When Did Growth Begin? New Estimates of Productivity Growth in England from 1250 to 1870

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Abstract

We provide new estimates of the evolution of productivity in England from 1250 to 1870. Real wages over this period were heavily influenced by plague-induced swings in the population. We develop and implement a new methodology for estimating productivity that accounts for these Malthusian dynamics. In the early part of our sample, we find that productivity growth was zero. Productivity growth began in 1600—almost a century before the Glorious Revolution. Post-1600 productivity growth had two phases: an initial phase of modest growth of 4% per decade between 1600 and 1810, followed by a rapid acceleration at the time of the Industrial Revolution to 18% per decade. Our evidence helps distinguish between theories of why growth began. In particular, our findings support the idea that broad-based economic change preceded the bourgeois institutional reforms of 17th century England and may have contributed to causing them. We also estimate the strength of Malthusian population forces on real wages. We find that these forces were sufficiently weak to be easily overwhelmed by post-1800 productivity growth.

JEL Classification: N13, O40, J10

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1 Introduction

When did economic growth begin? A traditional view holds that economic growth began with the Industrial Revolution around 1800. Recent work has challenged this view pushing the date of the onset of growth back. Crafts (1983, 1985) and Harley (1982) revised downward previous estimates of growth in Britain during the Industrial Revolution. These new estimates indicate that British output per capita was larger by mid-18th century than was previously thought implying that substantial growth must have occurred at an earlier date (see also Crafts and Harley, 1992). Acemoglu, Johnson, and Robinson (2005) argue that a First Great Divergence occurred starting around 1500 with Western Europe growing apart from other areas of the world following the discovery of the Americas and the sea route to India. They support this view with data on urbanization rates. Broadberry et al. (2015) argue that growth began even earlier than this. They present new estimates of GDP per person for Britain back to 1270. These data show slow but steady growth in GDP per person from the beginning of their sample. Finally, Kremer (1993) uses world population estimates to argue for positive but glacially slow growth for hundreds of thousands of years.

An important facet of the debate about when growth began is when *productivity* growth began. We contribute to this debate by constructing a new series for productivity growth in England back to 1250. Figure 1 plots our new productivity series (solid black line). Our main finding is that productivity growth in England began in 1600. Before that time our estimates indicate that productivity growth was zero.[1] Between 1600 and 1810, productivity growth was modest at about 4% per decade. Productivity growth then dramatically increased after 1810 to about 18% per decade. These results indicate that there was a two hundred year transition period—1600 to 1810—between the era of near total stagnation and the era of rapid modern growth that was ushered in by the Industrial Revolution.

Our results help distinguish between different theories of *why* growth began. They suggest that researchers should focus on developments proximate to the 16th, 17th and 18th centuries. An important debate regarding the onset of growth is whether economic change drove political and institutional change as Marx famously argued or whether political and institutional change kick-started economic growth (e.g., North and Thomas, 1973). Reality is likely more complex than either polar view. However, our result that productivity growth began almost a century before the Glorious Revolution and well before the English Civil War supports the Marxist view—articulated

[1] The positive but glacially slow productivity growth rate implied by Kremer's (1993) population data for the period 1200 to 1500 data lies within our credible set.

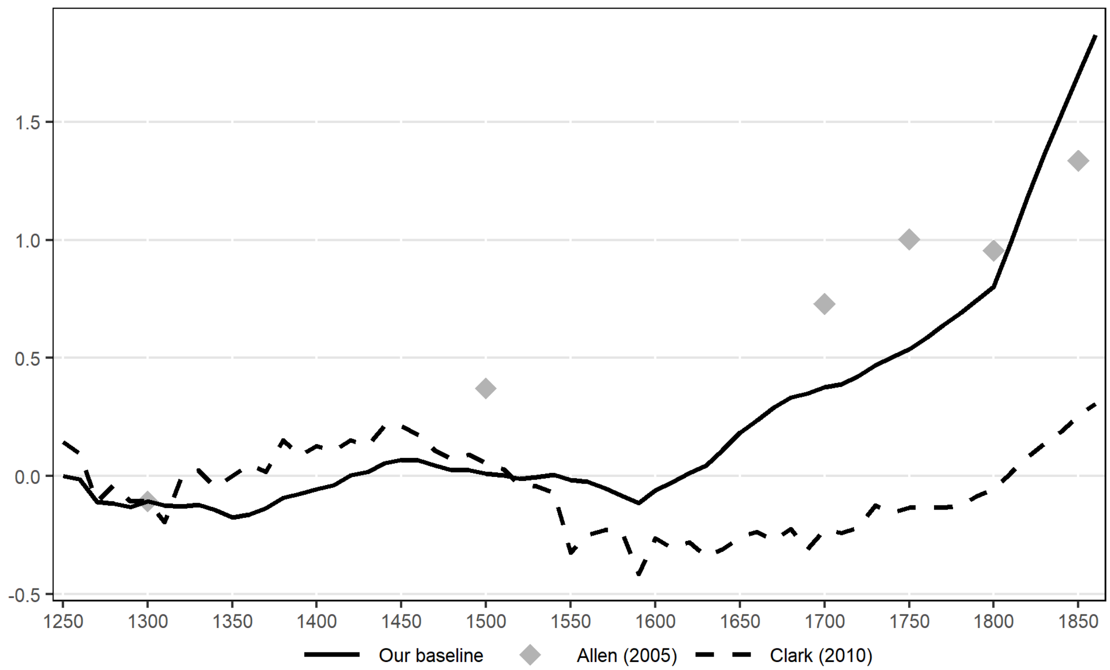

Figure 1: Estimates of Productivity in England

Note: Each series is the natural logarithm of productivity. Clark's (2010) series estimates TFP for the entire economy based on a dual approach. Allen's (2005) estimates are for TFP in the agricultural sector using a primal approach. Our baseline productivity series is normalized to zero in 1250. The other two periods are normalized to match our baseline series in 1300.

for example by Hill (1940, 1961)—that economic change contributed importantly to 17th century institutional change in England.

Our estimates also indicate that a very substantial increase in productivity growth – from essentially zero to modern levels – occurred over a relatively brief period of time. Our results therefore support the notion that the Industrial Revolution was revolutionary. In this sense, our results are closer to the traditional view of Deane and Cole (1962) and Feinstein (1988) than the 'revisionist' view of Crafts and Harley (1992).

The most comprehensive existing productivity series for England was constructed by Clark (2010). He estimated changes in TFP for the entire English economy from 1209 onward using the "dual approach"—i.e., as a weighted average of changes in real factor prices (e.g., Hsieh, 2002). Figure 1 plots Clark's series over our sample period (broken black line). A striking feature of this series is that it implies that productivity in England was no higher in the mid-19th century than in the 15th century. This result does not line up well with other existing (less comprehensive) measures of productivity in England or with less formal assessments of the English economy. For example, Allen (2005) estimates that TFP in agriculture was 162% higher in 1850 than in 1500 (grey diamonds in Figure 1).[2] Clark himself commented that if the fluctuations in his series are not measurement error "they imply quite inexplicable fluctuations in the performance of the preindustrial economy."

Our conclusions about productivity in England are clearly dramatically different from those of Clark (2010). According to our estimates, productivity in England was roughly 440% higher in 1850 than in 1500 rather than being essentially unchanged. These large differences arise from differences in the data and methodology we use. We take the labor demand curve as our starting point and estimate changes in productivity as shifts in the labor demand curve. This means that the key data series that inform our estimates are real wages and measures of labor supply (population and days worked per year). Real wages and population are arguably among the best measured series of all economic time series over our long sample period. In contrast, an important input into Clark's productivity series is a series for land rents that is essentially flat between 1250 and 1600 despite enormous fluctuations in the land-labor ratio in England over this period associated with plagues. We conjecture that mismeasurement in Clark's rent series contributes importantly to the differences in our results.

To get a better sense for how our approach works, consider the following simple labor demand curve for a pre-modern society

$$W_t = (1-\alpha)A_t\left(\frac{Z}{L_t}\right)^{\alpha},$$

where W_t denotes real wages, A_t denotes productivity, Z denotes land (which is fixed), and L_t denotes labor. The model we consider later in the paper is more general. But the basic idea can be grasped using this simple model. If we take logarithms, this equation becomes

$$w_t = \phi - \alpha l_t + a_t,$$

where lower case letters denote logarithms of upper case letters. Given this equation, a simple-minded empirical approach would be to regress wages on labor and equate the residual from that regression with productivity. In our context, however, this simple-minded approach is not likely to work well because of Malthusian population forces. In a Malthusian world, increases in productivity induce increases in the population and therefore the labor force. This means that in a Malthusian world l_t and a_t are likely to be correlated and an OLS estimate of α is likely to yield non-sense.

[2] Allen (2005) employs the familiar "primal approach," i.e., subtracts a weighted average of growth in factor inputs from output growth.

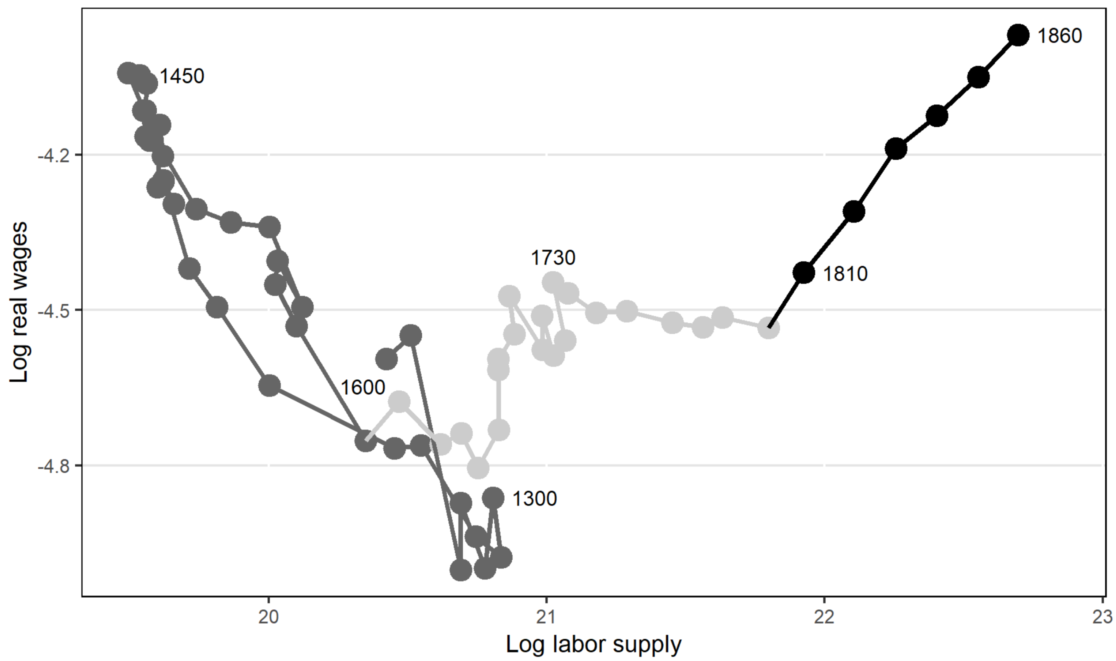

Figure 2: Real Wages and Labor Supply

Note: The figure presents a scatter plot of the logarithm of real wages in England against the logarithm of labor supply in England over the period 1250-1860. The data on real wages are from Clark (2010). Estimates of labor supply are based on our calculations. Labor supply varies mainly due to variation in the population, but also due to changes in days worked per person.

To illustrate this, Figure 2 presents a scatter plot of real wages in England against labor supply in England. Variation in labor supply in England is mostly driven by variation in the population, but also affected by variation in days worked per worker. The period from 1300 to 1450 was a period of frequent plagues—the most famous being the Black Death of 1348. Over this period, the population of England fell by a factor of two resulting in a sharp drop in labor supply. Over this same period, real wages rose substantially. Then from 1450 to 1600, the population (and labor supply) recovered and real wages fell. In 1630, the English economy was back to almost exactly the same point it was at in 1300. One way to explain these dynamics between 1300 and 1630 is as movements along a stable labor demand curve with no change in productivity. Then in the 17th century, something important seems to change. The points start moving off this labor demand curve. Specifically, they start moving up and to the right relative to the earlier curve. This suggests that productivity started growing in the 17th century in England.

This is the basic idea behind our approach to estimating productivity. We seek to estimate a labor demand curve for England and then back out productivity growth as shifts in this labor demand curve. Clearly, estimating the labor demand curve by an ordinary least squares regression will not work in this setting since the shifts in productivity induce increases in the population (see the points after 1630 in Figure 2). For this reason, we take a more structural approach. We write down a Malthusian model of the economy which includes both a labor demand curve and a model for the evolution of the population over time as a function of real incomes. We then estimate this full model. In other words, we model the endogeneity of population dynamics. This allows us to produce an estimate of the labor demand curve and an estimate of productivity growth that accounts for the endogeneity implied by Malthusian population dynamics.

Our estimates also shed light on the lack of real wage growth during the latter part of the 18th century, sometimes referred to as "Engel's Pause" (Engels, 1845, Allen, 2009b). Our Malthusian model suggests that during this period real wages were held back by very rapid increases in the population, which in a Malthusian world put downward pressure on the marginal product of labor. This explanation contrasts with the common idea that the absence of real wage growth during this period resulted from the lion's share of the fruits of technical change going to capital as opposed to labor.

In addition to estimates of productivity, our methodology yields estimates of the speed of Malthusian population dynamics in pre-modern England. Our estimates imply that Malthusian population dynamics were very slow: a doubling of real incomes led to a 6 percentage point per decade increase in population growth. Together with our estimate of the slope of the labor demand curve, this implies that the half-life of a plague-induced drop in the population was roughly 150 years. Earlier estimates of the speed of Malthusian population dynamics in England also indicate that they were slow. For example, Lee and Anderson (2002) find a half-life of 107 years, while Crafts and Mills (2009) find a half-life of 431 years. Chaney and Hornbeck (2016) document very slow population dynamics in Valencia after the expulsion of the Moriscos in 1609.

Finally, we can use our estimates to ask to what extent the speed of productivity growth after 1800 overwhelmed the Malthusian population force. In a Malthusian world, the steady state level of real income is increasing in the steady state level of productivity growth. Our estimates imply that the steady state level of real income associated with the post-1800 growth in productivity of 18% per decade was 28 times higher than the steady state level of real income associated with zero growth in productivity. This implies that even if the Demographic Transition—i.e., the rapid fall in birth and death rates and decoupling of these rates from real income—had not happened, the level of productivity growth post-1800 would have resulted in substantial growth in living standards before being choked off by population growth.

Our work builds on ideas in Clark (2005, 2007a). These works discuss informally how shifts in the labor demand curve of a Malthusian model can be informative about the timing of the onset of economic growth. The existing papers most closely related to ours from a methodological point of view are Lee and Anderson (2002) and Crafts and Mills (2009). These papers structurally estimate a Malthusian model of the English economy, as we do. However, their sample period is considerably shorter than ours (theirs starts in 1540 while ours start in 1250). This means that they cannot address the question of when growth began.

Our paper is also related to the literature in macroeconomics on the transition from pre-industrial stagnation to modern growth—often referred to as the transition "from Malthus to Solow." Important papers in this literature include Galor and Weil (2000), Jones (2001), and Hansen and Prescott (2002). Relative to these papers, our work is more empirical. We contribute detailed estimates of the evolution of productivity, while these papers propose theories of how productivity growth rose. Our work is also related to recent work by Hansen, Ohanian, and Ozturk (2020).

Our paper proceeds as follows. Section 2 presents our Malthusian model of the economy. Section 3 discusses the data we use and our estimation strategy. Section 4 presents our results on productivity. Section 5 presents our results on the strength of the Malthusian population force. Section 6 presents our estimates of the population. Section 7 concludes.

2 A Malthusian Model of the Economy

We now present a simple model meant to describe the pre-industrial English economy. The model is Malthusian in that diminishing returns to labor (the only variable factor of production) give rise to a downward-sloping labor demand curve and the rate of population growth is increasing in people's real income. We model time as discrete and denote it by a subscript t. Since we use decadal data later in the paper, each time period in the model is meant to represent a decade.

Output is produced with land and labor according to the following production function:

$$Y_t = A_t Z^{\alpha} L_t^{1-\alpha},$$

where Y_t denotes output, A_t denotes productivity, Z denotes land (which is fixed), and L_t denotes labor (in units of worker days). We assume that owners of land hire workers in a competitive labor market taking wages as given. Optimal behavior by land owners gives rise to the following labor demand curve:

$$W_t = (1-\alpha)A_t\left(\frac{Z}{L_t}\right)^{\alpha},$$

where W_t denotes the real daily wage. Taking logarithms of this equation yields

$$w_t = \tilde{\phi} - \alpha l_t + a_t, \tag{1}$$

where lower case letters denote logarithms of upper case letters and $\tilde{\phi} = \log(1-\alpha) + \alpha \log Z$. Our assumption above of a Cobb-Douglas production function is for expositional simplicity. Equation (1) holds as a log-linear approximation of labor demand for a generic production function. For instance, if the production function displays a constant elasticity of substitution between land and labor, α is not equal to the land share of output. Rather, its value depends both on the land share and the elasticity of substitution between labor and land. We present this more general derivation in appendix A.

As in all models, productivity is a catch-all variable capturing the influence of all variables that are not explicitly modeled in the production function. The simple form of our assumed production function—with only a single variable factor of production—implies that our measure of productivity is rather broad. For example, it captures improvements in land and capital over our sample period. In section 4.2, we extend our model to include capital. This allows us to estimate a productivity series that accounts for capital accumulation.

We assume that the labor force in the economy is proportional to the population and that each worker works D_t days per year. This implies that

$$L_t = \lambda D_t N_t,$$

where N_t denotes the population. Taking logs of this equation and using the resulting equation to eliminate l_t in equation (1) yields

$$w_t = \phi - \alpha(d_t + n_t) + a_t, \tag{2}$$

where $\phi = \log(1-\alpha) + \alpha \log Z - \alpha\lambda$.

A central aspect of our model is the law of motion for the population. Following Malthus

(1798), we assume that population growth is increasing in real income:

$$\frac{N_t}{N_{t-1}} = \Omega(W_{t-1}D_{t-1})^{\gamma}\Xi_t,$$

where Ω is a constant, γ is the elasticity of population growth with respect to real income, and Ξ_t denotes other (exogenous) factors affecting population growth. Taking logarithms of this equation yields

$$n_t - n_{t-1} = \omega + \gamma(w_{t-1} + d_{t-1}) + \xi_t. \tag{3}$$

Malthus argued that both the birth rate and the death rate varied with real income. He described "preventive checks" on population growth that lowered birth rates. These included contraception, delayed marriage, and regulation of sexual activity during marriage. Malthus also described "positive checks" on population growth that raised death rates. These include disease, war, severe labor, and extreme poverty. In our model, the parameter γ captured the elasticity of both birth rates and death rates with respect to income. This parameter therefore captures any tendency of either preventive or positive checks to lower population growth when income falls.

We assume that the logarithm of productivity is made up of a permanent and transitory component:

$$a_t = \tilde{a}_t + \epsilon_{2t}, \tag{4}$$

where

$$\tilde{a}_t = \mu + \tilde{a}_{t-1} + \epsilon_{1t}, \tag{5}$$

$\epsilon_{1t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_1})$, and $\epsilon_{2t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_2})$. Both ϵ_{1t} and ϵ_{2t} are independently distributed over time. Here, $\tilde{a}_t$ is the permanent component of productivity, which follows a random walk with drift, while ϵ_{2t} is the transitory component of productivity. The average growth rate of productivity is given by the parameter μ. This is a key parameter in our model. As we describe in more detail below, we allow for structural breaks in μ, i.e., changes in the average growth rate of productivity.

We allow for two types of exogenous population shocks:

$$\xi_t = \xi_{1t} + \xi_{2t}. \tag{6}$$

First, we allow for "plague" shocks:

$$\exp(\xi_{1t}) \sim \begin{cases} \beta(\beta_1, \beta_2), & \text{with probability } \pi \\ 1, & \text{with probability } 1-\pi \end{cases} \tag{7}$$

These plague shocks occur infrequently (with probability π) but when they occur they kill a (potentially sizable) fraction of the population. The fraction of the population that survives follows a beta distribution $\beta(\beta_1, \beta_2)$. The historical record indicates that plagues ravaged Europe frequently in the 14th and 15th centuries and continued to strike until the 17th century (Gottfried, 1983, Shrewsbury, 1970). Europe had no significant plague outbreak from the late 8th to the mid-14th century. This changed with the Black Death, which probably originated on the steppes of Mongolia, reached China in the 1330s, travelled west through trade routes, and landed in England in the summer of 1348. For three centuries after the Black Death, the plague would reappear every few decades, and wipe out a significant share of the population each time.[3] In England, the last major outbreak was the Great Plague of London in 1665-66. There is no shortage of potential explanations for the relatively sudden disappearance of plague—collective immunity, better nutrition, changes in the dominant rat species, improved quarantine methods, among others (Appleby, 1980). In addition to the "plague," smallpox, measles, typhus, and dysentery were frequent mass killers. While they were certainly catastrophic to those afflicted, these outbreaks are ideal from an identification standpoint: they are frequent, sizeable and plausibly unrelated to the state of the economy.

In addition to the plague shocks, we allow for a second type of population shocks: $\xi_{2t} \sim \mathcal{N}(0, \sigma^2_{\xi_2})$. Both of the population shocks are independently distributed over time. Together these population shocks are meant to capture a host of potential influences on population growth such as weather and wars, in addition to plagues.

It is useful to consider the dynamics of the model after a plague shock and a productivity shock. Figure 3 depicts the evolution of real wages and the population after a plague. The downward sloping curve in the figure represents the labor demand curve in the economy. Suppose the economy is initially in a steady state at point A, but then a plague strikes that kills a fraction of the population. The economy will then jump from point A to point B. At point B the population

[3] Plague is caused by a bacillus, *Yersinia pestis*, which naturally lives in the digestive tract of rodent fleas (although some epidemiologists contend that human fleas were the main carriers (Appleby, 1980, Dean et al., 2018)). Rat populations thus constitute a reservoir for plague. Periodically, the bacilli would multiply in the fleas' stomachs, causing them to regurgitate in the rat's bloodstream. This would kill the rats. Once the rats died, the fleas would turn to other hosts, including humans. This mechanism explains the recurring character of the disease.

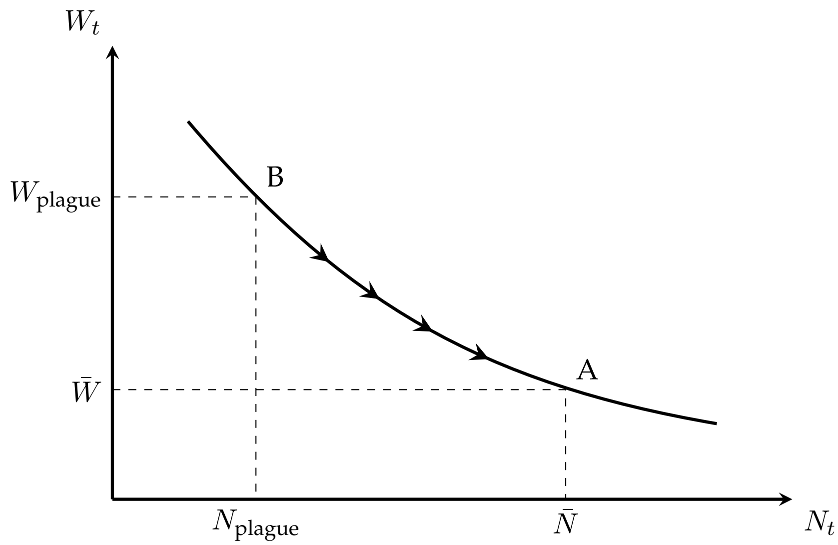

Figure 3: Effects of a Plague

is lower reflecting the death caused by the plague and real wages are higher reflecting the higher marginal product of labor of the surviving workers. After the plague, the economy will then gradually move down the labor demand curve until it reaches point A again. This movement occurs because higher real wages lead to positive population growth. As the population grows the economy moves along the labor demand curve and real wages fall. Once the economy is back at point A, real wages are again sufficiently low that population growth is zero.

Figure 4 depicts the evolution of real wages and the population after a productivity shock. Suppose again that the economy is initially in a steady state at point A. This time, however, suppose a permanent increase in productivity shock occurs. This shifts the labor demand curve out. In the short run, the population is fixed. The economy therefore jumps from point A to point B. Over time after the shock, the economy will then gradually move along the new labor demand curve until it gets to a new steady state at point C. These dynamics are again due to high wages causing positive population growth and a larger and larger population reducing wages until they are back at a point where population growth is zero.

Notice that the dynamics of the economy after these two shocks are quite different. In the case of a plague shock, the population moves sharply on impact but returns to its original level in the long run. In the case of a permanent change in productivity, however, real wages move on impact but not the population, while the population changes over time and ends up at different point than the economy started at. The main empirical challenge we face is distinguishing between labor demand (productivity) shocks and labor supply (plague) shocks. It is these differences in

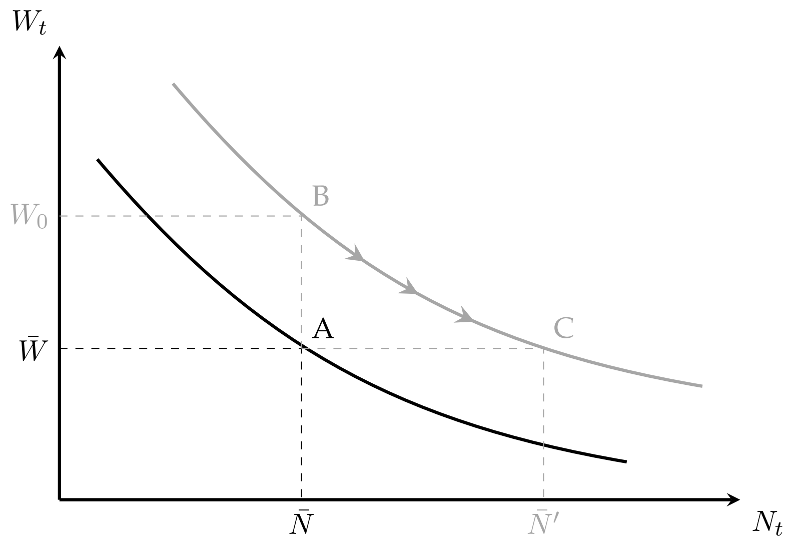

Figure 4: Effects of a Permanent Increase in Productivity

dynamics that help us to distinguish these two empirically.

3 Data and Estimation

We reproduce the equations and distributional assumptions of our full model here for convenience:

$$
\begin{aligned}
w_t = &\quad \phi + \tilde{a}_t - \alpha(n_t + d_t) + \epsilon_{2t} \\
n_t - n_{t-1} = &\quad \omega + \gamma(w_{t-1} + d_{t-1}) + \xi_{1t} + \xi_{2t} \\
\tilde{a}_t = &\quad \mu + \tilde{a}_{t-1} + \epsilon_{1t}
\end{aligned}
$$

$$
\exp(\xi_{1t}) \sim \begin{cases} \beta(\beta_1, \beta_2), & \text{with probability } \pi \\ 1, & \text{with probability } 1 - \pi \end{cases}
$$

$$
\epsilon_{1t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_1}), \quad \epsilon_{2t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_2}), \quad \xi_{2t} \sim \mathcal{N}(0, \sigma^2_{\xi_2})
$$

We estimate the model using data on wages w_t, the population n_t, and days worked d_t. We do this using Bayesian methods. In particular, we use a Hamiltonian Monte Carlo sampling procedure (Gelman et al., 2013, Betancourt, 2018).[4] The unobservables that we make inference about are the permanent component of productivity $\tilde{a}_t$, the shocks to this component ϵ_{1t}, the transitory shocks to productivity ϵ_{2t}, the plague shocks ξ_{1t}, the symmetric population shocks ξ_{2t}, and the parameters ϕ, α, ω, γ, μ, π, β_1, β_2, $\sigma^2_{\epsilon_1}$, $\sigma^2_{\epsilon_2}$, and $\sigma^2_{\xi_2}$. Below, we first describe the data we use in

[4] We implement this procedure using Stan (Stan Development Team, 2017).

Figure 5: Real Wages in England, 1250-1860

Note: The figure presents estimates of the real wages of unskilled building workers in England from Clark (2010).

more detail and then discuss the priors we assume for the parameters and structural breaks we allow for.

All the data we use are decadal averages. In our figures, a data point listed as 1640 refers to the decadal average from 1640 to 1649. We sometimes refer to a variable at a point in time (say 1640) when we mean the decadal average for that decade. In other words, we use 1640 and "the 1640s" interchangably.

Figure 5 plots the data series we use for real wages in England. This is the series for unskilled building workers from Clark (2010). The main features of this series are a large and sustained rise between 1300 and 1450, a large and sustained fall between 1450 and 1600, some recovery over the 17th century, stagnation during the 18th century, and finally a sharp increase after 1800. Figure A.1 compares this series with several other series for real wages in England. This comparison shows that the real wage series we use is quite similar to Clark's real wage series for farmers. It also largely shares the same dynamics as Clark's series for craftsman. We have redone our analysis with the farmers and craftsmen series and discuss this analysis in section 4.[5]

[5]Much controversy has centered on the behavior of real wages in England between 1770 and 1850. This debate revolves around the extent to which laborers shared in the benefits of early industrialization (see, e.g. Feinstein, 1998, Clark, 2005, Allen, 2007, 2009b). In Figure A.1, we also plot Allen's (2007) wage series (which starts in 1770). The figure shows that the differences discussed in the prior literature are modest from our perspective and therefore do not materially affect our analysis.

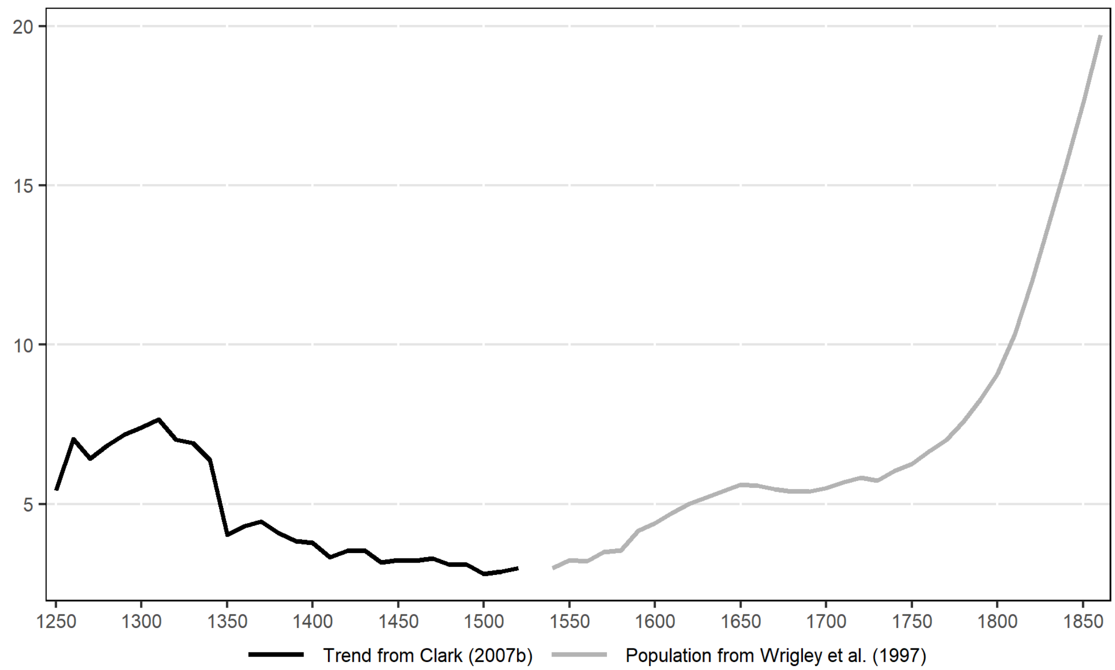

Figure 6: Population Data for England, 1250-1860

Note: The figure presents population estimates for England for the period 1540-1860 from Wrigley et al. (1997) (grey line) and the population trend estimates by Clark (2007b) for the period 1250-1520 (black line). In this figure, the black line is normalized for visual convenience such that its last point is equal to the first point of the grey line.

Figure 6 presents the population data that we use. For the period from 1540 onward, we use population estimates from Wrigley et al. (1997), which in turn build on the seminal work of Wrigley and Schofield (1981). Sources for population data prior to 1540 are less extensive. Clark (2007b) uses unbalanced panel data on the population of villages and manors from manorial records and penny tithing payments to construct estimates of the population prior to 1540. We build on Clark's work to construct an estimate of the population before 1540. We cannot directly use Clark's pre-1540 population series since Clark's method for constructing his series involves making assumptions about the evolution of productivity.[6] Since we aim to use the population series to make inference about the evolution of productivity in England, we cannot use a population series that already embeds assumptions about productivity growth. However, as an intermediate input into constructing his pre-1540 population series, Clark estimates a regression of his village and manor level population data on time and village/manor fixed effects. Clark refers to the time effects from this regression as a population trend. We plot this population trend in Figure 6 (normalized for visual convenience). We base our estimates of the population of England prior to 1540 on this population trend series. In section 5, we discuss how this series compares to (lower

[6]Appendix B discusses Clark's method in more detail.

frequency) population data reported in Broadberry et al. (2015).

The population data plotted in Figure 6 are missing information about the population in 1530 and are also missing a normalization for the population prior to 1540. We assume that the true population is unobserved and estimate it using the following equation

$$n_t = \psi + \hat{n}_t + \iota_t^n,$$

where n_t denotes the true unobserved population, $\hat{n}_t$ denotes our observed population series (Clark's population trend series prior to 1530 and the population series from Wrigley et al. (1997) after 1530), $\iota_t^n \sim t_{\nu_n}(0, \sigma_n^2)$ denotes measurement error, and ψ denotes a normalization constant. We normalize ψ to zero after 1530 and estimate its value for the pre-1530 Clark series.

The final variable to discuss is days worked d_t. We treat this variable as exogenous and present results for two assumptions about its evolution. Our baseline estimation is based on the series for days worked from Humphries and Weisdorf (2019). Figure 7 plots this series which indicates that days worked dropped sharply after the Black Death and then started a long upward march. Humphries and Weisdorf's series, thus, indicates that England experienced a large Industrious Revolution (de Vries, 1994, 2008). Since the extent of the Industrious Revolution is quite controversial, we also present results assuming that days worked remain constant throughout our sample period.[7] These two sets of results turn out to be quite similar (see section 4.3). In other words, it doesn't matter for our conclusions whether England experienced the type of Industrious Revolution indicated by the Humphries and Weisdorf series.

In our baseline analysis, we assume that Humphries and Weisdorf's series is measured with error

$$d_t = \tilde{d}_t + \iota_t^d,$$

where d_t denotes the true number of days worked per worker, which is unobserved, $\tilde{d}_t$ denotes Humphries and Weisdorf's estimates of days worked, and $\iota_t^d \sim t_{\nu_d}(0, \tilde{\sigma}_d^2)$ denotes the measurement error. Humphries and Weisdorf do not provide estimates for 1250, 1850, and 1860. We extrapolate from the series we have assuming that $d_t = d_{t-1} + \eta_t$ where $\eta_t \sim \mathcal{N}(0, \sigma_d^2)$.

Table 1 lists the priors we assume for the model parameters. In all cases, we choose highly

[7]Comparisons of direct estimates by Blanchard (1978) for 1400-1600 and Voth (2000, 2001) for 1760-1830 also support the idea of an Industrious Revolution. Earlier indirect estimates by Clark and Van Der Werf (1998), however, suggest modest changes in days worked over our sample. Humphries and Weisdorf (2019) argue that their new series on the income of workers on annual contracts represents an important improvement relative to the series used by Clark and Van Der Werf (1998).

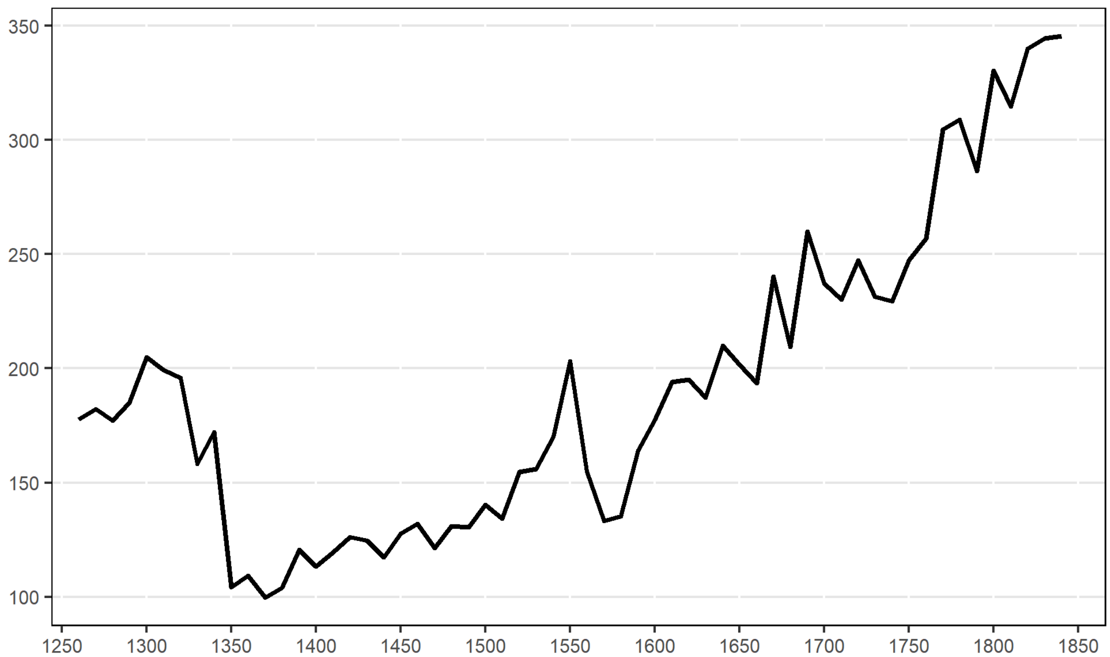

Figure 7: Days Worked per Worker in England, 1260-1840

Note: The figure presents an estimate of the evolution of days worked per worker in England from Humphries and Weisdorf (2019).

dispersed priors. Most of the priors are self-explanatory. But some comments are in order. The prior for ψ is set such that the peak population before the Black Death is between 4.5 and 6 million with 95% probability. This range encompasses the estimates of Clark (2007b) and Broadberry et al. (2015). Rather than specifying priors for β_1 and β_2, we specify priors for the mean of ξ_1 which we denote $\mu_{\xi_1} = \beta_1/(\beta_1 + \beta_2)$ and the pseudo sample size of ξ_1 which we denote $\nu_{\xi_1} = \beta_1 + \beta_2$. The priors we choose for these parameters follow the recommendations of Gelman et al. (2013, p. 110) for a flat prior for a beta distribution. Figure A.2 plots the prior densities for the standard deviations of ϵ_1, ϵ_2, and ξ_2. In section 4.3, we discuss how varying our priors affects our main results.

We allow for a structural break in the scale and degrees-of-freedom parameters of the measurement error in our population data—σ_n^2 and ν_n, respectively—in 1540. This date coincides with the change in the source of the population data. The post-1540 population data we use is higher quality than the earlier data. The structural break allows us to capture this change. We also allow for a structural break in the probability of a plague π in 1680. The timing of this break is chosen to immediately follow the Great London Plague of 1665.[8] This break is meant to capture the fact that

[8]Notice that the change in the population between the 1660s and the 1670s is affected by the Great London Plague. So, ξ_{1t} for $t = 1670$ will be affected by the Great London Plague. This is why we assume that ξ_{1t} for $t \geq 1680$ is governed by a different π than earlier values of ξ_{1t}.

Table 1: Priors for Model Parameters

Parameter	Prior	Parameter	Prior
α	$\mathcal{U}(0,2)$	γ	$\mathcal{U}(-2,2)$
ϕ	$\mathcal{N}(0,100^2)$	ψ	$\mathcal{N}(10.86,0.07^2)$
ω	$\mathcal{N}(0,1)$	μ	$\mathcal{N}(0,1)$
μ_{ξ_1}	$\mathcal{U}(0.5,0.9)$	ν_{ξ_1}	$\mathcal{P}_I(0.1,1.5)$
π	$\mathcal{U}(0,0.5)$		
$\sigma^2_{\epsilon_1}$	$\text{I}\Gamma(3,0.001)$	$\sigma^2_{\epsilon_2}$	$\text{I}\Gamma(3,0.005)$
$\sigma^2_{\xi_2}$	$\text{I}\Gamma(3,0.005)$	σ^2_n	$\text{I}\Gamma(3,0.005)$
σ^2_d	$\text{I}\Gamma(3,0.005)$	$\tilde{\sigma}^2_d$	$\text{I}\Gamma(3,0.005)$
ν_n^{-1}	$\mathcal{U}(0,1)$	ν_d^{-1}	$\mathcal{U}(0,1)$

plagues are less frequent in the latter part of our sample. The exact timing of this break does not affect our main results in a material way.

To be able to capture potential changes in average growth of productivity, we allow for two structural breaks in μ. Visual inspection of the data suggests that there is a large structural break around 1800 (see Figure 2). By allowing for two breaks, we allow for the possibility that there may be a second structural break earlier in the sample. We choose the timing of these structural breaks to maximize the marginal likelihood (Bayes factor) of the model. This is a widely used model selection method for Bayesian models (see, e.g. Sims and Zha, 2006). Consider two models M_t and M_t'. In our case, these are two different versions of our model from section 2 with different break dates for μ. Bayes rule implies that

$$\underbrace{\frac{p(M_t \mid y)}{p(M_{t'} \mid y)}}_{\text{posterior odds}} = \underbrace{\frac{p(y \mid M_t)}{p(y \mid M_{t'})}}_{\text{Bayes factor}} \times \underbrace{\frac{p(M_t)}{p(M_{t'})}}_{\text{prior odds}}$$

where y denotes the observed data and $p(\cdot)$ denotes a probability density. Using the Bayes factor as one's model selection criterion then implies that one is choosing the model with the largest posterior odds conditional on assuming that neither model is favored a priori (i.e., that the prior odds are one). To calculate the marginal likelihood of our models, we employ the bridge sampling method of Gronau, Singmann, and Wagenmakers (2020).

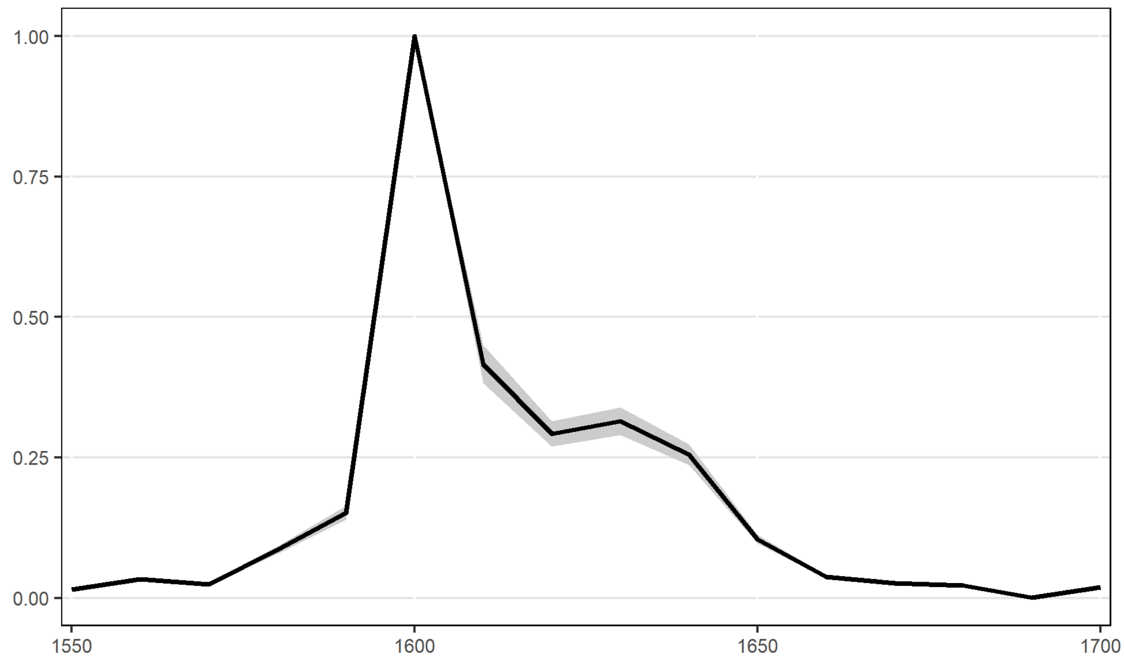

Figure 8: Bayes Factor for Different Productivity Growth Break Dates

Note: The figure plots the Bayes factor for models with different break dates for average productivity growth μ when compared to the model with breaks occurring in 1600 and 1810. In all cases, the models being considered also have a break in μ in 1810. The grey bands around our estimates represent computational uncertainty. They capture the range between the 5th and 95th quantile of the Bayes factor from 1000 draws of our bridge sampling procedure.

4 When Did Productivity Growth Begin in England?

Our primary object of interest is the evolution of productivity in England over our sample period. We therefore start the discussion of our empirical results by describing our results about productivity. As we discuss above, we allow for two structural breaks in average productivity growth μ over our sample. The pair of break dates that yields the highest marginal likelihood is 1600 and 1810. Figure 8 illustrates the statistical evidence favoring a break in 1600 by reporting the Bayes factors for models where the 1600 break date is shifted to other dates. We estimate a sharp rise in the Bayes factor from 1580 to 1600 and a somewhat more gradual fall from 1600 to 1650. Break dates before 1590 and after 1640 are clearly rejected.[9]

Table 2 presents our estimates of the average growth rate of productivity μ as well as the productivity shocks σ_{ϵ_1} and σ_{ϵ_2}. (Our estimates of the other parameters are presented in Table 4 and will be discussed in more detail in section 5.) We estimate that average productivity growth

[9]A common rule of thumb for Bayes factor analysis is to view a factor of $10^{1/2} \approx 3.2$ as substantial evidence and a factor of 10 as strong evidence favoring one model relative to another.

Table 2: Productivity Parameters

	Mean	St Dev	2.5%	97.5%
$\mu_{a,t<1600}$	-0.00	0.01	-0.02	0.01
$\mu_{a,1600\leq t<1810}$	0.04	0.01	0.02	0.06
$\mu_{a,t\geq 1810}$	0.18	0.03	0.12	0.23
σ_{ϵ_1}	0.04	0.01	0.02	0.07
σ_{ϵ_2}	0.05	0.01	0.03	0.07

Note: The table presents the mean, standard deviation, 2.5% quantile, and 97.5% quantile of the posterior distribution we estimate for average productivity growth μ in the three regimes and also for the standard deviation of the permanent and transitory productivity shocks ϵ_{1t} and ϵ_{2t}.

prior to 1600 was zero.[10] Sustained productivity growth began in 1600 (or around that time). At first, average productivity growth was modest. Our estimate of μ for the period from 1600 to 1810 is 4% per decade. In the early 19th century, productivity growth accelerated sharply to 18% per decade. The timing of this second break in average productivity growth is estimated quite sharply to be in 1810. The only other date that is not clearly rejected is 1800 (see Table A.1). We conclude from these estimates that the period from 1600 to 1810 was a period of transition in England from an era of total stagnation to an era of modern economic growth.

Figure 9 presents our baseline estimates of the time series evolution of the permanent component of productivity. These estimates indicate that the level of productivity in England was very similar in 1600 to what it had been in the late 13th century. In the intervening period, productivity fluctuated a slight bit. It reached its lowest point right before the Black Death in 1340, then increased and peaked a century later before receding slightly. After 1600, productivity began a sustained increase, which accelerated sharply in 1810.

Figure 10 compares our estimate of productivity with the data we use on real wages. This figure illustrates well the importance of accounting for Malthusian population forces when estimating productivity in the pre-industrial era. Through the lens of our Malthusian model, the large changes in real wages prior to 1600 are explained almost entirely by changes in labor supply—the

[10]Estimates of the world population presented in Kremer (1993) indicate that world population growth from 1200 to 1500 was 0.6% per decade. In our Malthusian model (as well as Kremer's model) steady state productivity growth is α times steady state population growth – α being the slope of the labor demand curve. Our baseline estimate of α is 0.53 (Table 4). This suggests that world productivity growth was 0.3% per decade over the period 1200 to 1500. This is well within the credible set of our pre-1600 estimate of μ.

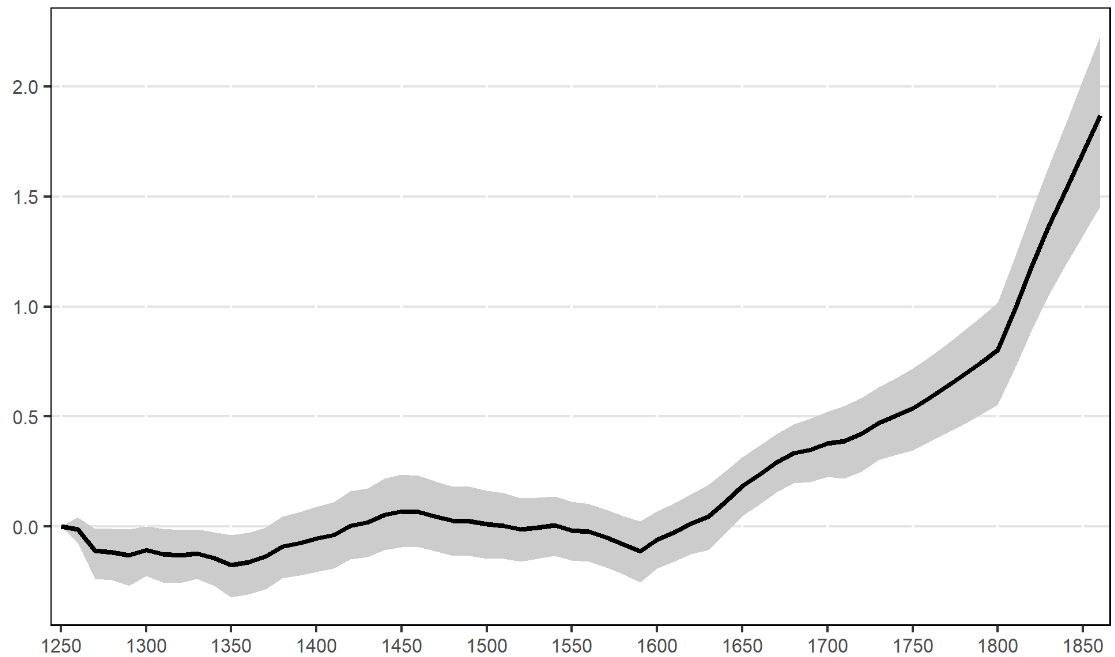

Figure 9: Permanent Component of Productivity

Note: The figure plots our estimates of the evolution of the permanent component of productivity $\tilde{a}_t$ over our sample period. The black line is the mean of the posterior for each period and the gray shaded area is the 90% central posterior interval.

economy was moving up and down a stable labor demand curve as suggested by Figure 2. As a result, changes in productivity were very substantially muted relative to changes in real wages over this period. In sharp contrast, after 1600 productivity increased much more rapidly than real wages. Over this period, the population in England grew rapidly as did days worked per worker. Our Malthusian model implies that this expansion of labor supply held back real wages relative to the change in productivity.

The period between 1730 and 1800 is particularly interesting in this context. Over this period, real wages in England fell slightly despite substantial productivity growth. One potential explanation of this is "Engels' Pause," i.e., the idea that the lion's share of the gains from early industrialization went to capitalists as opposed to laborers (Engels, 1845, Allen, 2009b). However, our Malthusian model provides an alternative explanation of this divergence between wages and productivity: Over this period, the population of England grew rapidly. In our Malthusian model, the growth in the population reduced the growth in wages relative to productivity.[11]

[11] The literature on Engels' Pause has typically focused on the first half of the 19th century as opposed to the second half of the 18th century. The real wage series we use—constructed by Clark (2010)—indicates that real wages actually started growing robustly around 1810. This contrasts with the real wage series of Feinstein (1998) and Allen (2007), which date the onset of rapid real wage growth a few decades later.

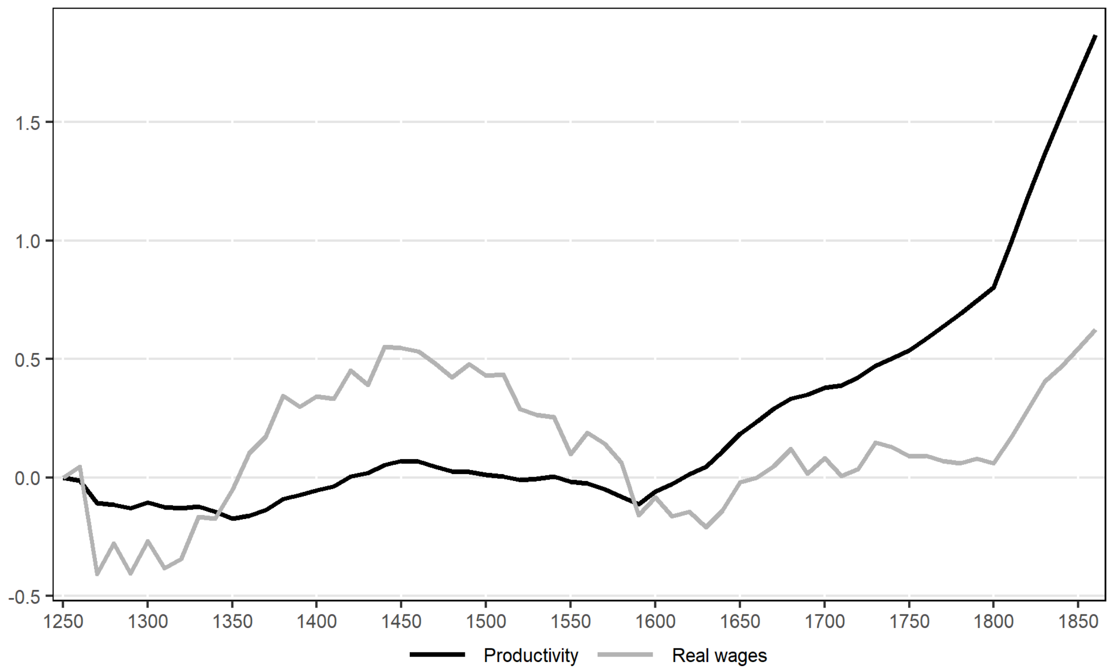

Figure 10: Comparison with Real Wages

Note: The figure plots our estimates of the evolution of the permanent component of productivity $\tilde{a}_t$ along with the real wage series we use.

4.1 From When to Why

By dating the onset of productivity growth, our results help discriminate between competing explanations for *why* growth began. First, the fact that we estimate sharp and sizable shifts in average growth from essentially zero to levels close to modern growth rates over a short period of time suggests the notion that "something changed." Galor and Weil (2000) show how the marriage of endogenous growth models and Malthusian models yields the conclusion that growth will transition from a low rate to modern growth rates. They argue that this transition can occur quite rapidly in such models. Our results indicate that this transition did in fact occur very rapidly.

Second, we estimate that sustained productivity growth began in England substantially before the Glorious Revolution of 1688. According to our estimates, productivity in England rose by 48% from 1600 to 1680. North and Weingast (1989) argue that the political regime that emerged in England after the Glorious Revolution—characterized by a power sharing arrangement between Parliament, the Crown, and the common law courts—resulted in secure property rights and rule of law and thereby laid the foundation for economic growth. While the institutional changes associated with the Glorious Revolution may well have been important for growth, our results contradict the view that these events preceded the onset of growth in England.

Our results support explanations of the onset of growth that focus attention on developments that occurred in the period surrounding 1600. The Reformation is an obvious candidate. In particular, Henry VIII's confiscation of monastic lands was a big shock to land ownership patterns and the land market in England. Also, London experienced an explosion of its population around this time—from 55,000 in 1520 to 475,000 in 1670 (Wrigley, 2010)—likely due to a rapid increase in international trade. English woolen exports expanded rapidly over this period (new draperies) as did intercontinental trade, colonization, and privateering. The British East India Company was founded in 1600 and the Virginia Company founded its first permanent settlement in North America in 1607.

Our finding that the onset of growth preceded both the Glorious Revolution and the English Civil War (1642-1651) lend support to the Marxist view that economic change propelled history forward and drove political and ideological change. Marx (1867) stressed the transition from feudalism to capitalism. He argued that after the disappearance of serfdom in the 14th century, English peasants were expelled from their land through the enclosure movement. That spoliation inaugurated a new mode of production: one where workers did not own the means of production, and could only subsist on wage labor. This proletariat was ripe for exploitation by a new class of capitalist farmers and industrialists. In that process, political revolutions were a decisive step in securing the rise of the bourgeoisie. To triumph, capitalism needed to break the remaining shackles of feudalism. As the *Communist Manifesto* puts it, "they had to be burst asunder; they were burst asunder" (Marx and Engels, 1848, pp. 40-41). Hill (1940, 1961) offers more recent treatments of the political revolutions in England in the 17th century that stress class conflict and their economic origins.

Acemoglu, Johnson, and Robinson (2005) synthesize the Marxist and institutionalist views. They argue that Atlantic trade enriched a merchant class that then demanded secure property rights and secured these rights through the Civil War and Glorious Revolution. This last narrative lines up well with our result that steady growth—perhaps driven by the Atlantic trade—began about half a century before the Civil War. However, we do not detect a radical increase of growth in the immediate aftermath of either the Civil War or Glorious Revolution: 4.4% (1600-1640), 5.7% (1640-1680) and 5.2% (1680-1810).

Allen (1992) argues that a long and gradual process of institutional change in England over the 600-year period from the Norman Conquest to the Glorious Revolution resulted in a situation in the 16th century where the yeoman class had acquired a substantial proprietary interest in the land, and thus an incentive to innovate. The timing of Allen's 'rise of the yeoman' lines up reasonably well with our estimate of the onset of growth. According to Allen, property rights, rule of law, and personal freedom gradually expanded and the social order was gradually transformed from a feudal to a capitalist order. From the 12th century, royal courts helped freeholders gain full ownership over their land. After the Black Death, serfdom collapsed as landlord competed for scarce labor. Early enclosures (15th and early 16th centuries) involved brutal evictions and depopulation of manors. The Crown reacted to this by increasing protection of tenant farmers.

The spread of movable-type printing across Europe after 1450 led to a large increase in literacy in England in the 16th and 17th centuries (Cressy, 1980, Houston, 1982), and a huge drop in the price of books (Clark and Levin, 2011). This likely had wide ranging effects on culture. Mokyr (2009, 2016) and McCloskey (2006, 2010, 2016) have argued that the crucial change that caused growth to begin was the emergence of a culture of progress based on the idea that mankind can improve its condition through science and rational thought. Others have stressed a Potestant ethic (Weber, 1905) and Puritanism (Tawney, 1926). The timing of these changes lines up reasonably well with our estimates although it is not straightforward to pinpoint precisely what these theories imply about the timing of the onset of growth.

Bogart and Richardson (2011) stress the importance of the post-Glorious Revolution regime's push to reorganize and rationalize property rights through enclosures, statutory authority acts, and estate acts. While our results contradict the notion that growth began with the Glorious Revolution, the fact that England underwent massive institutional change in the 17th and 18th century may have played an important role in laying the groundwork for the huge speedup of growth that we estimate occurred around 1810.

Allen (2009a) argues that the Industrial Revolution occurred in Britain around 1800 because innovation was uniquely profitable then and there. His theory relies on growth in the 17th century leading to high real wages in England in the 18th century as well as the development of a large coal industry. High wages and cheap coal made it profitable to invent labor saving technologies in textiles such as the spinning jenny, water frame, and mule, as well as coal burning technologies such as the steam engine and coke smelting furnace. Allen's theory lines up well with our results in that it provides an explanation for the second break in productivity growth that we estimate—the Industrial Revolution—and links it causally to the earlier break.

4.2 Incorporating Capital

Our baseline model abstracts from variation in physical capital. Since productivity is a catch-all residual, the fact that our baseline model does not incorporate capital implies that our baseline productivity estimates encompass capital accumulation. We can extend our model to explicitly allow for capital accumulation. Doing this allows us to estimate a measure of productivity that does not encompass capital accumulation.

The primary challenge associated with incorporating capital into our analysis is that—as far as we are aware—a time series measure of capital accumulation does not exist for England for the period prior to 1760. To overcome this challenge, we rely on data on rates of return on physical assets to infer the evolution of the stock of capital over time. Figure 11 plots data on rates of return on agricultural land and "rent charges" compiled by Clark (2002, 2010).[12] The rate of return on agricultural land is measured as R/P, where R is the rent and P is the price of a piece of land. As Clark (2010) explains, "rent charges" were perpetual nominal obligations secured by land or houses. Again, these are measured as R/P, where R is the annual payment and P is the price of the obligation. See Clark (2010) for more detail. We view each of these series as a noisy measure of the rate of return on capital in England over our sample period and use both in our analysis as described below.

We make capital accumulation explicit in our model by considering an economy in which output is produced using the production function:

$$Y_t = A_t Z^{\alpha} K_t^{\beta} L_t^{1-\alpha-\beta}, \tag{8}$$

where K_t denotes physical capital. As in our baseline analysis, our results for productivity accounting for capital accumulation are robust to assuming any production function with decreasing marginal returns to capital and labor (appendix A).

As before, we assume that factor markets are competitive. This implies that producers accumulate capital to the point where the marginal product of capital is equal to its user cost:

$$r_t + \delta = \beta A_t Z^{\alpha} K_t^{\beta-1} L_t^{1-\alpha-\beta}, \tag{9}$$

where r_t is the rental rate for capital and δ is the rate of depreciation of capital. Similarly, producers

[12] Clark's series on "rent charges" should not be confused with his series on rents. These are different series. "Rent charges" is a rate of return on an asset (i.e., measured in percent), while rents are a nominal series (i.e., pounds sterling per year).

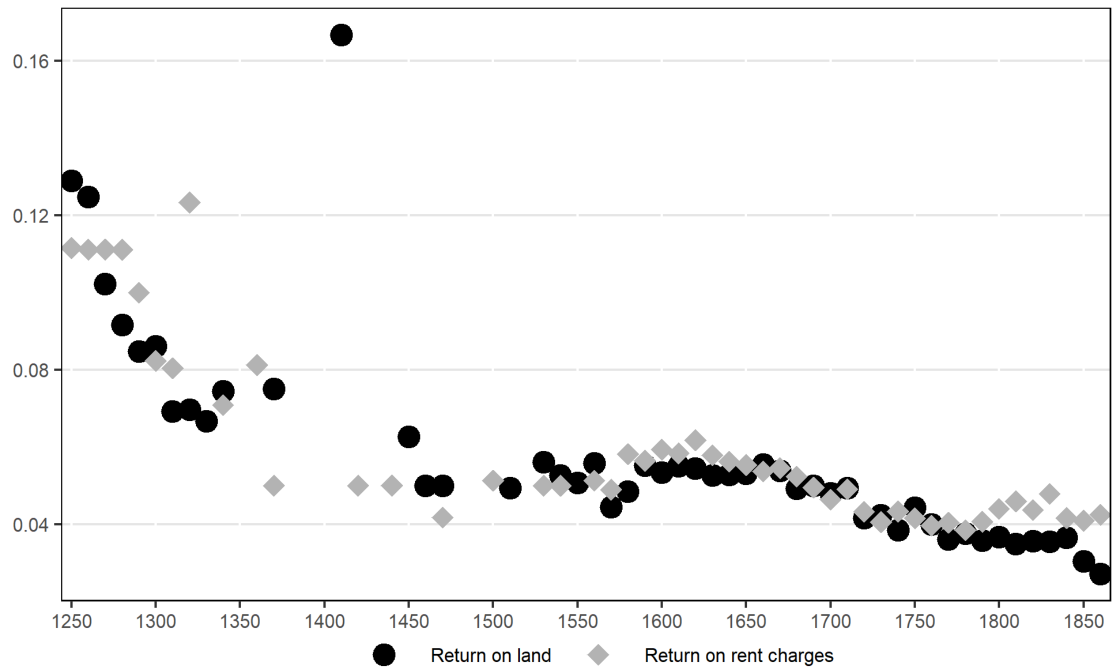

Figure 11: Rates of Return on Land and Rent Charges

Note: The figure plots the data we use on rates of return on land and rent charges. These data are from Clark (2002, 2010).

hire workers until the marginal product of labor is equal to the wage:

$$W_t = (1 - \alpha - \beta) A_t Z^{\alpha} K_t^{\beta} L_t^{-\alpha-\beta}. \tag{10}$$

Combining these two optimality conditions so as to eliminate K_t and taking logs of the resulting equation yields

$$w_t = \phi'' + \frac{1}{1-\beta} a_t - \frac{\alpha}{1-\beta} l_t - \frac{\beta}{1-\beta} \log\left(r_t + \delta\right). \tag{11}$$

We can now replace the labor demand curve in our baseline model—equation (1)—with this labor demand curve and proceed as before. As we mention above, we assume that each of Clark's series on rates of return represent a noisy measure of the true rate of return on capital in England. In other words, we assume that

$$r_t = \tilde{r}_{it} + \iota_{it}^r,$$

where r_t denotes the true rate of return on capital at time t, $\tilde{r}_{it}$ denotes noisy measure i (either land or rent charges), and $\iota_{it}^r \sim t_{\nu_{ir}}(0, \tilde{\sigma}_{ir}^2)$ denotes the measurement error. In periods when neither measure is available, we assume that the interest rate follows a random walk with truncated normal innovations: $r_t \sim \mathcal{N}_{(0,.2)}(r_{t-1}, 0.01^2)$. We assume that $\nu_{ir}^{-1} \sim U(0,1)$, $\tilde{\sigma}_{ir}^2 \sim I\Gamma(3, 0.005)$,

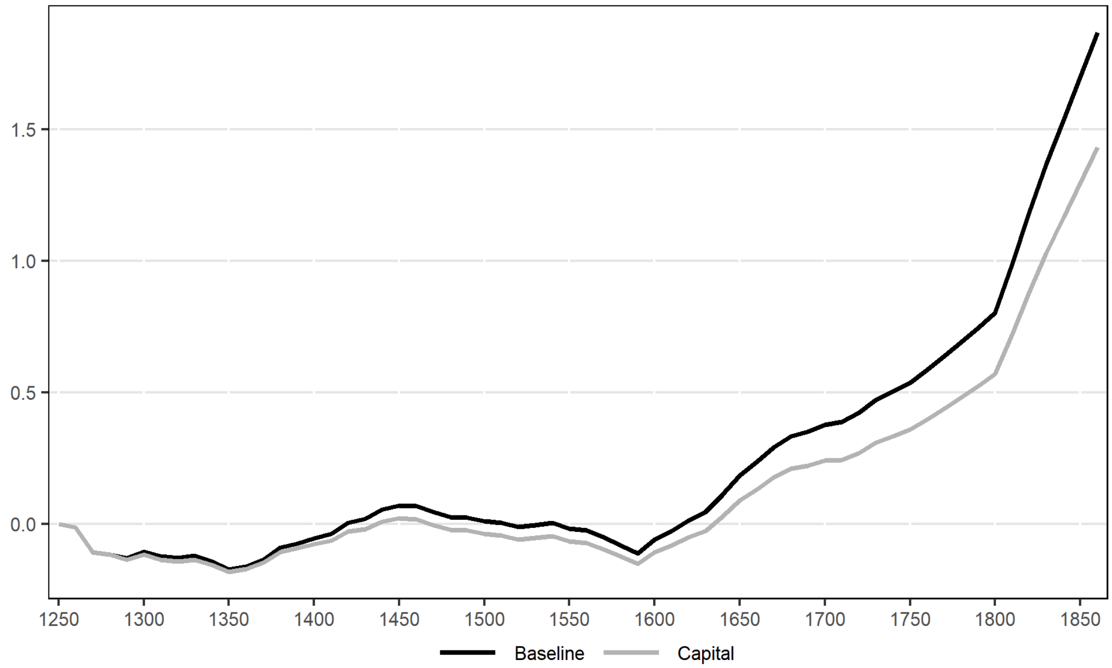

Figure 12: Estimate of Total Factor Productivity Incorporating Capital

Note: The figure plots our estimates of the evolution of the permanent component of productivity $\tilde{a}_t$ for our model with capital and our baseline model.

and $\delta \sim \mathcal{N}_{(0,0.2)}(0.1, 0.05^2)$.

The main results from this analysis are presented in Figure 12 and Table 3. Overall, we reach very similar conclusions to those in our baseline analysis. The main difference is that the average growth rate in productivity μ is estimated to be somewhat smaller. During the transition period from 1600 to 1810, we estimate average growth in productivity of 3% per decade, as opposed to 4% in our baseline model. After 1810, we estimate average growth in productivity of 14% per decade, as opposed to 18% in our baseline model. This is consistent with the notion that part of the growth in labor productivity after 1600 was due to the accumulation of physical capital. The modest difference between these two sets of results, however, suggests that the vast majority of growth from 1600 to 1870 was due to other developments than capital accumulation. It may seem surprising that accounting for capital accumulation does not make a bigger difference for productivity. One reason for this is that the role of capital in pre-industrial England was rather small. Our estimate of β—the exponent on capital in the production function—is only 0.18.

Our methods allow us to back out not only an estimate of the evolution of productivity but also an estimate of the evolution of the capital stock in England over our entire sample period. Figure 13 plots our estimate of the capital stock. We find that the level of the capital stock was similar in 1600 to what is had been in the late 13th century. But as productivity began increasing,

Table 3: Parameter Estimates with Capital

	Mean	St Dev	2.5%	97.5%
$\mu_{t<1600}$	-0.00	0.01	-0.02	0.01
$\mu_{1600\leq t<1800}$	0.03	0.01	0.01	0.06
$\mu_{t\geq 1800}$	0.14	0.03	0.08	0.20
α	0.42	0.10	0.22	0.62
β	0.17	0.11	0.01	0.42
$\alpha/(1-\beta)$	0.50	0.09	0.32	0.67

the capital stock also began increasing. From 1600 to 1860, the capital stock in England grew by a factor of five, or 8% per decade. Figure 13 also plots estimates of the gross and net capital stock from Feinstein (1998) from 1760 onward. Our estimates imply a somewhat more rapid rate of growth of capital than Feinstein's estimates over the period 1760-1860 (22% per decade versus 16% per decade).[13]

4.3 Further Robustness

Days Worked: Our baseline analysis uses Humpries and Weisdorf's (2019) estimates of the evolution of days worked per worker. These estimates imply a substantial Industrious Revolution. Since this conclusion is controversial, Figure A.3 presents an alternative set of estimates for productivity where we instead assume that days worked per worker were constant throughout our sample period. This yields a very similar time series pattern for productivity both qualitatively and quantitatively. Perhaps surprisingly, therefore, our conclusions about productivity are insensitive to whether England experienced an Industrious Revolution. Rather than changing our conclusions about productivity, assuming constant days worked changes the estimated slope of the labor demand curve. With constant days worked, the labor demand curve is estimated to be steeper than when days work follow the path estimated by Humphries and Weisdorf (2019).

Real Wage Data: Our baseline analysis uses real wage data for unskilled builders from Clark (2010). Figure A.4 presents five alternative estimated productivity series where we instead use other wage series. First, we present estimates of productivity using the following day wage series: 1) Clark's (2010) real wages series for farm laborers, 2) Clark's (2010) real wages series for building craftsmen, 3) Allen's (2007) real wage series for the period 1770 onward (with our baseline wage series before that time). We also present estimates of productivity based on the assumption that

[13]Unlike our results on productivity, our results on the evolution of the capital stock rely on the Cobb-Douglas assumption. See appendix A.2.

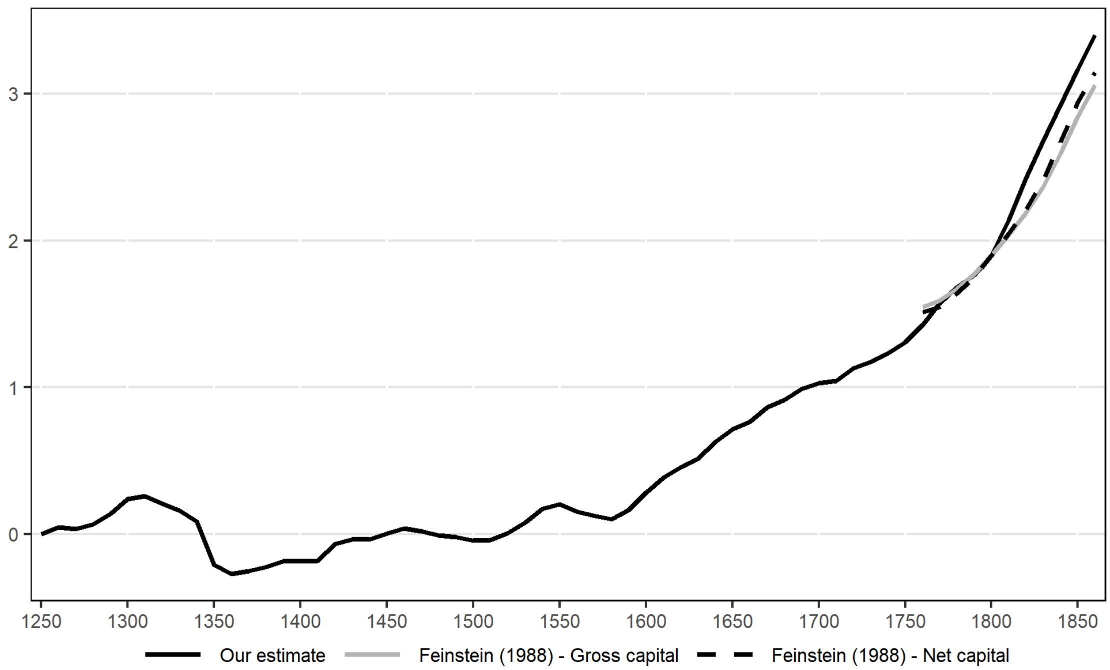

Figure 13: Estimate of the Capital Stock in England

Note: The figure plots our estimates of the evolution of the logarithm of the capital stock in England. The series is normalized to zero in 1250. We also plot Feldstein's (1998) estimates of the gross and net capital stock from 1760 onward. These series are normalized to be equal to our series in 1800.

the builders, farmers, and craftsmen series from Clark (2010) are all noisy signals of the underlying true wage. Finally, we use Humphries and Weisdorf's (2019) annual wage series along with the assumption that days worked are constant. All five of these alternative productivity series are quite similar to our baseline productivity series, although they differ somewhat for the earliest part of the sample.

Population Data: Our baseline analysis uses population data prior to 1540 from Clark (2010). Figure A.5 presents estimates of productivity using population data from Broadberry et al. (2015) for the period prior to 1540. Broadberry et al.'s (2015) estimates of the population are infrequent and irregular in their frequency. There are quite a few decades for which Broadberry et al. (2015) have no estimate, e.g., they present no estimate between 1450 and 1522. In this robustness analysis, we view the population as an unobserved variable in decades for which we do not have an estimate from Broadberry et al. (2015). Our results on the evolution of productivity for this case are very similar to our baseline case.

Priors: Figure A.6 presents estimates of productivity using different prior distributions than we use in our baseline analysis. First, we present results for a case where we change the prior on σ_{ϵ_1}—the variance of permanent productivity shocks—to be $\mathrm{I}\Gamma(3, 0.005)$, i.e., the same as the prior on the other productivity and population shocks. Second, we present results for a case where we change the prior on ψ—the level of the population prior to 1540—to be $\mathcal{N}(10.86, 10^2)$, i.e., much wider than in our baseline analysis. In both cases, the resulting productivity series are very similar to our baseline results. Other priors are quite dispersed. We find it unlikely that our results are sensitive to making any of these priors even more dispersed.

5 The Strength of the Malthusian Population Force

In a Malthusian world, real wages return to steady state after a shock has led them to either rise or fall. Consider, for example, a plague. Wages initially rise after a plague. But the higher wages lead the population to grow. As the population grows, wages fall back to steady state. We refer to these population dynamics and their effects on real wages as the Malthusian population force. In this terminology, it is the Malthusian population force that brings real wages back to steady state in a Malthusian world.

The strength or speed of the Malthusian population force is governed by two parameters in our model: 1) the elasticity of population growth with respect to per capita income—which we denote γ—and 2) the slope of the labor demand curve—which we denote α. To see this, we can use equation (2) to substitute for the real wage in equation (3). This yields the following equation for the dynamics of the population in our Malthusian model as a function of exogenous variables:

$$n_{t+1} = (1 - \alpha\gamma)n_t + \omega + \gamma(\phi + (1 - \alpha)d_t + \tilde{a}_t) + \xi_t + \gamma\epsilon_{2t}. \tag{12}$$

This equation implies that the speed of population recovery after a plague-induced decrease is governed by $1 - \alpha\gamma$. In particular, the half-life of the population dynamics, i.e., the time it takes the population to recover half of the way back to steady state after a plague-induced drop, is $\log 0.5/\log(1 - \alpha\gamma)$. The half-life of real wage dynamics is the same as that of the population.

Table 4 presents our estimates of α and γ (as well as all other parameters not presented in Table 2). Our estimate of α is 0.53. If we assume a Cobb-Douglas production function, $1 - \alpha$ is the labor share of output. As we discuss in appendix A.1, the interpretation of α is more complicated if the true production function is a CES function with an elasticity of substitution between labor and land that differs from one. In that case, α is equal to one minus the labor share divided by the elasticity of substitution between labor and land.

Our estimate of γ is 0.09. This relatively small estimate for γ implies that the strength of the

Table 4: Parameter Estimates

	Mean	St Dev	2.5%	97.5%
Main Parameters				
α	0.53	0.09	0.34	0.69
γ	0.09	0.02	0.05	0.14
ω	-0.03	0.02	-0.08	0.01
Population Parameters				
$\pi_{t<1680}$	0.14	0.11	0.01	0.42
$\pi_{t\geq 1680}$	0.11	0.10	0.00	0.38
μ_{ξ_1}	0.80	0.10	0.55	0.90
ν_{ξ_1}	3.38	12.94	1.02	14.05
σ_{ξ_2}	0.06	0.01	0.04	0.07
Population Measurement Error Parameters				
$\sigma_{n,t<1540}$	0.04	0.01	0.03	0.06
$\sigma_{n,t\geq 1540}$	0.03	0.00	0.02	0.04
$\nu_{n,t<1540}$	16.27	592.24	1.10	36.39
$\nu_{n,t\geq 1540}$	123.97	7825.18	2.17	284.51
Days Worked Parameters				
σ_h	0.08	0.01	0.06	0.10
$\tilde{\sigma}_h$	0.04	0.01	0.03	0.07
ν_h	6.08	271.63	1.21	14.63

Note: The table presents the mean, standard deviation, 2.5% quantile, and 97.5% quantile of the posterior distribution we estimate for the slope of labor demand α, the elasticity of population growth to income γ, the subsistence wage parameter ω, the probability of a plague shock π, the mean of the plague shock μ_{ξ_1}, the pseudo sample size of the plague shocks ν_{ξ_1}, the standard deviation of the normal population shock σ_{ξ_2}, the scale and degrees of freedom parameters of the population measurement error shocks, σ_n and ν_n, respectively, the standard deviation of inferred changes in days worked σ_h, and the scale and degrees of freedom parameters of the days worked measurement error shocks, $\tilde{\sigma}_h$ and ν_h, respectively.

Malthusian population force was rather weak in England over our sample period. Using this estimate of γ and our estimate of α, we get that the half-life of population and real wage dynamics after a shock was roughly 150 years.

Another way to gauge the quantitative magnitude of our estimate of γ is to calculate how much the changes in real per capita income in England over our sample prior to the 17th century affected population growth. Between 1270 and 1440, real per capita income in England rose by 70%. Our estimate of γ implies that this increase in per capita income stimulated population growth by a mere 5 percentage points per decade. A doubling of real per capita income would have stimulated population growth by only slightly more, 6 percentage points per decade.

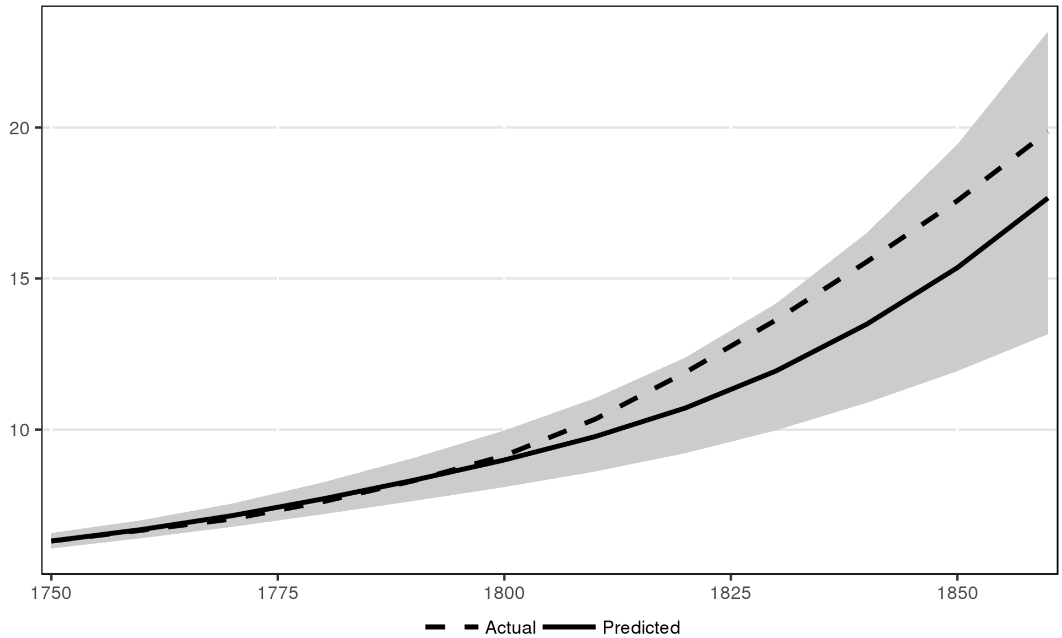

Figure 14: Actual and Predicted Population Dynamics after 1750

Note: The dashed line is the evolution of the population in England. The solid line is the predicted evolution of the population in England from our Malthusian model. In calculating this line, we take the evolution of real wages and days worked in England as given and simulate the evolution of the population using equation (3) starting from its actual value in 1740. The gray shaded area is the 90% central predictive interval given our estimates of α and γ.

5.1 Post-1750 Population Explosion

The modest strength of the Malthusian population force in our model begs the question whether our model can explain, with these parameter values, the large increase in the population of England that occurred after 1750 (see Figure 6). In 1740, the population of England was 6 million. By 1860, it had risen to almost 20 million. The population therefore grew at a compound rate of 10.4% per decade over this 120-year period.

Figure 14 compares the evolution of the population in England from 1750 to 1860 with the predicted evolution of the population from our model. We construct the predicted evolution by taking the evolution of real wages and days worked in England as given and simulating the evolution of the population using equation (3) starting from its actual value in 1740 and assuming no population shocks. This analysis shows that in fact our model can explain the vast majority of the rapid increase in the population between 1740 and 1860. This may seem surprising given the weak Malthusian population force and the somewhat modest increase in real wages over this period. However, per capita income in England over this period rose much more than real wages did because days worked increased quite substantially (see Figure 7).

5.2 Overwhelming the Malthusian Population Force

A simplistic view of Malthusian economics holds that wages are always stuck at subsistence in a Malthusian world. This is not the case. One reason for this is that steady productivity growth can result in a persistent force driving wages higher. As wages rise, the Malthusian population force put greater and greater downward pressure on wages and eventually chokes off further increases in wages. This means that for each level of average productivity growth there is a steady state real wage. But the steady state real wage is not necessarily at subsistence. Rather the steady state real wage is increasing in average productivity growth.

As we saw above, we estimate that the Malthusian population force was quite weak in England over our sample period. The average productivity growth we estimate for the latter part of our sample period may therefore have been large enough to raise per capita income quite substantially even in the presence of the Malthusian population force. We can use our estimated model to quantify the extent to which this is true. In appendix C, we show that the steady state level of wages in our Malthusian model is given by

$$\bar{w} = \frac{\mu}{\alpha\gamma} - \bar{d} - \frac{\omega}{\gamma} - \frac{\bar{\xi}_1}{\gamma},$$

where $\bar{\xi}_1$ is the average level of our plague shock ξ_{1t}. Notice that steady state wages are increasing in the speed of productivity growth μ. Faster productivity growth results in a stronger force pushing wages up and therefore a higher level of wages at which this force is exactly counteracted by the negative Malthusian population force.

Figures 15 and 16 present impulse responses to a change in productivity growth that show quantitatively how much changes in productivity growth increase wages over time according to our model. For each impulse response, we start the economy off in a steady state with zero productivity growth ($\mu = 0$). At time zero in the figures, productivity growth increases. In Figure 15, we show the evolution of the growth rate of wages (log change) over the subsequent 500 years. In Figure 16, we show the evolution of the level of wages relative to its earlier steady state level over the subsequent 1000 years. In both figures, we assume that all other shocks are constant at their mean values.

In Figure 15, we see that the growth rate of wages is initially equal to the change in productivity. As wages rise and the Malthusian population force kicks in, the growth rate of wages falls. This process takes a very long time due to the weakness of the Malthusian population force. The

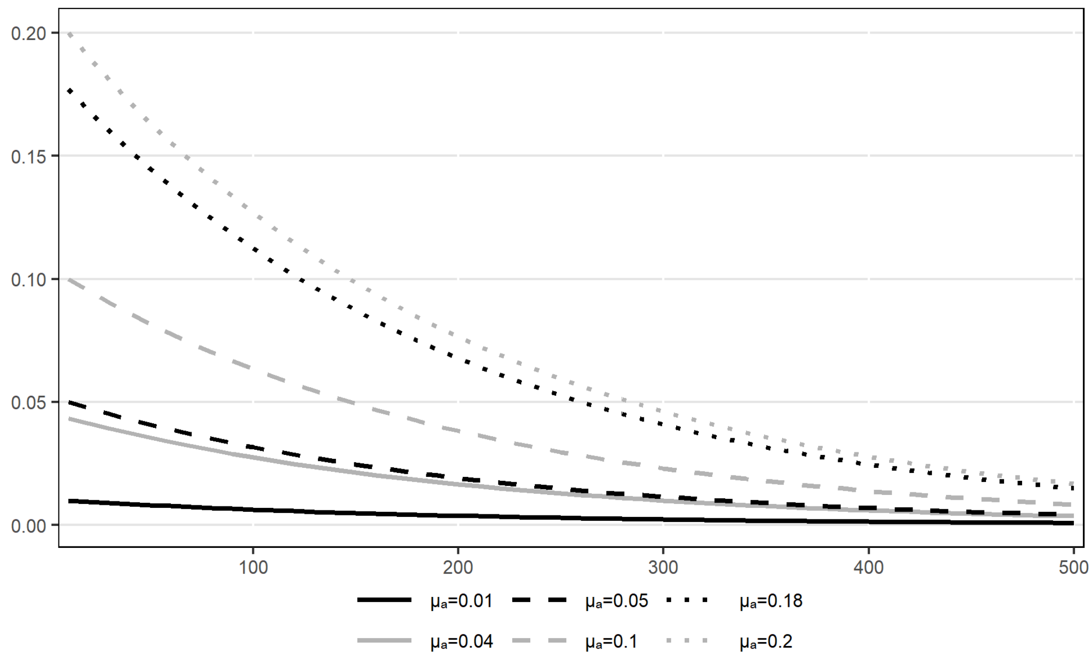

Figure 15: Real Wage Growth After an Increase in Productivity Growth

Note: Each line plots the growth rate of real wages over time after an increase in productivity growth from $\mu = 0$ to a higher value. These impulse responses are calculated assuming that all model parameters are at their posterior mean values.

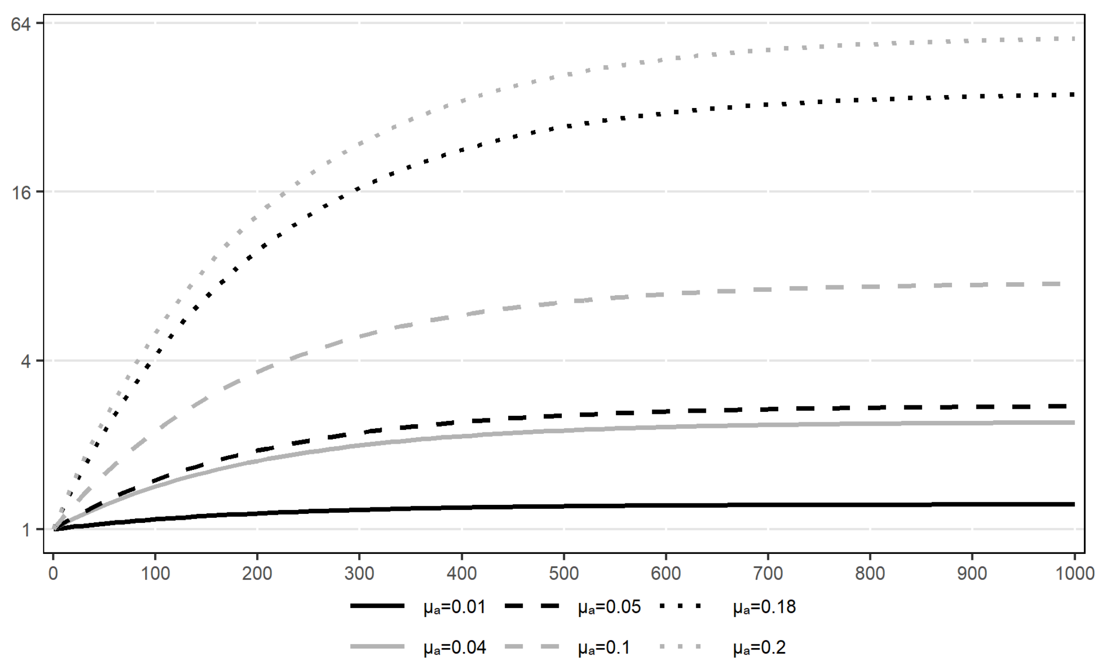

Figure 16: Evolution of Real Wages After an Increase in Productivity Growth

Note: Each line plots the evolution of real wages over time after an increase in productivity growth from $\mu = 0$ to a higher value. These impulse responses are calculated assuming that all model parameters are at their posterior mean values.

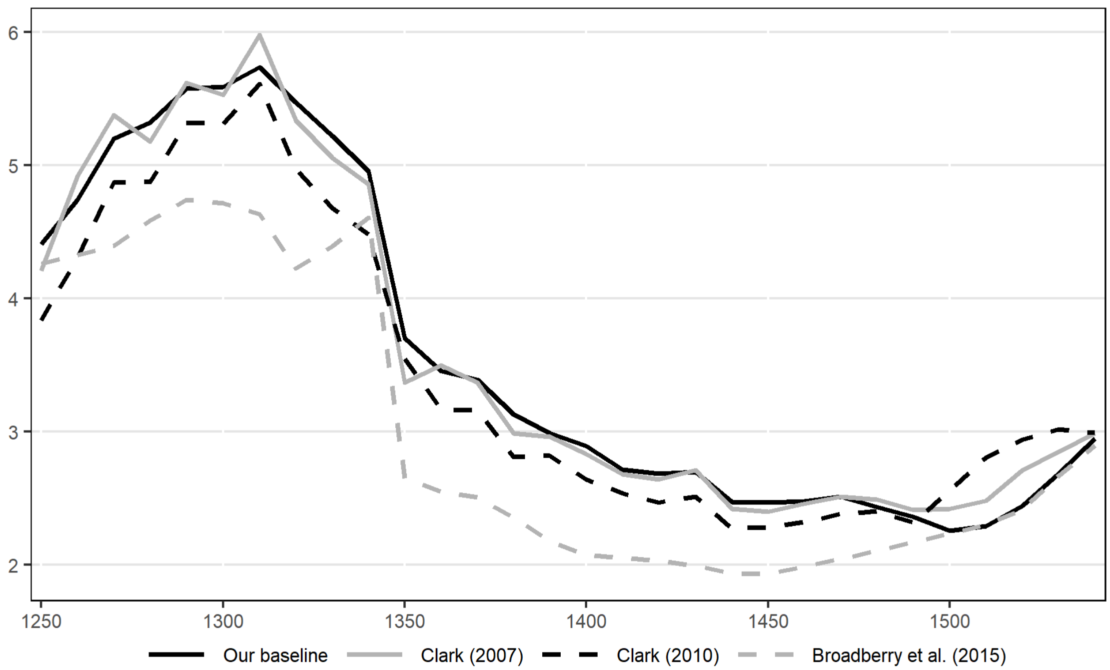

Figure 17: Comparison of Population Estimates for England

Note: The figure plots our estimates of the evolution of the population of England along with estimates from Clark (2007a), Clark (2010), and Broadberry et al. (2015)

half-life of wage growth is roughly 150 years.

The fact that wage growth continues for hundreds of years after a change in productivity implies that the cumulative increase in wages is substantial. In Figure 16, we can read off the long-run effect of higher productivity growth on wages. For a productivity growth rate of $\mu = 0.18$, which is what we estimate for the period after 1810, we find that the long-run effect on the level of wages is an increase of a factor of 28. In other words, the growth in productivity that we estimate for the period after 1810 would have eventually led to a 28-fold increase in real wages even if the Demographic Transition had not occurred and the Malthusian population force had continued.

6 Plagues and the Population

Figure 17 plots our estimate of the evolution of the population of England from 1250 to 1550 along with prior estimates from Clark (2007a), Clark (2010), and Broadberry et al. (2015). Our estimates are very similar to Clark's. This implies that our estimation procedure largely validates the assumptions Clark makes regarding the evolution of productivity in constructing his population estimates. The estimates of Broadberry et al. (2015) are substantially lower early in the sample period, but then gradually converge.

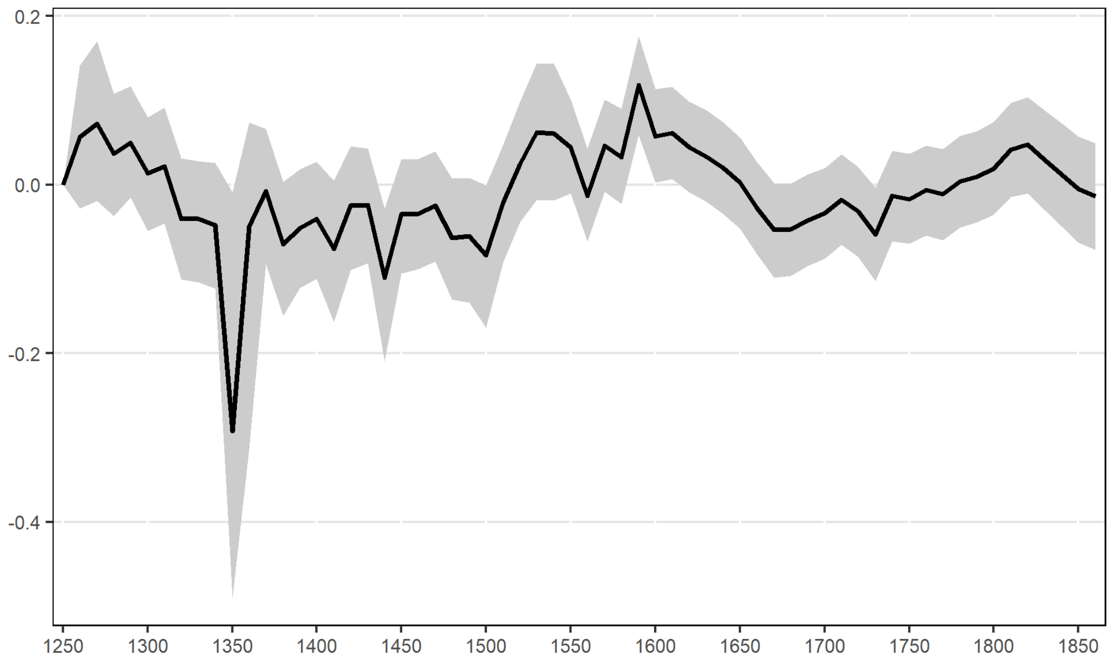

Figure 18: Population Shocks

Note: The figure plots our estimates of the population shocks hitting the English economy over our sample period, i.e., $\xi_{1t} + \xi_{2t}$. The black line is the mean of the posterior for each period and the gray shaded area is the 90% central posterior interval.

The evolution of the population in England over our sample period is heavily affected by plagues. Our model captures plagues (and other influences on the population other than changes in real income) through the shocks ξ_{1t} and ξ_{2t}. Figure 18 plots the evolution of the sum of these population shocks over our sample period. The largest population shock by far is the Black Death of 1348. We estimate that the population shocks associated with the Black Death lead the population of England to shrink by 25%. But Figure 18 also makes clear that England faced steady population headwinds—i.e., persistent negative population shocks—from the early 14th century until about 1500.

7 Conclusion

In this paper, we use a Malthusian model to estimate the evolution of productivity in England from 1250 to 1870. Our principal finding is that productivity growth began in 1600. Before 1600, productivity growth was zero. We estimate a growth rate of productivity of 4% per decade between 1600 and 1810. In 1810, the growth rate of productivity increases sharply to 18% per decade. These results indicate that sustained growth in productivity began well before the Glorious Revolution. They point in particular to the early 17th century as a crucial turning point for productivity growth in England, a result that helps distinguish between competing lines of thought regarding the ultimate causes of the emergence of growth.

We also use our model to estimate the strength of the Malthusian population force on wages in England prior to the Demographic Transition, i.e., the pressure that increases in population put on wages whenever they rose. We find that this force was relatively weak. A doubling of real income led to only about a 6 percentage point increase in population growth per decade. This implies that the half-life of the response of real wages after a plague induced decrease in the population was about 150 years and the long-run increase in wages associated with an increase in productivity growth was quite substantial.

A More General Production Function

Consider the concave production function

$$Y_t = A_t F(Z, L_t, K_t) \tag{13}$$

The first-order conditions are

$$\begin{aligned} W_t &= A_t F_L(Z, L_t, K_t) \\ r_t + \delta &= A_t F_K(Z, L_t, K_t) \end{aligned}$$

where δ is the depreciation rate of capital.

Taking logs in the FOC

$$w_t = a_t + \log\left(F_L(Z, L_t, K_t)\right) \approx \tilde{\phi}' + a_t + \frac{LF_{LL}}{F_L} l_t + \frac{KF_{LK}}{F_L} k_t \tag{14}$$

$$\log(r_t + \delta) = a_t + \log\left(F_K(Z, L_t, K_t)\right) \approx \tilde{\phi}'' + a_t + \frac{LF_{LK}}{F_K} l_t + \frac{KF_{KK}}{F_K} k_t \tag{15}$$

Solving for k_t in equation (15)

$$k_t = \tilde{\phi}''' + \frac{F_K}{KF_{KK}} \left(\log(r_t + \delta) - a_t\right) - \frac{LF_{LK}}{KF_{KK}} l_t \tag{16}$$

Substituting into equation (14)

$$w_t \approx \tilde{\phi} + \left(1 - \frac{F_K F_{LK}}{F_L F_{KK}}\right) a_t + \frac{L}{F_L F_{KK}} \left(F_{LL} F_{KK} - F_{LK}^2\right) l_t + \frac{F_K F_{LK}}{F_L F_{KK}} \log(r_t + \delta)$$

Which can be rewritten

$$w_t \approx \tilde{\phi} + \left(1 + \tilde{\beta}\right) a_t - \tilde{\alpha} l_t - \tilde{\beta} \log(r_t + \delta) \tag{17}$$

where

$$\begin{aligned} \tilde{\alpha} &= -\frac{L}{F_L F_{KK}} \left(F_{LL} F_{KK} - F_{LK}^2\right) \\ \tilde{\beta} &= -\frac{F_K F_{LK}}{F_L F_{KK}} \end{aligned}$$

Equation (17) shows that a_t is identified up to a first-order approximation. This result does not require a Cobb-Douglas production function, not even constant returns to scale.

A.1 CES Case

Consider the production function

$$Y_t = A_t \left[\alpha'^{\frac{1}{\sigma}} Z^{\frac{\sigma-1}{\sigma}} + (1-\alpha')^{\frac{1}{\sigma}} (L_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where σ denotes the elasticity of substitution between land and labor in production. Optimal choice of labor by land owners gives rise to the following labor demand curve

$$W_t = (1-\alpha')^{\frac{1}{\sigma}} A_t \left[\alpha'^{\frac{1}{\sigma}} \left(\frac{Z}{L_t} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha')^{\frac{1}{\sigma}} \right]^{\frac{1}{\sigma-1}}.$$

A log-linear approximation of this equation yields

$$w_t = \phi - \alpha l_t + a_t,$$

where

$$\alpha = \left[\sigma \left(1 + \left(\frac{1-\alpha'}{\alpha'} \right)^{\frac{1}{\sigma}} \left(\frac{L}{Z} \right)^{\frac{\sigma-1}{\sigma}} \right) \right]^{-1}$$

and L is the level of labor we linearize around. Notice that $\alpha \to \alpha'$ when $\sigma \to 1$.

It is furthermore easy to show that with the CES production function given above, the labor share of output is

$$\bar{LS} = 1 - \left[1 + \left(\frac{1-\alpha'}{\alpha'} \right)^{\frac{1}{\sigma}} \left(\frac{L}{Z} \right)^{\frac{\sigma-1}{\sigma}} \right]^{-1}.$$

Combining these last two equations, we get that

$$\alpha = \frac{1-\bar{LS}}{\sigma}.$$

A.2 Capital Stock

With a Cobb-Douglas production function, equations (16) and (17) are true without approximation, and simplify to

$$w_t = \phi + a_t + \beta k_t - (\alpha + \beta) l_t \tag{18}$$

$$k_t = \phi' + \frac{1}{1-\beta}\left(a_t - \log(r_t + \delta)\right) + \frac{1-\alpha-\beta}{1-\beta} l_t \tag{19}$$

where

$$\phi = \frac{\alpha}{1-\beta}\log(Z) + \log\beta + \log(1-\alpha-\beta)$$

$$\phi' = \phi + \log\left(\frac{\beta}{1-\alpha-\beta}\right)$$

Substituting equation (19) into equation (18), we obtain equation (11). Having estimated the latter, we can recover k_t through equation (19).

B Clark's Population Series

As we discuss in the main text, Clark (2007b) uses unbalanced panel data on the population of villages and manors from manorial records and penny tithing payments to construct estimates of the population prior to 1540. Clark starts by running a regression of this data on time fixed effects and manor/village fixed effects. He refers to the time fixed effects from this regression as as a population trend series.

Clark's population trend series does not provide information on the overall level of the population prior to 1540, only changes in the population (i.e., a normalization is needed). In addition, Clark's microdata is sufficiently unreliable for the 1530s that Clark does not make use of his estimated population trend for that decade. Clark uses the following procedure to surmount these problems. First, he regresses his population trend on real wages from 1250 to 1520, and separately regresses the Wrigley et al. (1997) population series on wages from 1540 to 1610. He observers that the R^2 in both regressions are high and that they yield similar slope coefficients. He concludes from this that (i) the English economy moved along stable labor demand curves during both subsamples and (ii) these two labor demand curves had similar slopes.

Clark next makes the assumption that there was no productivity growth between 1520 and

1540—the labor demand curve did not shift during this time. This allows him to extrapolate the relationship that he finds in the post-1540 data to the earlier sample, and infer both the population in 1530 and the missing normalization from the level of real wages. Clark also uses the fitted values for the population from his labor demand curve as an alternative estimate of the population and averages this with the trend series to get what he calls the "best" estimate of population before 1540.

C Dynamics After Change in Productivity Growth

Our Malthusian model implies that an increase in productivity growth will result in higher steady state wages. To see this, we first abstract for notational simplicity from all the shocks in our model. More precisely, we set the value of all shocks equal to their mean. The mean value of ϵ_{1t}, ϵ_{2t}, and ξ_{2t} is zero. The mean value of ξ_{1t}, however, is $\bar{\xi}_1 = \pi(\psi\beta_1) - \psi(\beta_1 + \beta_2))$, where $\psi(\cdot)$ is the digamma function. We furthermore, assume that days worked are constant at $\bar{d}$.

Given these assumptions, our model simplifies to:

$$w_t = \phi + \tilde{a}_t - \alpha(n_t + \bar{d}). \tag{20}$$

$$n_t - n_{t-1} = \omega + \gamma(w_{t-1} + \bar{d}) + \bar{\xi}_1. \tag{21}$$

$$\tilde{a}_t = \mu + \tilde{a}_{t-1}. \tag{22}$$

We can use equation (20) to eliminate w_t in equation (21). This yields:

$$n_t - n_{t-1} = \omega + \gamma\phi + \gamma\tilde{a}_{t-1} - \alpha\gamma n_{t-1} + \gamma(1-\alpha)\bar{d} + \bar{\xi}_1.$$

Next, we subtract α times this last equation from equation (22) and rearrange. This yields:

$$\tilde{a}_t - \alpha n_t = \mu - \kappa + (1 - \alpha\gamma)(\tilde{a}_{t-1} - \alpha n_{t-1})$$

where $\kappa = \alpha(\omega + \gamma\phi + \gamma(1-\alpha)\bar{d} + \bar{\xi}_1)$. This shows that $\tilde{a}_t - \alpha n_t$ follows an $AR(1)$ and therefore settles down to a steady state in the long run as long as $|1 - \alpha\gamma| < 1$. The steady state value of $\tilde{a}_t - \alpha n_t$ is $(\mu - \kappa)/(\alpha\gamma)$ and (using equation (20)) the steady state real wage is

$$\bar{w} = \frac{\mu}{\alpha\gamma} - \bar{d} - \frac{\omega}{\gamma} - \frac{\bar{\xi}_1}{\gamma}.$$

We see from this that the steady state real wage in our Malthusian economy is increasing in the productivity growth rate μ and the extent to which this is the case is influenced by the strength of the Malthusian population force as summarized by $\alpha\gamma$.

Table A.1: Bayes Factor for Different Productivity Break Dates

	1590	1600	1610	1620	1630
1790	0.04	0.07	0.07	0.07	0.05
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
1800	0.24	0.42	0.28	0.26	0.24
	(0.01)	(0.02)	(0.02)	(0.01)	(0.01)
1810	0.15	1.00	0.42	0.29	0.31
	(0.01)	—	(0.02)	(0.01)	(0.01)
1820	0.02	0.06	0.06	0.06	0.03
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: The table presents the Bayes factor for models with different break dates for average productivity growth μ when compared to the model with breaks occurring in 1600 and 1810. The numbers reported in parentheses represent computational uncertainty. They are the standard deviation of the Bayes factor from 1000 draws of our bridge sampling procedure.

Figure A.1: A Comparison of Real Wage Measures in England, 1250-1860

Note: The figure presents four estimates of the real wages in England. Three are from Clark (2010): builders, farmers, and craftsmen. The remaining series is from Allen (2007). The builders series is the series we use in our main analysis. The builders series is normalized to 100 in 1860. The levels of the farmers and craftsmen series indicate differences in real earnings relative to builders. The Allen (2007) series is normalized to equal the builders series in 1770.

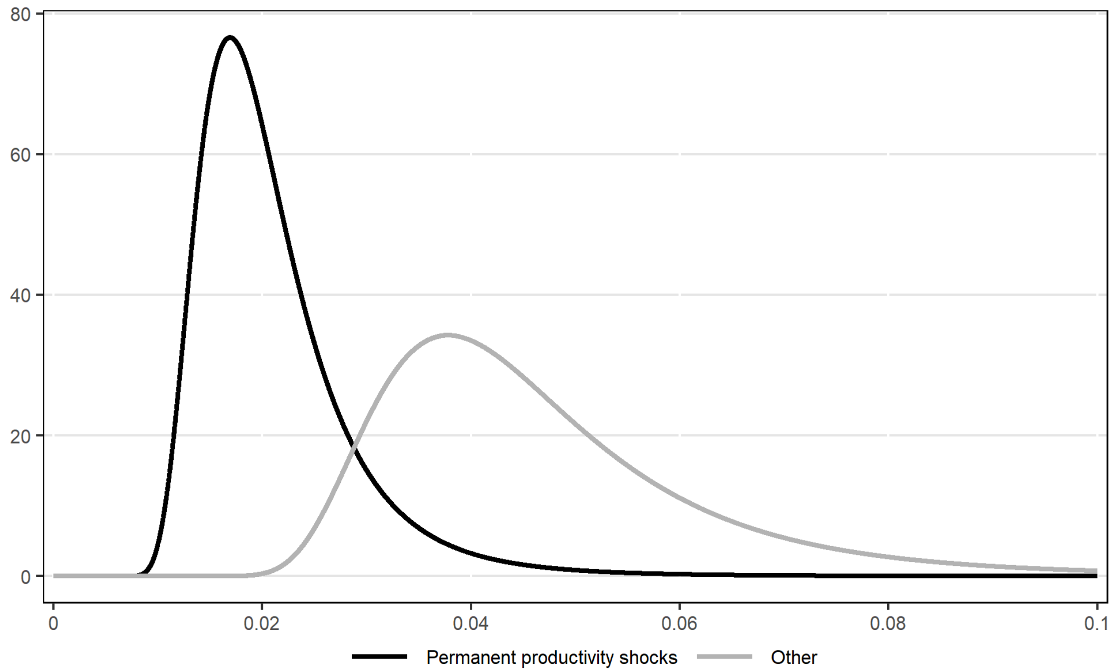

Figure A.2: Prior Densities for Standard Deviations

Figure A.3: Productivity with Constant Days Worked

Note: The figure compares our baseline estimates of the evolution of the permanent component of productivity $\tilde{a}_t$ with alternative estimates where we assume that days worked per workers were constant over our sample period.

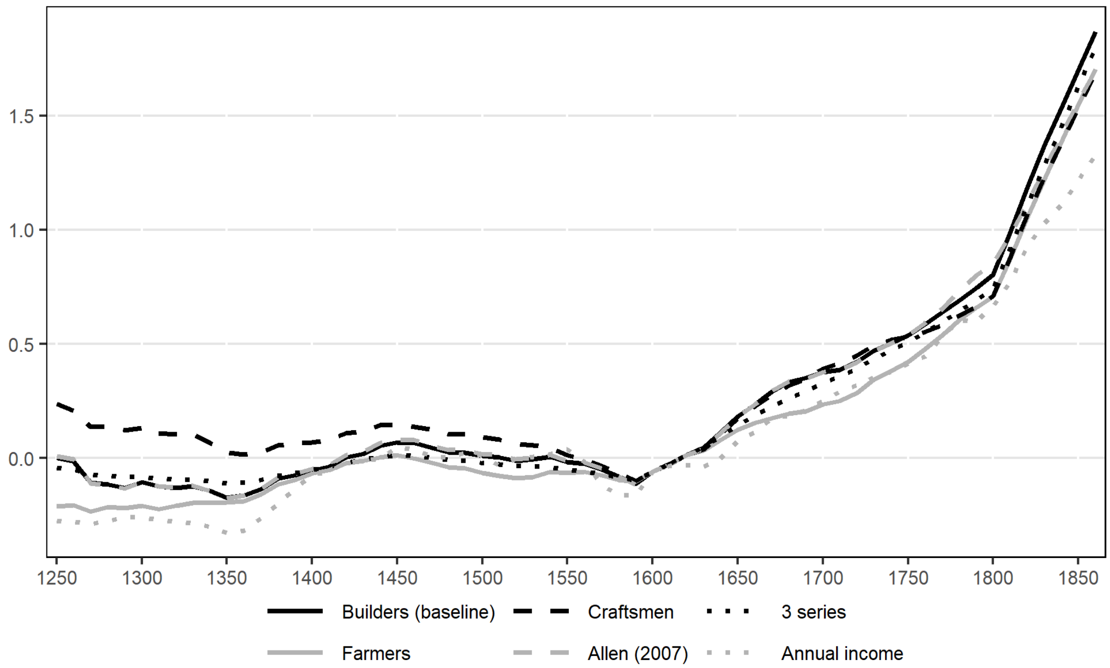

Figure A.4: Productivity using Alternative Wage Series

Note: The figure compares our baseline estimates of the evolution of the permanent component of productivity $\tilde{a}_t$ with estimates using different wage series. The "Farmers" series is the farm worker series from Clark (2010), the "Craftsmen" series is the building craftsmen series from Clark (2010), the "Allen (2007)" series uses Allen's (2007) series from 1770 onward (but our baseline wage series before that). Finally, we present estimates of productivity based on the assumption that the builders, farmers, and craftsmen series are all noisy signals of the true underlying wage. These estimates are labeled "3 series".

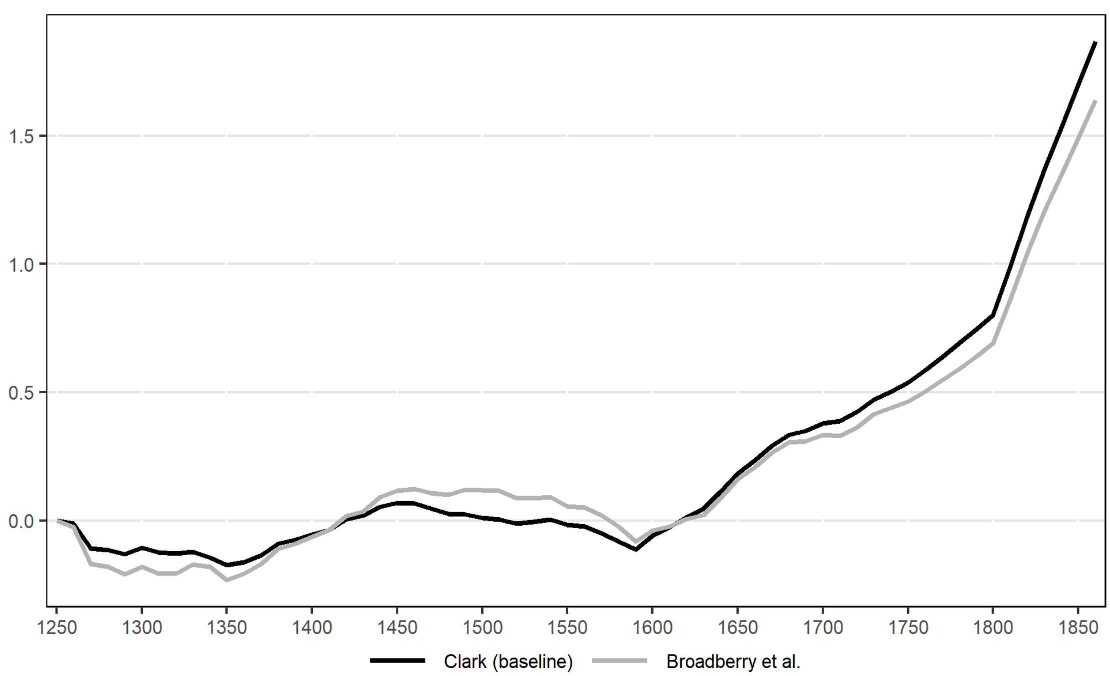

Figure A.5: Productivity using Different Population Data

Note: The figure compares our baseline estimates of the evolution of the permanent component of productivity $\tilde{a}_t$ with estimates using data on the population of England prior to 1540 from Broadberry et al. (2015).

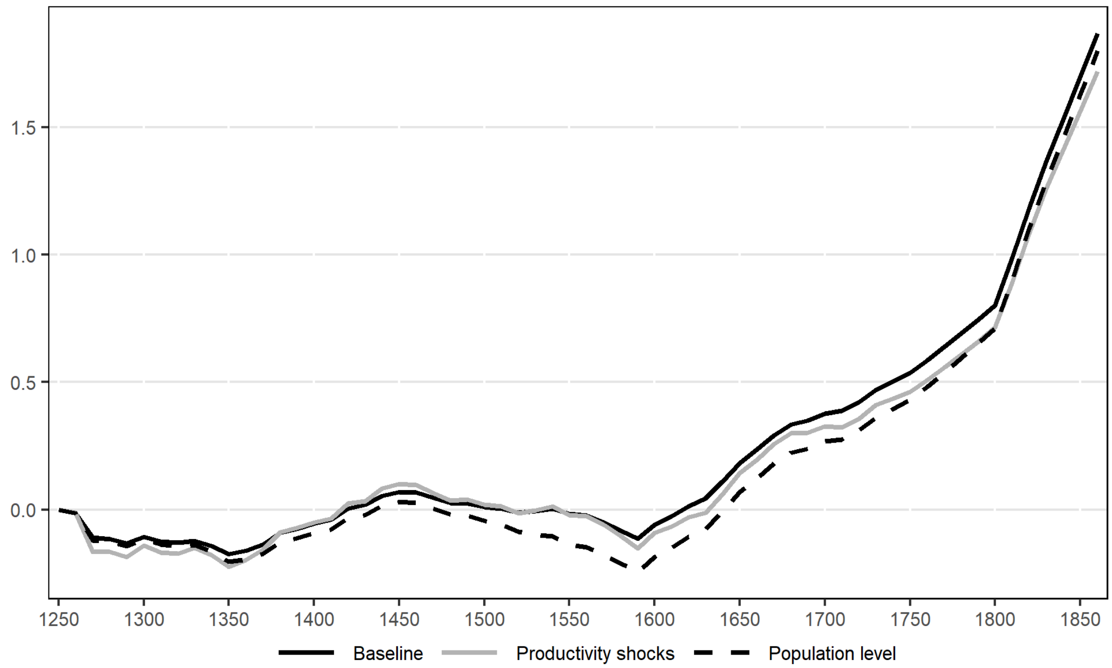

Figure A.6: Productivity using Different Priors

Note: The figure compares our baseline estimates of the evolution of the permanent component of productivity $\tilde{a}_t$ with estimates using different prior distributions. The "Productivity shocks" series changes the prior on σ_{ϵ_1} to be $\text{I}\Gamma(3, 0.005)$, i.e., the same as the prior on the other productivity and population shocks. The "Population level" series changes the prior on ψ to be $\mathcal{N}(10.86, 10.0)$.

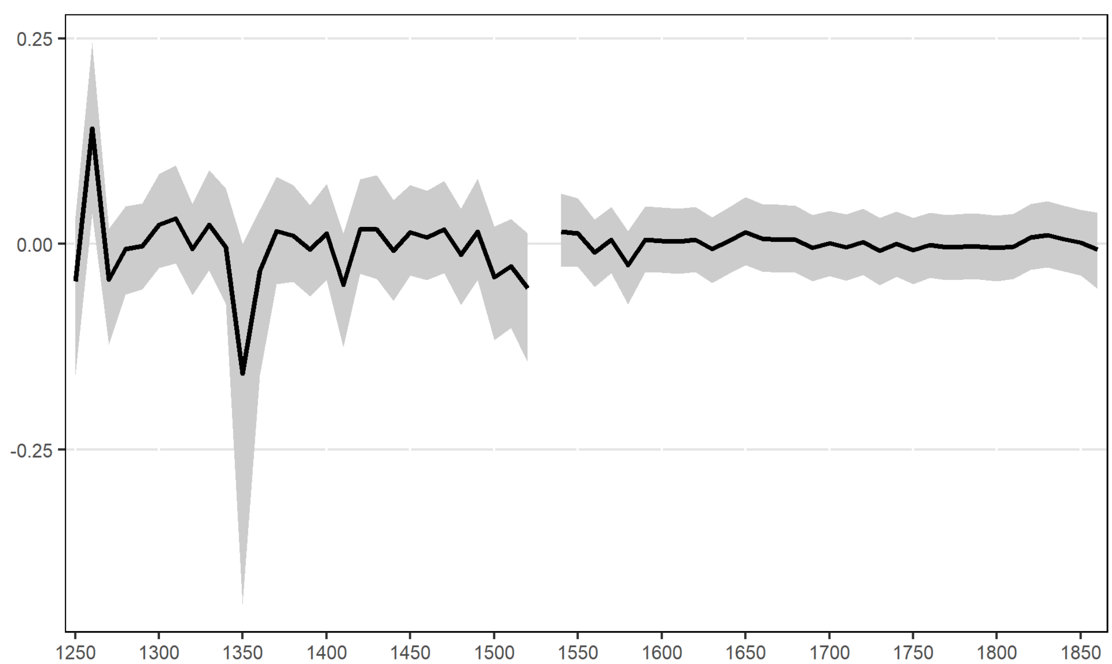

Figure A.7: Measurement Error in Population Data

Note: The figure plots our estimate of the measurement error in our population data ι_t^n.

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