Optimally Stubborn

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Motivation

The literature on reputation builds on the idea that some players may be limited in their choice of strategies – behavioral types:

- chain store game with an incumbent committed to fighting entry (Kreps and Wilson, 1982),
- finitely repeated prisoner's dilemma with for instance a grim-trigger type (KMRW, 1982).

Rational players may benefit from mimicking behavioral types.

Motivation

The results in this literature rely on the choice of behavioral types.

With the "right" behavioral (Stackelberg) type present, sharp predictions in terms of lower (or upper) bounds on payoff (Fudenberg and Levine, 1989).

Motivation

Aim of this paper: endogenize behavioral types by giving them some minimal flexibility in their behavior.

Questions

How reliant on exogenous behavioral types are the predictions of the reputation literature regarding

(i) behavior and

(ii) payoffs?

Is the “right” behavioral type present?

Which kinds of behavioral types occur in equilibrium?

Environment

I consider a bargaining game using the framework by Myerson (1991) and Abreu and Gul (2000).

Why bargaining?

- Baseline allows for characterization of equilibrium.
- Endogenizing behavioral types has a natural economic interpretation.

Application: private profit-sharing rules between CEOs and managers.

Model overview

Bargaining game with **two types of players** – rational and stubborn.

There are two stages: a **demand stage**, and a **concession game**.

Stubborn type: choose from the set of “insistent” strategies that always make the same demand and never concede to anything less.

Rational type: “flexible” both at the demand and concession stage.

Results

Strong behavioral predictions:

Equilibria in which both types randomize over multiple offers exist.

As the probability of stubbornness goes to 0, a "mixed" equilibrium must involve one or two offers in its support.

The "right" stubborn type may not be present.

Even in the limit, delay may not disappear.

Weak payoff predictions: There is a Folk theorem like payoff multiplicity (when no refinement is applied).

Set-Up

Timing

Time is continuous, horizon is infinite.

Two players decide on how to split a unit surplus.

At time 0, players 1 and 2 simultaneously announce demands, α_1 and α_2:

- if $\alpha_1 + \alpha_2 \leq 1$, game ends.
- if $\alpha_1 + \alpha_2 > 1$, a concession game starts. Game ends when one player concedes.

Concession means agreeing to the opponent's demand.

Types

Each player $i = 1, 2$ is

stubborn with probability z, and

rational with probability $1 - z$.

Type is private information.

A stubborn type can make any demand $\alpha \in [0, 1]$ at time 0, but cannot concede to his opponent.

A rational type can make any demand $\alpha \in [0, 1]$ at time 0, and concede to his opponent at any time.

Concession game

Why only allow concession rather than gradual adjustment of offers?

- Revising one's offer reveals rationality.
- Revealing rationality when the opponent is stubborn wpp leads to immediate concession (Myerson, 1991).
- Hence, revision is essentially equivalent to concession.

Payoffs

If $\alpha_1 + \alpha_2 \leq 1$, the demands are said to be *compatible*. Each player i receives α_i and $1 - \alpha_j$ with equal probability.

If $\alpha_1 + \alpha_2 > 1$, and i concedes to j at time t, then

$$v_i = e^{-\rho t} \left(1 - \alpha_j\right),$$
$$v_j = e^{-\rho t} \alpha_j,$$

where $\rho > 0$ is the common discount rate.

Strategies

A strategy for the *stubborn* type is the choice of the initial offer, $s_i \in \Delta([0,1])$.

A strategy for the *rational* type is a pair $\left(r_i, F_i^{\alpha_i,\alpha_j}\right)$, $\forall \alpha_i, \alpha_j$ with $\alpha_i + \alpha_j > 1$, where

> $r_i \in \Delta([0,1])$ is the choice of the initial offer, and
> $F_i^{\alpha_i,\alpha_j}$ is the cdf of concession given α_i, α_j.

$F_i^{\alpha_i,\alpha_j}(t)$ is the probability of player i conceding to player j by time t (inclusive), given α_i, α_j.

Solution concept

A Perfect Bayesian equilibrium is a profile of strategies $(s_i, (r_i, F_i))$, and a system of beliefs $\pi_i : [0,1] \to [0,1]$ for $i = 1,2$ such that,

- sequential rationality: the strategy is optimal given the beliefs, from any point on, and
- Bayes' rule is satisfied (where possible).

Note: here, only the initial updating is specified.

Benchmark

Exogenous distribution of offers of the stubborn type

Myerson (1991) and Abreu and Gul (2000)

My model

Two types of players: rational and stubborn.

Stubborn player i can choose his initial demand $\alpha_i \in [0, 1]$, but cannot concede to his opponent.

Abreu and Gul

$N + 1$ types of players: one rational type and N stubborn types.

Set of stubborn types: $C = \{\alpha^1, \alpha^2, \dots, \alpha^N\}$.

Stubborn player of type α^n always demands α^n, and cannot concede to his opponent.

Existence with exogenous types

Proposition (Abreu and Gul, 2000)

A PBE exists. All PBE are outcome-equivalent.

The equilibrium outcome is characterized by the two choices a rational player makes:

whom to mimic, and

when to concede.

Concession game

When to concede:

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(1) at most one player concedes wpp at time 0,

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(2) players initially concede at a rate that makes the opponent indifferent between waiting and conceding,

Concession game

When to concede:

(1) at most one player concedes wpp at time 0,

(2) players initially concede at a rate that makes the opponent indifferent between waiting and conceding,

(3) there is a finite time, call it T_0, by which the posterior probability of stubbornness reaches 1 simultaneously for both players.

Concession rate

Player i is indifferent between waiting and conceding if:

$$\underbrace{\rho(1-\alpha_j)}_{\text{Net cost of waiting}} = \underbrace{(\alpha_i - (1-\alpha_j))\,\frac{F_j'(t)}{1-F_j(t)}}_{\text{Net benefit of waiting}}.$$

Therefore, player j concedes at a rate:

$$\text{rate of concession}_j = \frac{\rho(1-\alpha_j)}{\alpha_i + \alpha_j - 1}.$$

Player's strength

The requirement that the probability of stubbornness reaches 1 *simultaneously* for both players pins down:

- the identity of the player who concedes at time 0, and
- the probability with which this happens.

Counterfactual: What is the time T_i at which a player *i would* be known to be stubborn if he *did not* concede wpp at time 0?

$\rightarrow$ A player i is *weak* if $T_i > T_j$.

The weak player has to concede with sufficient probability at time 0 for the posterior probability of stubbornness to reach 1 at the same time.

Player's strength

Strength $\mu_i(\alpha_i)$ as defined by:

$$\mu_i(\alpha_i) = \pi_i(\alpha_i)^{\frac{1}{1-\alpha_i}}$$

depends on

how likely a player is thought to be stubborn,

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- how likely a player is thought to be stubborn, and
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the value of the demand.

A player's payoff is increasing in his strength:

$$v_i = \underbrace{1-\alpha_j}_{\substack{\text{Payoff of the}\\ \text{weak player}}} \quad \text{vs.} \quad v_j = \underbrace{\alpha_j F_i(0) + (1-\alpha_i)(1-F_i(0))}_{\text{Payoff of the strong player}},$$

where

$$F_i(0) = 1 - \left(\frac{\mu_i(\alpha_i)}{\mu_j(\alpha_j)}\right)^{1-\alpha_j}.$$

Strength decreasing in demand

Lemma

In any symmetric PBE, strength is decreasing in the (equilibrium) demand.

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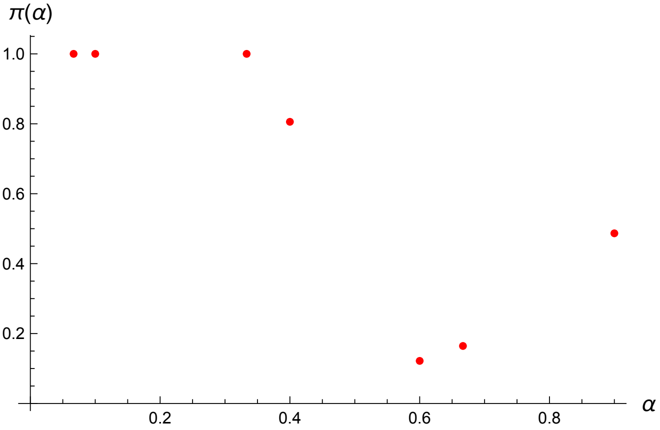
The probability with which the opponent concedes is increasing in a player's strength.

Fixing a player's demand, his payoff is increasing in the probability of immediate concession.

Demand stage

Whom to mimic: Any demand above some threshold is mimicked wpp.



Cond. probability of stubbornness, $\pi(\alpha)$, in a PBE with a given set of stubborn types.

Payoffs and delay with exogenous types

If the “right” stubborn type is present, the rational type receives a payoff of $1/2$ as $z \to 0$.

The right stubborn type is the type demanding $1/2$.

- In the limit, a higher offer which is played with non-negligible probability immediately concedes to a lower offer.
- By demanding $1/2$, a rational player receives at least $1/2$ regardless of the demand he faces.

There is no delay as $z \to 0$.

Summary predictions

Abreu and Gul

Weak behavioral predictions: Any demand above some threshold value is mimicked.

With the right behavioral type present, there is no delay in the limit.

Strong payoff predictions: The rational player receives $1/2$.

Summary predictions

My model

Strong behavioral predictions: In the limit, mixing over at most two demands by both types.

The right behavioral type may not be present.

Even in the limit, delay may not disappear.

Weak payoff predictions: Folk theorem like payoff multiplicity.

Abreu and Gul

Weak behavioral predictions: Any demand above some threshold value is mimicked.

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Endogenous stubborn types

Benchmark: A stubborn type *cannot* choose his initial demand.

From now on: A stubborn type **chooses** his initial demand.

Symmetric, mixed PBE

Analysis focuses on symmetric, mixed PBE.

- Symmetric: $r_1 = r_2$ and $s_1 = s_2$.
- Mixed: $\operatorname{supp} r = \operatorname{supp} s$.

Do other PBE exist?

- There can be at most one offer made exclusively by one type.
- Only the stubborn type can make a separating offer.
- Asymmetric PBE exist.

Existence – one offer

Proposition

Symmetric PBE, where players make one demand only exist. In such a PBE, there is either

- immediate agreement (with $\alpha = 1/2$), or
- infinitely long delay (with $\alpha = 1$).

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- immediate agreement (with $\alpha = 1/2$), or
- infinitely long delay (with $\alpha = 1$).

When $1/2 < \alpha < 1$, a stubborn type has an incentive to deviate to $1 - \alpha$.

When $\alpha < 1/2$, either type has an incentive to deviate to $1 - \alpha$.

When $\alpha \in \{1/2, 1\}$, no player has an incentive to deviate if a deviation is believed to be from a rational type.

Difference between rational and stubborn type

The difference between the rational and stubborn type is the payoff when

faced with an incompatible demand,

coming from a stubborn opponent.

A stubborn type does not have the **option value of concession**.

The difference vanishes if

the delay to agreement is infinitely long, or

agreement is immediate.

Necessary conditions for existence with multiple demands

Lemma

Fix any set of demands C, with $|C| \geq 2$. In any symmetric, mixed PBE with support C, the following holds:

1. the lowest and highest demand in C are incompatible;
2. if $\alpha < \alpha'$, with $\alpha, \alpha' \in C$, then there exists $\alpha'' \in C$ such that $\alpha + \alpha'' \leq 1 < \alpha' + \alpha''$.

Set of compatible demands decreasing

The rational type is indifferent between two offers if

- the increased gain from the higher offer is offset by
- the decrease in the option value from asking for a higher demand.

Suppose there was no $\alpha'' \in C$ such that $\alpha + \alpha'' \leq 1 < \alpha' + \alpha''$.

- When facing a compatible demand, stubborn type receives the same payoff as rational type.
- When facing an incompatible demand, rational type has the option value of concession, the stubborn type does not.
- The option value of concession is decreasing in the offer.

$\Rightarrow$ If the set of compatible offers is not decreasing, the two types cannot both be indifferent.

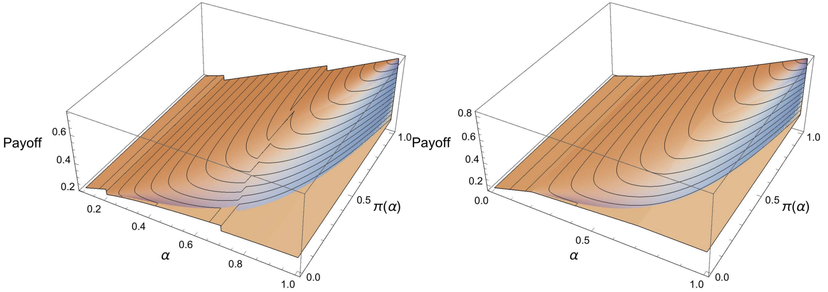
Existence – two offers

The necessary conditions are sufficient for existence of PBE with two demands:

Proposition

Fix any two demands α, β, such that $\alpha \leq 1 - \alpha < \beta < 1$. There exists $\bar{z} > 0$ such that for all $z < \bar{z}$, there exists a symmetric, mixed PBE, with support $\{\alpha, \beta\}$.

Preferences



3D payoff profile for stubborn (left) and rational (right) player i as a function of i's demand α and probability of stubbornness $\pi(\alpha)$. Parameters for player j: $(\alpha_j, \beta_j, z_j) = (3/10, 8/10, 1/4)$.

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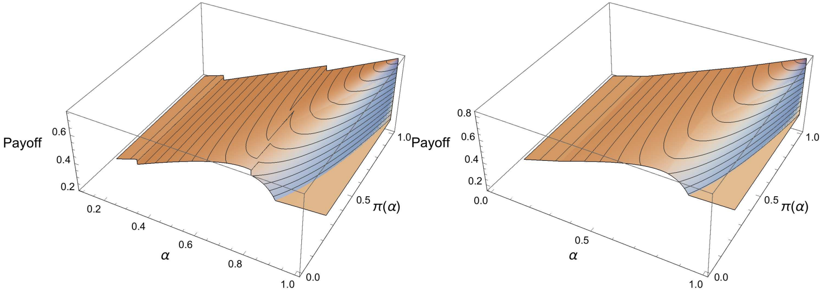

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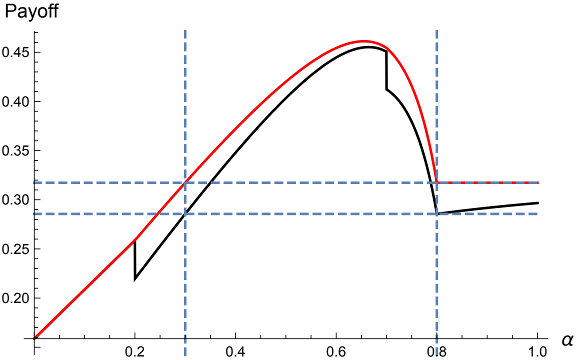
Cross-section



Cross-sections of the 3D payoff profile for rational (red) and stubborn (black) type.

Discontinuity

Suppose player 1 demands α, upon which he is believed to be stubborn with probability z. For small $\epsilon > 0$,

Player 2's demand:	$1-\alpha$	$1-\alpha+\epsilon$
v_2^r	$1-\alpha$	$\geq (1-\alpha)$
v_2^s	$1-\alpha$	$< (1-z)\,(1-\alpha+\epsilon)$

A stubborn type's payoff v_i^s is discontinuous in α_i because, unlike a rational type, a stubborn type does not have the option value of concession.

Payoffs and Delay

Corollary

Fix any $v \in (0, 1/2]$. Then there exists $\bar{z} > 0$ such that for any $z < \bar{z}$, a symmetric PBE exists such that the rational type's payoff is v.

The equilibrium payoff (with two offers) in the limit:

$$v = \frac{1}{2} - \frac{(1/2 - \alpha)^2}{\beta - 1/2},$$

where $\alpha + \beta > 1$ and $\alpha \leq 1/2$.

Even in the limit, delay may not disappear: the right stubborn type may not be present.

Existence – three offers

Proposition

(a) Fix $C = \{\alpha, \beta, \gamma\}$, with $\alpha \leq 1 - \alpha < \beta < \gamma \leq 1$. There exists $\bar{z} > 0$, such that for all $z < \bar{z}$, there exists no symmetric, mixed PBE, with support C.

(b) Fix $C = \{\alpha, 1 - \alpha, \gamma\}$, with $\alpha < 1 - \alpha < \gamma < 1$. There exists $\bar{z} > 0$, such that for all $z < \bar{z}$ there exists a symmetric, mixed PBE, with support C.

In such a PBE, as $z \to 0$, the offer $1 - \alpha$ is assigned probability 1.

Rational type – Indifference over three offers

Rational player i's payoff when facing a given demand of player j:

	α	β	γ
α	$\frac{1}{2}\alpha + \frac{1}{2}(1-\alpha)$	$\frac{1}{2}\alpha + \frac{1}{2}(1-\beta)$	$F^{\gamma\alpha}(0)\alpha + (1 - F^{\gamma\alpha}(0))(1-\gamma)$
β	$\frac{1}{2}\beta + \frac{1}{2}(1-\alpha)$	$1-\beta$	$F^{\gamma\beta}(0)\beta + (1 - F^{\gamma\beta}(0))(1-\gamma)$
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Recall: $\alpha < \beta$.

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Recall: $\alpha \leq 1 - \beta$.

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Recall: $\alpha < \beta$ and $\alpha + \gamma > 1$.

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$F^{\gamma\alpha}(0) \gg F^{\gamma\beta}(0)$ and $r(\gamma) \gg 0.$

Stubborn type – Indifference over three offers

$$F^{\gamma\alpha}(0) \gg F^{\gamma\beta}(0)$$

Stubborn type – Indifference over three offers

$$F^{\gamma\alpha}(0) \gg F^{\gamma\beta}(0) \Rightarrow \quad \mu_\alpha \gg \mu_\beta$$

Payoff difference between rational and stubborn type is the option value of concession:

	α	β	γ
α	0	0	$(1-\gamma)\mu_\gamma^{1-\gamma}\mu_\alpha^{\alpha+\gamma-1}$
β	0	$(1-\beta)\mu_\beta^\beta$	$(1-\gamma)\mu_\gamma^{1-\gamma}\mu_\beta^{\beta+\gamma-1}$

Stubborn type – Indifference over three offers

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$\Rightarrow$ When demanding β, option value of concession is low.

Stubborn type – Indifference over three offers

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$\Rightarrow$ When demanding β, option value of concession is low.

$\Rightarrow$ Stubborn type strictly prefers β over α.

Rational type – Indifference over three offers

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$$F^{\gamma\alpha}(0) \not\gg F^{\gamma\beta}(0) \Rightarrow \quad \beta \succ \alpha \text{ unless } \beta \approx 1-\alpha \text{ and } r(\beta) \approx 1.$$

Existence – More offers

Proposition

Fix C such that $|C| > 3$. Then there exists $\bar{z} > 0$ such that for all $z < \bar{z}$, there exists no symmetric PBE with support C.

Refinements – Passive beliefs

Passive beliefs: players do not update their beliefs' about their opponent's type when seeing an out-of-equilibrium demand.

Lemma

There is a unique passive belief PBE. In this PBE, players demand $1/2$.

Refinements – D1

Informally, D1 assigns probability 1 to the type who has the "strongest" incentive to deviate to a given demand. Formal definition

Lemma

In the set of symmetric PBE with support $|C| \leq 2$, there exists a unique PBE satisfying D1. In this PBE, players demand $1/2$.

Conjecture

In the set of symmetric PBE with support $|C| \leq 3$, there is a unique symmetric PBE satisfying D1.

Related literature – bargaining and reputation

Endogenizing behavioral types in bargaining

Abreu and Sethi (2003): evolutionary stability approach.

Kambe (1999) and Wolitzky (2012): players do not know at the demand stage whether they are behavioral or not.

Exogenous behavioral types in bargaining:

Abreu, Pearce and Stacchetti (2015): one-sided uncertainty about a player's patience.

Fanning (2016a): agreement can only be reached until a deadline arrives.

Fanning (2016b): uncertainty about the cost of delaying agreement.

Conclusion, To Dos and Extensions

Prove D1 with three or more offers.

Broaden the set of strategies available to stubborn types, for instance exit option.

Endogenous behavioral types in repeated games framework.

Supplementary slides

Divinity

Define $\Theta = \{R, S\}$, where R denotes rational and S stubborn.

$D(\theta, \Theta, d)$: set of best responses (BR) F_i to demand d for some arbitrary belief (with support in Θ) that make type θ strictly prefer d to his equilibrium strategy.

$D^0(\theta, \Theta, d)$: set of BR that make type θ exactly indifferent.

A type θ is deleted for demand d under criterion $D1$ if there is a θ' such that

$$\{D(\theta, \Theta, d) \cup D^0(\theta, \Theta, d)\} \subset D(\theta', \Theta, d).$$

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