus, the k-means algorithm converges when there is a stable set of nests such that no variety can be re-assigned. See Appendix E for estimation details.

Given that we know how to solve the optimization problem for subsets of the optimization variables, I proceed using an iterative algorithm based on k-means to solve for nest membership and a fixed effect regression to solve for the inverse demand elasticity. The proposed algorithm differs from Algorithm 1 proposed by Bonhomme and Manresa. One concern with k-means clustering algorithms is the sensitivity to initial nest centroids. My algorithm is able to exploit the performance of k-means++, which is an algorithm that improves the selection of the initial centroids (Arthur and Vassilvitskii 2007).[35]

To jointly identify the nesting structure and demand parameters, I make an additional assumption that the nest-level fixed effects are well-separated.

Assumption 3: On average, the nest-level fixed effects in each market are asymptotically different across nests

$$\text{plim}_{M\to\infty} \frac{1}{M} \sum_{m=1}^{M} \left(\tilde{\alpha}^0_{km} - \tilde{\alpha}^0_{\tilde{k}m} \right) = c^{FE}_{k\tilde{k}} > 0, \quad \forall k \neq \tilde{k}$$

where I subsume both MSAs and years into a market dimension m so that M denotes the total number of markets. $\tilde{\alpha}^0_{km}$ denotes the demean and detrended nest-level fixed effect and the superscript ^{0} denotes the true parameter value.[36] This adjustment is necessary due to the preliminary step where I demean and detrend prices and quantities to account for $(\alpha_{iv}, \alpha_{iv}t)$.

Given Assumptions 1-3, joint estimation of the lower-level elasticities of substitution and the nesting structure is both consistent and asymptotically well-behaved following Bonhomme and Manresa (2015).

variation in prices and quantities consistent with a nested CES demand system.

[35]For the size of my estimation data, the algorithm also reduces computation time as I do not need to repeatedly estimate the fixed effect regression at every step. To check sensitivity of my results to the algorithm, I use both the algorithm suggested by Bonhomme and Manresa as well as my proposed algorithm and choose the solution that minimizes the least squares criterion.

[36]Bonhomme and Manresa consider the case where there are only unit fixed effects. Since I consider both unit fixed effects and a unit time trend, $\tilde{\alpha}^0_{km}$ corresponds to the residual of $\alpha^0_{km} = \gamma_m + \gamma_m t + \tilde{\alpha}^0_{km}$. With only unit fixed effects, the nest-level fixed effects cannot be parallel. With both unit fixed effects and unit time trends, the difference between the nest-level fixed effects cannot be linear across time.

Number of Nests

To find K, the number of nests, I follow Bonhomme and Manresa (2015) and consider a BIC criterion based on Bai and Ng (2002). The BIC trades off the reduction in the least squares criterion in (14) as the number of nests increase, with the increase in the number of parameters that have to be estimated (corresponding to the nest-market level fixed effects). See Appendix H.1 for details on the BIC criterion.

IV.4 Estimation of Top-Level Elasticity

Using the estimated nesting structure and the lower-level elasticities of substitution, I can then proceed to estimate the top-level elasticity of substitution that governs the substitution across nests. I develop two estimation strategies for the top-level elasticity of substitution σ. Both approaches rely on the fact that the nest-level price index can be recovered up to the geometric mean of relative variety quality shocks.

The first approach builds upon the lower-level nest estimation and develops a panel estimation approach that assumes the nest-level quantity index is exogenous. The second approach uses the dispersion of shares within a nest as an instrument for the nest-level price index. The idea behind the instrument is that dispersion in shares within a nest will be uncorrelated with the nest-level quality shock.

I find that both approaches produce similar estimates of σ in Monte-Carlo simulations and in estimation with the housing transaction data. The similarity of the estimates supports my assumption that quantities are exogenous conditional on the fixed effects in the top-level estimation.[37]

For estimation, it is helpful to separate the variety quality shock into a nest-level component and a relative variety-level component. Let $\varphi_{ik(v)t}$ denote the component that is common to all varieties within a nest k. Without loss of generality,

$$\varphi_{ivt} \equiv \varphi_{ik(v)t}\tilde{\varphi}_{ivt}$$

where $\tilde{\varphi}_{ivt}$ is the within-nest relative quality shock for variety v.

Following Hottman et al. (2016), the nest-level price index can be expressed as

$$\ln \mathbb{P}_{ikt} = \ln \overline{P}_{ikt} + \frac{1}{1-\sigma_k} \ln \left[\sum_{v\in\Omega_{ikt}} \frac{S^K_{ivt}}{\overline{S}^K_{ikt}} \right] - \ln \overline{\tilde{\varphi}}_{ikt} \tag{15}$$

where S^K_{ivt} denotes the within-nest k expenditure share on variety v in market it. The line above a variable denotes the geometric mean across varieties within a nest, so

$$\ln \overline{P}_{ikt} = \frac{1}{N_{ikt}} \sum_{v\in\Omega_{kt}} \ln p_{ivt}, \qquad \ln \overline{S}^K_{ikt} = \frac{1}{N_{ikt}} \sum_{v\in\Omega_{kt}} \ln S^K_{ivt}, \qquad \ln \overline{\tilde{\varphi}}_{ikt} = \frac{1}{N_{ikt}} \sum_{v\in\Omega_{kt}} \ln \tilde{\varphi}_{ivt}$$

[37] I formally test this using a J-test with the share instrument from Hottman et al. (2016) and the quantity index as an additional instrument.

The first two components of the nest-level price index are computable given estimates of the lower-level elasticities of substitution. The first term in (15) is the geometric average of variety prices within nest k. The middle term in (15), $\frac{1}{1-\sigma_k}\ln\left[\sum_{v\in\Omega_{ikt}}\frac{S^K_{ivt}}{\tilde{S}^K_{ikt}}\right]$, is composed of 1) the inverse of one minus the nest-level elasticity, 2) the average of shares relative to the geometric mean of shares. This term reflects a love of variety. As shares become more dispersed, the nest-level price index decreases (as $\sigma_k > 1$). When varieties are symmetric, then $S^K_{ivt} = \overline{S}^K_{ikt}$ so that the second term is simply the log number of varieties in nest k. If the elasticity of substitution tends towards infinity, then the CES price index is the geometric mean of quality-adjusted housing variety prices.

The third term is the geometric mean of relative variety-level quality shocks. Note that it is not possible to separately identify the nest-level quality shock and the relative variety-level quality shock. Hence, a normalization is needed. Following Hottman, Redding, and Weinstein, I consider the following normalization of the relative variety-level quality shocks: $\prod_{v\in\Omega_{ikt}}\tilde{\varphi}_{ivt} = 1$. As a result, after taking logs the third term in (15) is zero.

Estimation Approach 1: Panel

I consider an estimating equation similar to the one for the lower nest that treats the nest-level quantity as exogenous. The inverse market-level nest demand equation can be written as

$$\ln\mathbb{P}_{ikt} = -\frac{1}{\sigma}\ln\mathbb{Q}_{ikt} + \frac{\sigma-1}{\sigma}\ln\mathbb{P}_{it} + \frac{\sigma-1}{\sigma}\ln\varphi_{ikt}$$

where

$$\mathbb{Q}_{ikt} \equiv \ln S_{ikt} - \ln\mathbb{P}_{ikt} + \ln E_{it}$$

where S_{ikt} denotes the share of nest k in market it and E_{it} is total expenditure at location i at time t. The price index for nest k is decreasing in the quantity index, increasing in the overall price index, and increasing in the nest-level φ_{ikt} quality shock.

If quantity supplied is exogenous after controlling for the set of fixed effects, I can estimate

$$\ln\mathbb{P}_{ikt} = -\frac{1}{\sigma}\ln\mathbb{Q}_{ikt} + \alpha_{it} + \alpha_{ik} + \alpha_{kt} + \epsilon_{ikt}$$

Estimation Approach 2: IV

The middle term in (15) can be used as an instrument for the nest-level price index. Specifically, it can be used as an IV in a regression of nest shares on the nest price index to identify the top elasticity of demand σ. I consider the structural equation of shares on prices

$$\ln S_{ikt} = (1-\sigma)\ln\mathbb{P}_{ikt} + (\sigma-1)\ln\mathbb{P}_{it} + (\sigma-1)\ln\varphi_{ikt}$$

where S_{ikt} can be directly computed from expenditure shares and $\mathbb{P}_{ikt}$ can be computed (up to an unobserved additive constant γ_{ik} that is specific to each MSA X nest) by (15).

The estimating equation is given as

$$\ln S_{ikt} = (1-\sigma)\ln \mathbb{P}_{ikt} + \alpha_{it} + \alpha_{ik} + \alpha_{kt} + \epsilon_{ikt}$$

The overall price index $\mathbb{P}_{it}$ is absorbed by the α_{it}, or MSA x year fixed effects, nest-level quality shocks common across all MSAs are absorbed by the α_{kt}, or nest X year fixed effects, and nest-level quality shocks persistent in an MSA are absorbed by α_{ik}, or MSA X nest fixed effects.

The identifying assumption of the IV is that the relative shares of each variety purchased within a nest only affect the share of the nest in overall expenditure through the nest price index. This holds when the IV is uncorrelated with the nest-level quality shock φ_{ikt} since the nest-level quality shock does not impact the relative within-nest share of variety A and variety B that both belong to the same nest.

IV.5 Monte Carlo Simulations

I perform Monte Carlo simulations to validate the proposed lower-level and top-level estimation strategies. I show that the lower-level estimation strategy is highly accurate in recovering the true nest assignment and lower-level elasticities of substitution. I then show that both the panel and IV approaches yield accurate estimates of the top-level elasticity of substitution.

I generate random nesting structures, quantity supplied, and qualities of each variety. I consider a two-layer nested CES for 1000 varieties in a single location for 14 periods. The top elasticity of substitution is drawn uniformly between 3 and 5 while the within nest-level elasticities of substitution are drawn uniformly between 7 and 15. To assign varieties to nests, I draw a quality shock from a log-normal distribution. I then generate cutoffs on this distribution to separate the varieties into 12 nests. For each variety, I further draw a persistent quantity and quality from log-normal distributions. For each period, I generate multiplicative i.i.d. shocks to quantity, quality, and expenditure. See Appendix E for further details.

Given the nesting structure, quantities, and qualities, I can solve for variety prices that equate quantity supplied and quantity demanded. I further set 20% of data to be randomly missing. I then use both the algorithm suggested by Bonhomme and Manresa as well as my proposed algorithm to recover $\hat{\sigma}_k$ and $\hat{\sigma}$ in 2000 simulations. I find similar accuracy between the two algorithms, with my proposed algorithm resulting in lower run-times.

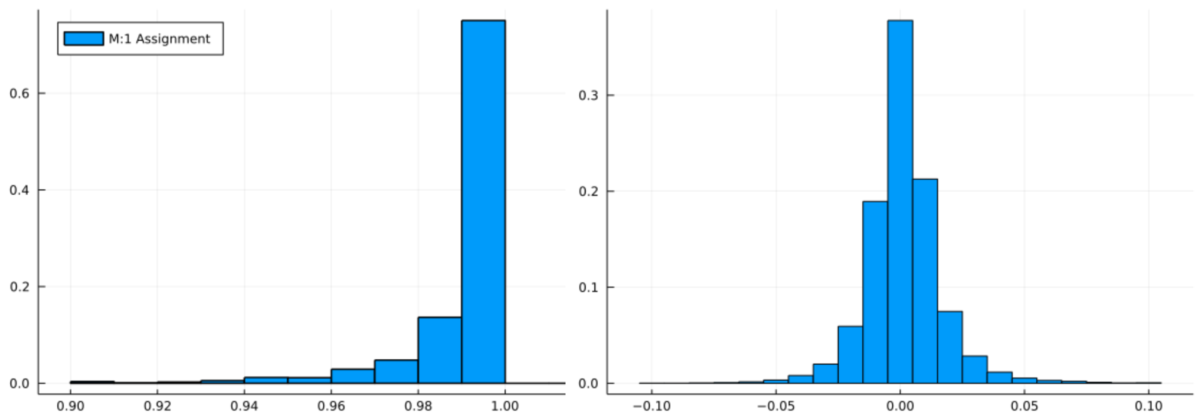
I find that the estimation approach is able to recover the nest assignment of each variety and yields precise estimates of σ_k. The top panel of Figure 5 shows percentage of varieties that are correctly assigned. I find a mean accuracy of 99.2%, a median of 99.7%, and a minimum accuracy (across 2000 simulations) of 84.8%. The bottom panel of Figure 5 shows the percent difference between the estimated nest elasticity and the true nest elasticity across all simulations. I find that almost all of the elasticities differ by less than 5% from their true value, with the mean of the absolute percentage difference equal to 1.1% and median of the absolute percentage difference equal to 0.7%.

The accuracy in the assigned mapping decreases when the top elasticity is larger: the correla-

Figure 5: Nest Level Monte Carlo Results

(a) Across Simulations: Fraction of Varieties Correctly Assigned

(b) Percent Error in Estimated Sigma at Variety Level



Notes: Details of Monte Carlos Simulations in text. Left figure presents the fraction of varieties assigned to the correct nest (allowing for multiple estimated nests to match with a single true nest). The right figure presents the estimated percentage error in the lower-level elasticity of substitution.

tion between the accuracy rate and top elasticity is -0.58. This suggests that when the elasticity of substitution is closer between the top level and the nest, it is harder to differentiate the nest-level fixed effects.[38] I further find that the accuracy of the assigned mapping is decreasing in the variance of the persistent quality shock φ_v and increasing in the variance of the persistent quantity shock q_v.

Using the estimated lower-level nesting structure and elasticities of substitution, I then estimate the top-level elasticity with both approaches described in Section IV.4 . I find that the panel and IV approaches yield almost identical results so I only present the IV results in Appendix Figure A4. I find a mean absolute percent error of 1.1% for both approaches. Although 95% of estimates have less than a 5% error, there is asymmetry in the error distribution. I find that there is a positive correlation of 0.42 between the true top elasticity and the absolute percent error, suggesting that the right tail of the histogram is due to the lower accuracy in the first step estimation of the nest elasticities when the top elasticity is close to the nest elasticities.[39]

[38] To see why, the coefficient on the nest-level price index in the estimating equation is given by $\frac{\sigma_{k(v)}-\sigma}{\sigma_{k(v)}}$, so that when the elasticity of substitution between the top level and the nests are more similar, the nest-level price index plays a smaller role in the nest-level fixed effect. This is related to Bonhomme and Manresa's group-separation property, or where the group fixed-effects have to be asymptotically different.

[39] When I use the true nest assignments and the true nest elasticities, the IV and panel approach for the top elasticity yield highly accurate and symmetric error distributions. The 1st percentile of the percent error distribution is -0.6% and the 99th percentile is 0.5% for the IV approach.

V Data

In this section, I describe the data sources that I use to measure housing price indices and real income across metropolitan areas. I use housing characteristics and transaction data from the Zillow Transaction and Assessment Dataset (ZTRAX), mortgage applicant data from the Home Mortgage Disclosure Act (HMDA), and household-level wage and demographic data from the American Community Survey (ACS) obtained from IPUMS. To verify the quality of the housing transaction data, I replicate the S&P Case-Shiller repeat sales index for 18 of the overlapping MSAs and show a high correlation in the time-series. Finally, I discuss how I aggregate from transactions to variety-level prices and expenditure shares.

V.1 Data Description

V.1.1 Housing Transaction and Characteristics

To construct housing price indices, I require arms-length data on housing transactions along with associated housing characteristics and square feet. I use data from ZTRAX, restricting attention to transactions between 2005 and 2019.[40] The Transaction data includes legal proceedings processed by county recorder's offices while the Assessment data includes data from county assessor offices. Note that alternative data sources on housing sales data from ACS or the American Housing Survey are not based on market transaction prices, typically do not include square feet, or lack broad geographic coverage.

To define housing varieties, I require detailed data on observable housing characteristics. ZTRAX is well suited for this as it contains unit and structure specific characteristics such as structure type, structure-built year, lot size, number of rooms, number of bedrooms, number of floors, number of bedrooms, location (property address), and square feet. To classify the unit's structure type, I follow categories commonly used in the housing literature based on the American Housing Survey (AHS) or American Community Survey (ACS). Further details are described in Appendix F.

I clean the transaction data in two main ways: 1) restricting to arms-length transactions, 2) dropping transactions that have outlier sales prices or square feet. Since housing deeds are used to record transactions between family members, I restrict attention to arms-length transactions by focusing on a select subset of deed types. As there may be several transactions associated with a single property, I keep one transaction per property per month and take the maximum sales price if there are multiple transfer records. I drop transactions where the sales price is missing, less than 30,000 or greater than 10 million, and properties where the square feet is less than 100 or greater than 20,000. Finally, I drop transactions where the year built (or the most recent year when there is new construction or major rehabilitation) is after the transaction date since property characteristics may have changed.[41]

[40]This ensures broad coverage as certain counties only report data back to the mid-2000s.

[41]This is similar to the data-cleaning process in Gindelsky et al. (2020) and Graham and Makridis (2020). The sales price cutoffs correspond to the 6th percentile and 99.97 percentile. The square feet cutoffs correspond to the 1st percentile and 99.8 percentile. Nowak and Smith (2020) show that there may be systematic differences in housing reno-

Finally, due to concerns of missing housing characteristics and the bias that may entail in the spatial price indices, I restrict attention to a subset of MSAs for which there is a high percentage of non-missing characteristics or for which there is a sufficient number of transactions with non-missing characteristics. Specifically, I restrict attention to 98 of the top 200 MSAs (by 2010 census population) that meet a minimum number of transactions with non-missing characteristics.[42]

I find that the restriction alleviates the concern that the variety effects I find are due to missing characteristics in smaller MSAs. Prior to the restriction, the correlation between the MSA population rank and the fraction of transactions with non-missing characteristics across the top 200 MSAs is -0.15. This means that larger MSAs have a lower fraction of non-missing characteristics. However, once I focus on the 98 MSAs in my final sample, I find that the same correlation is 0.36, alleviating the concern of systematically missing characteristic data in smaller cities.[43] The restricted sample includes 40 out of the top 50 MSAs and 70 out of the top 100 MSAs. A list of the MSAs and average number of transactions and transactions with non-missing characteristics are presented in Appendix G.

V.2 Variety and Summary Statistics

I provide details on the construction of a housing variety. My definition of a variety is based on several standard characteristics, such as the number of rooms, bathrooms, bedrooms, and floors. Table 1 contains the other categories used in the definition of a housing variety, including: the decade that the structure was built (or most recently renovated), the structure type, the distance from the city center, and quantiles of the lot size. Across my entire sample, these granular characteristics generate 152 thousand unique varieties. See Table A3 for additional summary statistics on the fraction of housing units with the different characteristics.[44]

Next, I present summary statistics number of varieties and transactions across markets, and distribution statistics on prices and square feet. The first two columns of Table 2 are statistics on the distribution of the number of varieties across MSA-years and the number of transactions for a variety-MSA-year. Columns 3 to 5 present statistics on the sales price, square feet, and price per square feet across transactions in the entire sample. There is an average of 2800 varieties within an MSA-year and an average of five transactions within a variety-MSA-year. The average unit transacted in the sample has a square foot of 2,100, with a sales price of $300,000.

vation and quality along the business cycle that may bias standard housing price indices including Case-Shiller. Since my focus are cross-sectional comparisons rather than time-series comparisons, unobserved quality change over the business cycle may not be as much of a concern.

[42]Out of the 98 MSAs, there are four that belong to non-disclosure states, where county governments are prohibited from disclosing sales price information to the public. These include Dallas, Houston, Memphis, and St. Louis. These 98 MSAs either have an average of 5,000 transactions per year with non-missing characteristics, or at least an average of 2,000 transactions per year with non-missing characteristics and for which at least 50 percent of all transactions have non-missing characteristics.

[43]Hence, if anything my results are potentially underestimating the impact of varieties. However, this concern is potentially alleviated by the robustness of the ACS sensitivity check in Figure A5. I also find that the estimated demand parameters are robust to only considering top 50 MSAs, where I have a more complete sample.

[44]The number of rooms is top-coded at 8, and the number of bathrooms, bedrooms, and floors are top-coded at 5.

Table 1: Example Characteristics and Categories

Decade Built	Structure Type	Distance from City Center	Lot Size
Pre 1900s	Single-family detached	0-5 miles	Quantile 1: 0 - 0.12 acres
1900 - 1910	Single-family detached	5-10 miles	Quantile 2: 0.12 - 0.17 acres
1910 - 1920	**Apartments**	10-20 miles	Quantile 3: 0.17 - 0.24 acres
1920 - 1930	2	20-30 miles	Quantile 4: 0.24 - 0.50 acres
1930 - 1940	3-4	30+ miles	Quantile 5: 0.50+ acres
1940 - 1950	5-9		
1950 - 1960	10-19		
1960 - 1970	20-49		
⋮	50+		
2010 - 2020			

Notes: Quantiles of lot size based off housing transactions in Zillow's ZTRAX data. Central Business Districts (CBDs) are obtained from Manduca (2020) who uses an algorithm based on relative employment to population densities.

Table 2: Summary Statistics: ZTRAX Data (98 MSAs from 2005-2019)

	# Varieties MSA-Year	# Transactions Variety-MSA-Year	Sales Price	Sq Ft	Price per Sq Ft
Mean	2,843	4.7	$301,944	2,113	$171.2
10th pctile	871	1.0	80,000	1,020	$50.5
25th pctile	1,336	1.0	$138,000	1,329	$79.2
50th pctile	2,150	1.0	$227,500	1,826	$119.4
75th pctile	3,694	3.0	$364,000	2,560	$182.6
90th pctile	5,961	8.0	$565,000	3,478	$292.9

Notes: A variety is defined as the full interaction of the decade the housing structure was originally built (or underwent substantial renovation), number of rooms, number of bedrooms, number of bathrooms, number of floors, structure type, lot size quintile, and distance categories from the MSA central business district (CBD). Summary statistics on sales price, square feet, and price per square feet are computed over all transactions.

V.3 ZTRAX Data Verification

To verify the quality of the ZTRAX data, I check to see whether constructed repeat-sales indices are similar to the S&P Case-Shiller Indices. I follow the S&P CoreLogic Case-Shiller Home Price Indices Methodology (2020) and construct arithmetic repeat-sales methodology for the set of overlapping MSAs. I find that across 18 MSAs, my constructed Case-Shiller tracks the S&P index closely, with a raw correlation of 0.96 (see Figure A6 for a further comparison).

V.4 Variety-level Prices

Since I observe housing sales at the transaction level, I aggregate sale prices and square feet to the variety level. Let Ω_{ivt} denote the set of the transactions (indexed by s) of a specific variety v in MSA i in year t. The price per square foot of a variety is defined as the total sales of a housing variety divided by the total quantity of square feet transacted

$$p_{ivt} = \frac{\sum_{s \in \Omega_{ivt}} \mathbb{V}_{ivts}}{\sum_{s \in \Omega_{ivt}} q_{ivts}} \equiv \sum_{s \in \Omega_{ivt}} \frac{q_{ivts}}{\sum_{s' \in \Omega_{ivt}} q_{ivts'}} p_{ivts}$$

where $\mathbb{V}_{ivts}$ is the sale price of transaction s and q_{ivts} is the square feet of transaction s. Thus, the variety price is a quantity-weighted average of the price per square feet of each transaction.

Analogously, the quantity of a housing variety is simply the sum of quantities across transactions, or

$$q_{ivt} = \sum_{s \in \Omega_{ivt}} q_{ivts}$$

The share of expenditure on a housing variety is

$$s_{ivt} = \frac{\sum_{s \in \Omega_{ivt}} \mathbb{V}_{ivts}}{\sum_{v' \in \Omega_i} \sum_{s \in \Omega_{iv't}} \mathbb{V}_{iv'ts}}$$

V.5 Housing Transactions with Mortgage Applicant Income

To investigate heterogeneity in the price indices by household income, I merge mortgage applicant data from the Home Mortgage Disclosure Act (HMDA) from 2005-2017 with the Zillow ZTRAX transaction data.[45] Enacted in 1975, the Home Mortgage Disclosure Act requires mortgage lenders to submit loan-level borrower characteristics, including mortgage lender, applicant gross income, census tract, year, and loan amount (rounded to the nearest thousands). To merge the data to the Zillow ZTRAX transaction data, I follow Bayer et al. (2007) and merge on the mortgage lender, loan amount, and census tract.

I find that out of my final sample of 17.0 million transactions from 2005-2017 for the 98 MSAs with non-missing characteristics, and transaction prices, I obtain a matched mortgage record with non-missing applicant income for 7.9 million observations (47% match rate). Table A1 presents summary statistics for the merged ZTRAX and HMDA data.[46] Table A2 presents summary statistics on applicant income in my sample.

V.6 Household Income

IPUMS ACS is a representative sample of households with key demographic information that allows me to adjust for composition differences when measuring average household income in each MSA and year. I regress log household income on observable household-head characteristics and MSA X year fixed effects. The MSA X year fixed effects are extracted and used as the average

[45] I do not merge 2018 and 2019 mortgage data since loan amounts were updated to be reported in $10,000 intervals rather than rounded to the nearest thousand. See Federal Register (2019) for details.

[46] The average number of varieties per MSA falls by 40%. The correlation between the MSA population rank and fraction of all transactions with non-missing transactions is 0.37 in the merged dataset, compared to 0.36 in the main dataset. As a result, the HMDA sample does not over-sample larger MSAs relative to smaller MSAs. I find that the average transaction price is 7% higher, driven by a higher price per square feet while the average size of houses are the same between the main and merged datasets.

household income in an MSA. I include the following set of household-head characteristics: education categories, potential experience, industry, occupation, marital status, veteran status, race, immigrant status, and English proficiency (all characteristics interacted with gender). As highlighted by Albouy (2011), it is important to account for federal and state taxes when measuring real consumption across US metropolitan areas. Since the ACS reports pre-tax income, I use the NBER TAXSIM program to compute post-tax income. See Appendix F for further details.

Following Diamond and Moretti (2021), IPUMS ACS allows me to map skill types defined by education categories to income categories in each MSA and year. Using the previously merged HMDA mortgage data with the ZTRAX transaction data, I construct income-specific price indices. These income-specific housing price indices are then used to compute skill-specific price indices based on the mapping from skill type to income categories.

VI Estimation Results

This section presents estimates of two key parameters in the housing price indices: the nest-level demand elasticities and nesting structure. I implement the methodology developed in Section IV on the ZTRAX transaction data. I estimate both the nest structure and demand parameters for $K = 2, ..., 15$ and select the optimal number of nests to minimize a BIC criterion described in Bonhomme and Manresa, based off Bai and Ng (2002). I find that the optimal number of nests is six. As a robustness check, I compare the nest-level analysis with the results from a single nest.

To validate the estimated nest structure, I show that there are systematic differences in the characteristics of housing varieties across nests. After establishing key differences across nests, I then present the estimated elasticities of substitution across nests. Since the elasticities of substitution vary across nests, this heterogeneity will subsequently impact the measured effect of variety across space.

VI.1 Multiple Nests

To validate the estimated nesting structure, I show that the housing characteristics systematically vary across nests. The joint estimation relies on variation in prices and quantities across markets. Because characteristics are used to define varieties but are not directly used in the joint estimation, characteristic differences that emerge across different nests are not mechanical. Rather, the characteristic differences that emerge show how households systematically substitute across housing varieties.

I find systematic patterns of characteristic differences across nests. I first sort the nests based on the average transaction price per square feet (see Table 4 for average prices in each nest). Prices range from \$160 in the cheapest nest to \$204 in the most expensive nest. There is also significant heterogeneity in average square feet. Nest 4 contains units that are on average 1,600 square feet, compared to nest 1 that contains units that are 2,500 square feet. Table 3 presents characterizations based on housing characteristics in the six nests. To support these characterizations, Figure 6 presents the density of characteristics for nests 1, 2, and 4.[47]

I am able to sharply characterize nests 1, 2, and 4, which account for 90% of expenditure (see Table 4 for nest expenditures). The first nest contains McMansions and covers 54% of expenditure. These are suburban large homes with two floors, three bathrooms, and built in the 1990s and 2000s. The second nest contains Suburban housing. Suburban housing accounts for 33% of expenditures, are smaller units than McMansions, have fewer bathrooms, and are built in the post-war era. Nest 4 contains Urban homes that are close to the city center, are composed of a mixture of single family detached and multi-unit structures, and are the smallest in terms of average square feet.[48]

The other three nests, 3, 5, and 6, are characterized as Other Urban homes. These nests account for 10% of expenditures and are close to the city center, with 1 to 2 bathrooms and are composed of a mixture of single family detached and multi-unit structures. Varieties in these three nests are similar to each other, but distinct from nests 1, 2, and 4.

Table 3: Nest Characteristics

Nest	Label	Characteristics
1	McMansions	Large SF Detached, post 1990s, 2 Floors, 3 Baths
2	Suburban	Small SF Detached, post 1950s, 1 Floor, 2 Baths
3	Other Urban	Medium Units, 1-2 Baths
4	Urban	Smallest Units, 1 Baths, 2-3 Bedrooms
5	Other Urban	Medium Units, 1-2 Baths
6	Other Urban	Small Units, 1-2 Baths

Table 4: Nest Summary Statistics

Nest	Label	Frac Varieties	Frac Expenditure	Mean Price	Mean Sq Ft
1	McMansions	22.0%	54.3%	\$160	2,522
2	Suburban	32.4%	32.6%	\$176	1,714
3	Other Urban	12.6%	4.3%	\$188	2,002
4	Urban	10.8%	2.9%	\$191	1,613
5	Other Urban	11.9%	3.2%	\$193	2,000
6	Other Urban	10.4%	2.7%	\$204	1,881

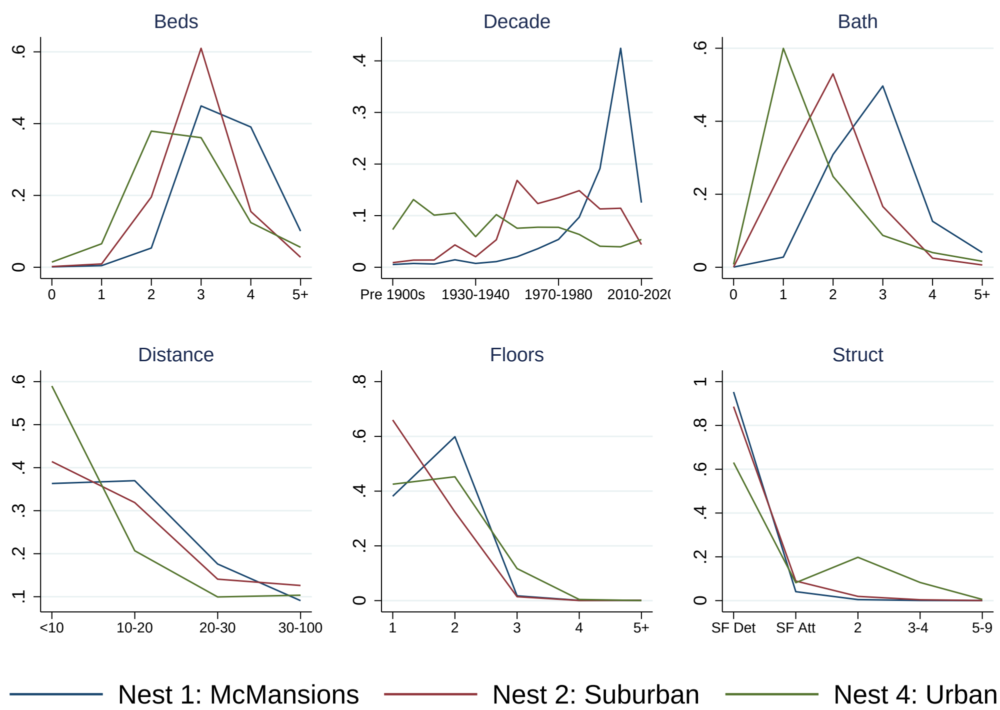
Notes: Square feet and prices are summarized at the transaction level within each nest.

Next, I assess the heterogeneity in nest-level elasticities of substitution. Table 5 presents the estimated lower-level demand elasticities. The Estimated Coefficients column presents the coefficient on log square feet for each nest obtained after solving the joint least squares criterion in (14). The demand elasticity, or elasticity of substitution, is the negative inverse of the estimated coefficient (presented in the second column). The estimated nest elasticities range from 6.5 in nest 5 to

[47]Figure A8 presents the same figure for all nests and all characteristics. Table A6 contains the full list of mean characteristics by nest. Table A7 provides mean and median price and square feet, averaged across transactions within each nest.

[48]Since I demean and detrend variety-level prices and square feet in the preliminary step, the estimation will not mechanically group varieties based on the level of prices or square feet.

Figure 6: Density of Characteristics Across Nests



9.9 in nest 1.[49] It is interesting that the McMansions nest has the highest elasticity of substitution of 9.9, consistent with the fact that McMansions are typically mass-produced with a lack of quality differentiation. To the extent that there are larger variety differences in nests 3 or 5 ($\hat{\sigma}_k < 7$) versus nest 1 ($\hat{\sigma}_k = 9.9$), then the aggregate welfare impact of variety differences will be larger.

A nested CES relaxes substitution patterns between housing varieties and allows the elasticity of substitution to vary by nest. As a robustness check, I compute the single nest demand elasticity. Table A8 presents estimates of demand elasticities for a single nest. My preferred specification is column (5) with MSA X year fixed effects, MSA X variety fixed effects, and MSA X variety time trends. The single nest elasticity is estimated to be 8.3. Thus, we omit important heterogeneity in the elasticities of substitution when using only a single nest.

VI.2 Top-Level Demand Elasticity

After estimating the nesting structure and lower-level nest elasticities, I estimate the upper-level elasticity of substitution across nests. I use the estimation strategy detailed in Section IV.4. Column (1) presents the estimated top-level demand elasticity with the panel approach under the assumption of a vertical supply curve. Column (2) presents the estimated top-level demand elas-

[49] The standard errors presented do not include the standard error from estimation of the group membership. Bonhomme and Manresa state that "in a large-T perspective standard errors are unaffected by the fact that group membership has been estimated." The T-dimension in my analysis is the combination of MSAs and years: 98 MSAs X 15 years = 1,470.

Table 5: Nested Demand Estimation using ZTRAX Data (2005-2019)

Nest	Label	Estimated Coefficients	Demand Elasticities $\hat{\sigma}_k$
1	McMansions	-0.101***	9.93***
		(0.004)	(0.39)
2	Suburban	-0.131***	7.65***
		(0.005)	(0.27)
3	Other Urban	-0.154***	6.48***
		(0.006)	(0.26)
4	Urban	-0.121***	8.24***
		(0.005)	(0.35)
5	Other Urban	-0.145***	6.90***
		(0.006)	(0.27)
6	Other Urban	-0.130***	7.68***
		(0.006)	(0.34)
N		3,900,948	
R2		0.865	
Within R2		0.033	
FE		iv,it,iv·t	

Notes: Estimated coefficients are presented in the first column from a regression of log prices on log square feet interacted with nest indicators. The estimated demand elasticities in the second column are the negative inverse of the estimated coefficients and standard errors are computed using the delta method. I use six nests following the BIC criterion from Bonhomme and Manresa, based off Bai and Ng (2002). Standard errors clustered at MSA X Year level in parentheses. $^{*}\ p < 0.05$, $^{**}\ p < 0.01$, $^{***}\ p < 0.001$

ticity with the IV approach based on Hottman et al. Since both estimated demand elasticities are similar, I use the IV estimate of 4.5 as the baseline.

The fact that both demand elasticities are similar supports the validity of treating the nest-level quantity as exogenous. I can formally test this using a Hansen J-test where I estimate a J-statistic of 0.037 and a p-value of 0.84 when both the quantity index as well as the dispersion term are used as instruments for nest-level prices.[50]

The estimated top-level demand elasticity is lower than all the nest-level demand elasticities, even though this was not enforced in the estimation itself. As pointed out by McFadden (1978), Goldberg (1995), and Hottman et al. (2016), this is a sufficient condition for the nesting structure to be consistent with utility maximization.

[50] This is also suggestive of the identifying assumption in the lower-level of treating variety quantities as exogenous.

Table 6: Top Nest Estimation using ZTRAX Data (2005-2019)

	Panel ln Nest Price Index	IV ln Nest Expenditure Share
ln Nest Quantity Index	-0.22***	
	(0.02)	
ln Nest Price Index		-3.5***
		(0.4)
Demand Elasticity $\hat{\sigma}$:	4.6***	4.5***
	(0.3)	(0.4)
N	8,820	8,820
R2	0.969	
Within R2	0.09	
FE	it,kt,ik	it,kt,ik
KP F-Stat		137.2

Notes: Table presents the estimated top coefficients and the corresponding demand elasticities. Standard errors clustered at MSA X year level in parentheses. Estimation of nest price and quantity indices follow Hottman et al. (2016) and use a normalization on the geometric average of relative-variety shocks in each nest. IV in the second column is a measure of the dispersion of shares within a nest. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

VII Spatial Price Indices

I compare housing prices across metropolitan areas by building upon the bilateral CES price index literature. Motivated by the micro-founded model of housing demand and location choice, the bilateral CES price index summarizes price, quality, and variety availability differences between two comparison points. The literature has traditionally considered the two comparison points as two points in time, but there exists an emerging literature that has considered different spatial locations as comparison points.

The standard CES price index approach by Sato (1976) and Vartia (1976) is developed for two comparison points with the same set of varieties and an assumption that the quality shocks are constant. Feenstra (1994) extends Sato-Vartia to account for differences in the set of varieties between the two comparison points (denoted CES-Feenstra).

Redding and Weinstein (2020) show that the Sato-Vartia assumption that quality shocks are constant between the comparison points is inconsistent with the use of observed expenditure shares from both comparison points. This is due to the fact that the observed expenditure shares reflect varying quality shocks. Redding and Weinstein show that when the geometric mean of the quality shocks for the common varieties are equal over time (equivalently, across spatial units), then there is a simple expression of the exact price index (denoted CES-RW).

Since the CES price indices are not transitive, the choice of the MSA as the comparison unit matters. In the subsequent analyses, I consider two measures. First, I consider all bilateral comparisons using a GEKS approach, where each MSA serves as the comparison unit. The Gini-Éltető-Köves-Szulc (GEKS) approach is widely used in cross-country and cross-time comparisons

(Deaton and Dupriez 2011, Diewert 2013). The GEKS price index is defined as

$$\mathbb{P}_i^{H,GEKS} = \left(\prod_{j=1}^{N} \frac{\mathbb{P}_j^H}{\mathbb{P}_C^H} \frac{\mathbb{P}_i^H}{\mathbb{P}_j^H} \right)^{\frac{1}{N}}$$

where C is the base MSA in the GEKS approach, which only affects the scale of the price indices.[51]

Second, I evaluate each MSA's housing price index against the Chicago-Naperville-Elgin MSA's housing price index. I choose the Chicago MSA since it has the third largest population, and has a high share of non-missing transactions relative to the New York and Los Angeles MSAs (the two largest populations). I show in the subsequent sections that the results are robust to either measure, with the first measure of all bilateral comparisons as my preferred specification.

VII.1 Exact Price Index Comparisons

In this section, I show how the exact nested CES price index is constructed. Under the assumption that the geometric mean of quality shocks within each nest and the geometric mean of nest-level quality shocks are equal between each MSA i and the comparison MSA C, the overall log CES-RW index is defined as

$$\ln \mathbb{P}_{i,C}^H = \ln \mathbb{P}_{i,C}^{Common} + \ln \mathbb{P}_{i,C}^{Variety}$$

where the common price index compares the price indices of nests and housing varieties that exist in both locations

$$\ln \mathbb{P}_{i,C}^{Common} = \left[\frac{1}{N_{i,C}^K} \sum_{k \in \Omega_{i,C}^{K*}} \frac{1}{N_{i,C,k}} \sum_{v \in \Omega_{i,C,k}^*} \ln \frac{p_{iv}}{p_{Cv}} \right] + \left[\frac{1}{N_{i,C}^K} \sum_{k \in \Omega_{i,C}^{K*}} \frac{1}{N_{i,C,k}} \sum_{v \in \Omega_{i,C,k}^*} \frac{1}{\sigma_k - 1} \ln \frac{s_{ivk}^*}{s_{Cvk}^*} \right] + \left[\frac{1}{N_{i,C}^K} \sum_{k \in \Omega_{i,C}^{K*}} \frac{1}{\sigma - 1} \ln \frac{s_{ik}^*}{s_{Ck}^*} \right]$$

and the adjustment measures nest-level availability differences and within-nest variety availability differences. It is defined as

$$\ln \mathbb{P}_{i,C}^{Variety} = \frac{1}{\sigma - 1} \ln \frac{\lambda_i^{Nest}}{\lambda_C^{Nest}} + \frac{1}{N_{i,C}^K} \sum_{k \in \Omega_{i,C}^{K*}} \frac{1}{\sigma_k - 1} \ln \frac{\lambda_{ik}}{\lambda_{Ck}}$$

where $\Omega_{i,C}^{K*}$ denotes the set of overlapping nests between MSA i and comparison MSA C, $N_{i,C}^K = |\Omega_{i,C}^{K*}|$, or the number of overlapping nests between MSA i and MSA C.[52] Analogously, $\Omega_{i,C,k}^*$

[51] Since $\prod_{j=1}^{N} \left(\frac{\mathbb{P}_j^H}{\mathbb{P}_C^H} \right)^{\frac{1}{N}}$ is the same across all i, in cross-sectional regressions this term will be absorbed by the constant, leaving $\left(\prod_{j=1}^{N} \frac{\mathbb{P}_i^H}{\mathbb{P}_j^H} \right)^{\frac{1}{N}}$.

[52] s_{ik}^* is MSA i's expenditure on nest k out of expenditure on overlapping nests between MSA i and MSA C and s_{Ck}^* is MSA C's expenditures on nest k out of expenditure on overlapping nests between MSA i and MSA C (see Appendix I.1

denotes the set of overlapping varieties that are present in both MSA i and comparison MSA C within nest k, and $N_{i,C,k}$ is the number of overlapping varieties between MSA i and MSA C in nest k, or $N_{i,C,k} = |\Omega^*_{i,C,k}|$.[53]

There are two variety adjustments in the price index: one at the nest level, $\frac{\lambda_{ik}}{\lambda_{Ck}}$, and one at the top level, $\frac{\lambda_i^{Nest}}{\lambda_C^{Nest}}$. To gain intuition for the variety adjustment, we can first focus on the variety adjustment at the nest-level. λ_{ik} is the MSA i expenditure share on housing varieties that exist in both MSA i and MSA C in nest k while λ_{Ck} is the MSA C expenditure share on housing varieties that exist in both MSAs in nest k. This comparison asks within nest k, how much households in each location prefer the common varieties over the unique varieties available in their location.

Consider the scenario where MSA C has a super-set of varieties in nest k compared to MSA i. If MSA i is missing varieties that MSA C has a substantial expenditure share on within nest k, then $\frac{\lambda_{ik}}{\lambda_{Ck}} = \frac{1}{\lambda_{Ck}}$ will be large since λ_{Ck} will be small, leading to a higher price index. The amount the price index increases is moderated by $\frac{1}{\sigma_k - 1}$. If there are substantial variety differences in nests with a smaller σ_k (where housing varieties are more differentiated or where preference draws are less correlated), then the welfare impacts of variety differences are larger.

For the general case where the set of varieties in MSA C are non-overlapping with the varieties of MSA i, consider the scenario when the share of expenditure on the common varieties is higher in MSA i than MSA C. By revealed preference, households in MSA i prefer the common varieties over MSA i's unique varieties more than how much households in MSA C prefer the common varieties over MSA C's unique varieties. As a result, the unique varieties in MSA C provide increased benefits compared to MSA i's unique varieties, leading to a lower price index from housing choice in MSA C compared to MSA i.

VII.2 Housing Price Indices and Population

To quantify availability differences over space, I compare how housing costs based on a CES price index and housing costs based on a hedonic price index vary with city size. In Table 7, I estimate the elasticity of the hedonic price index with respect to MSA population and the elasticity of CES price indices with respect to MSA population for 98 MSAs from 2005 to 2019

$$\ln \mathbb{P}^H_{it} = \beta \ln Pop_{it} + \gamma_t + \epsilon_{it}$$

The top panel considers the baseline nested CES while the bottom panel provides a sensitivity check with a single CES nest. The columns labeled *All Bilateral Comparisons* show the estimated population elasticities across all bilateral MSA comparisons with the GEKS approach, while the columns labeled *vs Chicago* show the estimated population elasticities with Chicago as the comparison MSA.

for the formulas of these shares).

[53] s^*_{ivk} is MSA i's expenditure on variety v out of expenditure on overlapping varieties in MSA i and comparison MSA C in nest k, and s^*_{Cvk} is MSA C's expenditure on variety v out of expenditure on overlapping varieties in MSA i and comparison MSA C in nest k (see Appendix I.1 for the formulas of these shares).

Table 7: ZTRAX Price Indices vs Population (2005-2019)

		Nested CES					
		All Bilateral Comparisons			vs Chicago		
	Hedonic	Variety	Common	CES-RW	Variety	Common	CES-RW
ln Pop	0.185***	-0.070***	0.161***	0.092***	-0.057***	0.198***	0.142***
	(0.017)	(0.003)	(0.018)	(0.017)	(0.004)	(0.019)	(0.020)
N	1470	1470	1470	1470	1470	1470	1470
r2	0.159	0.358	0.0693	0.0249	0.163	0.106	0.0651

		Single Nest CES					
		All Bilateral Comparisons			vs Chicago		
	Hedonic	Variety	Common	CES-RW	Variety	Common	CES-RW
ln Pop	0.185***	-0.052***	0.148***	0.096***	-0.037***	0.163***	0.126***
	(0.017)	(0.002)	(0.016)	(0.016)	(0.003)	(0.017)	(0.018)
N	1470	1470	1470	1470	1470	1470	1470
r2	0.159	0.325	0.0660	0.0302	0.117	0.0948	0.0688

Notes: All dependent variables are in logs. Population estimates are obtained from the Census Bureau. Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

I find that the standard hedonic housing price index overestimates housing costs in larger cities. A standard hedonic housing price index has an elasticity of 0.19 with respect to population compared to the nested CES-RW index, which has a population elasticity of 0.09. This 50% reduction is significant. The standard hedonic approach results in predicted housing prices that are 2.2 higher in New York (population 19.3 million) than Merced, CA (population 266,000). Utility-consistent price indices that account for variety differences imply housing prices that are only 1.5 times higher.

Omitting variety differences across space will then lead to an overestimate of housing costs in larger cities.[54] The variety adjustment accounts for the increased availability of housing varieties in larger cities.[55] As a result, the variety adjustment reduces the housing gradient by 38%. The common price index contributes the remaining 12% in reduction of the population elasticity. Differences that arise between the common price index and hedonic index reflect the effect of accounting for substitution across housing varieties.

The negative relationship between population and the variety adjustment is driven by the fact that expenditure shares on the common varieties (the set of varieties that exist in both locations)

[54]In Table A11, I repeat the analysis for the CES-Feenstra (that does not account for quality differences across space). I find that the nested CES-Feenstra variety adjustment for the Chicago comparison has a population elasticity of -0.04 and the overall index having a population elasticity of 0.13.

[55]See Figure A9 for a plot of the variety adjustment against population. The variety adjustment quantifies not only the positive welfare effect of a larger number of varieties available in larger cities, as documented in Figure 1, but also captures differences in the unique varieties available in larger cities compared to the unique varieties available in smaller cities. Consider the set of varieties available in smaller MSAs with a population rank above 50. I find that 11% of those varieties are ever transacted in Chicago while 77% of those varieties are ever transacted in larger MSAs with a population rank below 50. Larger MSAs have more varieties but there are unique varieties in both small and large MSAs.

are systematically higher in smaller MSAs than in larger MSAs.[56] By revealed preference, households in smaller MSAs prefer the common varieties more than how much households in larger MSAs prefer the common varieties. As a result, households in larger MSAs benefit more from having access to their location-specific unique varieties compared to households in smaller MSAs.

I find that the increased availability in larger cities is robust to using Chicago as a comparison unit and to the national household comparison from Handbury and Weinstein (2014). In the last two columns of Table 7, I estimate that the nested CES variety adjustment has a negative elasticity with respect to population of -0.057, slightly higher than the baseline estimate of -0.07. I further show that the increased availability in larger cities is robust to using a comparison against a national household that has access to all varieties. I estimate a population elasticity of -0.061 for the variety adjustment in the national household comparison in Table A10 (Handbury and Weinstein, 2014).

The nested CES increases the magnitude of the variety adjustment compared to a single nest CES. The population elasticity of the variety adjustment decreases from -0.052 with a single nest to -0.070 with the nested CES.[57] This is due to the fact that there are larger variety differences for nests with lower elasticities of substitution. In a regression of the nest demand elasticity σ_k on the log of the average nest-level expenditure shares on common varieties, $\ln \frac{\lambda_{ik}}{\lambda_{Ck}}$, the estimated coefficient is -0.16 (standard error of 0.02).[58] This means that there are larger differences in expenditure shares in nests with more inelastic demand, increasing the magnitude of variety adjustment relative to a single nest CES.

The difference between the nested and single CES index is related to Ossa (2015). Ossa shows that when there is heterogeneity in the elasticities of demand across nests, the equivalent aggregate elasticity is a weighted harmonic mean of the nest-level demand elasticities. In the case of CES-RW, the equivalent aggregate elasticity, σ_i^{Agg}, is defined as the solution to[59]

$$\left(\frac{\lambda_i}{\lambda_C}\right)^{\frac{1}{\sigma_i^{Agg}-1}} = \left(\prod_k \left(\frac{\lambda_{ik}}{\lambda_{Ck}}\right)^{\frac{1}{\sigma_k-1}}\right)^{\frac{1}{K}}$$

where λ_i is the expenditure share on common varieties in MSA i and λ_C is the expenditure share on common varieties in the comparison MSA. The formula for the equivalent aggregate elasticity

[56] How important is the number of unique varieties in determining the variety adjustment? I find that a regression of the nest variety adjustment on the number of unique varieties has a R^2 of 0.50 for the Chicago comparison and 0.92 for All Bilateral Comparisons. Appendix J.1 provides a further discussion comparing the variety adjustment to the number of varieties.

[57] The behavior of the overall price with respect to MSA population is similar for the nested and single CES. The increase in magnitude of the variety adjustment is offset by a lower common price index of the single nest CES in larger cities. I also find that the increase in the welfare impact from a nesting structure is especially strong for the national household comparison where the single nest variety adjustment has a population elasticity of -0.037 compared to -0.061 for the nested variety adjustment.

[58] See Table A12 for details. The regression includes MSA fixed effects.

[59] I omit nest-level differences at the top-level since empirically all of the variety differences are within nests.

is

$$\frac{1}{\sigma_i^{Agg}-1}=\frac{1}{K}\sum_k \frac{\ln\lambda_{ik}-\ln\lambda_{Ck}}{\ln\lambda_i-\ln\lambda_C}\frac{1}{\sigma_k-1}$$

There are two forces that generate a larger welfare impact from the nesting structure. First, since this is a harmonic mean of σ_k, smaller values of σ_k will be weighted more. Second, a negative relationship between σ_k and the nest-level variety indices, $\ln\frac{\lambda_{ik}}{\lambda_{Ck}}$, means that the smaller σ_k are weighted more, decreasing σ_i^{Agg} and increasing the welfare impacts of variety differences. Relative to the single nest demand elasticity of 8.3, I find that the median equivalent aggregate elasticity across all MSA x years is 5.7.[60]

VII.3 Robustness Checks

Alternative Nesting Structures

I consider two sensible alternative nesting structures but find that they produce inconsistent elasticities of substitution with utility maximization. The first alternative is nests determined by the number of bedrooms. I find that the bedroom nesting structure is inconsistent with utility maximization: the top-level elasticity of substitution across nests of 5.6 is greater than the lower-level elasticity of substitution within the one bedroom nest of 5.1 (McFadden 1978).[61]

The second alternative is nests formed by clustering on housing characteristics.[62] Using six nests, I find strong clustering on the decade the structure was built and number of rooms in Figure A10. Compared to the density of characteristics in my estimated nesting structure in Figure 6, the differences in the nests are evidence that the proposed joint estimation is not driven by clustering on housing characteristics. I find that the second alternative is inconsistent with utility maximization: the top-level elasticity of substitution of 8.1 across nests is greater than the lower-level elasticities of substitution across varieties in two of the six nests.

Income and Population

How do the estimated housing price indices and variety adjustment vary with both population and average household income? There are two takeaways: 1) the lower variety adjustment in larger cities is robust to the inclusion of income, 2) the hedonic price index and the common price index of the CES-RW index do not vary systematically with population after the inclusion of income. The first four columns of Table A16 present regressions of the housing price indices on income, while the last four columns presents regressions on both income and population.

[60] Since the expenditure share on common varieties differs for each MSA and each year, there is a different aggregate elasticity for each MSA and year. There are cases when the sign of $\ln\lambda_{ik}-\ln\lambda_{Ck}$ is different than $\ln\lambda_i-\ln\lambda_C$. As a result, the aggregate sigma is either negative or very large. As a result, the mean is computed over the equivalent aggregate elasticities between the 10th and 90th percentile. See Table A13 for the trimmed distribution.

[61] Using the alternate bedroom nest structure, I re-estimate the population elasticities of housing price indices in Table A15. I estimate a population elasticity of the variety adjustment of -0.067, similar to the baseline population elasticity of -0.7 in Table 7.

[62] I use the k-means to cluster housing varieties based on the characteristics used to define a variety: number of rooms, bedrooms, floors, bathrooms, structure type, decade built, and distance from the city center.

Distance from City Center and City Size

To investigate whether the variety effect is mechanically driven by the size of larger cities and the inclusion of distance categories in the variety definition, I estimate the housing price indices for housing transactions within varying distances from the MSA city center (i.e., circles of different radii around the city center). Importantly, I do not restrict to housing only located within the geographic boundaries of the MSA, but consider all housing within a certain distance from the MSA city center. Table A14 presents the comparison for 20, 30 and 50 mile bands around the city center. Both tables show that the variety effect is significant and robust even within 20 miles. The single nest variety adjustment is -0.036 within 20 miles, increasing to -0.047 for all transactions within 50 miles.[63]

VII.4 Long-Run Changes in Housing Costs and Population

Faster growing MSAs from 2005 to 2019 experienced both significant increases in housing prices as well as increased housing variety relative to slower growing MSAs. Increased housing variety decreases the variety adjustment, partially offsetting the increase in housing prices. As a result, not accounting for housing variety means that we would overestimate the relative cost of housing in faster growing MSAs by 20%. These long-run changes are consistent with the cross-sectional results, where variety adjustments offset 38% of higher housing prices in larger MSAs.

There are two ways of estimating relative price changes over time. In this analysis, I use the constructed spatial price indices that measures the variety adjustment and common price index for each MSA against all other comparison MSAs. An alternative is to construct over-time price comparisons for each MSA. I focus on the spatial price indices to ensure comparability with the results in the cross-section.[64] Using the spatial price indices, I regress long-run changes in housing prices on long-run changes in population from 2005 to $t \in 2015, ..., 2019$

$$\Delta \ln \mathbb{P}_{it}^{H} = \beta \Delta \ln Pop_{it} + \gamma_t + \epsilon_{it}$$

Since I include year fixed effects, γ_t, β measures relative changes in housing prices between faster growing MSAs and slower growing MSAs.

I find that faster growing MSAs experienced increased variety availability compared to slower growing MSAs. Figure 7 shows changes in the variety adjustment and the common price index over the change in population. To interpret the estimates, an MSA that had a 1% higher population growth than average experienced a relative increase in the common component of housing prices by 0.57% and a relative decrease in the variety adjustment by 0.12%. As a result, the relative increase in housing costs is overestimated by 20% if we do not account for changes in available

[63]The overall CES-RW price index is substantially lower compared to the hedonic index, with population elasticities below 0.1 compared to 0.17 for the hedonic index: this difference is driven by both the variety adjustment as well as a less steep common price index against population.

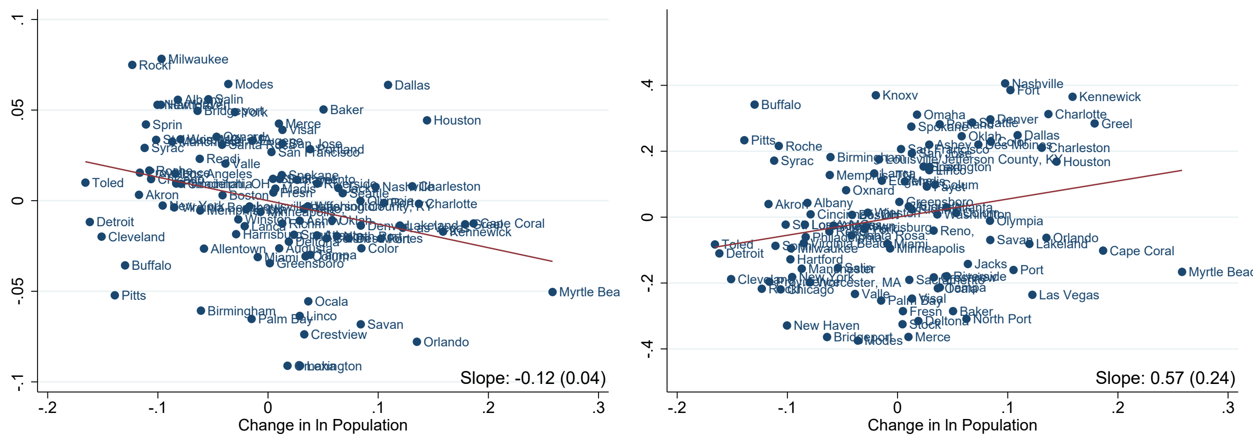
[64]Constructing and measuring housing cost inflation is beyond the scope of the paper, but is an important avenue for future research.

housing varieties.[65]

Figure 7: Long-Run Changes in Prices and Population

(a) Change in Variety Adjustment

(b) Change in Common Price Index



Notes: Figure presents regression of changes in components of the log price index on changes in log population from 2005 to the end of the period (2015-2019). For presentation purposes, the figure collapses the panel data into the cross-section by computing the average change in price index components on average change in population (demeaned). Figure is based on winsorized price changes at the 2.5% level. The regression estimates are robust to not winsorizing.

VIII Spatial Implications: Real Income and Amenities

Housing is a third of household expenditure so housing costs matter for measured real income across space. Standard hedonic approaches overestimate the cost of housing in larger MSAs by not accounting for the increased housing variety. As a result, standard hedonic approaches systematically underestimate real income in larger MSAs relative to smaller MSAs. These differences matter: I find that accounting for housing variety differences leads to real income increasing in population. In contrast, previous estimates in the literature by Albouy (2011, 2016) suggest that real income decreases in MSA population.

A spatial equilibrium requires marginal households to be indifferent across space. Thus, spatial models feature a compensating differential for real income differences: amenities. By not accounting for housing variety, standard hedonic approaches overestimate amenities in larger MSAs. Following Roback (1982), urban researchers have regressed amenities on observable MSA characteristics to understand drivers of non-consumption utility. I find evidence that key characteristics that covary positively with population, including commute time and worse air quality, have their valuation overestimated using a standard hedonic approach.

[65] I find that using a hedonic index results in an estimate of $\beta = 0.42$. Hence, the overall nested CES-RW index is similar to the hedonic for changes in population over time. However, this omits two important offsetting factors: an increase in the common prices and a decrease in the variety adjustment.

VIII.1 Real Income

Real income consistent with the location-choice model is defined in equation (8),

$$\text{Real Income}_i = \frac{w_i}{\left(\mathbb{P}_i^T\right)^{\mu_i}\left(\mathbb{P}_i^H\right)^{1-\mu_i}}$$

I combine housing price indices, housing expenditure share, and composition-adjusted average household income data. I estimate MSA-specific housing expenditure shares based on rental data from the IPUMS ACS data (average $\mu_i = 0.35$).[66]

I assume that other consumption prices do not vary across cities. The literature has found that local housing costs explain a significant fraction of local prices. As a result, I only consider spatial variation in housing prices and set $\mathbb{P}_i^T = 1$. Diamond and Moretti estimate that local housing rents explain 89% of the variation high-income local prices and 96% of low-income local prices. For middle-income workers, the elasticity of local prices with respect to housing rents is 0.36, which is the same elasticity that Albouy (2011) estimates. Thus, my estimated housing expenditure share (average $\mu_i = 0.35$) is similar to the extrapolation of housing expenditure shares to other local prices in the literature.

In contrast to previous results, real income is increasing in MSA population after accounting for housing variety. Figure 8 plots the population elasticity of nominal wages, real income implied by a hedonic approach, and real income implied by the CES-RW price index. The elasticity of nominal wages with respect to population is 0.05, consistent with previous estimates (see Rosenthal and Strange (2004) for a review). After accounting for housing prices using the standard hedonic approach, real income is decreasing in population (with an elasticity of -0.017). This elasticity is consistent with previous estimates, including Albouy (2011, 2016).

After accounting for increased housing variety availability in larger cities, real income is increasing in population with an elasticity of 0.017. Thus, households are systematically compensated in larger cities with higher real incomes.

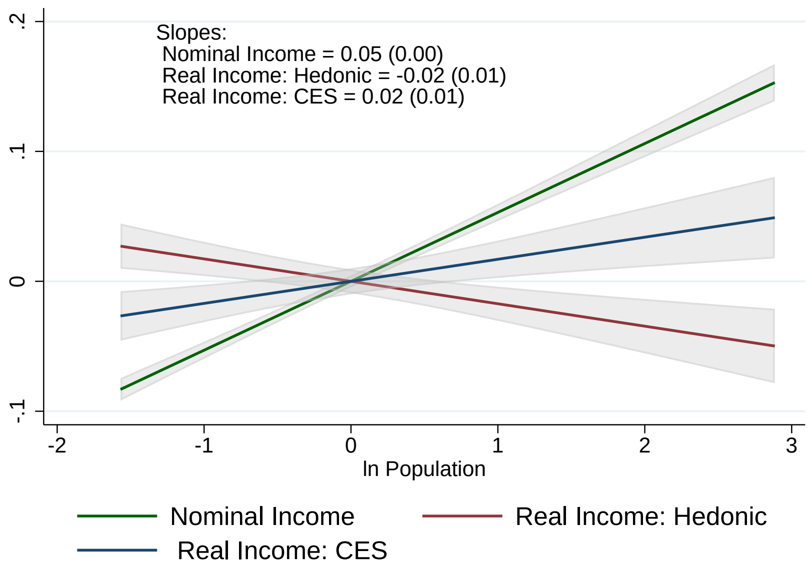
VIII.2 Amenities

Given population shares, wages, and the housing price indices, I can use the location-choice probabilities in the first stage of the model, (9) to recover the amenities of each MSA. An important parameter is ν that governs the idiosyncratic preference dispersion over locations. The relative amenity of MSA i against a comparison MSA C is

$$\ln B_{it} - \ln B_{Ct} = \frac{1}{\nu}(\ln L_{it} - \ln L_{Ct}) - (\ln w_{it} - \ln w_{Ct} - \mu_i(\ln \mathbb{P}_{it}^H - \ln \mathbb{P}_{Ct}^H)) \tag{16}$$

[66] This average is slightly higher than the previous literature as I only consider the household income of the head of household and spouse (if present). Monte, Redding, and Rossi-Hansberg (2018) use a housing expenditure share of 0.4 based off the Bureau of Economic Analysis. Poole, Ptacek, and Verbugge (2005) discuss how the BLS computes housing costs in the CPI. The largest component of CPI are rents and owner-occupied equivalent rents, accounting for a 0.29 share in 2004 and 0.33 in 2021. Since housing expenditure shares differ across locations, I use a Törnqvist index where the log difference in local prices between locations i and n is equal to $\frac{\mu_i+\mu_n}{2}(\ln \mathbb{P}_i - \ln \mathbb{P}_n)$.

Figure 8: Population Elasticity of Nominal Wages and Real Income



Notes: Figure presents the regression of nominal income, real income implied by hedonic, and real income implied by CES-RW on demeaned log population. 95% confidence intervals for the standard error of predicted outcome variables are presented.

This nests a Rosen-Roback spatial equilibrium with perfect mobility and no idiosyncratic preferences by letting $\nu \to \infty$. The Rosen-Roback spatial equilibrium will have amenities exactly compensate for real income differences across MSAs.[67] As the baseline scenario, I assume a Rosen-Roback equilibrium.

Since standard hedonic approaches overestimate amenities in larger MSAs, any MSA characteristics that covary positively with population will have their valuation overestimated. I follow Albouy (2011) and consider six characteristics: worse air quality (median AQI), commute time, violent and property crime, cooling days, heating days, and precipitation days.[68] Using the recovered amenities, I regress them on these six characteristics in Table 8:

$$\ln \mathbb{B}_{it} = \beta \ln X_{it} + \gamma_t + \epsilon_{it}$$

I estimate a counter-intuitive result that the valuation for commute time is positive under a hedonic approach. This result is rationalized by the fact that larger cities have higher commute times but also higher amenities under a hedonic approach. However, once I accounting for housing variety, the sign on commute time reverses. As a result, a 1% increase in commute time is in equilibrium valued as -0.1% of real income. The other major change after accounting for housing

[67] With $\nu < \infty$, population shares also inform us of amenity differences: for two MSAs with the same housing prices and wages, a larger population share in one of the MSAs will be the result of higher amenities.

[68] Worse air quality is measured by the median Air Quality Index from the Environmental Protection Agency. Section K.1 contains a list of data sources. Results are robust to also using

Table 8: Valuation of Amenities

	Amenities: Hedonic	Amenities: CES-RW	Difference
ln Median AQI	-0.08**	-0.17***	-0.09***
	(0.03)	(0.03)	(0.01)
ln Commute Time	0.09*	-0.10*	-0.19***
	(0.04)	(0.04)	(0.01)
PCA of Violent and Property Crime	0.005	0.0009	-0.00
	(0.009)	(0.010)	(0.003)
ln Cooling Days	-0.07***	-0.08***	-0.01***
	(0.005)	(0.006)	(0.002)
ln Heating Days	-0.001	0.003	0.004**
	(0.005)	(0.005)	(0.002)
ln Days with Precip > 0.1inch	-0.13***	-0.15***	-0.02***
	(0.008)	(0.010)	(0.003)
N	1179	1179	1179
R2	0.363	0.396	0.602

Notes: Table presents regressions of recovered amenities based on a Rosen-Roback spatial equilibrium on six MSA characteristics. Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

variety is worse air quality (median AQI), where the coefficient doubles from -0.08 to -0.17. Thus, previous approaches underestimate the cost of worse air pollution.[69]

IX Housing Prices: Heterogeneity by Income

In the last section of the paper, I ask how housing costs vary by household income. Given the mortgage data, I am able to measure transaction-level household income. Consistent with priors, I find that high-income households benefit almost three times more from the increased housing variety in larger MSAs than low-income households. In Appendix K.4, I use the income-specific price indices to measure updated real income by skill group that accounts of housing variety. In contrast to Diamond and Moretti (2021), I find that real income is underestimated in larger MSAs for households across all skill groups. The increase in the elasticity of real income with respect to MSA population for each skill group is similar to the aggregate change of 0.03.

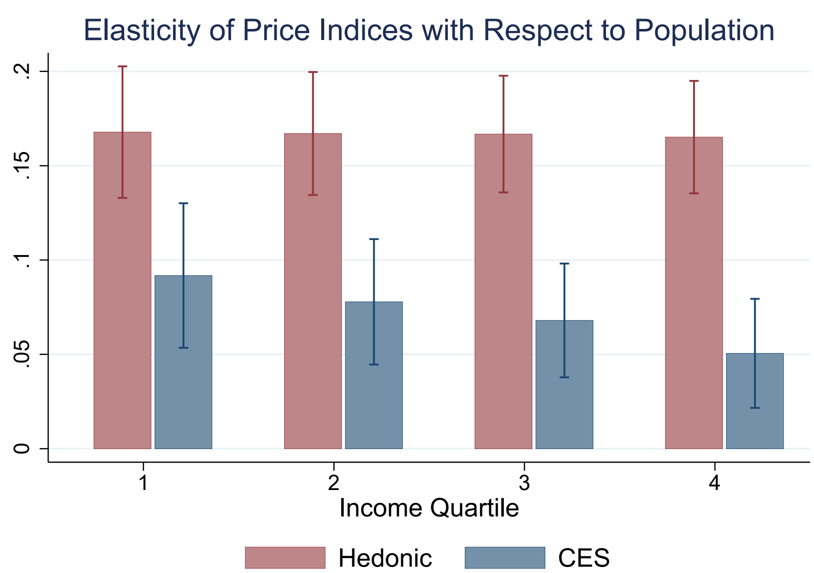
To maintain tractability, I separate transactions into quartiles of the income distribution each year and estimate income quartile-specific price indices. By using income quartile-specific expen-

[69]Consistent with Albouy (2011), I find that crime does not covary with recovered amenities. This points potentially to the highly local nature of crime. Heating days has a close to zero valuation, while cooling days has a negative valuation. Finally, precipitation days have a significant negative amenity value. Assuming a finite preference elasticity over MSAs will result in different levels of amenities and different valuations. However, the change in valuation (third column), will be the same for all values of ν.

diture shares, the price indices allow quality and taste shocks to differ for the same variety by income-quartile. As a robustness check, I also allow elasticities of substitution to vary by income-quartile and use income-specific variety prices. I find similar results as with my baseline approach. Thus, differences in housing costs by income quartile are driven by variation in the types of housing varieties that households in different income quartiles purchase.

I find that households in the fourth quartile of income benefit almost three times more from the availability of housing varieties in larger MSAs relative to households in the first quartile of income. Figure 9 shows that the increased variety benefit in larger cities for high income households leads to a lower elasticity of housing costs with respect to population. In contrast, a hedonic approach yields a similar elasticity of housing prices with respect to population for all income quartiles. These results are consistent with high income households being able to afford a wider variety of housing.[70]

Figure 9: Population Elasticity by Income Quartiles



Notes: Figure presents the estimated population elasticity for four income quartiles. 95% CI bars presented.

In Appendix K.4, I find that previous approaches underestimate real income in larger MSAs for households across all skill groups. To do so, I map skill groups to income quartiles and estimate income-quartile specific housing costs and housing expenditure shares. I find that my estimates of the population elasticity of real income implied by a hedonic index are consistent with estimates from Diamond and Moretti (2021). Accounting for housing variety differences then causes the population elasticity of real income to increase by 0.03 all three skill groups. As a result, high-skill households have even higher real incomes than previously measured in larger MSAs. At the same

[70]Results are robust to using income-quartile specific prices for the hedonic approach. Lower housing costs for high-skill households also has implications for how we think about changes in real income and utility from the increased high skill sorting into large cities since the 1980s (Moretti 2013, Diamond 2016).

time, low-skill households face a smaller decline in real income when moving from a smaller to a larger MSA than previously estimated.

X Conclusion

This paper estimates housing costs across metropolitan areas that account for differences in the set of available housing varieties. When households have idiosyncratic preferences over housing varieties, increased variety availability allow households to find a better match to their ideal type. I find that in the cross-section, the increased availability in larger MSAs substantially revises downwards housing costs in larger MSAs. Since housing is a third of household expenditure, housing costs that account for variety differences have significant impacts on real income and residual amenities recovered under a spatial equilibrium.

To generate flexible substitution patterns, this paper proposes a new method to jointly estimate both the nesting structure and elasticities of substitution. I find that a nested CES generates important heterogeneity and differences in the variety adjustment compared to the single nest. The estimation approach I develop can be applied to other settings, especially ones without rich data on product characteristics. In international trade, this is especially important since 1) trade flows are aggregated to categories that reflect the objectives of policy makers (Grant, 2022) and 2) the Armington assumption that varieties differentiated by country of origin belong to a single nest, which restricts how consumers substitute across import varieties.

Housing costs that account for variety differences have implications of a wide range of spatial and housing research. These include how we think about changes in spatial real inequality (Moretti 2013, Diamond 2016), the costs of zoning in restricting the variety of housing, and the production function of housing. The standard approach in the housing production literature has considered the production of single family homes and estimated constant returns to scale (see e.g., Epple, Gordon, and Sieg 2010, Combes, Duranton, and Gobillon 2021). If varieties are important, then the production of non-single family units has important implications. Furthermore, systematic new housing construction over time is an important dimension of housing variety, consistent with the link between durable housing and long-run population dynamics (Glaeser and Gyourko 2005).

An important avenue of future research is to measure variety changes over time. How did variety availability change during the housing downturn in 2006-2011 and the subsequent boom? Broda and Weinstein (2010) and Aghion et al. (2019) have pointed out the importance of changing varieties over time. In the aggregate and by region, an important avenue is to analyze how housing variety adjustments and quality changes over time impact measured inflation.

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Appendix A Stylized Model of Differentiated Housing Varieties

I develop a stylized model of housing construction with differentiated housing varieties. I show that the stylized model generates a higher number of varieties in higher population locations. To ensure tractability, the model focuses on a long-run symmetric equilibrium.[71]

Following Ahlfeldt et al. (2015) and Epple et al. (2010), construction firms create residential floor area by combining capital K and land M with a Cobb-Douglas production technology. The total square feet constructed in location i for variety m is equal to $q_{iv} = K_{iv}^{\alpha} M_{iv}^{1-\alpha}$, where $0 < \alpha < 1$.

The price of capital $\mathbb{K}$ is assumed to be constant across space, while the price of land $\mathbb{R}_i$ is location-specific. Given the constant-returns to scale Cobb-Douglas production technology, the location-specific marginal cost of producing is equal to $\mathbb{Q}_i = \kappa \mathbb{K}^{\alpha} \mathbb{R}_i^{1-\alpha}$, where κ is a scalar. Note that the marginal cost does not differ across varieties within a location.

In contrast to the previous literature, I assume that construction firms operate in an imperfectly competitive market and face a fixed cost F_{iv} of constructing a variety. These fixed costs entail the design of blueprints, structural safety design, and training of construction crews that are specific to each variety v.

For a firm that decides to construct a variety v, their profit maximization problem is

$$\max_{p_{iv}} \quad p_{iv} q_{iv} - \mathbb{Q}_i q_{iv} - w_i F_{iv}$$

where p_{iv} is the price per square feet. I have assumed that the fixed costs are incurred in wages rather than the numeraire.

I assume that households have symmetric CES preferences over housing varieties and spend a constant μ share of their household income on housing. For simplicity, I assume that varieties are symmetric and there is a single nest over all varieties. The demand for variety v is given by

$$q_{iv} = p_{iv}^{-\sigma} \mathbb{P}_i^{\sigma-1} \mu w_i L_i$$

Construction firms set prices such that the price of variety v is a constant mark-up over marginal costs

$$p_{iv} = p_i = \frac{\sigma}{\sigma - 1} \mathbb{Q}_i \tag{17}$$

With free entry and the assumption that fixed costs are the same for all varieties v, the zero-profit condition results in

$$q_{iv} = \bar{q}_i = \frac{(\sigma - 1) w_i F_i}{\mathbb{Q}_i}$$

Due to symmetry, the scale of each firm (i.e., square feet produced of each variety) is the same across all varieties. Higher land prices result in higher marginal costs, $\mathbb{Q}_i$, and higher housing prices, leading to a lower total supply of each variety.

To determine the number of equilibrium varieties N_i, we can combine the CES demand with

[71] As a result, I abstract from the secondary-home market.

the zero-profit condition (similar to Almagro and Dominguez-Iino 2022),

$$N_i = \frac{\mu L_i}{\sigma F_i} \tag{18}$$

This result uses the fact that the housing price index in a symmetric equilibrium can be expressed as a function of the number of varieties and the variety price, $\mathbb{P}_i = N_i^{\frac{1}{1-\sigma}} p_i$.

In equilibrium, larger populations result in a larger number of housing varieties. Notice that if we did not assume that fixed costs are incurred in wages, but instead in the numeraire, then $N_i = \frac{\mu w_i L_i}{\sigma F_i}$, so that the number of varieties is increasing in the total housing expenditure rather than only population size.

Any location-specific increases in fixed costs (for example location-specific zoning regulations that restrict building of multi-unit structures), will also reduce the number of varieties in a location.

A.1 Spatial Equilibrium

I now embed the construction sector in a spatial equilibrium. I consider a set of locations $i = 1, ..., I$. In each location, there is a tradable sector that produces a single homogeneous good in perfect competition. The tradable good is set as the numeraire, so the price of the tradable good is $\mathbb{T} = 1$. The tradable sector faces a common location-specific productivity A_i and uses labor as its only input. As a result, wages are pinned down by local productivity, $w_i = A_i$.[72]

In each location, there is a fixed quantity of land M_i. By cost-minimization, we know that the total expenditures on land must equal $(1-\alpha)$ share of variable costs in the production of residential floor space

$$M_i \mathbb{R}_i = (1-\alpha)\bar{q}_i \mathbb{Q}_i N_i$$

Solving this yields an expression for local land costs

$$\mathbb{R}_i = (1-\alpha)\mu \frac{\sigma - 1}{\sigma} \frac{w_i L_i}{M_i} \tag{19}$$

Local land prices are pinned down by expenditures on land divided by the stock of land. The construction sector receives μ share of local expenditure, $w_i L_i$. Out of this share, $\frac{\sigma-1}{\sigma}$ goes toward variable costs, out of which $(1-\alpha)$ share goes to land.[73]

I assume that household utility in location i is given by

$$\mathbb{U}_i = \frac{A_i}{\mathbb{P}_i^{\mu} \mathbb{T}^{1-\mu}} B_i$$

[72]Note that A_i may include both endogenous and exogenous factors. For example, if there are agglomeration forces in production, then $A_i = L_i^{\gamma_L} \tilde{A}_i$.

[73]I assume that capital and land-owners are absentee.

where B_i are location-specific amenities. The utility can be expressed in terms of fundamentals as

$$\mathbb{U}_i = \kappa_1 \frac{A_i^{1-\mu(1-\alpha)} B_i L_i^{\gamma}}{F_i^{\frac{\mu}{\sigma-1}} M_i^{(\alpha-1)\mu}}$$

where κ_1 is a scalar that does not differ across locations and $\gamma = \frac{\mu}{\sigma-1} - (1-\alpha)\mu$. A higher variety fixed cost lowers the utility from a location, while more of the fixed land factor increases the utility from a location.

γ reflects two forces: there is a positive agglomeration force from housing variety and a congestion force through higher land prices. When the share of land costs is high in the production of square feet of floor space and when housing varieties are more substitutable, then $\gamma < 0$ so that the net effect of population is negative.[74]

A spatial equilibrium is then defined such that for all locations $i \in 1, ..., I$

$$\kappa_1 \frac{A_i^{1-\mu(1-\alpha)} B_i L_i^{\gamma}}{F_i^{\frac{\mu}{\sigma-1}} M_i^{(\alpha-1)\mu}} = \bar{\mathbb{U}} \tag{20}$$

In each location, land prices, housing variety prices, the number of varieties, and the location housing price index can be determined from (17), (18), and (19).

A.2 Uniqueness and Stability

The sum of the populations across locations is equal to the total population

$$\sum_i L_i = \bar{L}$$

Note that the spatial equilibrium condition can be re-written as

$$L_i = (\bar{\mathbb{U}} \kappa_i^*)^{\frac{1}{\gamma}}$$

where $\kappa_i^* = \left(\kappa_1 \frac{A_i^{1-\mu(1-\alpha)} B_i}{F_i^{\frac{\mu}{\sigma-1}} M_i^{(\alpha-1)\mu}} \right)^{-1}$. $\mathbb{U}$ can then be obtained as the solution to

$$\sum_i (\bar{\mathbb{U}} \kappa_i^*)^{\frac{1}{\gamma}} = \bar{L}$$

for arbitrary $\kappa_i^* > 0$ and $\bar{L} > 0$. Returning to (20) results in the interior equilibrium

$$\frac{L_i}{\bar{L}} = \frac{(\kappa_i^*)^{\frac{1}{\gamma}}}{\sum_{i'} (\kappa_{i'}^*)^{\frac{1}{\gamma}}}$$

[74] Combes et al. (2021) estimate $1 - \alpha = 0.35$. As long as $\sigma > 2.54$, then $\gamma < 0$.

When $\gamma < 0$, the congestion forces outweigh the agglomeration force. As a result, higher productivity and higher amenity locations lead to both larger populations and higher land prices, which then equalize real utility across locations.[75]

However, following Krugman (1991) and Allen and Arkolakis (2014), the equilibrium when $\gamma > 0$ is not stable. When $\gamma > 0$, a small increase in population will increase the indirect utility of a location, leading to further population inflows and an unstable equilibrium. With $\gamma < 0$, $\frac{\partial \mathbb{U}_i}{\partial L_i} < 0$ so that the interior equilibrium is stable.

Appendix B Joint Location and Housing Choice

I show that the results of the main text are robust to a model where households jointly choose housing variety and location. Note that rather than the timing assumption of idiosyncratic draws, the parallel assumption that drives this result is that the idiosyncratic preference draws for a variety v in location i are not systematically correlated with preference draws for the same variety v in a different location i'.

A household j obtains the following indirect utility from choosing a housing variety v in a location i

$$u_{ivj} = \ln \mathbb{B}_i + \mu_i \left(\ln Q^H_{ivj} + \ln \varphi_{iv} \right) + (1 - \mu_i) \ln Q^T_{ivj} + \varepsilon_{ivj} \tag{21}$$

where Q^H_{ivj} denotes the quantity of square feet of housing obtained by household j

$$\ln Q_{ivj} \equiv \ln(\mu_i w_{ij}) - \ln p_{iv}$$

and Q^T_{ivj} denotes the quantity of other consumption obtained by household j

$$\ln Q_{ivj} \equiv \ln \left((1 - \mu_i) w_{ij} \right) - \ln \mathbb{P}^T_i$$

so that the utility from choosing a location i is increasing in the location-specific amenity and wage, decreasing in the housing price for variety v in location i, increasing in the quality of variety v, and decreasing in other consumption prices $\mathbb{P}^T_i$.

ε_{ivj} is distributed i.i.d across households from a three-layer nested logit

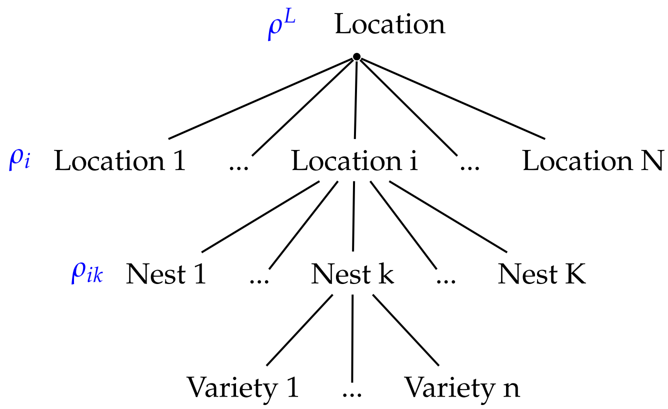
$$F(\vec{\varepsilon}) = \exp \left[- \sum_{i \in 1,\ldots,N} \left(\sum_{k \in \Omega^K_i} \left(\sum_{v \in \Omega_{ik}} e^{-\frac{\epsilon_{ivj}}{\rho_{ik}}} \right)^{\frac{\rho_{ik}}{\rho_i}} \right)^{\frac{\rho_i}{\rho^L}} \right]$$

where $0 \leq \rho_{ik} \leq \rho_i \leq \rho^L$.

This nested logit can be visualized as

[75] Notice that when $\gamma < 0$, the interior equilibrium will be the unique equilibrium since every location will be populated. In a location that has zero population, a household has an incentive to move there and consume the fixed land supply.

Figure A1: Nested Logit Demand System



Following standard nested logit derivations, the probability of picking variety v in location i is given as

$$Pr_j(iv) = \frac{\tilde{u}_{ivj}^{\frac{1}{\rho_{ik(v)}}}}{\sum_{v \in \Omega_{ik(v)}} \tilde{u}_{ivj}^{\frac{1}{\rho_{ik(v)}}}} \frac{\left(\tilde{u}_{ik(v)j}^K\right)^{\frac{\rho_{ik(v)}}{\rho_i}}}{\sum_{k \in \Omega_i^K} \left(\tilde{u}_{ij}^L\right)^{\frac{\rho_{ik(v)}}{\rho_i}}} \frac{\left(\tilde{u}_{ij}^L\right)^{\frac{\rho_i}{\rho^L}}}{\sum_{i \in 1,\dots,N} \left(\tilde{u}_{ij}^L\right)^{\frac{\rho_i}{\rho^L}}}$$

where

$$\tilde{u}_{ivj} = \left(\frac{\mathbb{B}_i w_{ij}}{\left(\mathbb{P}_i^T\right)^{1-\mu_i}}\right) \left(\frac{\varphi_{iv}}{p_{iv}}\right)^{\mu_i}$$

$$\tilde{u}_{ik(v)j}^K = \sum_{v \in \Omega_{ik(v)}} \tilde{u}_{ivj}^{\frac{1}{\rho_{ik(v)}}}$$

$$\tilde{u}_{ij}^L = \sum_{k \in \Omega_i^k} \left(\tilde{u}_{ikj}^K\right)^{\frac{\rho_{ik(v)}}{\rho_i}}$$

The probability of picking location i is then given as

$$\begin{aligned} Pr_j(i) &= \sum_{k \in \Omega_i^K} \sum_{v \in \Omega_{ik}} Pr_j(iv) \\ &= \frac{\left(\tilde{u}_{ij}^L\right)^{\frac{\rho_i}{\rho^L}}}{\sum_{i \in 1,\dots,N} \left(\tilde{u}_{ij}^L\right)^{\frac{\rho_i}{\rho^L}}} \\ &= \frac{1}{\sum_{i \in 1,\dots,N} \left(\tilde{u}_{ij}^L\right)^{\frac{\rho_i}{\rho^L}}} \left(\frac{\mathbb{B}_i w_{ij}}{\left(\mathbb{P}_i^T\right)^{1-\mu_i}}\right)^{\frac{1}{\rho^L}} \left(\sum_{k \in \Omega_i^K} \left(\sum_{v \in \Omega_{ik}} \left(\frac{\varphi_{iv}}{p_{iv}}\right)^{\frac{\mu_i}{\rho_{ik}}}\right)^{\frac{\rho_{ik}}{\rho_i}}\right)^{\frac{\rho_i}{\rho^L}} \end{aligned}$$

Let

$$\rho_{ik} = \frac{\mu_i}{\sigma_k - 1}$$

$$\rho_i = \frac{\mu_i}{\sigma - 1}$$

$$\rho^L = \frac{1}{\nu}$$

Then

$$Pr_j(i) = \frac{\left(\frac{\mathbb{B}_i w_{ij}}{\left(\mathbb{P}_i^T\right)^{1-\mu_i}\left(\mathbb{P}_i^H\right)^{\mu_i}}\right)^\nu}{\sum_{i\in 1,\dots,N}\left(\frac{\mathbb{B}_i w_{ij}}{\left(\mathbb{P}_i^T\right)^{1-\mu_i}\left(\mathbb{P}_i^H\right)^{\mu_i}}\right)^\nu}$$

This coincides with the choice probability generated with the timing assumption. Notice that unlike the timing assumption, this model does not nest a Rosen-Roback equilibrium since ν has satisfy $\nu\mu_i \leq \sigma - 1$ so that we cannot let $\nu \to \infty$.

What is the expenditure on each variety in the housing market? The probability of choosing variety v conditional on choosing location i is the same across all households

$$Pr_j(v;i) = Pr(v;i) = \frac{\tilde{U}_{iv}^{1-\sigma_{k(v)}}}{\sum_{v\in\Omega_{ik(v)}} \tilde{U}_{iv}^{1-\sigma_{k(v)}}} \frac{\left(\tilde{U}_{ik(v)}^K\right)^{\frac{1-\sigma}{1-\sigma_{k(v)}}}}{\sum_{k\in\Omega_i^K}\left(\tilde{U}_{ik(v)}^K\right)^{\frac{1-\sigma}{1-\sigma_{k(v)}}}}$$

where

$$\tilde{U}_{iv} = \left(\frac{p_{iv}}{\varphi_{iv}}\right)$$

$$\tilde{U}_{ik(v)}^K = \sum_{v\in\Omega_{ik(v)}} \tilde{U}_{iv}^{1-\sigma_k}$$

Since this probability is conditional on location, location-specific amenities and non-housing consumption prices drop out. The probability also do not depend on household income since income does not impact the choice of a variety v within location i (notice that $\ln w_{ij}$ is the same across varieties v in location i in the indirect utility).

As a result, the expected expenditure on variety v in location i is given as

$$E_{iv} = \sum_{i_j^*=i} Pr_j(v;i) p_{iv} Q_{ivj} = Pr(v;i) \sum_{i_j^*=i} \mu_i w_{ij}$$

This gives the exact same formulation as the market level expenditure in the main text.

Appendix C Fréchet Micro-Foundation

I now consider a two-stage decision process with a Fréchet micro-foundation that yields the same CES housing price index in the first stage. With a Fréchet micro-foundation, the exponential becomes unnecessary.

Second Stage

In the second stage, household j has chosen location i and housing expenditures E_{ij}^H. The household chooses a housing variety m and the quantity of square feet q_{im}

$$v_j^*, q_{ivj}^* = \arg\max_{v, q_{ivj}} \ u_{ivj} = \arg\max_{v, q_{ivj}} \ q_{ivj}\varphi_{iv}\varepsilon_{ivj} \tag{22}$$

$$s.t. \quad p_{iv}q_{ivj} \leq E_{ij}^H$$

where φ_{iv} represents the quality of housing variety v. u_{ivj} can be interpreted as a quality-adjusted square feet of housing services. ε_{ivj} is drawn from a max-stable multivariate Fréchet distribution following Lind and Romando (2021). A max-stable multivariate Fréchet distribution is defined as a vector of draws ε_{ivj} if for any $\alpha_v \geq 0$, the distribution of $\max_{v=1,...,M} \alpha_v \varepsilon_{ivj}$ is Fréchet with a shape parameter $\theta > 1$. Each housing variety's marginal distribution is Fréchet with shape parameter θ and scale parameter T_v.

Lind and Romando (2021) show that ε_{ivj} is a max-stable multivariate Fréchet distribution if and only if the joint CDF is given by

$$\mathbb{P}\left[\varepsilon_{i1j} \leq z_1, \ldots, \varepsilon_{iMj} \leq z_M\right] = \exp\left[-G_i\left(T_1 z_1^{-\theta}, \ldots, T_M z_M^{-\theta}\right)\right]$$

where G_i is the correlation function that satisfies the standard GEV properties and a normalization property:

$$G(0, \ldots, 0, 1, 0, \ldots, 0) = 1$$

The household will optimally choose $q_{ivj}^* = \frac{E_{ij}^H}{p_{iv}}$ so that the household's housing choice problem can be rewritten as

$$\arg\max_{v} \ \frac{E_{ij}^H}{p_{iv}}\varphi_{iv}\varepsilon_{ivj}$$

Let $T_1 = \ldots T_M = 1$. Notice that the CDF of the maximum of $u_{iv}(j)$ is also Fréchet

$$Pr\left[\max_m u_{ivj} \leq u\right] = Pr\left[\varepsilon_{i1j} \leq u\left(\frac{E_{ij}^H}{p_{i1}}\varphi_{i1}\right)^{-1}, \ldots, \varepsilon_{iM}(j) \leq u\left(\frac{E_{ij}^H}{p_{iM}}\varphi_{iM}\right)^{-1}\right]$$

$$= \exp\left[-G_i\left(\left(\frac{\varphi_{i1}}{p_{i1}}\right)^{\theta}, \ldots, \left(\frac{\varphi_{iM}}{p_{iM}}\right)^{\theta}\right) u^{-\theta}\left(E_{ij}^H\right)^{\theta}\right] \tag{23}$$

A specific G_i function that corresponds to the nested logit is

$$G_i(\vec{z}) = \sum_{k \in \Omega_i^K}\left(\sum_{v \in \Omega_{ik}} z_{iv}^{\frac{\theta_k}{\theta}}\right)^{\frac{\theta}{\theta_k}}$$

The expected value of the maximum utility will be equal to

$$E[\max_v u_{ivj}] = E_{ij}^H \Gamma \left(1 - \frac{1}{\theta}\right) \left(\sum_{k \in \Omega_i^K} \left(\sum_{v \in \Omega_{ik}} \left(\frac{\varphi_{iv}}{p_{iv}} \right)^{\theta_k} \right)^{\frac{\theta}{\theta_k}} \right)^{\frac{1}{\theta}} \propto \frac{E_{ij}^H}{\mathbb{P}_i^H}$$

where $\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx$. Let $\theta = \sigma - 1$ and $\theta_k = \sigma_k - 1$. Notice that the restriction of $\theta > 1$ is equivalent to $\sigma > 2$. The probability of purchasing any variety v will then coincide with the probability generated by the logit micro-foundation in the main paper.

First Stage

In the first stage, household j chooses a location i and decides how to allocate income between tradable and housing expenditures

$$\max_{i, E_{ij}^H, E_{ij}^T} \mathcal{U}_{ij} = \max_{i, E_{ij}^H, E_{ij}^T} \mathbb{B}_i \left(U_i^T(E_i^T) \right)^{\mu} \left(U_i^H(E_{ij}^H) \right)^{1-\mu} z_{ij}$$

$$s.t. \quad E_{ij}^H + E_{ij}^T \leq w_{ij}$$

where $U_i^T(E_{ij}^T)$ is the quantity of tradable consumption as a function of tradable expenditure, $U_i^H(E_{ij}^H)$ is the expected utility of housing services as a function of housing expenditure, $\mathbb{B}_i$ is the amenity of location i, and z_{ij} are idiosyncratic preference draws from a Fréchet distribution with shape parameter $\nu > 1$ over locations.

For tradable consumption, there is a tradable price index $\mathbb{P}_i^T$ in location i so that

$$U_i^T(E_i^T) \equiv \frac{E_i^T}{\mathbb{P}_i^T}$$

I assume that housing prices, quality, and the set of available varieties in each location are known to the household but the idiosyncratic housing draws ε are not yet realized. H_i can be interpreted as the expected quality-adjusted square feet of housing services since the utility in the second stage depends on the quantity of square feet and the quality of the housing variety (along with the idiosyncratic shock). Note that the order of expectation can be relaxed, as detailed subsequently.

Specifically, $U_i^H(E_{ij}^H)$ is equal to the expected value of the optimal housing choice:

$$U_i^H(E_{ij}^H) \equiv E[\max_v u_{ivj}] = \frac{E_{ij}^H}{\mathbb{P}_i^H}$$

Thus, the exponential transform is unnecessary in contrast to the logit formulation.

Expectation Order in Second Stage of Fréchet Micro-Foundation

Alternatively, I can consider the optimization problem where the expectation is taken over the entire indirect utility

$$\max_{i,E_i^H,E_i^C} E\left[\mathcal{U}_i(j)\right] = \max_{i,E_i^H,E_i^C} \mathbb{B}_i \left(T_i(E_i^T)\right)^{\mu} E\left[\max_m u_{im}(E_i^H)^{1-\mu}\right] z_i(j)$$

$$s.t. \quad E_i^H + E_i^C \leq w_i(j)$$

where $u_{im}(E_i^H)$ is

$$u_{im}(E_i^H) = \frac{E_i^H}{p_{im}} \varphi_{im} \varepsilon_{im}$$

corresponds to the indirect utility defined in (22). Thus, the CDF of the maximum housing utility $u_{im}(E_i^H)^{1-\mu}$ is

$$\begin{aligned} Pr\left(u_{im}(E_i^H)^{1-\mu} \leq u, \forall v \in \Omega_i\right) &= Pr\left(u_{im}(E_i^H) \leq u^{\frac{1}{1-\mu}}\right) \\ &= \exp\left[-G_i\left(\left(\frac{\varphi_{i1}}{p_{i1}}\right)^{\theta}, \ldots, \left(\frac{\varphi_{iM}}{p_{iM}}\right)^{\theta}\right) u^{-\frac{\theta}{1-\mu}} E_i^H(j)^{\theta}\right] \end{aligned}$$

where I used (23) to derive the second line. Thus, this is a Fréchet distribution with shape parameter $\frac{\theta}{1-\mu}$ (notice that this property also is used in the derivation of the price index in a standard Eaton-Kortum international trade model). The expected value of this random variable is then

$$E[\max_m u_{im}(j)^{1-\mu}] = E_i^H(j)\Gamma\left(1 - \frac{1-\mu}{\theta}\right)\left(\sum_{k\in\Omega_i^K}\left(\sum_{v\in\Omega_{ik}}\left(\frac{\varphi_{im}}{p_{im}}\right)^{\theta_k}\right)^{\frac{\theta}{\theta_k}}\right)^{\frac{1-\mu}{\theta}}$$

This leads to the same optimization problem as before

$$\max_{i,E_i^H,E_i^C} E\left[\mathcal{U}_i(j)\right] = \max_{i,E_i^H,E_i^C} \quad \mathbb{B}_i \left(\frac{E_i^C}{\mathbb{T}_i}\right)^{\mu} \left(\frac{E_i^H}{\mathbb{P}_i}\right)^{1-\mu} z_i(j)$$

$$s.t. \quad E_i^H + E_i^C \leq w_i(j)$$

Simplified CES Model

A simplified version of the model consists of a household j having a location, consumption, and housing decision given by

$$\max_{i,C_i,\{h_{iv}\}} B_i z_i(j) C_i^{\mu} \mathcal{H}_i^{1-\mu}$$

$$s.t. \quad \mathbb{T}_i C_i + \mathbb{P}_i \mathcal{H}_i \leq w_i(j)$$

where $\mathcal{H}_i$ is a composite over housing services from all varieties available, defined by

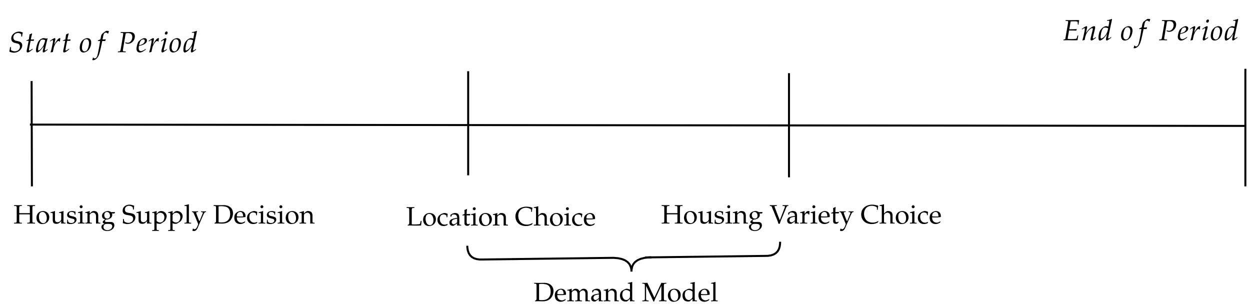
$$\mathcal{H}_i = \left(\sum_{k \in \Omega_i^K} \left(\sum_{v \in \Omega_{ik}} (\varphi_{iv} h_{iv})^{\frac{\sigma_k - 1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k - 1} \frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}}$$

where h_{iv} is square feet of housing variety v consumed in location i. Notice that with prices of each variety given by p_{iv} and the condition that $\sum_{v \in \Omega_i} p_{iv} h_{iv} = \mathbb{P}_i \mathcal{H}_i$, then the aggregate demand for a housing variety over all households is given by (10). Although this model is substantially simpler than the main model, in reality households do not consume a composite housing good. Rather, households choose to consume a certain variety, which the main model is able to capture.

Appendix D Market Equilibrium and Rational Expectations

Figure A2 summarizes the timing of the supply and demand decisions. At the beginning of the period, households that currently own a house decide whether to sell their house, determining the supply of each housing variety available in each location.

Figure A2: Model Timing



D.1 Supply

At the beginning of the period, households form an expected price of the housing variety v in MSA i at time t that is consistent with rational expectations of the market equilibrium. Given the household's information set at the beginning of the period t, denoted by $\mathcal{I}_t$, the supply of square feet of variety m in location i at time t is given by

$$\ln q^S_{ivt}(\mathcal{I}_t, \tilde{\omega}_{ivt}; \beta) = \beta_i E[\ln p^*_{ivt} | \mathcal{I}_t] + E[\ln \omega_{ivt} | \mathcal{I}_t] + \ln \tilde{\omega}_{ivt} \tag{24}$$

where the first term on the right hand side captures the endogenous quantity response to expected prices: p^*_{ivt} denotes the equilibrium variety price and β_i is the MSA-specific elasticity of quantity supplied with respect to expected prices. The second and third term on the right hand side capture supply shocks, where $E[\ln \omega_{ivt} | \mathcal{I}_t]$ are expected shocks to the quantity supplied of floor space and

$\ln \tilde{\omega}_{ivt}$ are unexpected shocks, so that

$$E[\ln \tilde{\omega}_{ivt} | \mathcal{I}_t] = 0$$

This reduced form supply equation nests a specific case where there is a fixed supply of q_{ivt} square feet of floor space available for sale within the short run (i.e., where $\beta_i = 0$).[76]

D.2 Demand

Total square feet demand for a housing variety v in an MSA i at time t is given by

$$q_{ivt}(\mathbf{p}_{it}, \varphi_{it}; \sigma) = \underbrace{\varphi_{ivt}^{\sigma_k - 1} \frac{p_{ivt}^{-\sigma_k}}{\mathbb{P}_{ikt}^{1-\sigma_k}}}_{P_{m|k(m)}} \underbrace{\frac{\mathbb{P}_{ikt}^{1-\sigma}}{\mathbb{P}_{it}^{1-\sigma}}}_{P_{k(m)}} E_{it}^{H} \tag{25}$$

$$\mathbb{P}_{ikt} = \left(\sum_{v \in \Omega_{ikt}} \left(\frac{p_{ivt}}{\varphi_{ivt}} \right)^{1-\sigma_k} \right)^{\frac{1}{1-\sigma_k}}, \mathbb{P}_{it} = \left(\sum_{k \in \Omega_{it}^{K}} \mathbb{P}_{ikt}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

D.3 Equilibrium

The market clearing prices in location i and time t for housing varieties are implicitly defined by the prices such that the entire square feet of floor space of each housing variety is purchased by households,

$$\ln q_{ivt}^{S}(\mathcal{I}_t, \tilde{\omega}_{ivt}; \beta) = \ln q_{ivt}(\mathbf{p}_{it}^{*}, \varphi_{it}; \sigma), \qquad \forall v \in \Omega_{it}$$

where q_{ivt} is given in (25).

The equilibrium prices are an implicit function of both the endogenous supply response and exogenous supply shocks to floor space. Even though the supply of floor space of each variety does not explicitly depend on quality shocks φ_{it}, it will depend on the homeowner's information set and expectations of the market equilibrium. As a result, the supply of housing will ultimately depend on the expected quality shock.

[76] The supply decision can be extended in two key ways. First, households can care about the relative expected price of their variety versus a location average, so that

$$\ln q_{ivt}(\mathcal{I}_t, \tilde{\omega}_{ivt}; \beta) = \beta_i \left(E[\ln p_{ivt}^{*} | \mathcal{I}_t] - E[\ln P_{it}^{*} | \mathcal{I}_t] \right) + E[\ln \omega_{ivt} | \mathcal{I}_t] + \ln \tilde{\omega}_{ivt}$$

where $E[\ln P_{it}^{*} | \mathcal{I}_t]$ is the expected price index of all housing in location i. Second, suppose that households care about the expected utility from the demand model (from choosing to move and buying a new house).

$$\ln q_{ivt}(\mathcal{I}_t, \tilde{\omega}_{ivt}; \beta) = \beta_i E[\ln p_{ivt}^{*} | \mathcal{I}_t] + \lambda_i E\left[\max_{\ell} \mathcal{U}_{\ell t} | \mathcal{I}_t \right] + E[\ln \omega_{ivt} | \mathcal{I}_t] + \ln \tilde{\omega}_{ivt}$$

where $E\left[\max_{\ell} \mathcal{U}_{\ell t} | \mathcal{I}_t\right]$ is the expected utility from moving and buying a new home and λ_i is the MSA-specific response. Due to the set of fixed effects I include, both extensions are consistent with my estimation approach.

Substituting in demand from (25) and supply from (24) yields

$$(\sigma_k - 1)\ln\varphi_{ivt} - \sigma_k \ln p_{ivt} + (\sigma_k - \sigma)\ln \mathbb{P}_{ikt} + (\sigma - 1)\ln \mathbb{P}_{it} + \ln E^H_{it} = \beta_i E[\ln p_{ivt}|\mathcal{I}_t] + E[\ln \omega_{ivt}|\mathcal{I}_t] + \ln\tilde{\omega}_{ivt}$$

Re-arranging yields

$$\ln p_{ivt} = \frac{1}{\sigma_k}\left[(\sigma_k - \sigma)\ln \mathbb{P}_{ikt} + (\sigma - 1)\ln \mathbb{P}_{it} + \ln E^H_{it} + (\sigma_k - 1)\ln\varphi_{ivt} - \beta_i E[\ln p_{ivt}|\mathcal{I}_t] - E[\ln\omega_{ivt}|\mathcal{I}_t] - \ln\tilde{\omega}_{ivt}\right]$$

The higher the aggregate price indices, expenditure, quality appeal shock, the higher the price of variety m in equilibrium. At the same time, higher supply reduces the equilibrium price.

Taking the expected value of both sides in terms of the household's information set at the beginning of the period t results in

$$\begin{aligned} E[\ln p_{ivt}|\mathcal{I}_t] = E\left[\frac{1}{\sigma_k}\left((\sigma_k - \sigma)\ln \mathbb{P}_{ikt} + (\sigma - 1)\ln \mathbb{P}_{it} + \ln E^H_{it}(\sigma_k - 1) + \ln\varphi_{ivt}\right)|\mathcal{I}_t\right] \\ - E[\frac{\beta_i}{\sigma_k}\ln p_{ivt}|\mathcal{I}_t] - E\left[\frac{1}{\sigma_k}\ln\omega_{ivt}|\mathcal{I}_t\right] - E\left[\frac{1}{\sigma_k}\ln\tilde{\omega}_{ivt}|\mathcal{I}_t\right] \end{aligned} \tag{26}$$

Households thus anticipate the effect of their quantity supplied (based on their information set) on the equilibrium price at time t. Consistent with rational expectations, I assume $E[\ln\tilde{\omega}_{ivt}|\mathcal{I}_t] = 0$. As a result, I can re-arrange the expression to yield

$$\begin{aligned} E[\ln p^*_{ivt}|\mathcal{I}_t] =& \frac{\sigma_k - \sigma}{\sigma_k + \beta_i}E\left[\ln \mathbb{P}_{ikt}|\mathcal{I}_t\right] + \frac{\sigma - 1}{\sigma_k + \beta_i}E\left[\ln \mathbb{P}_{it}|\mathcal{I}_t\right] + \frac{1}{\sigma_k + \beta_i}E\left[\ln E^H_{it}|\mathcal{I}_t\right] \\ &+ \frac{\sigma_k - 1}{\sigma_k + \beta_i}E\left[\ln\varphi_{ivt}|\mathcal{I}_t\right] - \frac{1}{\sigma_k + \beta_i}E\left[\ln\omega_{ivt}|\mathcal{I}_t\right] \end{aligned} \tag{27}$$

The expected price given the information set at time t is increasing in the nest and overall price index, increasing in housing expenditures, increasing in the expected quality shock, and decreasing in the expected supply shock.

D.4 Identification

To see why the strategy alleviates simultaneity bias, I can combine the supply equation (24) and the rational expectation consistent price equation (27) to obtain the quantity supplied in terms of fundamentals

$$\begin{aligned} \ln q_{ivt}(\mathcal{I}_t, \tilde{\omega}_{ivt}; \beta) = & \frac{(\sigma_{k(v)} - \sigma)\beta_i}{\sigma_{k(v)} + \beta_i}E\left[\ln \mathbb{P}_{ik(m)t}|\mathcal{I}_t\right] + \frac{(\sigma - 1)\beta_i}{\sigma_{k(v)} + \beta_i}E\left[\ln \mathbb{P}_{it}|\mathcal{I}_t\right] + \frac{\beta_i}{\sigma_{k(v)} + \beta_i}E\left[\ln E^H_{it}|\mathcal{I}_t\right] \\ &+ \frac{(\sigma_{k(v)} - 1)\beta_i}{\sigma_{k(v)} + \beta_i}E\left[\ln\varphi_{ivt}|\mathcal{I}_t\right] + \frac{\sigma_{k(v)}}{\sigma_{k(v)} + \beta_i}E[\ln\omega_{ivt}|\mathcal{I}_t] + \ln\tilde{\omega}_{ivt} \end{aligned} \tag{28}$$

In the main text I discuss how the $\alpha_{ik(m)t}$, or the MSA X nest X year fixed effects absorb the first three terms. Next, I assume that households currently living in a unit of variety m will generate an expectation for the quality based on the average nest quality, the variety's average quality and the variety's average quality growth in their MSA. Specifically,

$$E[\ln \varphi_{ivt}|\mathcal{I}_t] = \ln \tilde{\varphi}^E_{ik(v)t} + \ln \tilde{\varphi}^E_{iv} + \ln \tilde{\varphi}^E_{iv} t$$

where $\ln \tilde{\varphi}^E_{ik(v)t}$ denotes the expected nest k quality, $\ln \tilde{\varphi}^E_{iv}$ denotes the expected variety quality that is constant over time, and $\ln \tilde{\varphi}^E_{iv} t$ denotes the expected appeal growth over time for variety v in MSA i.

I make an analogous assumption that the expectation of supply shocks is based on a nest component, variety component, and variety average growth in their MSA

$$E[\ln \omega_{ivt}|\mathcal{I}_t] = \ln \tilde{\omega}^E_{ik(v)t} + \ln \tilde{\omega}^E_{iv} + \ln \tilde{\omega}^E_{iv} t$$

Plugging (28) into the estimating equation (12) and having the fixed effects absorb the relevant components means that the identifying assumption can be rewritten as $E\left[(\ln \tilde{\omega}_{ivt})(\ln \tilde{\varphi}_{ivt})\right] = 0$, or that demand (i.e., quality) and supply shocks are uncorrelated (conditional on the fixed effects). For the quantity term in (13), after the inclusion of the three sets of fixed effects, the remaining variation are the exogenous supply shocks, $\ln \tilde{\omega}_{ivt}$. For the error term in (13), the structural residual is

$$\epsilon_{ivt} \equiv \frac{\sigma_{k(v)} - 1}{\sigma_{k(v)}} \ln \varphi_{ivt}$$

where φ_{ivt} are the variety quality shocks.

D.5 Structural Fixed Effects

First, I exactly decompose the quality shocks, φ_{ivt} and quantity of floor space shocks, ω_{ivt} into housing variety X MSA terms ($\tilde{\varphi}_{iv}, \tilde{\omega}_{iv}$), MSA x nest X year terms ($\tilde{\varphi}_{ik(v)t}, \tilde{\omega}_{ik(v)t}$), terms that capture the linear time trend for each housing variety in each MSA ($\tilde{\varphi}_{imt} t, \tilde{\omega}_{iv} t$), and residual terms ($\tilde{\varphi}_{ivt}, \tilde{\omega}_{ivt}$).

$$\ln \varphi_{ivt} \equiv \ln \tilde{\varphi}_{iv} + \ln \tilde{\varphi}_{ik(v)t} + \ln \tilde{\varphi}_{iv} t + \ln \tilde{\varphi}_{ivt}$$

$$\ln \omega_{ivt} \equiv \ln \tilde{\omega}_{iv} + \ln \tilde{\omega}_{ik(v)t} + \ln \tilde{\omega}_{iv} t + \ln \tilde{\omega}_{ivt}$$

Based on these assumptions, I can derive the fixed effects in terms of the structural demand and supply terms by substituting (28) into (11)

$$\begin{aligned}
\alpha_{ik(v)t} \equiv & \frac{\sigma_{k(v)} - 1}{\sigma_{k(v)}} \ln \tilde{\varphi}_{ik(v)t} + \frac{\sigma_{k(v)} - \sigma}{\sigma_{k(v)}} \ln \mathbb{P}_{ik(v)t} + \frac{\sigma - 1}{\sigma_{k(v)}} \ln \mathbb{P}_{it} + \frac{1}{\sigma_{k(v)}} \ln E^H_{it} \\
& - \frac{1}{\sigma_{k(v)}} \frac{\beta_i}{\sigma_{k(v)} + \beta_i} E\left[(\sigma_{k(v)} - \sigma) \ln \mathbb{P}_{ik(v)t} + (\sigma - 1) \ln \mathbb{P}_{it} + \ln E^H_{it} |\mathcal{I}_t \right] \\
& - \frac{1}{\sigma_{k(v)}} \frac{\beta_i(\sigma_{k(v)} - 1)}{\sigma_{k(v)} + \beta_i} \ln \tilde{\varphi}^E_{ik(v)t} - \frac{1}{\sigma_{k(v)} + \beta_i} \ln \tilde{\omega}^E_{ik(v)t}
\end{aligned}$$

$$\alpha_{iv} \equiv \frac{\sigma_{k(v)}-1}{\sigma_{k(v)}} \ln \tilde{\varphi}_{iv} - \frac{1}{\sigma_{k(v)}} \ln \tilde{\omega}_{iv} - \frac{1}{\sigma_k} \frac{\beta_i(\sigma_{k(v)}-1)}{\sigma_{k(v)}+\beta_i} \ln \tilde{\varphi}^E_{iv} - \frac{1}{\sigma_{k(v)}+\beta_i} \ln \tilde{\omega}^E_{iv}$$

$$\alpha_{iv} t \equiv \frac{\sigma_{k(v)}-1}{\sigma_{k(v)}} \ln \tilde{\varphi}_{iv} t - \frac{1}{\sigma_{k(v)}} \ln \tilde{\omega}_{iv} t - \frac{1}{\sigma_{k(v)}} \frac{\beta_i(\sigma_{k(v)}-1)}{\sigma_{k(v)}+\beta_i} \ln \tilde{\varphi}^E_{iv} t - \frac{1}{\sigma_{k(v)}+\beta_i} \ln \tilde{\omega}^E_{iv} t$$

The expressions for the fixed effects make it clear how they absorb the supply response to household expected prices in (24) and hence address simultaneity bias. The error term in the regression can also be written as

$$\epsilon_{ivt} = \frac{\sigma_{k(m)}-1}{\sigma_{k(v)}} \ln \tilde{\varphi}_{ivt}$$

Then after the inclusion of the fixed effects, the structural estimation equation can be re-written as

$$\ln p_{ivt} = -\frac{1}{\sigma_{k(m)}} \ln \tilde{\omega}_{ivt} + \alpha_{ik(v)t} + \alpha_{im} + \alpha_{im} t + \epsilon_{ivt}$$

Appendix E Estimation

Since there is missing data as not all varieties appear in all MSAs in all time periods, I adapt the k-means algorithm for missing data by setting the difference to be zero whenever either the cluster mean for a specific MSA and year is missing (i.e., if all varieties assigned to a nest have a missing observation for the specific MSA and year), or if the variety has no transactions for the specific MSA and year. Since this will tend to draw varieties to clusters with missing centroids, I develop an alternative distance calculation where I set the distance between a data point and a cluster to be the value of the data point when the data point is non-missing and the cluster is missing.[77]

k-means Algorithm

Given a set of observations $x_i \in \mathbb{R}^d, i = 1, ..., N$, the objective is to partition observations into K sets, denoted by $\mathbf{S} = \{S_1, ..., S_K\}$ that solve

$$\min_{\mathbf{S}} \sum_{k=1}^{K} \sum_{x_i \in S_k} ||x_i - \mu_k||^2, \qquad \mu_k = \frac{1}{|S_k|} \sum_{x_i \in S_k} x_i$$

Equivalently, if we let $k(i) \in \{1, ..., K\}$ denote the nest assignment of observation i, the problem is equivalent to

$$\min_{\mu_k, k(i)} \sum_{i=1}^{N} ||x_i - \mu_{k(i)}||^2$$

[77] I estimate using both approaches and tend to find better performance using the latter approach.

Bonhomme and Manresa's Algorithm 1

Consider the general fixed effects regression

$$y_{ivt} = \beta_{k(v)} x_{ivt} + \alpha_{ki(v)t} + \epsilon_{ivt}$$

1. Let $\left(\beta^{(0)}, \alpha^{(0)}\right)$ be some starting value. Set $s = 0$

2. Find nearest nest for $v \in \Omega$

$$k(v)^{(s+1)} = \underset{k \in \{1,\ldots,K\}}{\text{argmin}} \sum_{i=1,t=1}^{N,T} \sum_{v \in \Omega_{it}} \left(y_{ivt} - \beta_k^{(s)} x_{ivt} - \alpha_{ikt}^{(s)}\right)^2$$

3. Given nest membership for each variety, compute:

$$\left(\beta^{(s+1)}, \alpha^{(s+1)}\right) = \underset{(\beta_k,\alpha)}{\text{argmin}} \sum_{i=1,t=1}^{N,T} \sum_{v \in \Omega_{it}} (y_{ivt} - \beta_{k(v)^{(s+1)}} x_{ivt} - \alpha_{ik(v)^{(s+1)}t})^2$$

4. Set $s = s + 1$ and go to Step 2 (until numerical convergence).

Proposed Algorithm

1. Let $\beta^{(0)}$ be some starting value. Set $s = 0$

2. Run k-means to convergence, initialized with k-means++ (fixing the data)

$$(k(v)^{(s+1)}, \alpha^{(s+1)}) = \underset{k,\alpha}{\text{argmin}} \sum_{i=1,t=1}^{N,T} \sum_{v \in \Omega_{it}} \left(y_{ivt} - \beta_{k(v)}^{(s)} x_{ivt} - \alpha_{ik(v)t}\right)^2$$

3. Given nest membership $k(v)^{(s+1)}$ for each variety, run FE regression to estimate $\left(\beta^{(s+1)}, \tilde{\alpha}^{(s+1)}\right)$

4. Set $s = s + 1$ and go to Step 2 (until numerical convergence).

Monte Carlo Simulation

E.1 Details

The nest-level quality is drawn from a log-normal distribution, where $\varphi_v^{Nest} \sim LN(0, 1)$. Cutoffs are then drawn based on this distribution to separate the varieties into nests. There is a random number of varieties per bottom nest (minimum 10). The persistent quantity for each variety is drawn from $q_v \sim LN(4, 4)$ and the persistent quality is drawn from $\varphi_v \sim LN(0, 0.1)$. Even though the quality variance is much smaller than the quantity variance, the contribution to the variation in prices is scaled by $\frac{\sigma_k - 1}{\sigma_k}$ compared to quantity which is scaled by $\frac{1}{\sigma_k}$. To combine the persistent quality shock and the nest quality shock, I set $\varphi_v^{Combined} = (\varphi_v^{Nest})^{\frac{1}{\sigma_{k(v)}-1}} \varphi_v$. Quantity in each period is the persistent draw times the shock, or $q_{vt} = q_v \tilde{q}_{vt}$. Quality in each period is $\varphi_{vt} = \varphi_v^{Combined} \tilde{\varphi}_{vt}$ and expenditure in each period is $E_t = E\tilde{E}_t$. I use $\tilde{q}_{vt} \sim LN(0, 0.1)$, $\tilde{\varphi}_{vt} \sim LN(0, 0.1)$, and $\tilde{E}_t \sim LN(0, 0.2)$.

For estimation, I focus on a single location over time and include variety fixed effects to absorb the persistent quality and quantity shock.

Figure A3: Simulation Nesting Structure

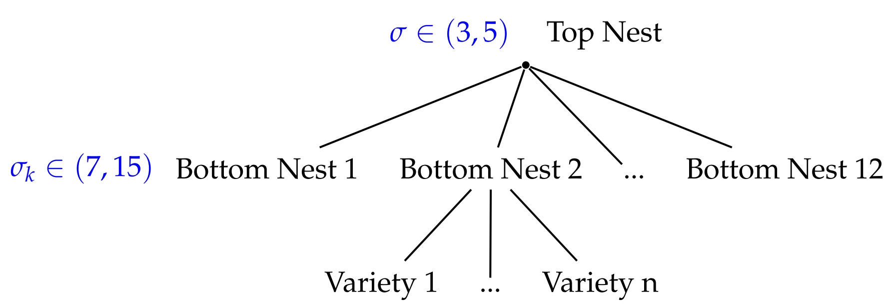
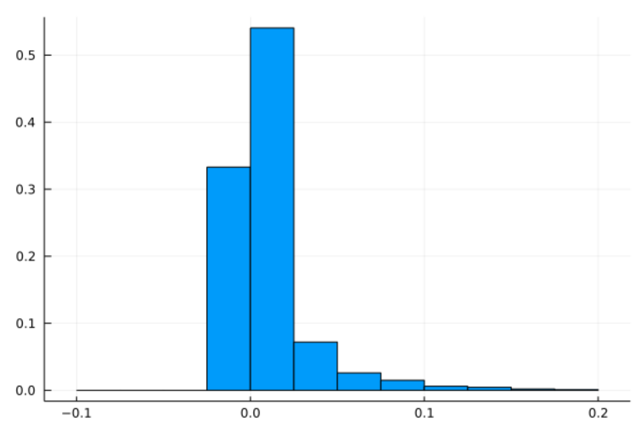
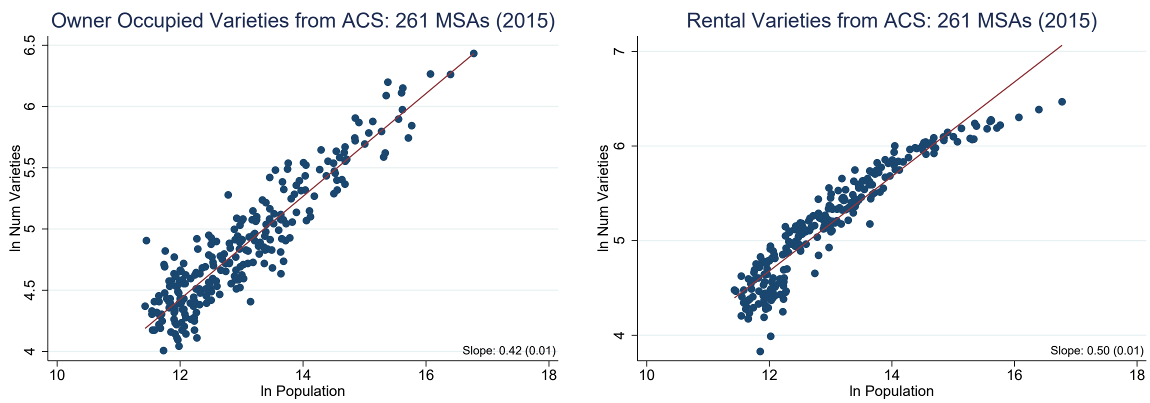


Figure A4: Percent Error in Estimated Top Sigma at Variety Level



Appendix F Data Appendix

Figure A5: Number of Varieties Across MSAs in the American Community Survey



Notes: Varieties are defined as full interaction of number of bedrooms, number of rooms, structure type, lot size category, and decade built. Sample excludes vacant units, mobile homes/trailers, boat/RVs, if any of the housing characteristics are missing, or if the rent or owner-reported valuation is missing.

F.1 ZTRAX Data Details

Since the characteristic information is reported at the structure level for multi-unit structures (since these are from county assessor offices), I compute unit-level characteristics by dividing total characteristics with the number of units. I take care to verify that the data for multi-unit structures are reported consistently at the structure level.

I focus on Deed Transfers and exclude transfers in the following categories: partial ownership transfer, exempt from a transfer tax, intrafamily transfer (flag based on language on document and programmatic buyer/seller names).[78] To identify the transaction date, I first find the document date. If missing, I use the signature date or recording date.

For square feet, I follow the recommendation of Zillow: I first take the maximum of either the total building area or the sum of all the building areas (excluding the total category). For properties missing building area data, I then take the sum of the non-building areas.[79] Since the square feet reported for multi-unit structures are at the structure level, I divide the total square feet by the number of units.[80]

[78]For deed transfers, I restrict attention to transactions where the document type belongs to one of the following categories: deed, condominium deed, warranty deed, trustee's deed (foreclosure sale transfer), special warranty deed, grant deed, bargain & sale deed, other, co-op deed, non-categorized deed, re-recorded deed, correction deed, individual deed, limited warranty deed, cash sale deed, contract of sale deed, and agreement deed. If more than 10% of deeds are uncategorized, I drop those transfers. Finally, I only include vendor's lien deeds for Texas.

[79]I exclude the ST categories as these would double count the square feet.

[80]I confirm that the square feet after this modification are reasonable.

F.2 Household Income

I define household income as the sum of household wages and business income. I use the ACS samples from 2005-2019. Results are robust to restricting sample to full time workers aged 25-55, which include workers who had more than 27 weeks worked and at least 30 hours on average in the past year. Implementation details to measure composition-adjusted household income follow Albouy (2011) Appendix B.1. To measure post-tax household income, I follow Diamond and Moretti (2021) and use NBER TAXSIM. I input head of household marital status, age, income, number of dependents, state, year, and spousal age and income (if relevant). I subtract federal, state, and FICA taxes from pre-tax income (defined as sum of wage income and self-employment income) to obtain post-tax income.

Table A1: Summary Statistics: ZTRAX and HMDA Data

	# Varieties MSA-Year	# Transactions Variety-MSA-Year	Sales Price	Sq Ft	Price per Sq Ft
Mean	1,740	3.6	$325,898	2,165	$182.8
10th pctile	352	1.0	115,000	1,046	%65.6
25th pctile	685	1.0	$165,000	1,363	$90.8
50th pctile	1,264	1.0	$253,300	1,876	$130.2
75th pctile	2,226	3.0	$390,000	2,619	$195.8
90th pctile	3,937	7.0	$596,200	3,546	$309.7

Notes: A variety is defined as the full interaction of the decade the housing structure was originally built (or underwent substantial renovation), number of rooms, number of bedrooms, number of bathrooms, number of floors, structure type, lot size quintile, and distance categories from the MSA central business district (CBD). Summary statistics on sales price, square feet, and price per square feet are computed over all transactions with non-missing applicant income.

Table A2: Summary Statistics for Applicant Gross Income (HMDA)

Mean	109,835
p1	22,000
p5	31,000
p10	38,000
p25	54,000
p50	82,000
p75	126,000
p90	194,000
p95	260,000
p99	524,000
N	7,919,236

Notes: Summary statistics on applicant income are computed over all transactions with non-missing characteristics, sales price, and applicant income.

Table A3: Fraction of Housing Units with Characteristics

Distance (miles)		**Rooms**		**Structure Type**	
0-5	14.4%	1	0.7%	Single detached	90.2%
5-10	25.6%	2	7.0%	Single attached	6.2%
10-20	33.3%	3	24.0%	**Apartments**	
20-30	16.1%	4	15.9%	2	2.7%
30-100	10.6%	5	11.6%	3-4	0.8%
		6	13.1%	5-9	0.1%
		7	9.0%	10-19	0.0%
		8+	18.6%	20-49	0.0%
				50+	0.0%
Decade Built		**Bathrooms**		**Bedrooms**	
Pre 1900s	1.4%	0	0.2%	0	0.3%
1900 - 1910	2.1%	1	18.6%	1	1.4%
1910 - 1920	1.7%	2	39.2%	2	14.3%
1920 - 1930	3.8%	3	31.4%	3	49.9%
1930 - 1940	1.9%	4	8.0%	4	27.1%
1940 - 1950	3.7%	5+	2.7%	5+	6.9%
1950 - 1960	8.9%				
1960 - 1970	8.1%	**Floors**		**Lot Size**	
1970 - 1980	9.2%	1	50.2%	Quantile 1	21.4%
1980 - 1990	11.5%	2	46.5%	Quantile 2	19.6%
1990 - 2000	14.2%	3	2.7%	Quantile 3	20.7%
2000 - 2010	22.9%	4	0.1%	Quantile 4	21.7%
2010 - 2020	10.4%	5+	0.1%	Quantile 5	16.5%

Figure A6: Comparison to Case-Shiller with ZTRAX data

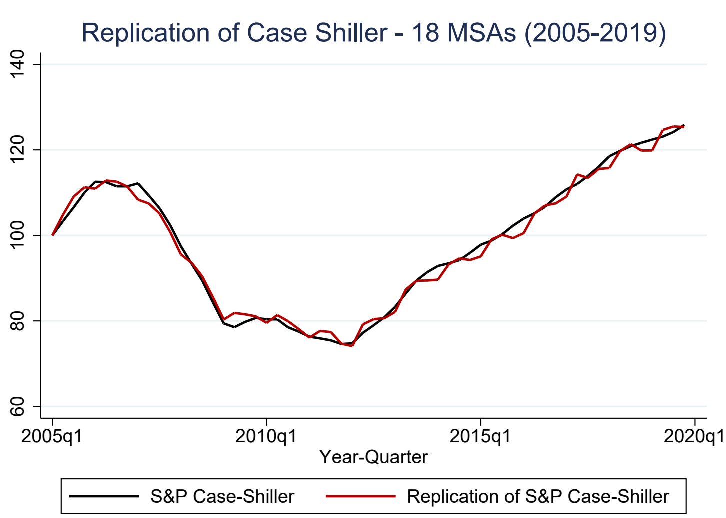
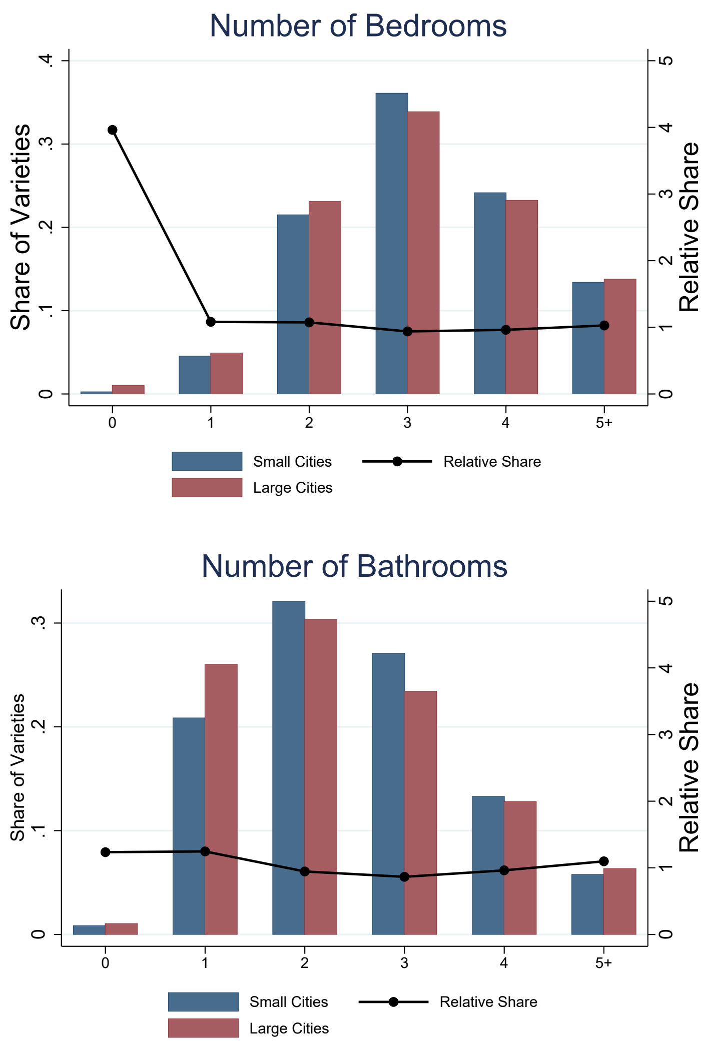
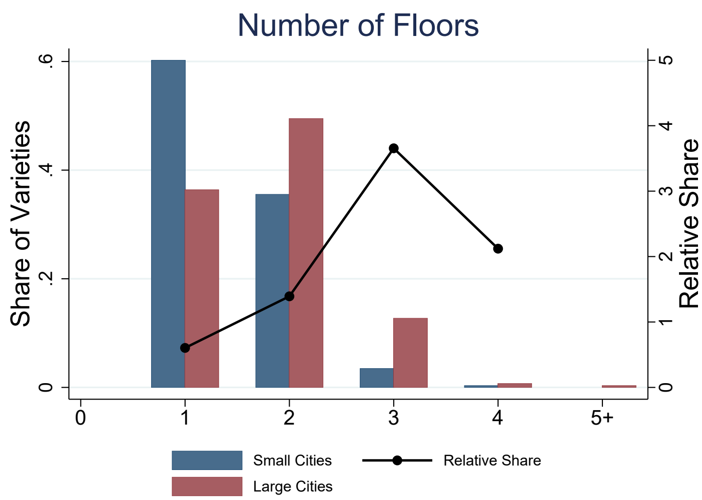
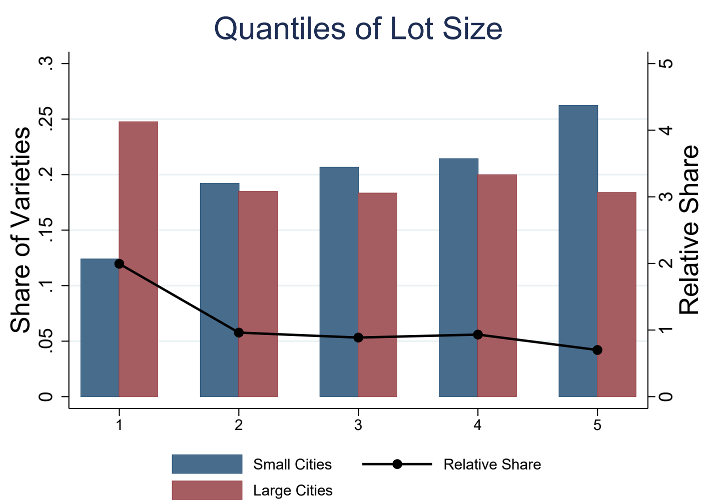


Figure A7: Characteristic Differences Between 10 Smallest and 10 Largest Cities




Number of Floors
Share of Varieties
Relative Share
0
.2
.4
.6
0
1
2
3
4
5
0
1
2
3
4
5+
Small Cities
Large Cities
Relative Share


Quantiles of Lot Size
Share of Varieties
Relative Share
0
.05
.1
.15
.2
.25
.3
0
1
2
3
4
5
1
2
3
4
5
Small Cities
Large Cities
Relative Share

Appendix G MSA List

Table A4: Final Sample of MSAs with Average Number of Transactions (2005-2019)

Pop Rank	MSA	CBSA Code	Unique Varieties	Average Frac Non-miss	Average #	Average # Non-miss
1	New York-Newark-Jersey City, NY-NJ-PA	35620	169,372	9%	132,403	12,132
2	Los Angeles-Long Beach-Anaheim, CA	31080	118,320	17%	96,255	16,455
3	Chicago-Naperville-Elgin, IL-IN-WI	16980	140,873	49%	84,909	41,545
4	Dallas-Fort Worth-Arlington, TX	19100	29,114	64%	9,672	6,347
5	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	37980	164,071	68%	70,481	47,579
6	Houston-The Woodlands-Sugar Land, TX	26420	31,524	78%	7,862	6,278
7	Washington-Arlington-Alexandria, DC-VA-MD-WV	47900	159,899	57%	77,363	44,032
8	Miami-Fort Lauderdale-Pompano Beach, FL	33100	85,255	71%	65,809	46,592
9	Atlanta-Sandy Springs-Alpharetta, GA	12060	174,814	59%	97,018	57,177
10	Boston-Cambridge-Newton, MA-NH	14460	208,276	93%	38,127	35,483
11	San Francisco-Oakland-Berkeley, CA	41860	153,382	86%	45,752	39,598
12	Detroit-Warren-Dearborn, MI	19820	166,061	42%	44,487	18,720
13	Riverside-San Bernardino-Ontario, CA	40140	128,000	97%	80,462	78,169
15	Seattle-Tacoma-Bellevue, WA	42660	112,548	95%	51,678	49,018
16	Minneapolis-St. Paul-Bloomington, MN-WI	33460	161,911	54%	45,912	24,428
18	St. Louis, MO-IL	41180	105,457	61%	28,631	17,640
19	Tampa-St. Petersburg-Clearwater, FL	45300	49,717	38%	47,519	18,405
21	Denver-Aurora-Lakewood, CO	19740	116,758	95%	46,862	44,718
22	Pittsburgh, PA	38300	99,547	75%	24,293	18,219
23	Charlotte-Concord-Gastonia, NC-SC	16740	67,110	73%	44,164	32,189
24	Portland-Vancouver-Hillsboro, OR-WA	38900	86,172	53%	35,289	18,717
25	Sacramento-Roseville-Folsom, CA	40900	100,584	79%	41,132	32,824
27	Cincinnati, OH-KY-IN	17140	94,384	77%	21,848	16,664
28	Orlando-Kissimmee-Sanford, FL	36740	49,885	84%	41,623	34,841
29	Cleveland-Elyria, OH	17460	69,508	93%	17,190	15,900
31	Las Vegas-Henderson-Paradise, NV	29820	34,077	96%	53,899	51,795
32	Columbus, OH	18140	74,954	93%	17,383	16,140
34	San Jose-Sunnyvale-Santa Clara, CA	41940	50,498	92%	15,200	13,925
36	Virginia Beach-Norfolk-Newport News, VA-NC	47260	92,375	58%	23,876	13,878
37	Nashville-Davidson–Murfreesboro–Franklin, TN	34980	60,018	30%	34,716	10,434
38	Providence-Warwick, RI-MA	39300	108,930	95%	16,124	15,396
39	Milwaukee-Waukesha, WI	33340	41,080	46%	16,696	7,817
40	Jacksonville, FL	27260	48,776	87%	25,214	21,918
41	Memphis, TN-MS-AR	32820	43,765	85%	15,389	13,168
42	Oklahoma City, OK	36420	57,810	91%	18,350	16,714
43	Hartford-East Hartford-Middletown, CT	25540	71,796	94%	11,439	10,787
44	Louisville/Jefferson County, KY-IN	31140	44,354	54%	12,704	6,775
46	Richmond, VA	40060	70,189	71%	17,876	12,721
47	Buffalo-Cheektowaga, NY	15380	59,985	96%	11,517	11,101
50	Rochester, NY	40380	68,355	98%	13,921	13,668
51	Birmingham-Hoover, AL	13820	47,944	47%	11,799	5,594
56	Fresno, CA	23420	28,138	95%	12,568	11,987

Pop Rank	MSA	CBSA Code	Unique Varieties	Average Frac Non-miss	Average #	Average # Non-miss
57	Worcester, MA-CT	49340	77,562	94%	10,042	9,398
58	Bridgeport-Stamford-Norwalk, CT	14860	55,366	94%	7,897	7,421
60	Albany-Schenectady-Troy, NY	10580	59,042	95%	10,008	9,545
61	Omaha-Council Bluffs, NE-IA	36540	41,171	84%	8,875	7,866
62	New Haven-Milford, CT	35300	58,427	94%	7,603	7,165
63	Bakersfield, CA	12540	34,442	99%	14,582	14,406
66	Oxnard-Thousand Oaks-Ventura, CA	37100	32,068	96%	8,409	8,110
67	Allentown-Bethlehem-Easton, PA-NJ	10900	56,016	82%	10,229	8,317
68	Knoxville, TN	28940	41,451	51%	12,869	6,613
72	Columbia, SC	17900	26,392	53%	10,689	5,534
73	Greensboro-High Point, NC	24660	24,658	60%	9,110	5,465
74	Akron, OH	10420	35,928	99%	5,634	5,571
75	North Port-Sarasota-Bradenton, FL	35840	43,603	95%	14,824	14,109
77	Springfield, MA	44140	58,381	95%	6,662	6,359
78	Stockton, CA	44700	29,650	99%	13,033	12,962
80	Charleston-North Charleston, SC	16700	33,851	57%	13,184	7,431
81	Syracuse, NY	45060	36,930	97%	7,350	7,109
82	Toledo, OH	45780	34,075	94%	4,408	4,144
83	Colorado Springs, CO	17820	31,795	98%	14,533	14,284
84	Winston-Salem, NC	49180	25,627	83%	8,125	6,717
86	Cape Coral-Fort Myers, FL	15980	17,319	99%	18,299	18,076
88	Des Moines-West Des Moines, IA	19780	58,588	84%	10,415	8,727
89	Madison, WI	31540	27,201	51%	6,848	3,500
90	Lakeland-Winter Haven, FL	29460	22,788	84%	13,150	11,008
92	Deltona-Daytona Beach-Ormond Beach, FL	19660	21,276	65%	11,657	7,618
95	Augusta-Richmond County, GA-SC	12260	32,108	65%	8,504	5,526
98	Harrisburg-Carlisle, PA	25420	46,927	79%	7,059	5,549
99	Palm Bay-Melbourne-Titusville, FL	37340	16,077	93%	8,609	8,026
103	Lancaster, PA	29540	44,264	98%	6,542	6,380
104	Spokane-Spokane Valley, WA	44060	59,217	83%	9,263	7,716
105	Modesto, CA	33700	23,726	63%	9,743	6,130
107	Santa Rosa-Petaluma, CA	42220	39,395	98%	5,827	5,724
108	Fayetteville, NC	22180	15,930	77%	7,547	5,860
109	Lexington-Fayette, KY	30460	31,935	64%	7,501	5,088
112	Visalia, CA	47300	27,848	88%	5,819	5,115
115	York-Hanover, PA	49620	36,346	99%	6,019	5,943
118	Reno, NV	39900	23,519	88%	10,358	9,133
119	Asheville, NC	11700	27,993	72%	6,190	4,474
120	Port St. Lucie, FL	38940	28,598	97%	10,130	9,818
124	Salinas, CA	41500	23,634	96%	4,185	4,046
125	Vallejo, CA	46700	23,882	97%	7,628	7,393
126	Reading, PA	39740	30,296	98%	4,713	4,613
131	Manchester-Nashua, NH	31700	29,216	91%	4,417	4,029
139	Myrtle Beach-Conway-North Myrtle Beach, SC-NC	34820	27,581	81%	8,793	7,077
146	Eugene-Springfield, OR	21660	18,367	96%	4,485	4,290
147	Rockford, IL	40420	27,095	84%	4,054	3,393
148	Savannah, GA	42340	19,285	80%	5,945	4,787

Pop Rank	MSA	CBSA Code	Unique Varieties	Average Frac Non-miss	Average #	Average # Non-miss
150	Ocala, FL	36100	7,933	99%	5,227	5,183
159	Lincoln, NE	30700	17,925	79%	4,080	3,258
160	Fort Collins, CO	22660	40,018	96%	6,551	6,267
165	Spartanburg, SC	43900	12,428	70%	4,042	2,821
176	Merced, CA	32900	18,136	98%	4,505	4,426
178	Kennewick-Richland, WA	28420	19,296	65%	4,582	2,959
179	Greeley, CO	24540	33,416	97%	5,696	5,483
182	Olympia-Lacey-Tumwater, WA	36500	14,662	92%	4,931	4,544
191	Crestview-Fort Walton Beach-Destin, FL	18880	18,992	95%	5,613	5,334

Appendix H Estimation Results

H.1 Optimal Number of Nests

I follow Bonhomme and Manresa (2015) and choose the number of nests K so that it minimizes a BIC criterion

$$\min_{K\in 1,\ldots,K_{\max}} BIC(K) = \frac{1}{NTM}\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{v\in\Omega}\underbrace{\left(\ln\overline{p}_{ivt} + \frac{1}{\sigma_{k(v)}}\ln\overline{q}_{ivt} - \alpha_{ik(v)t}\right)^2}_{\epsilon_{ivt}} + Kh_{vnt}$$

where M is the number of varieties, or $M = |\Omega|$ and h_{vnt} is a penalty term

$$Kh_{vnt} = \widehat{\sigma}^2\frac{KNT + M + P}{NTM}\ln(NTM)$$

where P is the number of other parameters: in the case with linear time trends for each variety in each MSA and variety X MSA fixed effects, $P = 2MN$. Notice that the M in the numerator $KNT + M + P$ is added to account for the fact that I am also estimating the nest assignment for each variety, while KNT accounts for the number of nest X MSA x year fixed effects I am estimating.

To estimate $\widehat{\sigma}^2$, I run the algorithm with the maximum number of nests (K_{max})

$$\widehat{\sigma}^2 = \frac{1}{NTM - K_{\max}T - M - P}\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{v\in\Omega}\left(\ln\overline{p}_{ivt} + \frac{1}{\sigma_{k(v)}}\ln\overline{q}_{ivt} - \alpha_{ik(v)t}\right)^2$$

In practice, I calculate $\widehat{\sigma}^2$ as the RSS divided by the remaining degrees of freedom (observations minus number of estimated parameters) with a maximum number of nests, $K_{max} = 15$.

In general since there is missing data, for the first term in the BIC (column (1) in Table A5), I use the RSS divided by the number of observations. For the penalty term (shown in column (2)), for the numerator $KNT + M + P$, I use the number of estimated parameters. For the NTM term, I use the number of observations. The resulting BIC sum is displayed in column (3) of Table A5, which shows that the BIC is minimized at six nests.

To check sensitivity, I compute the first term in the BIC as the RSS divided by NMT and for the penalty term, I use the potential number of estimated parameters plus the number of varieties $KNT + M + 2MN$ (see columns (4) and (5)). This sensitivity is going to be punitive because of the missing data relative to the baseline. I find that in this sensitivity check, the optimal number of nests is 3.

Table A5: BIC: Optimal Number of Nests

	(1)	(2)	(3)	(4)	(5)	(6)
Nests	RSS/n_{obs}	Kh_{nt}	BIC = (1) + (2)	RSS/NMT	Kh_{nt} ALT	BIC ALT = (4) + (5)
2	0.084365	0.659912	0.744277	0.001984	0.306877	0.308862
3	0.083115	0.660604	0.743719	0.001955	0.306897	0.308852
4	0.082131	0.661297	0.743428	0.001932	0.306918	0.30885
5	0.081358	0.661989	0.743347	0.001914	0.306938	0.308852
6	0.080662	0.662681	0.743344	0.001897	0.306958	0.308856
7	0.080102	0.663373	0.743475	0.001884	0.306979	0.308863
8	0.07959	0.664066	0.743656	0.001872	0.306999	0.308871
9	0.079069	0.664758	0.743826	0.00186	0.307019	0.308879
10	0.078657	0.665434	0.744091	0.00185	0.307039	0.30889
11	0.07829	0.66614	0.74443	0.001842	0.30706	0.308901
12	0.077883	0.666824	0.744706	0.001832	0.30708	0.308912
13	0.077567	0.667521	0.745088	0.001825	0.3071	0.308925
14	0.077245	0.66821	0.745455	0.001817	0.307121	0.308938
15	0.076899	0.668889	0.745788	0.001809	0.307141	0.30895

H.2 Additional Nest Summary Statistics

Table A6: Mean of Characteristics by Nest

Nest	Decade Built	Bedrooms	Rooms	Struct Type	Baths	Floors	Lot Size	Distance
1	10.91	3.52	5.08	0.05	2.84	1.64	3.13	15.61
2	8.73	2.99	4.81	0.14	1.96	1.36	2.64	15.48
3	8.06	3.07	4.85	0.28	2.13	1.51	2.88	14.69
4	6.09	2.68	4.44	0.75	1.60	1.69	2.13	12.88
5	7.69	2.99	5.24	0.33	2.12	1.60	2.76	14.55
6	7.37	2.93	4.91	0.47	1.99	1.62	2.52	15.69

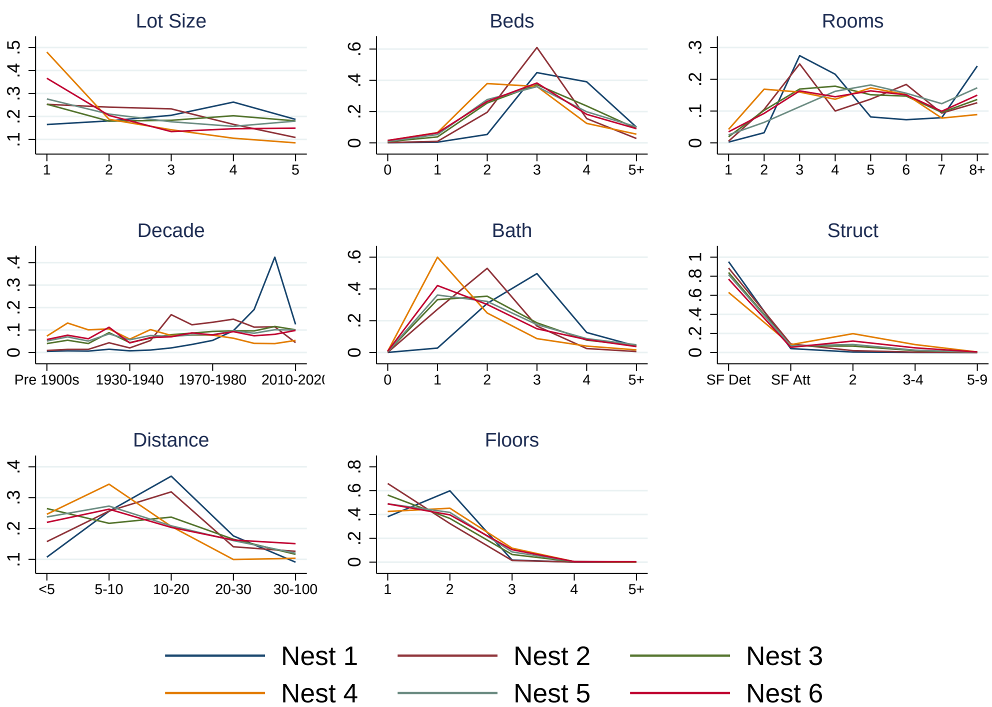
Notes: The mean characteristics are computed over all transactions within each nest. Lot Size is coded as quintiles so a larger number means on average, varieties are in a larger quintile. Decade Built is coded so that larger numbers represent more recent buildings (equal to 1 for pre 1890s and 13 for after 2010). Struct Type is coded so that the 1 corresponds to single family detached, 2 corresponds to single family attached, and 3-8 correspond to structures with multiple units (larger numbers represent buildings with more units). Distance measures the average number of miles from the city center.

Table A7: Summary Statistics For Each Nest

Nest	Number	Frac Expend Varieties	Frac Sq Ft	Mean Sq Ft	Median Sq Ft	Mean Price per Sq Ft	Median Price per Sq Ft
1	24,779	54%	56%	2,522	160.3	2,251	119.6
2	36,568	33%	33%	1,714	175.6	1,495	118.4
3	14,162	4%	4%	2,002	187.6	1,641	119.2
4	12,187	3%	3%	1,613	190.9	1,354	119.5
5	13,417	3%	3%	2,000	193.3	1,626	122.1
6	11,700	3%	2%	1,881	204.4	1,560	127.5

Notes: Square feet and prices are summarized at the transaction level within each nest.

Figure A8: Density of Characteristics Across Nests: All Characteristics and Nests



H.3 Single Nest Estimation

Table A8 presents estimates of demand elasticities for a single nest. In column (1), I follow regress log square feet on log price. The estimated demand coefficient is highly inelastic and suggests that the demand of housing varieties is complementary. Since market-clearing prices will reflect the current-period quality shock, then the omitted variable bias will be positive. This leads to an upward bias in the estimated coefficient.

Under my identifying assumptions, I regress prices on quantities with a rich set of fixed effects in columns (2)-(5). I use several different sets of fixed effects aimed at absorbing structural demand and supply terms. *iv* denotes MSA X variety fixed effects, so that the estimating variation are deviations in quantity supplied and prices over time. *it* denotes MSA x year fixed effects, absorbing the overall price index and so the demand elasticity is estimated from variation within each market. $v \cdot t$ denote linear time trends for each housing variety v and $iv \cdot t$ denote linear time trends for each housing variety X MSA, absorbing correlated demand and supply trends over time at the variety level. The single nest demand elasticities range from 7.8 to 9.2.

Table A8: Single Nest Demand Estimation using ZTRAX (2005-2019)

	(1) ln Sq Feet	(2) ln Price	(3) ln Price	(4) ln Price	(5) ln Price
ln Price	-0.245***				
	(0.006)				
ln Sq Feet		-0.109***	-0.127***	-0.129***	-0.120***
		(0.004)	(0.004)	(0.004)	(0.004)
Single Nest Elasticity $\hat{\sigma}$	-0.25***	9.21***	7.86***	7.78***	8.30***
	(0.006)	(0.33)	(0.24)	(0.24)	(0.30)
N	3,900,949	3,900,949	3,900,949	3,900,949	3,900,949
R2	0.847	0.630	0.807	0.817	0.855
Within R2	0.0315	0.0394	0.0307	0.0315	0.0296
FE	iv,it,v·t	it,v·t	iv,it	iv,it,v·t	iv,it,iv·t

Notes: Estimated coefficients are presented in the first panel and the estimated sigma for each nest is presented in the second panel. The estimated sigmas are the negative inverse of the coefficients for columns (2)-(5) and standard errors are computed using the delta method. Standard errors clustered at MSA X year level in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Appendix I Spatial Price Indices

I.1 Comparison vs a Comparison MSA C

The overall price index for either CES-RW or CES-Feenstra can be written as

$$\frac{\mathbb{P}_i}{\mathbb{P}_C} = \underbrace{\left(\frac{\lambda_i^{Nest}}{\lambda_{CH}^{Nest}}\right)^{\frac{1}{\sigma-1}}}_{\text{Variety Index}} \qquad \underbrace{\frac{\mathbb{P}_i^*}{\mathbb{P}_C^*}}_{\text{Common Nest: Sato-Vartia or Redding-Weinstein}}, \tag{29}$$

I.1.1 Definition of Shares

s_{ik}^* is MSA i's expenditure on nest k out of expenditure on overlapping nests between MSA i and comparison MSA C, and $s_{C,k}^*$ is comparison MSA C's expenditure on nest k out of expenditure on overlapping nests between MSA i and comparison MSA C.

$$s_{ik}^* = \frac{\sum_{v \in \Omega_{ik}} \mathbb{V}_{iv}}{\sum_{k' \in \Omega_{i,C}^{K*}} \sum_{v \in \Omega_{ik'}} \mathbb{V}_{iv}} \qquad s_{C,k}^* = \frac{\sum_{v \in \Omega_{Ck}} \mathbb{V}_{Cv}}{\sum_{k' \in \Omega_{i,C}^{K*}} \sum_{v \in \Omega_{Ck'}} \mathbb{V}_{Cv}}$$

where $\Omega^{K*}_{i,C}$ denotes the set of overlapping sets between MSA i and Chicago, Ω_{ik} denotes the set of varieties available in MSA i from nest k, and $\Omega_{C,k}$ denotes the set of varieties available in comparison MSA C from nest k.

s^*_{ivk} is MSA i's expenditure on variety v out of expenditure on overlapping varieties in MSA i and comparison MSA C in nest k, and $s^*_{C,vk}$ is comparison MSA C's expenditure on variety v out of expenditure on overlapping varieties in MSA i and comparison MSA C in nest k, so that

$$s^*_{ivk} = \frac{\mathbb{V}_{iv}}{\sum_{m' \in \Omega^*_{i,C,k}} \mathbb{V}_{iv'}} \qquad s^*_{C,mk} = \frac{\mathbb{V}_{Cv}}{\sum_{v' \in \Omega^*_{i,C,k}} \mathbb{V}_{Cv'}}$$

where $\Omega^*_{i,C,k}$ denotes the set of overlapping varieties between MSA i and comparison MSA C within nest k.

I.1.2 CES-Feenstra

The Feenstra nested CES price index (CES-Feenstra) assumes that the quality shocks are constant across the comparison periods. The price index has a recursive structure, where the top level is composed of two terms. The first is a nest-level variety adjustment term, which measures the impact of nest differences between MSA i and comparison MSA C. The second term is a weighted geometric mean across the set of common (i.e., overlapping) nest-level price indices, where the weights are a function of the nest expenditure shares in both locations. The second term measures the relative prices of nests that are available in both locations.

Following the recursive nature, there are two analogous terms within each nest: a variety-level adjustment term that measures differences in housing varieties within each nest and a weighted geometric mean of relative prices over the set of common varieties, where the weights are a function of the variety expenditure shares in both locations.

The Feenstra CES price index (CES Feenstra) is given by

$$\frac{\mathbb{P}_i}{\mathbb{P}_C} = \left(\frac{\lambda_i^{Nest}}{\lambda_C^{Nest}}\right)^{\frac{1}{\sigma-1}} \prod_{k \in \Omega^{K*}_{i,C}} \left(\left(\frac{\lambda_{ik}}{\lambda_{Ck}}\right)^{\frac{1}{\sigma_k - 1}} \prod_{v \in \Omega^*_{i,C,k}} \left(\frac{p_{iv}}{p_{Cv}}\right)^{\omega(s^*_{ivk}, s^*_{C,vk})} \right)^{\omega(s^*_{ik}, s^*_{C,k})} \tag{30}$$

where the ω function defines weights that are a logarithmic average of the input shares. For instance,

$$\omega(s^*_{ivk}, s^*_{C,vk}) \equiv \frac{\frac{s^*_{ivk} - s^*_{C,vk}}{\ln s^*_{ivk} - \ln s^*_{Cvk}}}{\sum_{\ell \in \Omega^*_{i,C,k}} \frac{s^*_{i\ell k} - s^*_{C\ell k}}{\ln s^*_{i\ell k} - \ln s^*_{C\ell k}}}$$

Focusing first on the variety correction, λ_{ik} is MSA i's expenditure share on housing varieties that exist in both MSA i and comparison MSA C in nest k. Similarly, λ_{Ck} is comparison MSA C's expenditure share on housing varieties that exist in both MSA i and comparison MSA C in nest k.

Notice that both the nest-level ($\frac{\lambda_{ik}}{\lambda_{Ck}}$) and top-level variety indices ($\frac{\lambda_i^{Nest}}{\lambda_C^{Nest}}$) are the same for the CES-Feenstra and CES-RW price indices. However, the nest-level variety index will have a different impact on the overall price index since the weight for the nest-level variety index is $\frac{1}{N_{i,C}^{K}}$ in the CES-RW price index compared to $\omega(s_{ik}^{*}, s_{Ck}^{*})$ in the CES-Feenstra index.

I.1.3 Single Nest CES Price Index

In this subsection I provide the formulas for a single nest CES price index in a comparison of each MSA against a comparison MSA C. The CES-Feenstra single nest index is given as

$$\frac{\mathbb{P}_i}{\mathbb{P}_C} = \left(\frac{\lambda_i^{Nest}}{\lambda_C^{Nest}}\right)^{\frac{1}{\sigma_{single}-1}} \prod_{v\in\Omega_{i,C}^{*}} \left(\frac{p_{iv}}{p_{Cv}}\right)^{\omega(s_{iv}^{*}, s_{Cv}^{*})} \tag{31}$$

where $\Omega_{i,C}^{*}$ denotes the set of overlapping varieties for MSA i and comparison MSA C, Ω_i denotes the set of available varieties in MSA i, and Ω_C denotes the set of available varieties in comparison MSA C.

The shares in (31) are defined as

$$\lambda_i^{Nest} = \frac{\sum_{v\in\Omega_{i,C}^{*}} \mathbb{V}_{iv}}{\sum_{v\in\Omega_i} \mathbb{V}_{iv}} \qquad \lambda_C^{Nest} = \frac{\sum_{v\in\Omega_{i,C}^{*}} \mathbb{V}_{C,v}}{\sum_{v\in\Omega_C} \mathbb{V}_{C,v}}$$

$$s_{iv}^{*} = \frac{\mathbb{V}_{iv}}{\sum_{v'\in\Omega_{i,C}^{*}} \mathbb{V}_{iv'}} \qquad s_{C,v}^{*} = \frac{\mathbb{V}_{C,v}}{\sum_{v'\in\Omega_{i,C}^{*}} v_{C,v'}}$$

The CES-RW single nest index is given as

$$\frac{\mathbb{P}_i}{\mathbb{P}_C} = \left(\frac{\lambda_i^{Nest}}{\lambda_C^{Nest}}\right)^{\frac{1}{\sigma_{single}-1}} \prod_{v\in\Omega_{i,C}^{*}} \left(\frac{p_{iv}}{p_{Cv}}\right)^{\frac{1}{N_{i,C}}} \prod_{v\in\Omega_{i,C}^{*}} \left(\frac{s_{iv}^{*}}{s_{Cv}^{*}}\right)^{\frac{1}{N_{i,C}}} \tag{32}$$

where $N_{i,C}$ is the number of overlapping varieties for MSA i and comparison MSA C, or $N_{i,C} = |\Omega_{i,C}^{*}|$.

I.2 Comparison to a Representative National Household

Following Handbury and Weinstein (2014), I consider a comparison for each MSA versus a representative national urban household who has access to all varieties and all nests. The representative urban household faces a price (a tilde above a variable denotes the national analogue and the subscript N denotes the national household)

$$\tilde{p}_{Nv} = \frac{\sum_i \mathbb{V}_{iv}}{\sum_i q_{iv}} \equiv \sum_i \frac{q_{iv}}{\sum_j q_{jv}} p_{iv} \tag{33}$$

where v_{iv} is the total expenditure in MSA i on variety v, p_{iv} is the price of variety v in MSA i, and q_{iv} is the total square feet of housing variety v in MSA i. To define expenditure shares for the national urban household, it is helpful to define total expenditures on variety v, $\tilde{\mathbb{V}}_{Nv} = \sum_i \mathbb{V}_{iv}$, so that the national expenditure share on variety v is

$$\tilde{s}_{Nv} = \frac{\tilde{\mathbb{V}}_{Nv}}{\sum_{v'} \tilde{\mathbb{V}}_{Nv'}} \equiv \frac{\sum_i \mathbb{V}_{iv}}{\sum_{i,v'} \mathbb{V}_{iv'}}$$

Under the assumption that the geometric mean of quality shocks within each nest and the geometric mean of nest-level quality shocks are equal between each MSA and the representative urban household, the overall CES-RW index is given by

$$\frac{\mathbb{P}_i}{\mathbb{P}_N} = \left(\frac{1}{\lambda_N}\right)^{\frac{1}{\sigma-1}} \left(\prod_{k\in\Omega_i^K} \frac{s_{ik}^*}{\tilde{s}_{Nk}^*}\right)^{\frac{1}{(\sigma-1)N_i^K}} \left(\prod_{k\in\Omega_i^K} \left[\left(\frac{1}{\lambda_{Nk}}\right)^{\frac{1}{\sigma_k-1}} \left(\prod_{v\in\Omega_{ik}} \frac{s_{ivk}^*}{\tilde{s}_{Nvk}^*}\right)^{\frac{1}{(\sigma_k-1)N_{ik}}} \left(\prod_{v\in\Omega_{ik}} \frac{p_{iv}}{\tilde{p}_{Nv}}\right)^{\frac{1}{N_{ik}}}\right]\right)^{\frac{1}{N_i^K}} \tag{34}$$

As in the Chicago comparison, there are two variety indices: one at the nest level, $\frac{1}{\lambda_{Nk}}$ and one at the top level, $\frac{1}{\lambda_N}$.[81] In contrast to the Chicago comparison, since the representative household has access to all varieties, variety or nest differences between MSA i and the national household are due to varieties and nests that are not present in MSA i. As a result, the set of common varieties and nests are simply the set of varieties available in MSA i, so that the expenditure share on common varieties and common nests in MSA i is always one.

Focusing on the nest-level variety index, λ_{Nk} is the national expenditure share on housing varieties that exist in MSA i in nest k, or

$$\lambda_{Nk} = \frac{\sum_{v\in\Omega_{ik}} \tilde{\mathbb{V}}_{Nv}}{\sum_{v\in\Omega_k} \tilde{\mathbb{V}}_{Nv}}$$

where the set Ω_{ik} denotes the set of varieties within nest k available in MSA i and Ω_k denotes the set of all varieties within nest k. If MSA i is missing varieties that have a significant national expenditure share within nest k, then λ_{Nk} will be small, leading to a higher relative price index. The amount the price index increases is moderated by $\frac{1}{\sigma_k-1}$. If there are substantial variety differences in nests with a smaller σ_k (where goods are more differentiated or where preference draws are less correlated), then the welfare impact of not having those varieties is larger.

Focusing on the top variety index, λ_N is the national expenditure share on nests available in MSA i, or

$$\lambda_N = \frac{\sum_{k\in\Omega_i^K} \sum_{v\in\Omega_k} \tilde{\mathbb{V}}_{Nv}}{\sum_{k\in\Omega^K} \sum_{v\in\Omega_k} \tilde{\mathbb{V}}_{Nv}}$$

where the set Ω_i^K denotes the set of nests available in MSA i and Ω^k denotes the set of all nests. If an MSA is missing certain nests that are a significant part of national expenditure, then λ_N will be

[81] In the first line, N_i^K is the number of available nests in MSA i, or $N_i^K = |\Omega_i^K|$, s_{ik}^* is the share of MSA i expenditure on nest k, and s_{kN}^* is the national expenditure share on nest k out of all nests that exist in MSA i. In the second line, N_{ik} is the number of varieties available in nest k in MSA i, or $N_{ik} = |\Omega_{ik}|$, λ_{Nk} is the national expenditure share on housing varieties that exist in MSA i in nest k, s_{ivk}^* is the share of expenditure in MSA i on variety v within nest k, and s_{Nvk}^* is the national share of expenditure on variety v of varieties available in MSA i in nest k. The formula of these share are provided in Appendix I.2.1.

small (note that the expenditure share will always be less than or equal to 1), driving the variety index up.

I.2.1 Definition of Shares

The overall price index for either CES-RW or CES-Feenstra can be written as

$$\frac{\mathbb{P}_i}{\mathbb{P}_N} = \underbrace{\left(\frac{1}{\lambda_N}\right)^{\frac{1}{\sigma-1}}}_{\text{Variety Index}} \underbrace{\frac{\mathbb{P}_i^*}{\mathbb{P}_N^*}}_{\substack{\text{Common Nest: Sato-Vartia or} \\ \text{Redding-Weinstein}}}, \tag{35}$$

s_{ivk}^* is the share of expenditure in MSA i on variety v within nest k and $\tilde{s}_{Nvk}^*$ is the national share of expenditure on variety v of varieties available in MSA i in nest k, so that

$$s_{ivk}^* = \frac{\mathbb{V}_{iv}}{\sum_{v' \in \Omega_{ik}} \mathbb{V}_{iv'}} \qquad \tilde{s}_{Nvk}^* = \frac{\tilde{\mathbb{V}}_{Nv}}{\sum_{v' \in \Omega_{ik}} \tilde{\mathbb{V}}_{Nv'}}$$

s_{ik}^* is the share of MSA i expenditure on nest k and $\tilde{s}_{Nk}^*$ is the share of national expenditure on nest k out of all nests that exist in MSA i. These are given by

$$s_{ik}^* = \frac{\sum_{v \in \Omega_{ik}} \mathbb{V}_{iv}}{\sum_{k' \in \Omega_i^K} \sum_{v \in \Omega_{ik'}} \mathbb{V}_{iv}} \qquad \tilde{s}_{Nk}^* = \frac{\sum_{v \in \Omega_k} \tilde{\mathbb{V}}_{Nv}}{\sum_{k' \in \Omega_i^K} \sum_{v \in \Omega_{k'}} \tilde{\mathbb{V}}_{Nv}}$$

I.2.2 CES-Feenstra

The overall CES-Feenstra price index is given by

$$\frac{\mathbb{P}_i}{\mathbb{P}_N} = \left(\frac{1}{\lambda_N}\right)^{\frac{1}{\sigma-1}} \prod_{k \in \Omega_i^K} \left(\left(\frac{1}{\lambda_{Nk}}\right)^{\frac{1}{\sigma_k - 1}} \prod_{v \in \Omega_{ik}} \left(\frac{p_{iv}}{\tilde{p}_{Nv}}\right)^{\omega(s_{ivk}^*, \tilde{s}_{Nvk}^*)} \right)^{\omega(s_{ik}^*, \tilde{s}_{Nk}^*)} \tag{36}$$

λ_{Nk} is the national expenditure share on housing varieties that exist in MSA i in nest k and λ_N is the national expenditure share on nests that exist in MSA i.

I.2.3 Single Nest CES Price Index

In this subsection I provide the formulas for a single nest CES price index in a comparison of each MSA against a national representative household. The CES-Feenstra single nest index is defined as

$$\frac{\mathbb{P}_i}{\mathbb{P}_N} = \left(\frac{1}{\lambda_N}\right)^{\frac{1}{\sigma_{single}-1}} \prod_{v \in \Omega_i} \left(\frac{p_{iv}}{\tilde{p}_{Nv}}\right)^{\omega(s_{iv}^*, \tilde{s}_{Nv}^*)} \tag{37}$$

where Ω_i denotes the set of available varieties in MSA i. The national expenditure share on varieties available in MSA i is defined as

$$\lambda_N = \frac{\sum_{v \in \Omega_i} \tilde{\mathbb{V}}_{Nv}}{\sum_{v \in \Omega} \tilde{\mathbb{V}}_{Nv}},$$

where Ω denotes the set of all available varieties.

The CES-RW single nest index is defined as

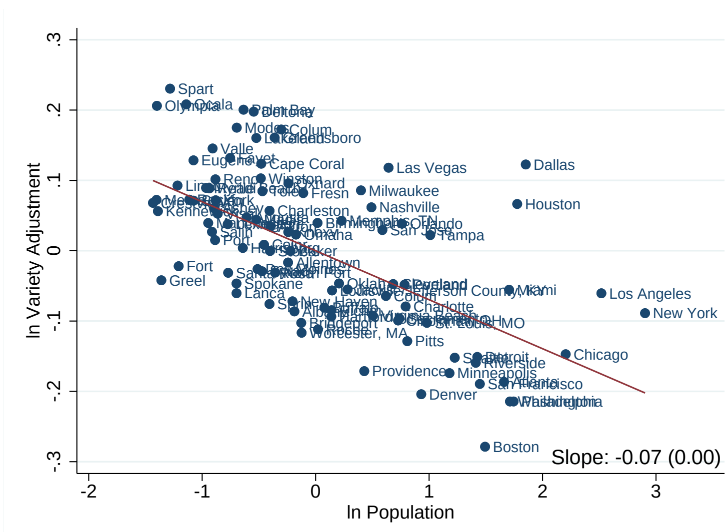
$$\frac{\mathbb{P}_i}{\mathbb{P}_{CH}} = \left(\frac{1}{\lambda_N}\right)^{\frac{1}{\sigma_{single}-1}} \prod_{v \in \Omega_i} \left(\frac{p_{iv}}{\tilde{p}_{Nv}}\right)^{\frac{1}{N_i}} \prod_{v \in \Omega_i} \left(\frac{s^*_{iv}}{\tilde{s}^*_{Nv}}\right)^{\frac{1}{N_i}} \tag{38}$$

where N_i is the number of varieties in MSA i , or $N_i = |\Omega_i|$.

Appendix J Reduced-Form Results

In Figure A9, I show the nested CES-RW Variety Adjustment against population for All Bilateral Comparisons. The population and nest variety adjustment are averaged over time for each MSA. In Table A9, I decompose the overall nested CES price index into its different components.

Figure A9: Nested CES Variety Index: Chicago Comparison



Notes: Figure plots the regression of the variety adjustment on demeaned log population.

J.1 Comparison to Number of Varieties

How much of the variation in the variety index is explained by the number of varieties? To answer this, I regress the single-nest variety adjustment for the Chicago comparison on the log number of

Table A9: Decomposition of Nested CES-RW Price Index vs Population (2005-2019)

	Hedonic	Variety Adj	Common			Nested CES-RW
			Price	Share	Nest Share	
ln Pop	0.185***	-0.0697***	0.159***	0.00958***	-0.00701***	0.0918***
	(0.0166)	(0.0029)	(0.0166)	(0.0018)	(0.0005)	(0.0174)
N	1470	1470	1470	1470	1470	1470
R2	0.159	0.358	0.0725	0.0165	0.102	0.0249

Notes: All dependent variables are in logs. Population estimates are obtained from the Census Bureau. Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A10: National Household Comparison: Price Indices vs Population (2005-2019)

		Nested CES					
		All Bilateral Comparisons			vs National Household		
	Hedonic	Variety Adj	Common	CES-RW	Variety Adj	Common	CES-RW
ln Pop	0.185***	-0.0697***	0.161***	0.0918***	-0.0610***	0.0487***	-0.0123
	(0.0166)	(0.0029)	(0.0176)	(0.0174)	(0.0024)	(0.0123)	(0.0123)
N	1470	1470	1470	1470	1470	1470	1470
R2	0.159	0.358	0.0693	0.0249	0.430	0.0150	0.00456

		Single Nest CES					
		All Bilateral Comparisons			vs National Household		
	Hedonic	Variety Adj	Common	CES-RW	Variety Adj	Common	CES-RW
ln Pop	0.185***	-0.0523***	0.148***	0.0955***	-0.0374***	0.0375**	0.0000606
	(0.0166)	(0.0022)	(0.0164)	(0.0162)	(0.0021)	(0.0121)	(0.0125)
N	1470	1470	1470	1470	1470	1470	1470
r2	0.159	0.325	0.0660	0.0302	0.334	0.00999	0.00234

Notes: All dependent variables are in logs. Population estimates are obtained from the Census Bureau. Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

varieties and find that the R2 is 0.5, so that half of the cross-sectional variation can be explained by the number of varieties.

Does the number of varieties predict the welfare impact of variety availability differences across MSA size? I find evidence that the number of varieties over-predicts the welfare impact of variety differences for large MSAs versus small MSAs.

I construct a predicted variety index by using the number of varieties multiplied by $-\frac{1}{\sigma-1}$. To see why this index is relevant, suppose that Chicago has a super-set of varieties relative to each comparison MSA and that the expenditure on each variety is the same. Then the variety index can be re-written as

$$\frac{1}{\sigma-1}\ln\frac{\lambda_{it}}{\lambda_{CHt}} = \frac{1}{\sigma-1}\ln\frac{1}{\lambda_{CHt}} = -\frac{1}{\sigma-1}\ln\frac{N_{it}}{N_{CHt}}$$

where N_{it} is the number of varieties in MSA i at time t. Note that λ_{it} is the expenditure on common

Table A11: CES-Feenstra Price Indices vs Population (2005-2019)

		Nested CES-Feenstra					
		All Bilateral Comparisons		vs Chicago		vs National Household	
	Hedonic	Variety Adj	CES-Feenstra	Variety Adj	CES-Feenstra	Variety Adj	CES-Feenstra
ln Pop	0.185***	-0.0518***	0.0993***	-0.0398***	0.133***	-0.0379***	0.00848
	(0.0166)	(0.00216)	(0.0156)	(0.00256)	(0.0167)	(0.00205)	(0.0124)
N	1470	1470	1470	1470	1470	1470	1470
R2	0.159	0.347	0.0358	0.149	0.0746	0.345	0.00304

		Single Nest CES-Feenstra					
		All Bilateral Comparisons		vs Chicago		vs National Household	
	Hedonic	Variety Adj	CES-Feenstra	Variety Adj	CES-Feenstra	Variety Adj	CES-SV
ln Pop	0.185***	-0.0398***	0.133***	-0.0379***	0.00848	-0.0482***	0.113***
	(0.0166)	(0.00256)	(0.0167)	(0.00205)	(0.0124)	(0.00181)	(0.00624)
N	1470	1470	1470	1470	1470	67620	67620
R2	0.159	0.149	0.0746	0.345	0.00304	0.462	0.391

Notes: All dependent variables are in logs. Population estimates are obtained from the Census Bureau. Robust standard errors in parentheses. Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A12: Relationship between Elasticity of Substitution and Nest-level Variety Adjustment

	All Bilateral	All Bilateral	National Household	National Household
ln Nest Variety Adj	-0.155***	-0.213***	-0.656***	-0.701***
	(0.0235)	(0.0322)	(0.0102)	(0.00890)
N	8820	8820	8820	8820
R2	0.00303	0.00416	0.205	0.219
FE	t	it	t	it

Notes: Observations are at the nest-MSA-year level. Standard errors clustered at the MSA X year level.

Table A13: Equivalent Aggregate Sigmas

	Sigma
Mean	6.3
p5	1.4
p10	2.6
p25	4.8
p50	6.4
p75	8.1
p90	9.8
p95	11.0
N	1176

Notes: Statistics are computed on the equivalent sigmas between the 10th and 90th percentile of the initial distribution.

varieties in MSA i, and since Chicago has a super-set of varieties relative to MSA i, then $\lambda_{it} = 1$.

For a single nest, the predicted population elasticity using the number of varieties is -0.054, in contrast to the estimated -0.052 with a single nest and -0.07 with multiple nests.

J.2 Distance Analysis

Table A14: Price Indices vs Population (2005-2019)

	All Bilateral Comparisons		
	Hedonic	Variety Index	CES-RW
Panel A: Within 20 miles			
ln Pop	0.181***	-0.0361***	0.0962***
	(0.0158)	(0.00245)	(0.0148)
Panel B: Within 30 miles			
ln Pop	0.170***	-0.0445***	0.0824***
	(0.0155)	(0.00215)	(0.0143)
Panel C: Within 50 miles			
ln Pop	0.169***	-0.0474***	0.0793***
	(0.0155)	(0.00209)	(0.0141)

Notes: Robust standard errors in parentheses. A single nest is used. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

J.3 Robustness Checks

Table A15: Price Indices vs Population: Alternative Nesting Structure Bedrooms

		Nested CES-RW		
		All Bilateral Comparisons		
	Hedonic	Variety	Common	CES-RW
ln Pop	0.185***	-0.0667***	0.167***	0.0999***
	(0.0166)	(0.0027)	(0.0168)	(0.0166)
N	1470	1470	1470	1470
R2	0.159	0.378	0.0795	0.0317

Notes: All dependent variables are in logs. Population estimates are obtained from the Census Bureau. Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Appendix K Spatial Implications

K.1 Amenity Data Sources

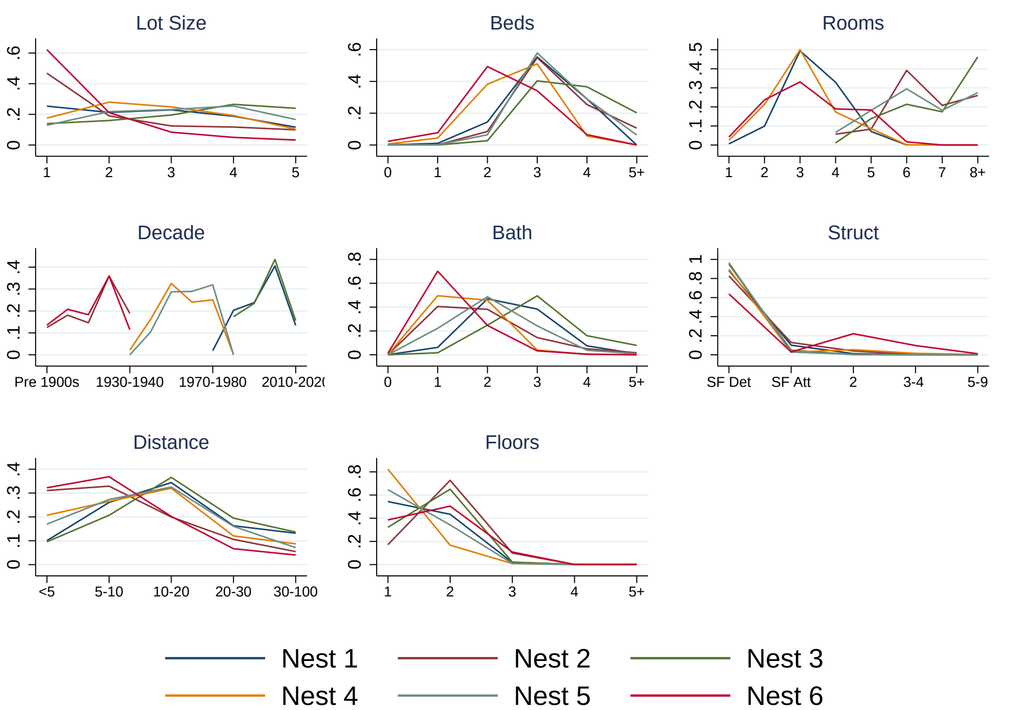
1. AQI is obtained at the MSA level from from the EPA (runs from 0 to 500). Higher levels indicate worse air pollution.

Table A16: Price Indices vs Income and Population (2005-2019)

		Nested CES-RW				Nested CES-RW		
	Hedonic	Variety	Common	CES-RW	Hedonic	Variety	Common	CES-RW
ln MSA HH Income	3.223***	-0.470***	3.062***	2.592***	3.061***	-0.153***	3.010***	2.857***
	(0.153)	(0.0308)	(0.167)	(0.166)	(0.171)	(0.0267)	(0.193)	(0.200)
ln Pop					0.0325	-0.0635***	0.0103	-0.0532*
					(0.0187)	(0.00316)	(0.0202)	(0.0207)
N	1366	1366	1366	1366	1366	1366	1366	1366
R2	0.352	0.167	0.249	0.198	0.354	0.377	0.249	0.203

Notes: All dependent variables are in logs. Population estimates are obtained from the Census Bureau. t denotes year fixed effects. Ln MSA HH Income is the post-tax average household income in each MSA, residualized with household-head demographic variables. Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Figure A10: Density of Characteristics Across Nests: Alternative Nesting Structure



Notes: Figure presents density of characteristics in six nests based on k-means estimated on only housing characteristics.

2. Average commute time is measured from IPUMS ACS.

3. Violent and Property Crime are obtained from NACJD from University of Michigan's ICPSR at the county-level for 2005-2014, 2016, while 2015 and 2017-2019 data are obtained from FBI UCR MSA level reports. This allows me to define a constant geography over time.

4. "Heating degree-days are the number of degrees that the daily average temperature falls below 65° F. Cooling degree-days are the number of degrees that the daily average temperature rises above 65° F." Precipitation is measured as days with more than 0.1 inch of precipitation. Data obtained from the National Center for Environmental Information Global Summary of the Year dataset.

K.2 Difference in Recovered Amenities

To rationalize the observed spatial equilibrium, urban and regional models include local amenities that compensate for differences in market consumption.

Figure A11 compares the average implied amenities from the hedonic index and the nested CES-RW index under three different values of ν: $4, 8, \infty$. Since the 45-degree line is close to the linear fit, amenities implied by the hedonic index are not systematically underestimated or overestimated.

However, there is significant dispersion around the line of best fit. Consider the absolute difference between the amenities implied by the hedonic index and the nested CES-RW index. I find that the mean difference for the Rosen-Roback calibration is 0.14 log points, with a standard deviation of 0.10 log points. The 75% percentile is 0.2 log points, so that that 25% of MSAs have amenities that change by at least 22% after using the CES-RW index.[82]

K.3 Household Heterogeneity

[82] These estimates are obtained by normalizing average amenities to zero under both calibrations.

Figure A11: Amenity Comparison Implied by Hedonic versus Nested CES-RW

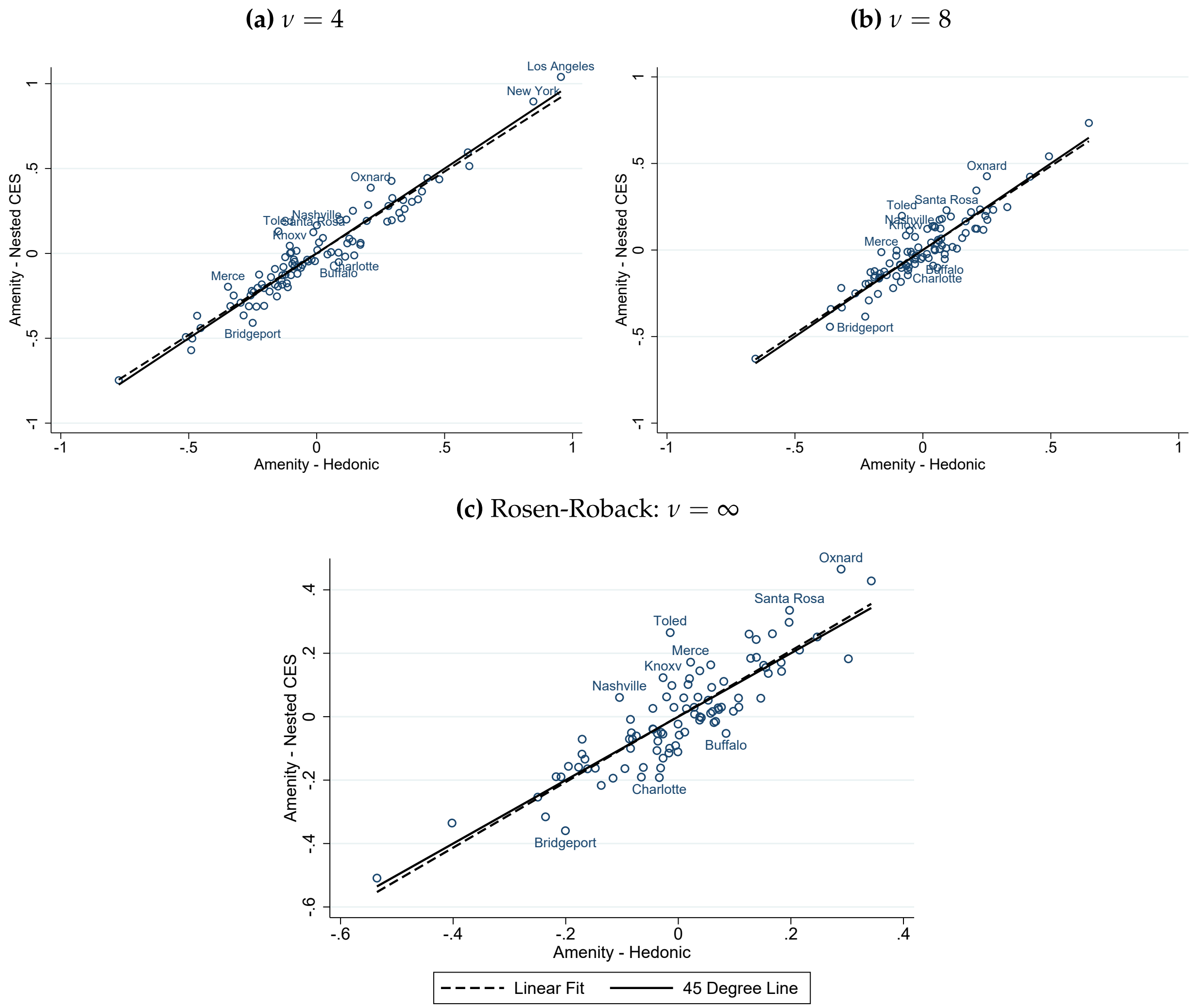
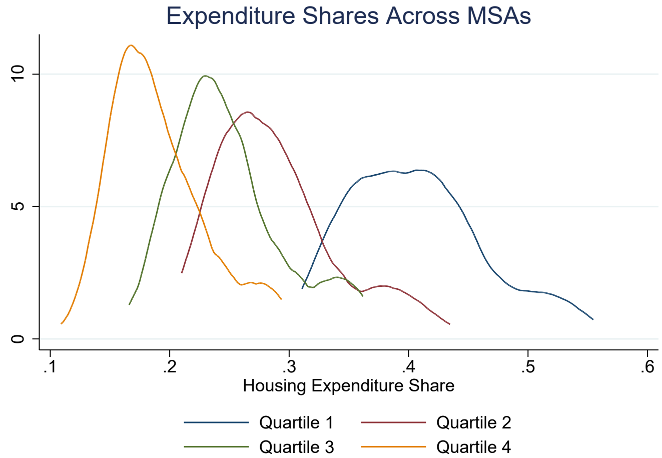


Table A17: Price Indices vs Population (2005-2019)

		Chicago Comparison		All Bilateral Comparisons	
	Hedonic	Variety Index	CES-RW	Variety Index	CES-RW
Quartile 1					
ln Pop	0.156***	-0.033***	0.115***	-0.047***	0.061***
	(0.016)	(0.008)	(0.022)	(0.003)	(0.009)
Quartile 2					
ln Pop	0.165***	-0.065***	0.091***	-0.065***	0.059***
	(0.016)	(0.008)	(0.018)	(0.002)	(0.008)
Quartile 3					
ln Pop	0.160***	-0.072***	0.063***	-0.075***	0.051***
	(0.015)	(0.007)	(0.016)	(0.002)	(0.008)
Quartile 4					
ln Pop	0.164***	-0.058***	0.103***	-0.075***	0.096***
	(0.015)	(0.005)	(0.016)	(0.006)	(0.017)

Notes: All dependent variables are in logs. Population estimates are obtained from the Census Bureau. t denotes year fixed effects. Robust standard errors in parentheses for the Chicago Comparison and clustered standard errors at the comparison MSA level for All Bilateral Comparisons. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Figure A12: Heterogeneity in Expenditure Shares



K.4 Skill Heterogeneity in Real Income

In this section, I measure how market consumption differs among three skill groups. Compared to Diamond and Moretti (2021), I find that standard approaches substantially underestimate real income in larger cities for all three skill types. Following Diamond and Moretti, I define the low skill group is where the household head has less than a high school degree, the middle skill group is where the household head has completed high school degree or partial college, and the high skill group is where the household head has a college degree or higher.

To flexibly account for differing expenditure shares, I compute average housing expenditure shares by MSA and income quartile, μ_{iq}, where i indexes MSAs and q indexes the income quartile. Figure A12 shows that expenditure shares on housing is more than twice as high in the lowest income quartile compared to the highest income quartile.

Using the estimated expenditure shares, I then construct relative local prices by using a Törnqvist upper-level price index (which uses the bilateral average expenditure share on housing).[83] I also compute the average household post-tax income for each skill group in each MSA, w_{ic}, where c indexes skill group.[84]

To map skill groups to income quartile, I measure the share of skill group that belongs to each quartile q. The local price index for skill group s is defined as

$$\mu_{ci} \ln \mathbb{P}^H_{ci} = \sum_q s^H_{cqi} \mu_{qi} \ln \mathbb{P}^H_{qi}$$

where s^H_{cqit} is the share of skill group c that belongs to income quartile q in MSA i

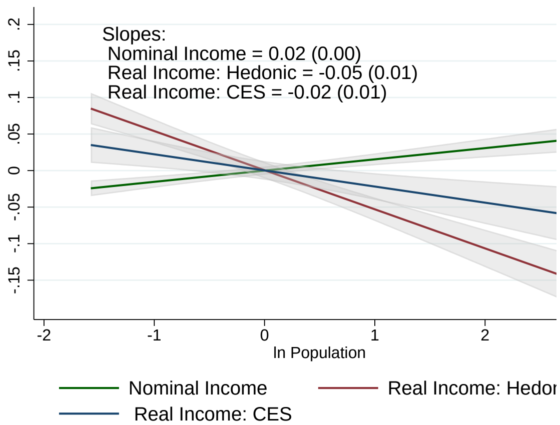
Figure A13 presents the estimated population elasticities of nominal wages, real income im-

[83] Specifically, the relative local price index for locations i and n for income quartile q is given by $\frac{\mu_{iq}+\mu_{nq}}{2}\left(\ln \mathbb{P}_{iq} - \ln \mathbb{P}_{nq}\right)$.

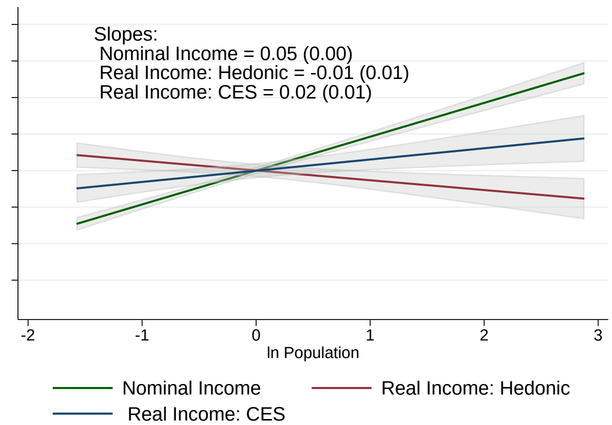
[84] Post-tax household income are residualized using the same set of variables in the main analysis by skill group.

Figure A13: Real Income: Skill Heterogeneity

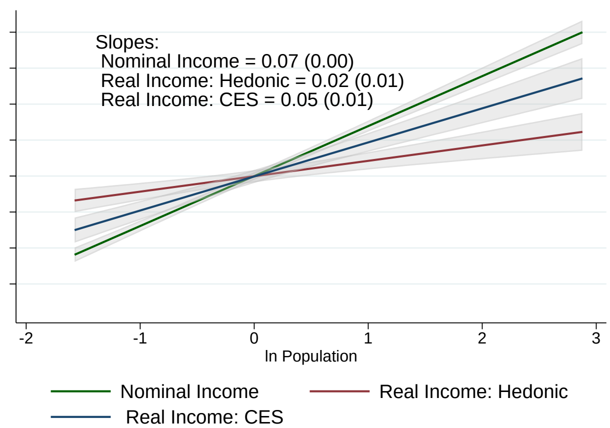
(a) Low Skill



(b) Medium Skill



(c) High Skill



plied by hedonic price index, and real income implied by the CES price index for three skill groups. The substantially higher wage profile for the high-skill group relative to the low-skill group reflects the well-documented urban skill premium.

I first focus on the change in nominal income to real income implied by a hedonic price index. The decrease in the population elasticity when going from nominal to real income is highest for low-skill households. This is due to the fact that low-skill households have the highest expenditure shares on housing. As a result, real income is substantially declining in MSA population for low-skill households (population elasticity of -0.05), compared to high-skill households that see an increasing real income in MSA population (population elasticity of 0.02).

Next, what happens to real income after accounting for housing variety? I find that the lower housing cost for high-income households in larger MSAs is offset by lower expenditure shares. As a result, the increase in the elasticity of real income for all three skill groups is similar (elasticity of real income increases by 0.03). As a result, high-skill households have even higher real incomes than previously measured in larger MSAs. At the same time, low-skill households face a smaller decline in real income when moving from a smaller to larger MSA than previously measured.