Informality, Risk Premia, and the Business Cycle

Livio Maya Gustavo Pereira

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Abstract

The share of informal jobs increases in downturns and decreases in booms. We provide a novel channel to explain this stylized fact. The key insight is that the breakdown of the workforce into formal and informal jobs over the business cycle behaves like the weights of an aggregate portfolio. Hiring a worker formally has advantages in terms of productivity, but is inherently risky due to severance payment requirements that prevent the termination of labor contracts that become unprofitable during recessions. The fact that the share of informal jobs increases in downturns reflects portfolio rebalancing towards the less risky asset: informal labor. We embed the risk structures of formal and informal jobs in a search and matching model with aggregate uncertainty, which we calibrate with data from Brazil. We find that a *countercyclical price of risk* is an essential ingredient for producing predictions about the cyclicality of labor market variables in line with the data. We simulate a reduction in severance payments in economies with different degrees of time variation in the price of risk (TVPR). We find substantial differences in the sensitivity of formalization to the reduction in the severance requirements across specifications. Moreover, reducing severance payments reduces unemployment volatility when the TVPR degree is low, and increases for sufficiently high TVPR. The results show that correctly specifying TVPR is consequential for policy analysis.

1 Introduction

The share of workers in informal jobs in developing countries comoves negatively with the business cycle. That fact has been documented by a large body of research and appears to hold for many countries, sample periods and is robust to alternative definitions of an informal job.[1] The typical interpretation of that stylized fact is that it provides evidence of the role of informality as a *buffer against unemployment* in countries where formal job opportunities are scarce. This view made its way into structural work in informality either by modeling informal work as frictionless self employment in a low productivity activity, or as work in a separate sector with abundant job opportunities due to low regulatory costs. Either way, workers avoid income losses from unemployment by working informally.

[1]See Bosch & Maloney (2008), Fernández & Meza (2015), Alcaraz et al. (2015), Leyva & Urrutia (2020) in the case of Mexico. In Brazil, the same fact is documented by Bosch & Maloney (2008), Ulyssea (2010) and Bosch & Esteban-Pretel (2012). In Colombia, see Mondragón-Vélez et al. (2010), Fernández & Villar (2016). See Loayza & Rigolini (2006) for cross-country analyses.

In this paper, we offer an alternative view of informality that can also explain the countercyclical nature of the share of informal workers (henceforth simply “informal share”). Our main idea is that some of the rules and regulations that fall upon employers in the formal sector impose a burden that *covaries with the business cycle*. Consider for example the requirement of severance pay. If the magnitude of severance pay is sufficiently high, there exists a large range of possible realizations for aggregate shocks such that a firm finds it optimal not to terminate a contract that has a negative net present value. Within that range, the worse the aggregate shock, the bigger the losses incurred by the firm. In the case of informal employers, the option of costlessly laying off a worker is a form of insurance against that aggregate risk.

The presence of a countercyclical burden reduces the value of hiring formally in recessions even if all agents in the economy are risk neutral. However we find that merely replicating the cost structure and its cyclicality in a quantitative model is not sufficient to generate a countercyclical informal share. We are only able to obtain a countercyclical informal share by letting the price of risk be sufficiently countercyclical. We find that to be in striking similarity with conclusions from the asset pricing literature dating back at least to Campbell & Shiller (1988). In order to generate a sufficiently large rebalancing of hiring towards (less risky) informal labor during a recession, we need to assume enough variation in discount rates.

Our technical approach in the paper is to use a combination of data and a structural model. We analyze a Brazilian household panel in the period between 2012 and 2019, which contains one sharp recession. We find the informal share to increase substantially after the onset of the contraction, adding one more piece of evidence the stylized fact we started from. The recession coincides with a *reduction* in the transitions of unemployed workers to both formal and informal jobs. We find the reduction in flows from unemployment to formal jobs to be much more pronounced than the informal counterpart.

We use moments obtained from the data to inform a search and matching model of the labor market with informality and aggregate uncertainty. The model includes two sources of countercyclical burden on formal jobs: large severance payments and a reduced degree of wage flexibility relative to informal jobs. We allow agents in the model to discount future payoffs with a flexible, exogenously specified stochastic discount factor, partly informed by data from the Brazilian financial sector. That specification has the advantage of being tractable and nesting assumptions on discounting commonly used in prior literature. We show in a quantitative exercise that in the absence of a time-varying price of risk, the model predicts the informal share to be procyclical, in the sense that medium and long run responses of the informal share to aggregate shocks are negative. With the arrival of a negative shock, separations in informal sector jobs spike. Separations in the formal sector remain flat, due to large regulatory costs. The increase in informal separations has the effect of bringing the informal share down on impact. Job creation in both sectors decreases, and while new matches are more likely to be informal, that is not quantitatively sufficient to revert the effect of the large mass of separations.

Allowing for time-variation in the price of risk reverts that conclusion. In that case, the arrival of the negative shock still causes the spike in informal separations, bringing down the informal share on impact. However, the greater price of risk exacerbates the impact of the negative shock on formal job creation, in line with the intuition laid out above. When the price of risk is sufficiently sensitive to the business cycle, the reduction in the hiring of formal workers relative to informal ones is large enough that the informal quickly overtakes the steady-state level, slowly reverting in subsequent periods.

We also study the impact of two policies under different values of the parameters that govern the cyclicality of the price of risk. We begin by simulating an increase in the *monitoring probability* of new matches. When a new match gets monitored, the firm cannot offer an informal contract, having then to choose between a formal one or to not offer a contract at all. We increase the monitoring probability so that the mean level of informality shrinks by half, going from 30% of the employed population to around 15%. For all parameter combinations, the model predicts output to increase, and unemployment to become less volatile and less persistent, though slightly higher. On the other hand, the informal share becomes more volatile and covaries more negatively with unemployment.

The other policy we investigate is a reduction of severance requirements. We find that in order to shrink informality by half, one needs to reduce severance requirements in about 80% when there is no time-variation in the price of risk (TVPR). Once we allow the price of risk to be countercyclical, the same reduction in severance payment requirements achieves a greater reduction in informality. That happens through an amplified impact on the relative likelihood of finding a formal job. That points to a greater sensitivity of the *level* of informality to severance policy relative to the constant price of risk specification. Moreover, different assumptions about TVPR also generate different predictions about the behavior of unemployment volatility after the change in severance policy. With a sufficiently countercyclical TVPR, the policy causes unemployment volatility to *increase*. When TVPR is shut down altogether, reducing severance payments has the effect of *reducing* unemployment volatility.

1.1 Related Literature

Recent literature has sought to understand the causes and implications of a large informal sector through the lens of structural models. Meghir et al. (2015) estimate a wage posting model with an informal sector using steady-state moments from Brazilian micro-data. Their estimation exercise seeks to realistically capture worker flows and the distribution of wages in the formal and informal sector. They find that an increase in enforcement on informal firms reduces unemployment and increases welfare. Ulyssea (2018) proposes a model that allows for an intensive margin of informality, and where fixed costs of operating and firm scale play a key role in determining the size of the informal sector. He estimates the model by targeting moments concerning the cross-sectional distribution of informality over firm size. He finds that increasing enforcement on the extensive margin of informality leads to a reduction in welfare. In a separate paper (see Maya & Pereira, 2020), we highlight the role labor income risk in understanding the impact of increasing government monitoring of informal activity. All those papers build into their models different perspectives on informal activity and highlight their policy implications.

While this paper is concerned with understanding the nature of informal activity, it differs from the papers above in that it focuses on business cycle dynamics. The dynamics of informality over the business cycle has been the topic of an extensive literature. Restrepo-Echavarria (2014) shows how excess consumption volatility in emerging markets can be partly explained by the presence of an unmeasured informal sector. She considers a small open economy (SOE) model in which labor freely moves between a competitive market for formal hours and a self-employment technology. Fernández & Meza (2015) document stylized facts about informality in Mexico and use their empirical analysis to inform an SOE model with informality. They find that the inclusion of an informal sector amplifies the impact of growth shocks, a mechanism that arises due to the imperfect pass-through of shocks from the formal to the informal sector. In both models, informality behaves as a buffer to which labor frictionlessly reallocates. In that sense, informality in their model resembles self-employment, which we abstract from in our model and empirical analysis. Bosch & Esteban-Pretel (2012) document the cyclical properties of worker flows between unemployment and jobs in both sectors, as well as flows between sectors. They build a search and matching model with informality where both sectors are subject to search frictions. Their theoretical framework is the closest to ours. Relative to their paper, we abstract from transitions between informal and formal jobs, focusing instead on flows to and from unemployment. We also relax their assumption of constant discount rates and show the quantitative importance of doing so. To the best of our knowledge, ours is the first paper to highlight the role of *risk to employers* in explaining the cyclical movements of informality and to draw attention to the connection between informality and the price of risk.

The idea that hiring workers can be seen as a risky investment has been the topic of recent developments in the macro-labor literature. In the context of the United States, Hall (2017) incorporates a reduced-form discount factor estimated from stock market data into a labor search model and shows that it generates a realistic volatility of labor market variables, therefore mitigating the Shimer (2005) puzzle. Kehoe et al. (2019) also take on the Shimer puzzle in a similar setting by considering several formulations for the stochastic discount factor that generate realistic patterns of asset prices. They are able to amplify the cyclicality of labor market variables by interacting a *time-varying price of risk* with human capital accumulation on the job. Borovička & Borovičková (2018) provide restrictions that stochastic discount factors must satisfy in order to rationalize the labor market fluctuations observed in the data, given the volatility of expected cash flows. Our paper does not attempt to solve the Shimer puzzle. Instead, we draw on the analogy between hiring and risky investments and use it to explain empirical patterns pertaining to informality.

The view that financial frictions matter for the business cycle of emerging markets has gained traction since the work of García-Cicco et al. (2010). In the context of informality, Leyva & Urrutia (2020) consider a model with search frictions in the formal sector, but interpret informal sector work as frictionless self-employment. They allow for shocks to international interest rates that are similar in spirit to shocks in the risk free rate that we consider in our model. They find that the shocks to the international interest rate are key in producing realistic predictions for the dynamics of job creation. Although our settings differ substantially, we believe that our analyses are complementary in reinforcing the importance of realistic discount rates in the context of informality.

2 Empirical analysis

We use microdata from the biggest survey of Brazilian households to document patterns in the cyclical dynamics of informality in Brazil. We choose Brazil as our country of analysis because it is widely regarded as a representative emerging economy, and there is a number of reliable sources of data on informality.

Our sample period is 2012-2019, covering the end of a boom period and one strong recession. In agreement with a large previous literature, we find that the share of informal workers to be counter-cyclical, increasing substantially after the onset of the recession in 2015. In terms of worker flows, we find a substantial reduction in transitions from unemployment to formal jobs, and an increase in transitions from informal jobs to unemployment.

2.1 Data description

We base our empirical analysis on the micro-data from the Continual National Household Survey (PNADc), conducted by the Brazilian national statistical bureau (IBGE). That sample has been used as the source for the official unemployment statistics since 2016. We collect all interviews conducted in the available period, 2012-2019. Households are interviewed five times, with one quarter interval between interviews. The interview provides information about each member of the household.[2]

The criterion we use for classifying workers as formal and informal is based on the Brazilian legislation that requires all employers to sign their employees' *labor booklet*, a document that allows workers to claim legal benefits. One of the questions PNADc asks surveyed participants is, conditional on the employment, whether their employer signed their labor booklet. We restrict our sample to non-military, non-civil service, wage earners in the private sector, and classify those workers who do not have a signed labor booklet as informal. That allows us to tag all individuals in the sample as one of `formal`, `informal`, or `unemployed`, which we call *employment status*. In order to create worker flows, we track whether individuals changed status from one quarter to the following one.[3]

Importantly, that classification leaves self-employed households out of our sample. While self-employed households are often classified as informal in empirical work, we leave them out of our sample because we believe that our theoretical framework does not properly capture the incentives that drive self-employment. Our model emphasizes the informal workers who experience unemployment and need to engage in costly search in order to find job opportunities.

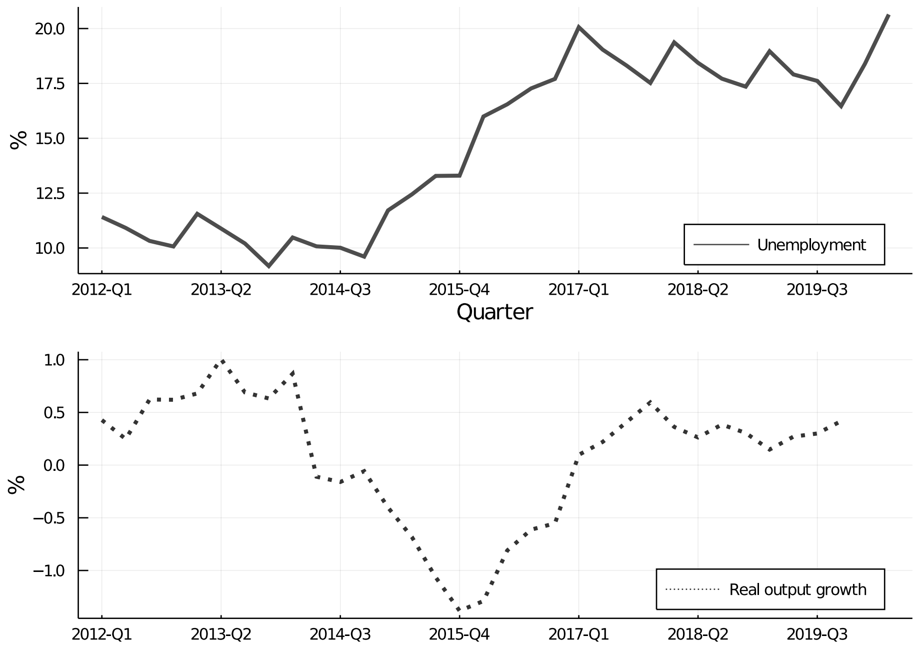
2.2 The dynamics of informality

We begin by computing the unemployment rate in our sample, and compare that with GDP growth in Figure 1. We observe a substantial, persistent increase in unemployment starting in the last quarter of 2014 accompanied by a sharp contraction in real GDP.

In Figure 2, we contrast the increase in unemployment with the evolution of the share of informal workers (henceforth informal share). The informal share is generally high, with a trough of approximately 28% in 2014-Q4. The informal share spikes after the onset of the recession, peaking at around 33% in the third quarter of 2019. Importantly, as stated above, that share refers to salaried workers and does not include self-employed individuals. The fact around a

[2] PNADc provides an identifier that allows us to identify a household in different quarters. They do not provide a person identifier. We use gender and date of birth – or age, when date of birth is not available – to identify people within a household.

[3] We do not observe whether individuals that maintain labor status remain in the same job.



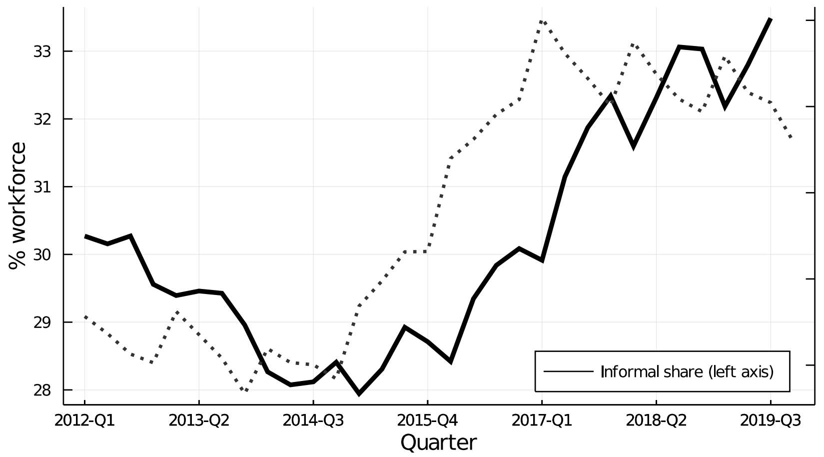
Notes: The solid line shows the proportion of unemployed individuals as a fraction of the labor force. The dotted line shows the quarter on quarter growth rate in real GDP measured by the Brazilian statistical bureau. Note that the units are **not** annualized.

Figure 1: Business cycle

third of *salaried workers* have informal sector jobs is striking. The share also displays a strong cyclical movement, correlating starkly with unemployment, even if in a lagged fashion. We turn to analyzing the drivers of that cyclical movement. The dynamics of the informal share can be broken down into worker flows involving formality, informality, employment, unemployment, and movements in and out of the labor force. Guided by our theoretical analysis, which does not allow for search on-the-job or the participation margin, we focus our analysis of worker flows to $\mathbb{u} \rightarrow \mathbb{i}$, $\mathbb{u} \rightarrow \mathbb{f}$, $\mathbb{i} \rightarrow \mathbb{u}$ and $\mathbb{f} \rightarrow \mathbb{u}$ transitions (where $\mathbb{f}$ denotes formal job, $\mathbb{i}$ denotes informal job, and $\mathbb{u}$ denotes unemployment). We also report flows from formality to informality and vice-versa for completeness.

Figure 3 contains the results of the analysis of worker flows. Panel (a) shows that transitions from formal jobs to unemployment remain relatively flat throughout the sample period, consistent with large regulatory costs of separation in the formal sector. On the other hand, the same figure shows a spike in the separation of informal jobs after the recession period, consistent with a story of lower regulation costs allowing for the dissolution of unprofitable labor contracts during downturns in the informal sector.

The flows from unemployment to formal and informal jobs display interesting patterns. Perhaps surprisingly, at the beginning of the sample the formal sector absorbs more workers from unemployment than the informal sector. That pattern reverses near the end of the sample. More importantly for our point, flows to formal are more strongly cyclical than flows to informal jobs. Transitions from unemployment to formal jobs decrease from a peak of around 27% at the beginning of the sample to a trough of around 13% near the end, whereas transitions to informal



Notes: The solid line shows the proportion of salaried workers who are employed in informal jobs (informal share). The series corresponds to the Y-axis on the left. The dotted line, with values on the right Y-axis, represents the rate of unemployment in the period.

Figure 2: Share of informal workers

jobs display less pronounced decrease, from a peak of around 20% to a low of around 14%.

In Figure 2, we plot the flows from formal to informal employment and vice-versa. The absorption of informal workers by the formal sector is more frequent than the reciprocal flow, as well as more cyclical. While for tractability we do not allow for transitions from informal to formal jobs in the model, a time varying price of risk would also affect the ability of the formal sector to absorb informal workers, hence the cyclicality of that variable is consistent with the story we propose. Hence, while considering those flows would be an interesting path for future research, we do not address them in this paper.

In summary, we find the behavior of $\mathbb{u} \to \mathbb{f}$ and $\mathbb{i} \to \mathbb{u}$ flows to be the most cyclically pronounced among the flows under scrutiny. It is important to note that the increased flows from informality to unemployment after the recession onset contribute to the *procyclicality* of the informal share. In practice, that means that in order for the informal share to be countercyclical, the flows that work in the opposite direction need to be strong enough to counteract that effect. In other words, we need the creation of formal jobs to respond even more strongly and persistently to the business cycle than the separation of informal workers.

2.3 Financial data

Our model assumes a stochastic discount factor that prices all assets in the economy. In order to discipline it, we match model implied statistics with moments from the Brazilian labor market. To that end, we obtain the closing prices of the *Ibovespa* benchmark index in Brazil, which we consider to be a proxy to a market portfolio in that country. We also obtain a time series of the average interest rate on banks savings deposits, which we consider to be a proxy for the risk free rate. Our analysis covers the period between 2003 and 2019, in which inflation is stable and there are no major changes in the country's monetary regime. The statistics we obtain from the data and that we use to discipline the model are listed below.

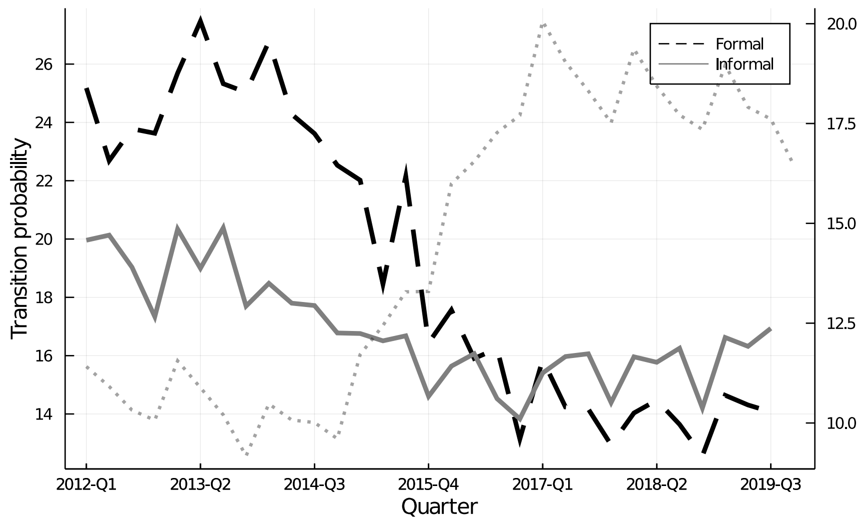
Variable	Value
Average risk-free rate	1.85%
Risk-free rate volatility	0.34%
Sharpe ratio	18.14%
Excess return volatility	11.93%

2.4 Discussion

The main finding from our analysis is that in the 2015-2016 recession episode, the share of informal workers increased substantially, driven in most part by a sharp contraction in transitions from unemployment to formal jobs. That pattern is similar to the one found by Bosch & Maloney (2008) and Leyva & Urrutia (2020) in Mexico, and by Bosch & Esteban-Pretel (2012) in Brazil using a different survey.

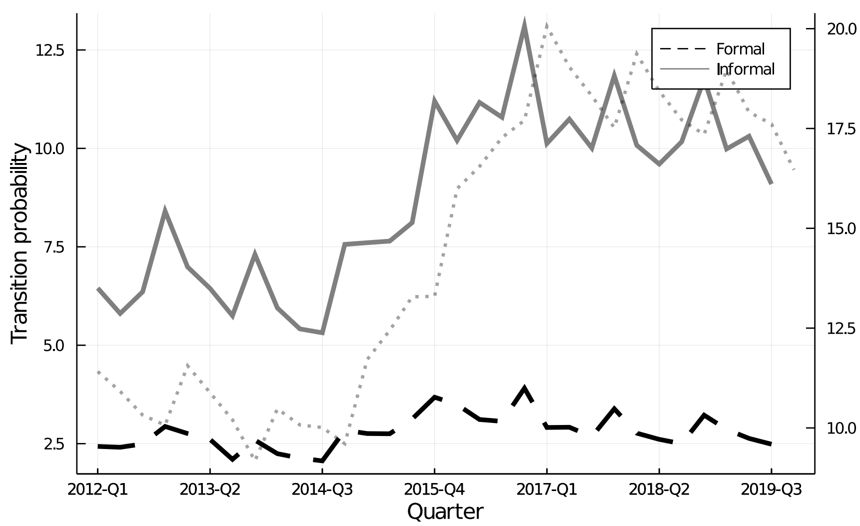
Together, Figures 2, 3 and 4 indicate that the cyclicality of the formal labor market is driven by hiring – from unemployment **and** from informal jobs – whereas the cyclicality of the informal labor market is driven by separations. That is in line with the mechanism we propose for the cyclicality of the informal share. Upon the arrival of a recession, a large number of informal employers with labor contracts at the margin of profitability exercise the option to freely separate. Formal employers who are also near that margin are not able to do that, due to the costs of laying off workers arising from regulations such as the large severance pay requirements. The bad aggregate shock reduces the incentive to open *any* new vacancies, driving down overall job creation. However, due to the countercyclical regulatory burden, those new vacancies that open are biased towards informality.

The question that remains to be answered is whether the cost structure itself is sufficient to generate the patterns we observe in the episode surrounding the 2015-2016 recession. To that end, we build a quantitative model that includes the mechanisms discussed above, and simulate the impact of a large negative aggregate shock. The details of the model are discussed in section 3, and the results of our quantitative exercise are presented in section 4.



(a) Transitions rates: unemployment to employment

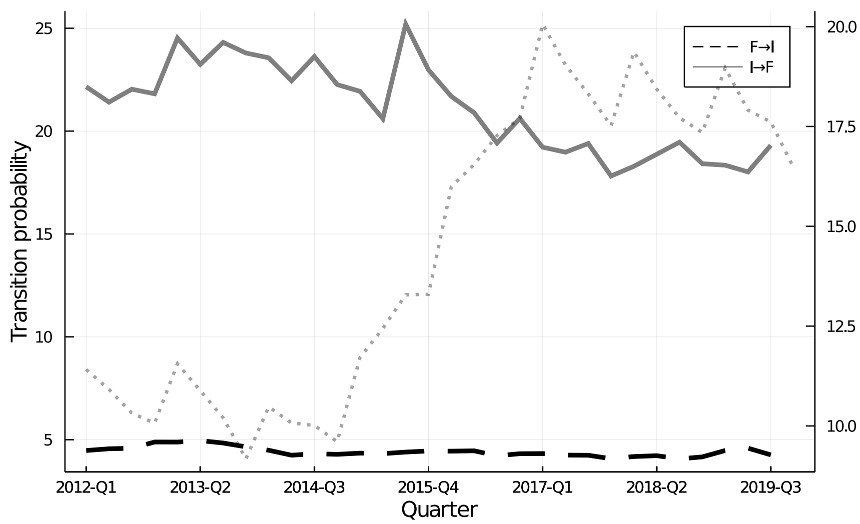
Notes: The solid gray line shows the transition rate from unemployment to informal jobs. The dashed black line shows the transition rate from unemployment to formal jobs. Both lines have values on the left Y axis. The dotted line shows unemployment in the period and has values on the Y axis on the right.



(b) Transitions rates: employment to unemployment

Notes: The solid gray line shows the transition rate from informal jobs to unemployment. The dashed black line shows the transition rate between formal jobs and unemployment. Both lines have values on the left Y axis. The dotted line shows unemployment in the period and has values on the Y axis on the right.

Figure 3: Transitions rates



Notes: The solid line shows the proportion of informal workers that transition to formal jobs. The dashed line represents the proportion of formal workers that transition to informal employment. Both series have values on the Y-axis on the left. The dotted line, with values on the right Y-axis, represents the rate of unemployment in the period.

Figure 4: Formal-informal and informal-formal flows

3 Theoretical framework

In this section, we develop a model that highlights the differential risk profile of formal and informal jobs, and that allows for discount rates as well as the price of risk to vary over the business cycle. The backbone of our model is a search and matching of unemployment with endogenous separations, similar in spirit to Mortensen & Pissarides (1994). We add formality choice in that setting following Bosch & Esteban-Pretel (2012, 2015), and time variation in discount rates and the price of risk comes from an exogenously specified affine stochastic discount factor.

As in the typical search and matching setting, our model concerns the extensive margin of employment. There is a continuum of workers that at each point in time can have one of three possible labor market states: *unemployed* ($\mathtt{u}$), *formally employed* ($\mathtt{f}$), or *informally employed* ($\mathtt{i}$). Employed workers supply a fixed amount of hours, which we normalize to unity. Unemployed workers transition to employment by meeting with vacant jobs, a process that is modeled in a simplified fashion by assuming an exogenous matching function. Production in this economy has constant returns to scale, and a free-entry condition determines the creation of vacancies in our economy.

We assume that vacancies and unemployed workers are identical, but every match is ex-post heterogeneous due to the draw of an idiosyncratic productivity shock. After the shock is observed, the firm has the option of offering a formal contract, an informal one, or to leave without offering a contract. If the firm chooses to be formal or informal, it cannot change its formality status in future periods. We set up the payoff structure of formal and informal contracts in such a way that the model predicts contract offering policies to satisfy a cutoff rule: for sufficiently low draws of the productivity shock, the firm chooses not to offer a contract; for intermediate values, the firm offers an informal contract, and the firm chooses to offer a formal contract when the match draws a high enough shock.

We capture aggregate risk in the economy by means of a shock that multiplies all firms' productivities in a given period. The interaction of the aggregate shocks with the cutoffs described above will drive the cyclicality of the informal share in our model. The time varying nature of the risk free rate and the price of risk will be key in explaining this interaction.

3.1 Formal description

We cast the model in discrete time. Each time period is divided into two subperiods: (i) the *labor market stage* and (ii) the *production stage*. In the labor market stage, new matches are formed, and ongoing matches decide whether to separate or not. In the production stage, new matches and those ongoing matches that did not dissolve in stage (i) engage in production. Consumption takes place after production, at the end of the period.

New matches are formed as follows. Vacancies and unemployed workers meet randomly according to an exogenous matching function $M(u, v) = au^{\eta}v^{1-\eta}$. After a vacancy and a worker meet, the match draws an idiosyncratic productivity level $z_t \sim \bar{F}$, which follows the Markov process below in subsequent periods,

$$z_{t+1} = \begin{cases} \tilde{z}_{t+1} & \text{with probability } \lambda \\ z_t & \text{with probability } 1-\lambda \end{cases} \tag{1}$$

where $\tilde{z}_{t+1} \sim \bar{F}$ are iid. We take $\bar{F}$ to be the CDF of a log normal distribution with parameters ν and σ_z^2, where ν is normalized so that $\int z d\bar{F}(z) = 1$. The idiosyncratic match productivity process is thus fully specified by λ and σ_z, which capture respectively the persistence[4] and dispersion of the process. The aggregate component of productivity $\{y_t\}$ satisfies an auto-regressive process in logs,

$$\log y_{t+1} = \rho \log y_t + \sigma_y \epsilon_{y,t+1} \tag{2}$$

where $\{\epsilon_{y,t+1}\}$ is iid and satisfies $\epsilon_{y,t+1} \sim N(0,1)$. Parameters ρ and σ_y capture respectively the persistence of aggregate productivity and the dispersion of innovations.

After new matches observe the draw of the first idiosyncratic shock, firms have the option of offering a formal contract, an informal one, or to leave without offering any contract. Not offering a contract has the value of zero.[5] Formal and informal contracts pay dividends according to

$$\pi^{\mathbb{f}}(z, y; w) = zy - w(1+\tau) \tag{3}$$

$$\pi^{\mathbb{i}}(z, y; w) = zy - w - (1-\alpha)z \tag{4}$$

where z is idiosyncratic productivity, y denotes the aggregate shock, and w is the wage paid to the worker. Informal contracts have a productivity penalty of $(1-\alpha) \in [0, 1]$ that scales only with the idiosyncratic shock, but not with the aggregate one. This parameter summarizes in a simplified fashion the empirical fact that, conditional on worker observables, workers in the informal sector are less productive than their formal counterparts. It captures factors such as potential fines from government monitoring, lower scale and selection into low productivity activities. To capture tax avoidance by informal firms, the tax rate τ only appears in the expression for formal dividends.

Ongoing matches have the *option to separate*. As will be detailed in a later part of this section, firms and workers renegotiate wages every period, and separation happens when no wage can make the surplus of both parties positive simultaneously. Importantly, whenever a formal match is dissolved, the firm must transfer a fixed amount χ to the worker. This captures the legal requirement of severance pay present in many countries with a large informal sector, and it is a key parameter for our theory. We do not allow for severance pay when informal matches separate.[6]

All agents in this economy discount one-period-ahead payoffs according to the exogenously

[4] Higher values of λ imply lower persistence.

[5] Alternatively, one could assign the discounted value of being in the vacancy pool next period. That is irrelevant in this case because of free entry condition (13).

[6] We could not find empirical evidence in favor or against the existence of (voluntary) severance payments in informal matches. We believe the more realistic assumption would be the existence of positive but relatively small firing costs for informal firms, not necessarily a transfer to the worker. For parsimony, we do not include that in the model.

specified, affine stochastic discount factor (henceforth SDF)

$$\log Q_{t,t+1} = -r - \mu_1 \log y_t - \tfrac{1}{2}\lambda_t^2 - \lambda_t \epsilon_{y,t+1} \tag{5}$$

where r is a steady-state risk-free rate, μ_1 is a parameter that allows for time variation in the risk free rate, and λ_t captures a potentially time-varying price of risk. The expression for λ_t is

$$\lambda_t = \gamma_0 + \gamma_1 \log y_t \tag{6}$$

where γ_0 introduces a constant risk premium, and γ_1 controls the extent to which the price of aggregate risk λ_t reacts to the contemporaneous value of the aggregate shock, y_t. This one of the specifications considered by Kehoe et al. (2019) and is a common assumption in the finance literature.[7] We defer a detailed discussion of the exogenous SDF to section 3.5. We are now in a position to rigorously define the problem of firms and workers.

To alleviate notation, we break down the problem of a matched firm of type $s \in \{\mathbb{f}, \mathbb{i}\}$ into their value at the search stage, $J^s(y,z)$, and the at the production stage, $\Pi^s(y,z)$. We also consider $\tilde{\Pi}^s(z,y;w)$ the value of a firm at production stage taking as given the current wage w. We begin by describing the latter value function,

$$\tilde{\Pi}^s(z,y;w) = \pi^s(z,y;w) + \mathbf{E}_{y'|y}\left\{Q(y',y)[(1-\lambda)J^s(z,y') + \lambda \int_{z'} J^s(z',y')d\bar{F}(z')]\right\} \tag{7}$$

The firm that decides to produce when facing wage w gets current dividends, plus the discounted value of continuing. The continuation value depends on the draw of the idiosyncratic shock, and the possibility of separation is incorporated in J^s.

We defer a discussion of wage determination to a later point in this section, but it is convenient to introduce the wage function $w^s(z,y)$ that results from that process. That allows us to state the value of a firm at the production stage,

$$\Pi^s(z,y) = \tilde{\Pi}^s(z,y;w^s(z,y)). \tag{8}$$

The value at the labor market stage of a formal firm is

$$J^{\mathbb{f}}(z,y) = \max\{-\chi, \Pi^{\mathbb{f}}(z,y)\} \tag{9}$$

and similarly

$$J^{\mathbb{i}}(z,y) = \max\{0, \Pi^{\mathbb{i}}(z,y)\} \tag{10}$$

in the case of the informal firms. We denote by $\delta^{\mathbb{f}}(z,y) \in \{0,1\}$ and $\delta^{\mathbb{i}}(z,y) \in \{0,1\}$ the policy functions arising from (9) and (10).

At the point of entry, after observing the idiosyncratic shocks, firms decide which type of contract to offer, or to not offer any contract. That decision is denoted by $\varsigma(z,y)$, and satisfies

$$\varsigma(z,y) = \arg\max\{\Pi^{\mathbb{f}}(z,y), \Pi^{\mathbb{i}}(z,y), 0\} \in \{\mathbb{f}, \mathbb{i}, \mathbb{n}\} \tag{11}$$

[7] List references here: Cochrane and Piazzesi, Ang and Piazzesi, etc.

where of course $\varsigma = \mathbb{f}$ means a formal contract is optimal, $\varsigma = \mathbb{i}$ means an informal contract is optimal, and $\varsigma = \mathbb{n}$ means it is optimal to offer no contract.[8]

Let u and v denote respectively the measure of unemployed workers searching for jobs, and vacancy openings. We denote by $\theta = v/u$ the market tightness. Given our random matching assumption and the matching function $M(u,v) = au^{\eta}v^{1-\eta}$, the probability that a vacancy meets a worker is

$$q = \frac{M(u,v)}{v} = M\left(\frac{u}{v}, 1\right) = M\left(\frac{1}{\theta}, 1\right) = a\theta^{-\nu}$$

In order to search for a worker, vacancies must pay a cost κ. When market tightness is θ, the value of searching for a worker is given by

$$\begin{aligned} V(y) = -\kappa + q(\theta)\int_z \left\{(1-\phi)\max\{\Pi^{\mathbb{f}}(z,y), \Pi^{\mathbb{i}}(z,y), 0\} + \phi\max\{\Pi^{\mathbb{f}}(z,y), 0\}\right\} d\bar{F}(z) \\ +(1-q(\theta))\mathbf{E}_{y'|y}[Q(y,y')V(y')] \end{aligned} \tag{12}$$

Vacancies entail a flow cost κ. With probability q that depends on market tightness, the vacancy meets a worker and draws an idiosyncratic shock z, choosing afterwards which kind of contract to offer, if it chooses to offer a contract. The firm can only offer an informal contract with probability $(1-\phi)$. The parameter ϕ is interpreted as the probability of a *monitoring shock* that occurs after the shock is observed and forces the firm to choose between a formal contract or no contract. With probability $1-q$, the vacancy goes to the next period unfilled.

We assume that vacancies enter up to the point where the value of a vacancy is zero at every possible history of shocks. That results in the free entry condition below,

$$q(\theta(y))\int_z \max\{\Pi^{\mathbb{f}}(z,y), \phi\max\{0, \Pi^{\mathbb{f}}(z,y)\} + (1-\phi)\Pi^{\mathbb{i}}(z,y), 0\}d\bar{F}(z) = \kappa \tag{13}$$

We now turn to the problem of the worker. The flow value of unemployment is b. Workers are hand to mouth and discount monetary units according to the same SDF $Q_{t,t+1}$. The present discounted value of a job of type $s \in \{\mathbb{f}, \mathbb{i}\}$ *given wage* w is simply

$$\tilde{W}^s(z,y;w) = w + \mathbf{E}\left\{Q(y,y')\int_{z'}[\delta^s(z',y')U^{sep,s}(y') + (1-\delta^s(z',y'))W^s(z',y')]d\bar{F}(z'|z)\right\} \tag{14}$$

where we use the conditional notation $F(z'|z)$ for simplicity. In the continuation value above, δ^s denotes the indicator of separation; $W^s(z',y') = W^s(z',y',w^s(z',y'))$ incorporates the bargained wage; and $U^{sep,s}(y')$ is the value of unemployment, possibly accounting for a severance payment, as described below.

The value of an unemployed worker at the production stage – when no severance pay is due – is given by

$$U(y) = b + \mathbf{E}\left\{Q(y',y)\left[p(\theta(y'))\int_{z'} W^{\varsigma(z',y')}(z',y')d\bar{F}(z') + (1-p(\theta(y')))U(y')\right]\right\} \tag{15}$$

the function $p(\theta) = a\theta^{1-\eta}$ is the probability that a worker meets a vacancy implied by the matching function M. The worker rationally expects the job to be of type $\varsigma(z',y')$, in accordance

[8] We resolve ties numerically in favor of production vs. non-production, and in favor of formal vs. informal.

with (11). Having defined the value of unemployment, we can recover the value of separating from a formal job: that is simply $U^{sep,\mathbb{f}}(y) = \chi + U(y)$. Separating from an informal job has the same value of unemployment, $U^{sep,\mathbb{i}}(y) = U(y)$. It is important to remember the assumption that workers that lose their jobs cannot search the same period.

Inspired by Tortorice (2013), we adopt a protocol for wage determination in which the prevailing wage is a convex combination between the outcome of Nash bargaining and a wage norm. That allows us to capture potentially different degrees of wage flexibility for formal and informal contracts. Let $\hat{w}^s(z, y)$ be the outcome of Nash bargaining, with disagreement points being respectively the value of separation $U^{sep,s}$ for the worker, and the value of a vacancy for the firm. Since the value of a vacancy is zero in equilibrium, Nash bargaining prescribes

$$\hat{w}^{\mathbb{f}}(z, y) = \max_{w \in \mathcal{B}^{\mathbb{f}}(z,y)} \left[\tilde{W}^{\mathbb{f}}(z, y; w) - U^{sep,\mathbb{f}}(z, y)\right]^{\beta} \left[\tilde{\Pi}^{\mathbb{f}}(z, y; w) + \chi\right]^{1-\beta} \tag{16}$$

$$\hat{w}^{\mathbb{i}}(z, y) = \max_{w \in \mathcal{B}^{ii}(z,y)} \left[\tilde{W}^{\mathbb{i}}(z, y; w) - U^{sep,\mathbb{i}}(z, y)\right]^{\beta} \tilde{\Pi}^{\mathbb{i}}(z, y; w)^{1-\beta} \tag{17}$$

where the constraints $\mathcal{B}^s(z, y)$ for $s \in \{\mathbb{f}, \mathbb{i}\}$ essentially impose that the surplus must be non-negative for both parties involved in the bargain. In the event that $\mathcal{B}^s(z, y) = \emptyset$, we set $\hat{w}(z, y)$ to satisfy $\tilde{W}(z, y, \hat{w}(z, y)) = U^{sep,s}(z, y)$. Since $\mathcal{B}^s(z, y) = \emptyset$, that implies $\tilde{\Pi}^f(z, y, \hat{w}(z, y)) < -\chi$, which necessarily triggers a separation according to (9) and (10). When $\mathcal{B}^s(z, y) \neq \emptyset$, one can show that an interior solution to the problem necessarily exists.

Whenever separation is not optimal, the prevailing wage is a convex combination of $\hat{w}^s$ and the wage $\bar{w}^s(z)$ that implements Nash bargaining in a deterministic steady state (i.e., when $y_t \equiv 1$). The expression for wage determination is then

$$w^s(z, y) = \psi^s \bar{w}^s(z) + (1 - \psi^s)\hat{w}^s(z, y) \tag{18}$$

The parameter ψ^s governs the degree of wage stickiness – in the sense that higher values of ψ imply a more dampened response of wages to movements in the aggregate shock. It is possible that $w^s(z, y)$ falls out of the set of feasible wages $\mathcal{B}^s(z, y)$. If $w^s(z, y)$ is higher than the highest wage that the firm is willing to accept, we reset it to the firm's indifference threshold numbers. Conversely, if $w^s(z, y)$ is so low that the worker strictly prefers to be unemployed, we reset it to the indifference wage for the worker.

3.2 Model timing

The timing of the model is as follows. At the beginning of period t, all agents observe the aggregate shock y_t. Subsequently, for ongoing matches:

1. Idiosyncratic shock z is revealed
2. Wage renegotiation takes place; when no wage brings about positive surpluses for worker and firm, match dissolves
3. Surviving matches engage in production.

The events in the search stage have the following timing:

1. Vacancies and unemployed meet

2. Nature draws idiosyncratic match shock
3. Firm decides between offering formal contract, informal contract, or leaving
4. Formal and informal contracts produce.

At the end of the period, agents consume the output due.

3.3 Equilibrium definition and solution method

Throughout our analysis, we focus on recursive equilibria of this economy in which the value and policy functions described above do not depend on the distribution of agents over match productivities and employment states. A *recursive equilibrium* in this economy consists of time invariant value functions $\mathcal{W}$ – comprising all the value functions defined on section 3.1 – as well as the policy functions ς, $\delta^{\mathbb{f}}, \delta^{\mathbb{i}}$, the bargained wages $w^{\mathbb{f}}, w^{\mathbb{i}}$, that solve equations (7), (8), (9), (10), (11), (13), (14), (15), (16), (17), (18), given the stochastic process for aggregate shocks (2), the exogenous stochastic discount factor (5), and the market tightness map $\theta(y)$.

Let $m_0(z, s)$ be an initial distribution of households over idiosyncratic states and labor market status at the *end* of a period (therefore after separations and search occur), with $s \in \{\mathbb{u}, \mathbb{f}, \mathbb{i}\}$. We can compute the evolution of that distribution given a history of shocks $y^t = \{y_\tau\}$, implied the shock structure and the policy functions from the recursive equilibrium. Denote that by $m_t(z, s; m_0)$, where for notation simplicity we omit the particular history of aggregate shocks. Then we can define the *ergodic measure* $m^*(z, s; m_0) = E[m_t(z, s; m_0)]$, and it follows from an ergodic theorem that there exists $m^*(z, s)$ such that

$$m^*(z, s) = m^*(z, s; m_0) = \lim_T \sum_{t=0}^{T} \frac{m_t(z, s; m_0)}{T} \tag{19}$$

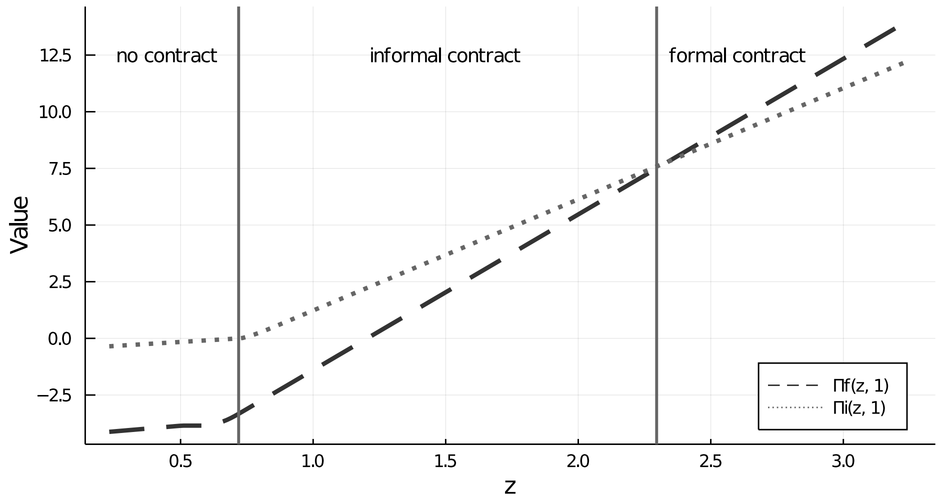
for almost every history $\{y_\tau\}$ and initial measure m_0.

As the model is very non-linear, we compute numerical approximations to the equilibrium and the ergodic measure. For that, we use a Rouwenhorst (1995) method in order to discretize the aggregate auto-regressive shock, and approximate the Log Normal idiosyncratic shock discretely, with grid points having equal weights. The equilibrium approximation is then computed with value function iteration.

3.4 Model mechanisms

In this section, we illustrate the main decisions that take place in our setting. They are essentially the decision of which contract to offer, and the decision to terminate a contract. Throughout this section, we use the value and decision functions we obtain numerically from the baseline calibration, as described in sections 3.3 and 4.1.

Firms that enter each period need to decide which contract to offer the contacted unemployed worker. That is done in accordance with equation (11), which is illustrated in Figure 5. In that figure, the aggregate state is fixed at the steady state level and the value functions $\Pi^{\mathbb{f}}(z, 1)$ and $\Pi^{\mathbb{i}}(z, 1)$ are represented respectively by the dashed and dotted line. The points z such both functions are below zero are such that no contract is offered. For intermediate values of the idiosyncratic shock, it is optimal to offer an informal contract. The formal contract is



Notes: Dotted line represents $z \mapsto \Pi^{\mathrm{i}}(z, 1)$. Dashed line represents $z \mapsto \Pi^{\mathrm{f}}(z, 1)$.

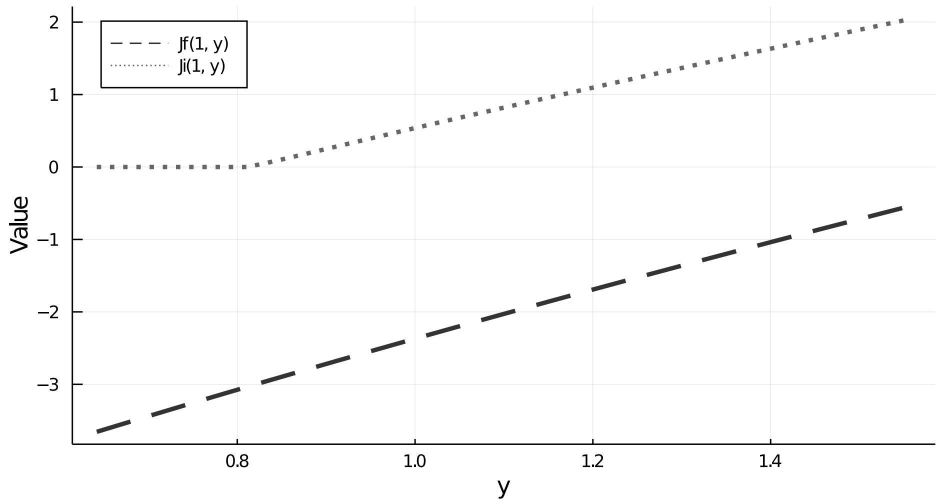
Figure 5: Determination of contracts

offered when the productivity is high enough. Note that for informal firms, the threshold that determines whether the firm offers no contract or an informal contract is the same as the one for the termination of ongoing contracts. That is not the case for formal firms. The severance pay parameter χ introduces a disconnect between the point at which the firm becomes indifferent between offering or not a contract − which in Figure 5 happens somewhere between $z = 1$ and $z = 1.5$ − and the point at which it is optimal to terminate the contract, which happens slightly to the left of the first bar. That disconnect is essential for our mechanism. The fact that the *margin for entering* is different than the *margin for separation* in the case of formal firms means that there will be a positive mass of firms that oscillate in equilibrium between positive and negative value depending on the value of the aggregate shock.[9]

One important factor regarding contract choice is that, due to the monitoring shock, a fraction ϕ of the contracts in the intermediate z region becomes formal even though it would be optimal to offer an informal contract. That parameter is important for avoiding a *selection effect*. If there was no monitoring, all new formal contracts would occur only at the very top of the distribution of idiosyncratic shocks, unless a very low parameter for the informal contract productivity α were to be chosen. That would anyway imply a disjoint set of productivities for new matches associated with formal and informal jobs, in contradiction with the existing empirical evidence (see La Porta & Shleifer, 2014; Meghir et al., 2015; Ulyssea, 2018).

One way of thinking of Figure 5 is that it illustrates the determination of contracts in the steady-state. It does not touch our main concern in this paper: the impact of *aggregate shocks*. Aggregate shocks affect the model in two main ways: (i) ex-ante vacancy creation, and (ii) shifts in the above curves. Channel (i) concerns variations in market tightness in response to aggregate shocks, and it is the topic of a large literature since the paper by Shimer (2005). Kehoe et al. (2019); Hall (2017); Borovička & Borovičková (2018) all demonstrate how changing the

[9]It is important to keep in mind for this argument that there is sufficient persistence in the idiosyncratic shock to reason from the perspective of a fixed z.



Notes: Dotted line represents $y \mapsto \Pi^{\mathtt{i}}(1, y)$. Dashed line represents $y \mapsto \Pi^{\mathtt{f}}(1, y)$.

Figure 6: Firm value and aggregate shocks

assumptions on discounting – relative to stochastic discount factors based on risk neutral or CRRA assumptions – can help generate realistic movements in market tightness over the business cycle. This paper is not concerned with this type of variation. We are focused on channel (ii): how changes in discounting affect the split between types of contracts, conditional on vacancy creation. In other words, while the aforementioned papers focus on *portfolio value*, this paper focus on *portfolio weights*.

We illustrate the variations of firm value given possible values of the aggregate shock in Figure 6. It shows the value of ongoing jobs at the median idiosyncratic shock point. A formal firm with a median idiosyncratic shock has negative value, meaning it would like to dissolve the match, but due to the presence of large severance payments, this is not feasible. For informal firms, the option of separating costlessly – which appears in Figure 6 as the flat part near the lower region of aggregate shocks – works as *insurance against bad realizations of the aggregate shock*. That is the key reason why formal contracts are risky: bad realizations of the aggregate shock (low ϵ_y) are more correlated with negative value due to the effective lack of the option to separate.

In order to illustrate how profits of formal and informal firms correlate with the business cycle, and the role of the severance pay requirement, we compute profit trajectories of formal and informal firms. We pick $z = 0.8422$ for formal firms and $z = 0.78$ for informal firms, which generate similar cash flow levels when y is at the steady state level, and are low enough that the contract is near the margin of separation for the informal firm. We then 200 possible histories of length $T = 15$ for the aggregate shock,[10] starting at two different levels: steady state and three standard deviations below steady state.

Figure 7 shows the results. Panel (a) contains the profit trajectories when the aggregate shock starts at the steady state level. Note that many trajectories in the informal case have jumps to

[10]Since new idiosyncratic shock arrives with probability $\lambda = 0.0645$ in our calibration, that simulation length is close to the average duration of an idiosyncratic shock.

zero, indicating a separation. That does not happen in the case of the formal firm. When we start from the bad shock, we see the trajectories in panel (b). The negative shock causes profits to decrease, but the large cost of separating forces the firm to remain with the unprofitable trajectories. The same does not hold for informal contracts. For many histories of the aggregate shock, the contract optimally separates and the profit jumpts to zero.

Together, those graphs show that informal contracts are less exposed to aggregate risk due to the option to separate. When the price of risk gets higher during downturns, the lower exposure to aggregate shocks translates into a higher value relative to formal contracts. This is why vacancy creation will relatively favor informal contracts during a recession.

3.5 Discussion on the *ad-hoc* stochastic discount factor

Our model allows for endogenous job destruction and heterogeneous matches, which are two sources of technical complications absent from other papers in the intersection between asset pricing and macroeconomic models of the labor market, such as Kehoe et al. (2019) and Hall (2017). While Kehoe et al. (2019) include human capital accumulation, they devise the structure of the model so that human capital aggregates and hence they do not have to keep track of the distribution, but only the average value of human capital. That allows them to easily compute aggregate consumption and to use stochastic discount factors that obtain from standard preferences in the Asset Pricing literature, such as Campbell & Cochrane (1999).

In our case, computing aggregate consumption at any period requires knowledge of the distribution of workers over idiosyncratic productivity and labor market states, (z, s). If we were to make the stochastic discount factor endogenous, we would need to work with Bellman equations such as

$$\mathcal{V}_t(z, s, y) = \pi(z, s, y) + \mathbf{E}\{Q_{t,t+1}\mathcal{V}_{t+1}(z, s)\} \tag{20}$$

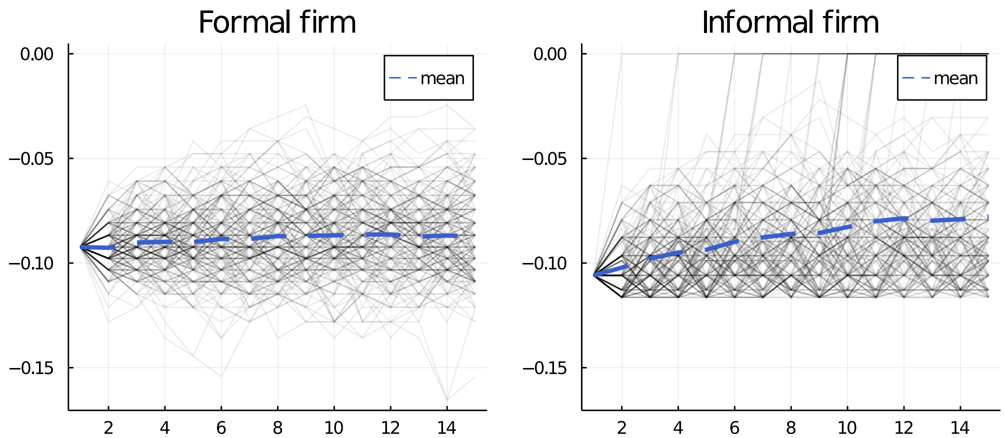
where the extra condition,[11]

$$Q_{t,t+1} = \beta\frac{u'(C_{t+1})}{u'(C_t)}$$

where C_t is aggregate consumption. However, in order to compute total consumption, one needs the distribution of households in order to aggregate wages $w^{\mathbb{f}}(z, y)$, $w^{\mathbb{i}}(z, y)$, profits, and unemployment benefits. On the other hand, these variables are outcomes of problems such as (20). The distribution of workers becomes another state variable, which renders this particular problem nearly impossible to solve.

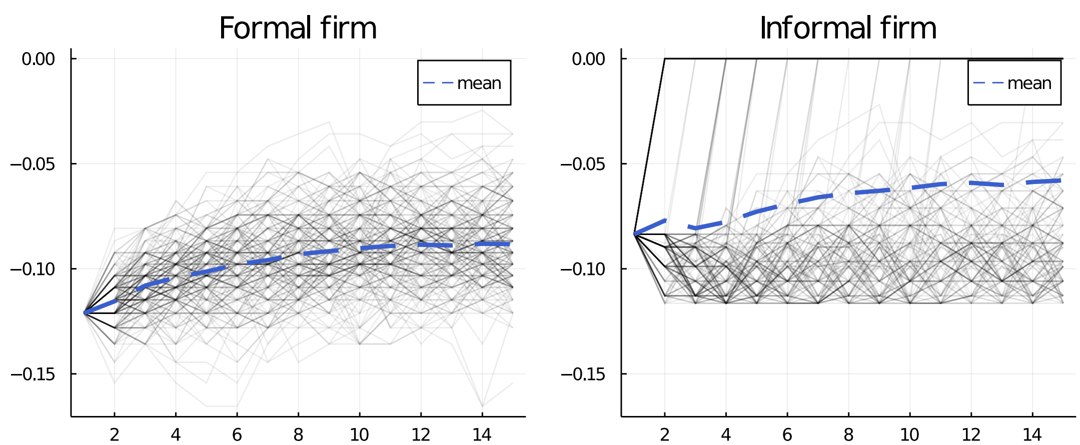
An exogenous stochastic discount factor allows us to circumvent that technical complication. Moreover, to our knowledge all papers on informality that employ search models make the assumption of risk neutral discounting, which is a particular case of our specification with $\gamma_0 = \gamma_1 = \mu_1 = 0$. In that sense, we are not giving up generality with respect to for example Albrecht et al. (2009), Meghir et al. (2015), Narita (2020), Bosch & Esteban-Pretel (2012) and Bosch & Esteban-Pretel (2015). Other examples of papers that use exogenous stochastic discount factors include Arellano & Ramanarayanan (2012) and Bianchi et al. (2018) in the International Macroeconomics literature.

[11]In here we assume that agents belong to a continuum of families that share risk through a complete market for one period contingent claims.



Notes: Each gray line represents a possible trajectory of period profits given a history of shocks $y_1, y_2, \ldots, y_{15}$. At time 0: aggregate shock at steady state value. X-axis is time. Y-axis units are consumption goods. Negative values denote period losses. Dashed lines denote the mean over the 200 histories.

(a) Aggregate shock at steady state value



Notes: Each gray line represents a possible trajectory of period profits given a history of shocks $y_1, y_2, \ldots, y_{15}$. At time 0: aggregate shock three standard deviations below steady state. X-axis is time. Y-axis units are consumption goods. Negative values denote period losses. Dashed lines denote the mean over the 200 histories.

(b) Negative aggregate shock

Figure 7: Trajectories of period profits

4 Quantitative Exercise

We now calibrate the economy described in section 3 based on the empirical analysis provided in section 2. We begin with an economy with a risk premium that does not feature a time varying price of risk. The parameters of the stochastic discount factor are chosen to match facts about the financial market in Brazil, namely: (i) the average interest of savings accounts, which we take to be a proxy of the risk free rate in Brazil, (ii) the volatility of that interest rate, and (ii) the sharpe ratio from *Ibov*, the main benchmark index in Brazil. The average interest rate is directly set by the parmeter μ_0. The volatility of the risk free rate is implied by the stochastic discount factor via

$$r_t = \mathbf{E}_t[-\log(Q_{t,t+1})],$$

whose variance can be estimated using the invariant distribution of the discretized aggregate shocks. We calibrate γ_0 to equate the Sharpe ratio from our sample with the implied maximum Sharpe ratio from the SDF model, $\sigma_t(\log Q_{t,t+1})$.[12]

The main result from that exercise is that the baseline specification with a *constant* price of risk fails to generate counter-cyclicality of the informal share. We then introduce gradually introduce a time varying price of risk in this economy by changing γ_1, adjusting the parameter γ_0 so as to keep the implied Sharpe ratio constant. In essence, what we do is to change the conditional volatility of the SDF under the constraint that the implied maximum Sharpe ratio remains constant.

The exercise shows that the correlation between unemployment and the informal share is increasing in $|\gamma_1|$. Moreover, the higher countercyclicality of the informal share is driven by a sharper response of hiring of formal workers, and a less pronounced response of separations of informal workers. These findings confirm the intuition that introducing a countercyclical price of risk reduces the incentives for hiring formal workers in recessions, and show quantitative relevance of this channel, provided that the extent of countercyclicality of the price of risk is large enough.

4.1 Calibration

We set the payroll tax $\tau = 0.3$, an approximation of statutory rules in Brazil. The target for severance pay is based on the Brazilian legislation, which prescribes a severance pay that is a function of tenure and the history of wages. Brazilian law requires that every month, employers deposit an amount that corresponds to 8% of the employee's compensation into a government run account which is linked to that worker. If the firm fires the worker, it must pay the worker an amount equivalent to 40% of the total deposits made to that account.[13] In order to approximate that, we target the following relationship:

$$\chi = 0.08 \times \text{mean duration of formal contract } \times 0.4 \times \text{average formal sector wage}$$

[12]The bound on the Sharpe ratio follows from straightforward manipulations. A detailed discussion can be found in Campbell (2017).

[13]Note that the amount is not *withdrawn* from the account. It is paid above and beyond what is already there.

Symbol	Interpretation	Source/Target	Parameter value
Shocks			
ρ	Aggregate shock persistence	Unemployement autocorrelation	0.78
σ_y	Aggregate shock st. dev.	Unemployement volatility	0.045
λ	Idiosync. shock arrival rate	Formal separation probability	0.065
σ_z	Idiosync. shock dispersion	Formal TFP dispersion	0.54
Informality			
α	Informal contract productivity	Average informality rate	0.7125
ϕ	Monitoring probability	Ratio of finding rates ≈ 1	0.67
$\psi^{\mathtt{f}}$	Wage norm (formal)	Rigid	1.0
$\psi^{\mathtt{i}}$	Wage norm (informal)	Flexible	0.0
Labor Market			
χ	Severance payment	Legislation	3.85
τ	Payroll tax	Legislation	0.30
b	Unemployment flow value	Informal separation probability	0.299
κ	Vacancy cost	Market tightness = 1	0.40
a	Matching function efficiency	Unemployment rate = 14.6%	0.23
η	Job-finding rate elasticity	Literature	0.5
β	Worker's bargaining weight	Literature	0.5
Stochastic discount factor			
r	Steady state risk-free rate	Average savings account rate	0.006
μ_1	Risk-free rate: time variation	Volatility of savings account rate	0.037
γ_0	Risk price: constant term	Maximum sharpe ratio	0.09
γ_1	Risk price: time variation	-	0.0

Table 1: Baseline Calibration

The mean duration and the average formal sector wages we use when computing the target are the model implied ones.

We take the worker's bargaining weight and the elasticity of the matching function to be 0.5, following Bosch & Esteban-Pretel (2012). We set $\psi^{\mathtt{f}}$ and $\psi^{\mathtt{i}}$, the parameters governing frictions in wage adjustment on both sectors, to be respectively 1 and 0. That captures an informal sector that adjusts wages entirely in accordance with Nash bargaining, and a formal sector in which wages do not adjust after aggregate shocks – with the exception of readjustments that avoid inefficient separations. We set the mean risk free rate r equal to the average monthly interest rate on savings deposits, $r = 0.6\%$.

Parameters ρ, σ_y, λ, σ_z, α, ϕ, χ, b, κ, a, μ_1, γ_0 are set jointly to target the 12 moments shown in Table 1. The aggregate shock parameters ρ and σ_y are set to match the persistence and volatility of unemployment observed in the data. The parameters governing the idiosyncratic shock σ_z and λ target respectively: the distribution of formal TFP that we infer from estimates by Meghir et al. (2015), and the rate of separation of informal workers.

Parameters α and ϕ govern the productivity of informal firms and the monitoring of new contracts. They affect informality in different ways. Parameter α works mainly by changing the slope of $z \mapsto \Pi^{\mathtt{i}}(z, y)$, so that for example a reduction in α would have the effect in Figure 5 of reducing the value where the two lines intersect, thereby reducing the distance between the bars which determine the region of idiosyncratic shocks where it is optimal to offer an informal contract. Conditional on the productivity thresholds that determine optimal contracts, parameter ϕ controls the fraction of new informal contracts that are forced to either formalize or leave. When $\phi = 0$, the split between new formal and informal contracts is entirely based on productivity via a cutoff rule. In that case, generating a sizable informal sector requires setting a sufficiently high productivity punishment $1 - \alpha$, which makes informal firms particularly susceptible to separations given idiosyncratic shocks. In order for the unemployment rate to be stable, that implies a high probability of landing an informal job relative to a formal job. We take advantage of this insight and set α and ϕ to target the mean informal share, and the ratio of new formal jobs to new informal jobs. In the data, that likelihood ratio of landing an informal job is close to 1, even though it varies strongly over the business cycle.

Parameter χ follows the legislation target discussed above. The flow value of unemployment b targets the separation rate of informal workers. Parameters κ and a set average market tightness equalt to one and the mean value of unemployment to that observed in the data.

Parameter μ_1 targets the volatility of the risk free rate and γ_0 sets the maximum Sharpe ratio to the Sharpe ratio obtained from the *Ibovespa* index relative to the savings deposits. We set $\gamma_1 = 0$ in our baseline specification, and in section 4.2 we compare that with alternative values that allow for a time varying price of risk.

In order to compute model implied moments, we construct a large story for the aggregate shock and use the model implied average for the considered statistics. Because the model is fairly non-linear and requires a large number of grid points for precision, each run (equilibrium plus measure) takes about six seconds. That renders the use of a derivative based solver for the calibration unfeasible. We use a simulated annealing algorithm to minimize the Euclidean distance between data and model moments. The value of calibrated parameters in our baseline specification are shown in Table 1. The model and data implied moments are shown in Table 2. Overall, despite the high degree of nonlinearity, the large number of moments and parameters to be matched, we obtain a reasonable fit.

We turn to evaluating different specifications for the time varying price of risk in terms of their ability to match the observed cyclicality of the labor market variables in the data.

4.2 Simulations: the role of a time-varying price of risk

In this section, we present the results of our main simulation. We consider an increasing the degree of time variation in the price of risk, adjusting γ_0 – the steady state price of risk – to keep the Sharpe ratio constant. The parameters we use are the following:

γ_0	γ_1	Maximum Sharpe ratio
0.108	0	18.71%
0.108	-0.2	18.71%
0.106	-0.8	18.72%
0.055	-1.6	18.72%

All other parameters remain unchanged. Table 7 in the appendix shows the effect of the changes in parameters on the values of the moments that were targeted in the baseline case. The main message is that the business cycle averages do not move much in response to the

Moment	Model	Data	Calibrated parameter
Average unemployment	14.3%	14.6%	a
Unemployment autocorrelation	0.9206	0.907	ρ
Unemployment volatility	0.029	0.037	σ_y
Share of informal workers	0.3087	0.303	α
Separation probability ($\mathbb{f}$)	3.02%	2.8%	λ
Separation probability ($\mathbb{i}$)	7.23%	8.9%	b
Finding $\mathbb{f}$ / Finding $\mathbb{i}$	0.93	1.08	ϕ
Interquartile ratio TFP ($\mathbb{f}$)	68.6%	70.3%	σ_z
Tightness	1.02	1	κ
Implied severance pay	3.85	3.8	χ
Sharpe ratio	18.71%	18.44%	γ_0
Risk free rate volatility	0.41%	0.34%	μ_1

Notes: Worker separation probabilities, volatility of risk free rate, Sharpe ratio and unemployment volatility are converted to quarterly so as to match data frequency. "Finding $\mathbb{f}$ / Finding $\mathbb{i}$" denotes the probability of transitioning from unemployment to a formal job divided by the probability of transitioning from unemployment to an informal job.

Table 2: Model fit

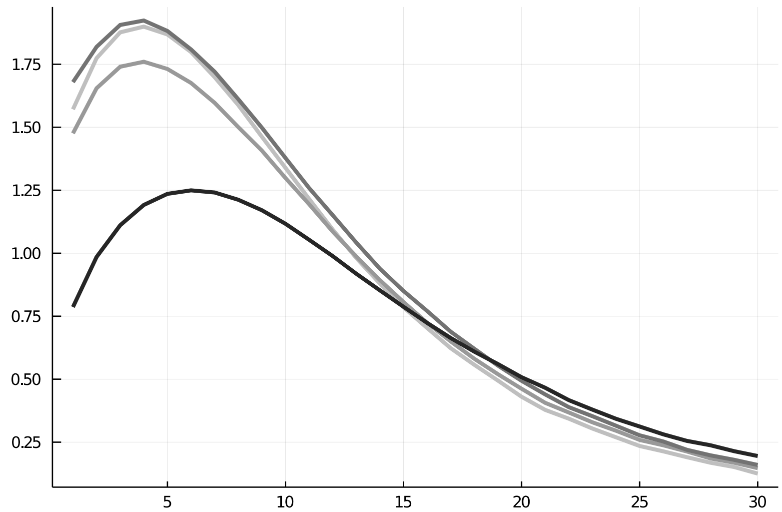
reparametrization. As we will show, the constancy in averages hides important changes in business cycle features of the different specifications.

We construct impulse response functions (IRFs) to a negative three standard deviation aggregate shock on all of the four economies above. The way we construct the IRFs is the following. We start from the ergodic measure $m^*(z, s)$, then simulate the paths of five thousand histories for the aggregate shock of length $T = 30$, all of which *start* from three standard deviations below the steady state value. We then take the average over stories of each of the aggregate variables in the model.

The response of unemployment to the negative shock is shown in Figure 8. The different assumptions on risk premia are represented by different shades of gray. Darker lines correspond to a higher degree of time variation in prices of risk, and the units are percentage points deviations from the steady state value. Even though we calibrate a relatively volatile aggregate shock, the responses of unemployment are small. A model with a flexible informal sector and endogenous separations does not generate a large volatility of unemployment, in spite of our deviation from typical Nash bargaining in formal jobs, and the relatively high value of the standard deviation of aggregate shocks obtained in our calibration.

The lowest response of unemployment to the aggregate shock happens when the SDF features the highest degree of time variation in the price of risk. This is not in contradiction with the findings of Kehoe et al. (2019) because, in addition to a countercyclical risk price, those authors find that human capital accumulation on the job needs to interact with the more conditionally volatile SDF in order to generate an emprically valid response in unemployment.

The focus of this paper, however, is not on fixing the Shimer puzzle, but on understanding the relationship between the cyclicality of risk prices and the informal share. In Figure 9 we show the response of the informal share to the same shock on aggregate TFP under different assumptions about risk prices. On all scenarios, the share of informal workers drops on impact. As we will see, that reflects a large but temporary increase in the dissolution of informal contracts. The



Notes: Time period is a month. Units are percentage points relative to the steady state. Each line represents a different parameter combination. Darker lines correspond to higher time variation in the price of risk.

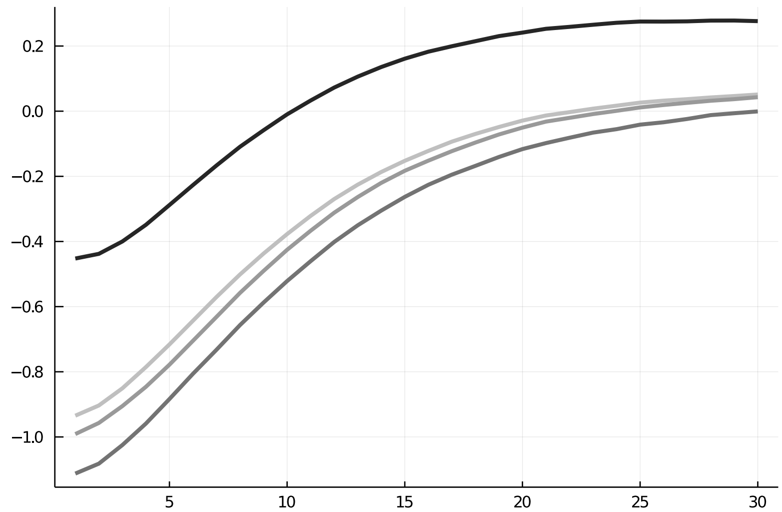
Figure 8: Response of unemployment to aggregate shock

darkest line, corresponding to the highest value of γ_1, features the smallest impact decline of the informal share, and in that case the informal share increases above the steady state value ten months after the shock. For the lower values of the price of risk, the informal share does not go significantly above the steady state value.

We turn to investigating what drives that pattern. To do so, we also compute the response of flows to and from unemployment. The flows from unemployment to formal and informal jobs are the content of Figure 10. Increasing risk price countercyclicality causes transitions *to formal jobs* to worsen, and transitions *to informal jobs* to improve. That is exactly the exactly the effect we anticipated: while vacancy creation generally decreases, the new vacancies that are posted tilt in favor of hiring informal workers. That illustrates the mechanism we propose. For a sufficiently high time variation in the price of risk, the downturn reduces the value of formal workers more than informal ones because they are more risky.

As in the data, the model also predicts separations in the informal sector to increase after the bad shock. That is evident in Figure 11: while separations of formal workers remain nearly unchanged, on all specifications the model predicts separations of informal workers to initially spike. That effect is large, but short lived relative to the flows of workers from unemployment to employment. Another noteworthy finding is that increasing time variation in the price of risk reduces the rate of informal separations. The reason for this is that the behavior of the risk premium has the effect of reducing the average wage in the informal sector, as well as making it more responsive to the business cycle. Because wages lower with the negative shock, fewer matches need to separate.

Tables 3 contains the standard deviation of relevant aggregates, obtained through a stochastic simulation with fifty thousand periods. For low values of γ_1, the model predicts the informal share to present little variability, around a fourth of what is observed in the data. With the highest value, the variability increases, slightly surpassing the data. The variability of the separation



Notes: Time period is a month. Units are percentage points relative to the steady state. Each line represents a different parameter combination. Darker lines correspond to higher time variation in the price of risk.

Figure 9: Response of informal share to aggregate shock

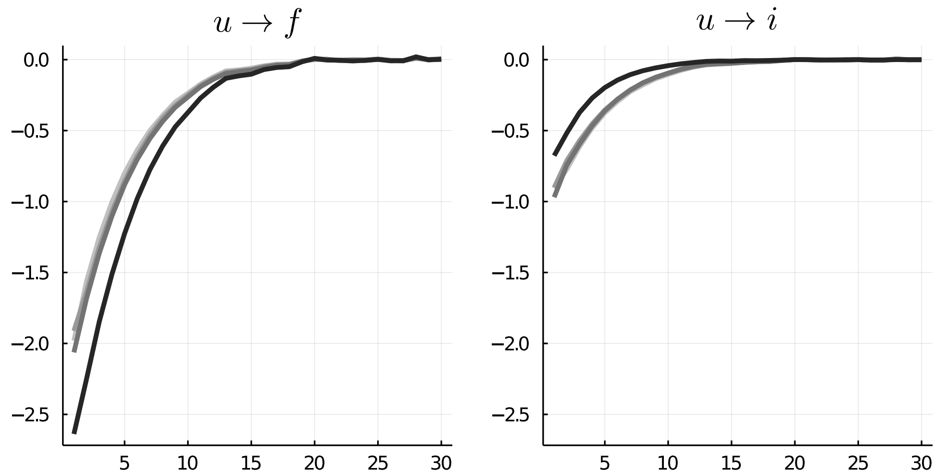
		Standard Deviation			
Variable	Data	$\gamma_1 = 0$	$\gamma_1 = -0.2$	$\gamma_1 = -0.8$	$\gamma_1 = -1.6$
Share of informal workers	0.004	0.001	0.001	0.003	0.007
Separation $\mathbb{f}$	0.055	0.009	0.008	0.008	0.004
Separation $\mathbb{i}$	0.224	0.013	0.013	0.015	0.021
Finding $\mathbb{f}$	0.026	0.005	0.005	0.005	0.002
Finding $\mathbb{i}$	0.0177	0.013	0.013	0.013	0.019

Table 3: Standard deviations: data and model

of informal workers is also nearly matched in that case, in contrast with lower values of γ_1. All models fail to generate sufficiently variable finding rates for formal workers.

In Table 4 we report the correlation of aggregates with unemployment in the data and in the stochastic simulation. Adding time substantial variation to the price of risk brings the correlation of the informal share with unemployment to around 50% from the -16.5% obtained when $\gamma_1 = 0$. The channel highlighted in the impulse responses presented in Figures 3a and 3b are not immediately translated in to the correlation between unemployment and worker flows to and from unemployment.

In summary, the exercise shows that assuming countercyclicality in the price of risk causes the absorption of workers from unemployment to tilt towards informal jobs after a recession. That effect is strong enough to generate a positive response of the informal share in the medium run to a negative aggregate shock. That is in contrast to the case where we assume an acyclical risk price, where the response of the informal share is more negative on impact and does not revert back to steady state levels until nearly two years after the shock.



Notes: Time period is a month. Left figure shows transition rate from unemployment to formal jobs. Figure on the right shows transition rates from unemployment to informal jobs. Units are percentage points relative to the steady state. Each line represents a different parameter combination. Darker lines correspond to higher time variation in the price of risk.

Figure 10: Response of *unemployment to employment flows* to aggregate shock

		Correlation(*Variable, Unemployment*)			
Variable	Data	$\gamma_1 = 0$	$\gamma_1 = -0.2$	$\gamma_1 = -0.8$	$\gamma_1 = -1.6$
Share of informal workers	0.71	-0.165	-0.094	-0.145	0.495
Separation probability $\mathbb{f}$	0.64	-0.036	-0.039	-0.015	0.143
Separation probability $\mathbb{i}$	0.95	0.173	0.164	0.165	0.121
Job finding $\mathbb{f}$	-0.96	-0.691	-0.678	-0.592	-0.293
Job finding $\mathbb{i}$	-0.85	-0.702	-0.692	-0.613	-0.354

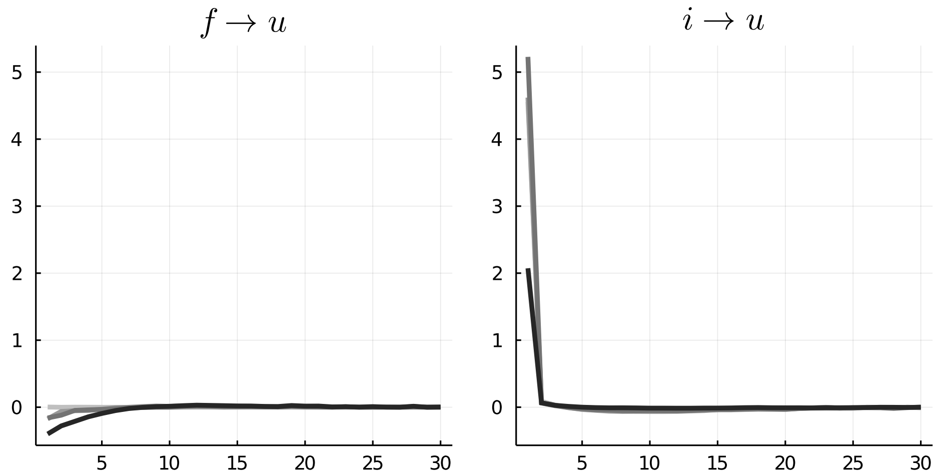
Table 4: Correlations with unemployment: data and model

4.3 Counterfactual analyses

We now turn to investigating the policy implications of a counter-cyclical price of risk in our model of informality. Our approach is to compare stochastic steady states under different policy parameters, hence we do not make claims about transition dynamics.[14] The point of the exercise is not only to study the predicted effects of different policies, but to understand how different specifications affect those predictions. We are interested in the consequences of misspecifying the channels that drive business cycle movements in labor market variables.

We compare two different policies in terms of their outcomes on labor market variables: increasing government monitoring and lowering severance requirements. We set both policies so as to reduce the size of the informal sector by half in the baseline specification, and repeat the exercise on all specifications for the stochastic discount factor. We find that predictions for both policies differ substantially.

[14] We refer the reader to Maya & Pereira (2020) for an analysis of the transition dynamics following a repression of informal activity.



Notes: Time period is a month. On the left, transition rates from formal jobs to unemployment. On the right, transition rates from unemployment to informal jobs. Units are percentage points relative to the steady state. Each line represents a different parameter combination. Darker lines correspond to higher time variation in the price of risk.

Figure 11: Response of *employment to unemployment flows* to aggregate shock.

4.3.1 Effects of increased monitoring

In this section we compare equilibria where the monitoring probability of new contracts ϕ increases from 68% to 85% so as to halve the informality rate in the baseline specification. We report the outcome for a subset of aggregates in Table 5. The same parameter change reduces the informal share by about the same amount in all specifications.

The first salient outcome is that unemployment also *increases* following the increased monitoring, irrespective of the calibration. There are two effects taking place. On the one hand, more workers are find formal jobs, which last longer due to a lower separation rate and that pushes down the rate of unemployment. On the other hand, some firms that would offer informal jobs under lower monitoring do not find it worthwhile to offer a formal contract and instead decide to go back to the vacancy pool. The decrease in new informal job opportunities more than compensates the increase in average job duration due to a greater proportion of formal workers. In spite of the increase in unemployment, output increases after monitoring for all specifications. The fact that formal firms do not pay the productivity penalty, and that employment is now more concentrated on those firms, more than compensates for the higher number of unemployed workers.

All calibrations predict an increase in government revenue following from the greater monitoring. That increase is muted by the fact that the greater rate of formality is biased towards less productive firms, hence in regions of the idiosyncratic shock that have lower associated wages. Also, tax revenues are predicted to be more procyclical, except when $\gamma_1 = -1.6$. All specifications predict tax revenues to be about equally volatile before and after the policy.

For this particular policy, different assumptions about the price of risk do not generate markedly different predictions. In that sense, this policy is robust to that particular type of model miss-specification. One important caveat about this policy is that we assume costless monitoring. In reality, the amount of government resources required to monitor the extensive

Calibration	$\gamma_1 = 0$		$\gamma_1 = -0.2$		$\gamma_1 = -0.8$		$\gamma_1 = -1.6$	
Monitoring probability (ϕ)	68%	85%	68%	85%	68%	85%	68%	0.89
Unemployment	0.141	0.147	0.142	0.146	0.141	0.145	0.151	0.156
Unemployment vol.	0.029	0.026	0.028	0.025	0.026	0.025	0.021	0.023
Output	0.967	1.006	0.966	1.005	0.963	1.003	0.978	1.016
Output vol.	0.152	0.151	0.152	0.150	0.152	0.151	0.154	0.153
Informal share	0.309	0.151	0.311	0.150	0.315	0.150	0.301	0.148
Separation $\mathbb{f}$	0.030	0.031	0.030	0.031	0.029	0.030	0.037	0.037
Separation $\mathbb{i}$	0.072	0.073	0.072	0.072	0.071	0.072	0.073	0.074
Finding $\mathbb{f}$	0.123	0.147	0.122	0.146	0.120	0.144	0.139	0.162
Finding $\mathbb{i}$	0.133	0.064	0.132	0.064	0.133	0.064	0.122	0.059
Tax revenue	0.154	0.188	0.153	0.188	0.152	0.188	0.157	0.190
Tax revenue vol.	0.004	0.005	0.004	0.005	0.004	0.005	0.005	0.005
σ(Unemp., Revenue)	-0.926	-0.949	-0.925	-0.932	-0.913	-0.915	-0.895	-0.899

Notes: Volatility is abbreviated to "vol.". Separation $\mathbb{f}$ and $\mathbb{i}$ refer to the transition rates from the respective type of job to unemployment, in quarterly terms. Finding $\mathbb{f}$ and $\mathbb{i}$ refers to transitions rates from unemployment to the respective type of job, in quarterly terms. $\sigma(X, Y)$ denotes the time series correlation between X and Y.

Table 5: The aggregate effects of increasing monitoring of new contracts

margin of formality might be large. The is especially true given the fact that informal jobs tend to be more common in smaller firms, which might be harder to monitor. The correct way to interpret our result is therefore that if the (unmodeled) *marginal cost of increasing monitoring* is low, increasing it will have benefits in terms of output and revenue collection without much loss in terms of increased unemployment or its volatility, regardless of whether countercyclical risk prices play a role in the economy.

4.3.2 Reducing severance pay

We now implement the *reduction in severance payment requirements* needed in order to achieve the same outcome of halving the informality rate in the baseline specification. That implies reducing the severance payment parameter by around 80%, from $\chi = 3.85$ to $\chi = 0.89$.

All specifications predict unemployment to increase after the policy. That is a direct consequence of increased separations of formal contracts, which becomes cheaper when severance requirements are lower. That reduces the average duration of formal contrats and hence the average unemployment rate. The effect is compounded with the marked increase in the separation of informal workers. All specification predict increases in output and its volatility, more so than with increased monitoring. That effect is the most pronounced for the highest $|\gamma_1|$ case.

The first and perhaps most striking difference between specifications concerns the prediction about the effect of the policy on the informal share itself. While in the baseline specification the informal share shrinks by half, the same change in the severance requirement causes the informal share to shrink to 7.3% in the highest $|\gamma_1|$ case. The reason for that can be inferred by the relative behavior of separation rates and finding rates between formal and informal jobs. While the impact of the severance requirement on separations is relatively similar in all economies, the ratio of new formal jobs to new informal jobs is starkly larger in the case with the most variable price of risk. I compute those ratios below:

	Ratio of new $\mathbb{f}$ jobs to new $\mathbb{i}$ jobs	
Calibration	Before policy	After policy
$\gamma_1 = 0$	0.92	3.22
$\gamma_1 = -0.2$	0.92	3.11
$\gamma_1 = -0.8$	0.90	2.9
$\gamma_1 = -1.6$	1.14	7.25

In the economy with highest $|\gamma_1|$, reducing the severance requirement has a much larger impact than in the other cases in terms of the ratio of new formal jobs to new informal jobs. That translates into an informal share more than fifty percent lower than the one predicted by the models with lower variation in the price of risk.

Another difference between specifications that is salient from Table 6 is the behavior of unemployment volatility. On all economies, except the one with the highest $|\gamma_1|$, the volatility of unemployment *decreases* after the policy. That conclusion reverts for $\gamma_1 = -1.6$. That reflects a more pronounced cyclicality in both job finding and job separation in the formal sector following the policy. That difference in cyclicality has consequences for the volatility and cyclicality of tax revenues. While tax revenue volatility remains nearly stable in the previous policy exercise, changes in severance requirements have an effect of doubling the tax revenue volatility in the baseline specification and nearly tripling it when $\gamma_1 = -1.6$.

Our results indicate that lowering severance requirements would bring the economy to a region where it is *more* sensitive to the price of risk. That is not in contradiction with the mechanism we propose. There are two sources of countercyclical burdens that make formal workers risky: severance payments *and* wage rigidity. What this exercise shows is that with the lower severance requirement, the imperfect adjustment of wages in the formal sector plays a relatively stronger role in determining the cyclicality of the labor market variables. Put differently, the relatively high severance requirement in the baseline economy dampens the amplifying effect of wage rigidity.

In terms of insurance, a reduced magnitude of severance payments has two effects. On the one hand, it lowers the insurance of each individual formal worker. On the other hand, it increases coverage due to the increased fraction of formal workers. The fact that we do not take a stand on how risk is shared between different economic agents in the economy prevents us from meaningfully addressing the welfare trade-off between the extensive and intensive margin of insurance provided by the severance payment. We leave that important question for future research.

5 Conclusion

In this paper we addressed the stylized fact that the rate of informality correlates negatively with the business cycle. We focused on one episode from recent Brazilian history, in which informality spiked after a sharp recession, driven mostly by reduced flows from unemployment to formal jobs.

Our main contribution is to interpret this event through the lens of a search and matching model of the labor market in which discount rates used to bring future cash flows to present value reflect a time varying price of risk. Our paper is the first to consider that the higher risk of hiring workers formally interacts with a countercyclical price of risk. In recessions, when the price of risk is high, new matches split between formal and informal jobs in a way that favors the less risky type of job, i.e., informal jobs.

We calibrate the model, targeting moments from the Brazilian economy, and we show this channel to be quantitatively relevant in explaining the episode observed around the 2015-2016 recession. We analyse policy implications of our proposed channel. We find that policies aimed at reducing the rate of informality have potentially different outcomes under different specifications discount rates. While in the case of a constant risk price a reduction in the severance pay policy aimed at halving the rate of informality *reduces* the volatility of unemployment, that conclusion reverses if the price of risk is sufficiently countercyclical.

Recent literature has shown that the discounting behavior of economic agents matters for the cyclical behavior of labor market in the United States. Our findings expand the reach of that conclusion, suggesting that the dynamics of labor markets in developing economies is also affected in important ways by the behavior of discount rates.

Calibration	$\gamma_1 = 0$		$\gamma_1 = -0.2$		$\gamma_1 = -0.8$		$\gamma_1 = -1.6$	
Severance payment (χ)	3.85	0.89	3.85	0.89	3.85	0.89	3.85	0.89
Unemployment	0.141	0.166	0.142	0.164	0.141	0.160	0.151	0.178
Unemployment vol.	0.029	0.025	0.028	0.024	0.026	0.024	0.021	0.041
Output	0.967	1.041	0.966	1.039	0.963	1.037	0.978	1.076
Output vol.	0.152	0.159	0.152	0.158	0.152	0.158	0.154	0.170
Informal share	0.309	0.152	0.311	0.155	0.315	0.159	0.301	0.073
Separation $\mathbb{f}$	0.030	0.054	0.030	0.053	0.029	0.051	0.037	0.066
Separation $\mathbb{i}$	0.072	0.087	0.072	0.087	0.071	0.087	0.073	0.106
Finding $\mathbb{f}$	0.123	0.216	0.122	0.215	0.120	0.211	0.139	0.261
Finding $\mathbb{i}$	0.133	0.067	0.132	0.069	0.133	0.073	0.122	0.036
Tax revenue	0.154	0.193	0.153	0.193	0.152	0.192	0.157	0.212
Tax revenue vol.	0.004	0.009	0.004	0.010	0.004	0.011	0.005	0.015
σ(Unemp., Revenue)	-0.926	-0.898	-0.925	-0.869	-0.913	-0.738	-0.895	-0.805

Notes: Volatility is abbreviated to "vol.". Separation $\mathbb{f}$ and $\mathbb{i}$ refer to the transition rates from the respective type of job to unemployment, in quarterly terms. Finding $\mathbb{f}$ and $\mathbb{i}$ refers to transitions rates from unemployment to the respective type of job, in quarterly terms. $\sigma(X, Y)$ denotes the time series correlation between X and Y.

Table 6: The aggregate effects of reducing severance requirements

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A Tables and Figures

A.1 Simulation: the behavior of targeted moments

		Calibration			
Moment	Data	$\gamma_1 = 0$	$\gamma_1 = -0.2$	$\gamma_1 = -0.8$	$\gamma_1 = -1.6$
Average unemployment	0.146	0.1413	0.1423	0.1412	0.1508
Unemployment autocorrelation	0.907	0.9206	0.9207	0.9173	0.8878
Unemployment volatility	0.037	0.029	0.0283	0.0263	0.0215
Share of informal workers	0.303	0.3087	0.3114	0.3147	0.3014
Separation probability $\mathbb{f}$	0.028	0.0302	0.0302	0.0295	0.0366
Separation probability $\mathbb{i}$	0.089	0.0723	0.0721	0.0711	0.0734
Interquartile ratio TFP $\mathbb{f}$	0.703	0.6858	0.6856	0.6906	0.6419
Tightness	1.0	1.0204	1.0188	1.0213	1.0185
Implied severance pay	3.85	3.8	3.79	3.81	3.78
Sharpe ratio	0.1844	0.1871	0.1871	0.1872	0.1872
Risk free rate volatility	0.0034	0.0041	0.0041	0.0041	0.0041

Table 7: Targeted moments: simulations